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ASYMPTOTIC BEHAVIOUR OF THE SCATTERING PHASE IN LINEAR ELASTICITY II

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The purpose of this note is to improve in the case of odd space dimension the result in [1] associated to the Neumann problem of linear elasticity in the exterior of a strictly convex body. Let $\mathcal{O} \subset \mathbf{R}^n$, $n \geq 2$, be a strictly convex obstacle with C^∞-smooth boundary Γ and denote $\Omega = \mathbf{R}^n \backslash \mathcal{O}$. Consider in Ω the elasticity operator L defined by

$$Lv = \mu_0 \Delta v + (\lambda_0 + \mu_0)\nabla(\nabla \cdot v),$$

$v = {}^t(v_1, \ldots, v_n)$, λ_0, μ_0 being the Lamé constants assumed to satisfy

$$\mu_0 > 0,\ n\lambda_0 + 2\mu_0 > 0.$$

Denote by $-L_0$ the self-adjoint realization of $-L$ on the Hilbert space $H_0 = L^2(\mathbf{R}^n; \mathbf{C}^n)$, and by $-L_D$ (resp. $-L_N$) the Dirichlet (resp. Neumann) realization of $-L$ on $H = L^2(\Omega; \mathbf{C}^n)$. Recall that the Neumann boundary conditions in this case are of the form $Bv = 0$ on Γ, where

$$(Bv)_i = \sum_{j=1}^{n} \sigma_{ij}(v)\nu_j, \qquad i = 1, \ldots, n,$$

$\sigma_{ij}(v) = \lambda_0 \nabla \cdot v\, \delta_{ij} + \mu_0(\partial v_j/\partial x_i + \partial v_i/\partial x_j)$ is the stress tensor, and $\nu = (\nu_1, \ldots, \nu_n)$ is the unit outer normal to Γ. Then the scattering phase $s_j(\lambda)$ associated to $-L_j$, $j = D, N$, is defined by

$$\frac{ds_j}{d\lambda}(\lambda) = (2\pi)^{-1}\hat{u}_j(\lambda), \quad s_j(0) = 0,$$

where $\hat{u}_j(\lambda)$ is the Fourier transform of the distribution $u_j(t) \in S'(\mathbf{R})$ defined by

$$u_j(t) = \mathrm{tr}_{H_0}(e_\Omega U_j(t) r_\Omega - U_0(t)),$$

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where $U_j(t)$ is the wave group associated to $-L_j$, $j = D, N, 0$, and e_Ω denotes the extension by 0 outside Ω and r_Ω denotes the restriction to Ω. It was proved in [9] that $ds_D(\lambda)/d\lambda$ admits a complete asymptotics, and as a consequence

$$(1)\qquad s_D(\lambda) = \sum_{k=0}^{n-1} a_k \lambda^{n-k} + a_n \log\lambda + O(1), \quad \lambda \to +\infty,$$

for all $n \geq 2$, where $a_0 = \tau_n((n-1)c_1^{-n} + c_2^{-n})\mathrm{Vol}(\mathcal{O})$, $\tau_n = (2\pi)^{-n}\mathrm{Vol}\{x \in \mathbf{R}^n : |x| \leq 1\}$. Here $c_1 = \sqrt{\mu_0}$ and $c_2 = \sqrt{\lambda_0 + 2\mu_0}$ are the speeds of propagation of the elastic waves.

The strictly convex obstacles, however, are no longer nontrapping for $-L_N$ because of the existence of a characteristic variety $\Sigma = \{\zeta \in T^*\Gamma : ||\zeta|| = c_R^{-1}\}$ for the parametrix of the Neumann operator in the elliptic region $\mathcal{E} = \{\zeta \in T^*\Gamma : ||\zeta|| > c_1^{-1}\}$ (e.g. see [15], [16]). This fact is interpreted as existence of Rayleigh waves on the boundary Γ moving with a speed $c_R < c_1$. It was shown in [15], [16], [20] that for odd n the Rayleigh waves generate infinitely many resonances (poles of the meromorphic continuation of the cutoff resolvent $\chi(L_N + \lambda^2)^{-1}\chi$, $\chi \in C_0^\infty(\mathbf{R}^n)$, $\chi = 1$ near Γ) converging rapidly to the real axis. On the other hand, it was proved in [13] that if $n \neq 4$ the counting function $N(\lambda) = \#\{\lambda_j : \mathrm{Im}\,\lambda_j \leq 1, 0 < \mathrm{Re}\,\lambda_j \leq \lambda\}$ of Rayleigh resonances satisfies

$$(2)\qquad N(\lambda) = \tau_{n-1} c_R^{-n+1}\mathrm{Vol}(\Gamma)\lambda^{n-1} + O(\lambda^{n-2}),\ \lambda \to +\infty.$$

So, it is not natural to expect that the scattering phase $s_N(\lambda)$ associated to $-L_N$ admits an asymptotic similar to (1). However, we showed in [1] that

$$(3)\qquad s_N(\lambda) = a_0\lambda^n + a_1'\lambda^{n-1} + r(\lambda),\ \lambda \to +\infty,$$

where $r(\lambda) = O(\log\lambda)$ if $n = 2$, $r(\lambda) = O(\lambda^{n-2})$ if $n > 2$. Note that the second terms in (1) and (3) are of the form $b_j\mathrm{Vol}(\Gamma)$, $j = D$, N, where b_j depend only on the Lamé constants and n but in a very complicated way. The purpose of this work is to prove the following

Theorem *If $\mathcal{O}$ is strictly convex with C^∞-smooth boundary and if $n \geq 3$ is odd, then there exists a function of the form*

$$g(\lambda) = \sum_{k=0}^{n-1} b_k\lambda^{n-k} + b_n \log\lambda,$$

where $b_0 = a_0$, such that for every $p \gg 1, 0 < \delta \ll 1$, we have

$$(4)\quad N(\lambda - \lambda^{-p}) - O_{p,\delta}(1) - O(\lambda^\delta) \leq s_N(\lambda) - g(\lambda) \leq N(\lambda + \lambda^{-p}) + O_p(1),$$

Moreover, if $\mathcal{O}$ is of analytic boundary, then (4) *holds with λ^{-p} replaced by $e^{-(\gamma-\varepsilon)\lambda}$ for some $\gamma > 0$ and any $0 < \varepsilon \ll 1$, and $O_p(1)$ replaced by $O_\varepsilon(1)$.*

REMARK. Clearly, in the case of odd n, the asymptotics (3) follows from (4) and (2). The advantage of this approach is that it allows to extend the result to more general obstacles for which the relations (6) and (7) bellow hold (see [4]).

Proof. We are going to take advantage of the Poisson formula proved in [6], [14] for compactly supported perturbations of the Laplacian but it is clear from the proof that it extends to perturbations of L as well. So, we have

$$u_N(t) = \begin{cases} \sum e^{it\lambda_j}, & t > 0, \\ \sum e^{it\bar{\lambda}_j}, & t < 0, \end{cases} \tag{5}$$

where $\{\lambda_j\}_{j=1}^{\infty}$ are the resonances of L_N repeated according to multiplicity. According to the result in [15] (see also [4]), $\{\lambda_j\}_{j=1}^{\infty} = \Lambda_1 \cup \Lambda_2$, where Λ_1 is the set of Rayleigh resonances which satisfy

$$0 < \operatorname{Im}\lambda_j \le C_m |\operatorname{Re}\lambda_j|^{-m}, \quad \forall m \gg 1, \tag{6}$$

and Λ_2 is a set of resonances satisfying

$$\operatorname{Im}\lambda_j \ge m \log|\lambda_j| - C'_m, \quad \forall m \gg 1. \tag{7}$$

Define the distributions $u_k(t) \in S'(\mathbf{R}), k = 1, 2,$ as follows

$$\langle u_k, \rho\rangle = \sum_{\lambda_j \in \Lambda_k} \left(\int_{-\infty}^{0} e^{it\bar{\lambda}_j}\rho(t)dt + \int_0^{\infty} e^{it\lambda_j}\rho(t)dt \right), \quad \rho \in C_0^{\infty}(\mathbf{R}),$$

and define $s_k(\lambda)$ by

$$\frac{ds_k}{d\lambda}(\lambda) = (2\pi)^{-1}\hat{u}_k(\lambda), \quad s_k(0) = 0.$$

Lemma 1. *For $\lambda \gg 1$ we have*

$$N(\lambda - \lambda^{-p}) - O_{p,\delta}(1) - O(\lambda^{\delta}) \le s_1(\lambda) \le N(\lambda + \lambda^{-p}) + O_p(1), \tag{8}$$

for every $p \gg 1$, $0 < \delta \ll 1$.

Proof. It is easy to see that

$$s_1(\lambda) = \sum_{\lambda_j \in \Lambda_1} (2\pi)^{-1} \int_0^{\lambda} \frac{2\operatorname{Im}\lambda_j}{|\sigma - \lambda_j|^2} d\sigma.$$

Since, in view of (6),

$$\sum_{\lambda_j\in\Lambda_1,\operatorname{Re}\lambda_j\le 0}(2\pi)^{-1}\int_0^\lambda \frac{2\operatorname{Im}\lambda_j}{|\sigma-\lambda_j|^2}d\sigma \le \sum_{\lambda_j\in\Lambda_1,\operatorname{Re}\lambda_j\le 0}(2\pi)^{-1}\int_0^\lambda \frac{2\operatorname{Im}\lambda_j}{\sigma^2+|\lambda_j|^2}d\sigma$$
$$\le \sum \frac{2\operatorname{Im}\lambda_j}{|\lambda_j|} \le C'\sum |\lambda_j|^{-n-1} \le C,$$

we have, using (6) again,

$$s_1(\lambda) = \sum_{\lambda_j\in\Lambda_1,\operatorname{Re}\lambda_j>0}(2\pi)^{-1}\int_{-\operatorname{Re}\lambda_j/\operatorname{Im}\lambda_j}^{(\lambda-\operatorname{Re}\lambda_j)/\operatorname{Im}\lambda_j}\frac{2d\tau}{\tau^2+1}+O(1)$$
$$\le N(\lambda+\lambda^{-p}) + \sum_{\lambda_j\in\Lambda_1,\operatorname{Re}\lambda_j>\lambda+\lambda^{-p}}(2\pi)^{-1}\int_{-\infty}^{(\lambda-\operatorname{Re}\lambda_j)/\operatorname{Im}\lambda_j}\frac{2d\tau}{\tau^2+1}+O(1)$$
$$\le N(\lambda+\lambda^{-p}) + \sum_{\lambda_j\in\Lambda_1}(2\pi)^{-1}\int_{-\infty}^{-C_p|\lambda_j|^{n+1}}\frac{2d\tau}{\tau^2+1}+O(1)$$
$$\le N(\lambda+\lambda^{-p}) + \widetilde{C}_p\sum|\lambda_j|^{-n-1}+O(1). \tag{9}$$

Similarly,

$$s_1(\lambda) \ge \sum_{\lambda_j\in\Lambda_1,\lambda^\delta<\operatorname{Re}\lambda_j\le\lambda-\lambda^{-p}}(2\pi)^{-1}\int_{-\operatorname{Re}\lambda_j/\operatorname{Im}\lambda_j}^{(\lambda-\operatorname{Re}\lambda_j)/\operatorname{Im}\lambda_j}\frac{2d\tau}{\tau^2+1}$$
$$= N(\lambda-\lambda^{-p}) - N(\lambda^\delta)$$
$$-\sum_{\lambda_j\in\Lambda_1,\lambda^\delta<\operatorname{Re}\lambda_j\le\lambda-\lambda^{-p}}(2\pi)^{-1}\left(\int_{-\infty}^{-\operatorname{Re}\lambda_j/\operatorname{Im}\lambda_j}\frac{2d\tau}{\tau^2+1}+\int_{(\lambda-\operatorname{Re}\lambda_j)/\operatorname{Im}\lambda_j}^{+\infty}\frac{2d\tau}{\tau^2+1}\right)$$
$$\ge N(\lambda-\lambda^{-p}) - O(\lambda^{\delta(n-1)})$$
$$-\sum_{\lambda_j\in\Lambda_1,\lambda^\delta<\operatorname{Re}\lambda_j\le\lambda-\lambda^{-p}}(2\pi)^{-1}\left(\int_{-\infty}^{-C'_p|\lambda_j|^{n+1}}\frac{2d\tau}{\tau^2+1}+\int_{C''_{p,\delta}|\lambda_j|^{n+1}}^{+\infty}\frac{2d\tau}{\tau^2+1}\right)$$
$$\ge N(\lambda-\lambda^{-p}) - O(\lambda^{\delta(n-1)}) - \widetilde{C}_{p,\delta}\sum|\lambda_j|^{-n-1}, \tag{10}$$

Now (8) follows from (9) and (10). □

Lemma 2. *We have $u_2 \in C^\infty(\mathbf{R}\setminus 0)$ and for every integer $k\ge 0$,*

$$|\partial_t^k u_2(t)| \le C_k e^{-C|t|}, \quad |t|\gg 1, \tag{11}$$

with some constants $C_k, C>0$.

Proof. We have in the sense of distributions for $t > 0$,

$$\partial_t^k u_2(t) = \sum_{\lambda_j \in \Lambda_2} (i\lambda_j)^k e^{it\lambda_j},$$

where, in view of (7), the series is upper bounded by

$$\sum_{\lambda_j \in \Lambda_2} |\lambda_j|^k e^{-t \operatorname{Im} \lambda_j} \leq e^{C'_m t} \sum_{\lambda_j \in \Lambda_2} |\lambda_j|^{k-mt}$$

for every $m \gg 1$. Since the counting function of $\{\lambda_j\}$ is $O(\lambda^n)$ (e.g. see [19]), it suffices to choose $m \geq (k-n-1)/t$ in order that the series above be absolutely convergent. Hence $u_2 \in C^\infty(0, +\infty)$. The case of $t < 0$ is treated similarly. Let now $t \gg 1$ and choose m above equal to 1. We clearly have, for any $q > 0$,

$$\begin{aligned} |\partial_t^k u_2(t)| &\leq \sum_{\lambda_j \in \Lambda_2, |\lambda_j| \leq q} |\lambda_j|^k e^{-t \operatorname{Im} \lambda_j} + e^{C' t} \sum_{\lambda_j \in \Lambda_2, |\lambda_j| \geq q} |\lambda_j|^{k-t} \\ &\leq A_q e^{-A'_q t} + e^{t(C' - \log q) + (k+n+1)\log q} \sum |\lambda_j|^{-n-1}, \end{aligned}$$

where $A_q, A'_q > 0$. Choosing now q so that $C' - \log q = -1$, we obtain (11) for $t \gg 1$. Clearly, the case of $t \ll -1$ is similar. □

Let $\phi(t) \in C_0^\infty(\mathbf{R})$, $\phi(t) = 1$ in a neighbourhood of $t = 0$.

Lemma 3. *For $\lambda \gg 1$ we have*

$$s_2(\lambda) - s_2 * \hat{\phi}(\lambda) = O(1). \tag{12}$$

Proof. For any integer $k \gg 1$ we have

$$(-i\lambda)^k \left(\frac{ds_2}{d\lambda}(\lambda) - \frac{ds_2}{d\lambda} * \hat{\phi}(\lambda) \right) = \mathcal{F}_{t \to \lambda} \left(\partial_t^k \{(1 - \phi(t)) u_2(t)\} \right).$$

By Lemma 2, $\partial_t^k \{(1 - \phi(t)) u_2(t)\} \in L^1(\mathbf{R})$ for every integer $k \geq 0$, and hence

$$\frac{ds_2}{d\lambda}(\lambda) - \frac{ds_2}{d\lambda} * \hat{\phi}(\lambda) = O(\lambda^{-\infty}).$$

Clearly, this implies (12) at once. □

In view of (5), $u_N(t) - u_1(t) - u_2(t)$ is a distribution supported at $t = 0$, and hence $s_N(\lambda) - s_1(\lambda) - s_2(\lambda)$ is a polynomial. Therefore, by (8) and (12) we get

$$\begin{aligned} N(\lambda - \lambda^{-p}) - O_{p,\delta}(1) &\leq s_N(\lambda) - s_N * \hat{\phi}(\lambda) + N * \hat{\phi}(\lambda) \\ &\leq N(\lambda + \lambda^{-p}) + O_p(1). \end{aligned} \tag{13}$$

On the other hand, an analogue of Ivrii's result [3] for the elasticity system yields

$$\frac{d}{d\lambda}(s_N * \hat{\phi})(\lambda) = \sum_{j=0}^{\infty} \alpha_j \lambda^{n-j-1}, \tag{14}$$

where $\alpha_0 = n\alpha_0$. This can be proved by the methods developed in [7] or [2], or [11], [12], [17], [18]. So, to prove (4) it suffices to show that

$$N * \hat{\phi}(\lambda) = \sum_{j=0}^{n-2} \beta_j \lambda^{n-j-1} + \beta_{n-1} \log \lambda + O(1). \tag{15}$$

It is proven in [13] that modulo a constant, $N(\lambda)$ is equal to the number $\widetilde{N}(\lambda)$ of the nonpositive eigenvalues of a matrix-valued self-adjoint $\lambda - \Psi DO$, $P(\lambda)$, with a principal symbol having $n-1$ strictly positive eigenvalues and one eigenvalue vanishing on Σ, negative in $\{\zeta \in T^*\Gamma : |\zeta| < c_R^{-1}\}$ and positive in $\{\zeta \in T^*\Gamma : |\zeta| > c_R^{-1}\}$. Hence, it suffices to prove (15) with N replaced by $\widetilde{N}$. It follows from the analysis in [13] that

$$\widetilde{N}(\lambda) = (2\pi i)^{-1} \mathrm{tr} \lim_{\varepsilon \to 0^+} \int_0^{\lambda} \left(\dot{P}(\sigma - i\varepsilon) P(\sigma - i\varepsilon)^{-1} - \dot{P}(\sigma + i\varepsilon) P(\sigma + i\varepsilon)^{-1} \right) d\sigma.$$

where $\dot{P}(\lambda) = dP(\lambda)/d\lambda$. Hence,

$$\begin{aligned} &\frac{d}{d\lambda}(\widetilde{N} * \hat{\phi})(\lambda) \\ &= (2\pi i)^{-1} \mathrm{tr} \lim_{\varepsilon \to 0^+} \int_{-\infty}^{\infty} \Big(\dot{P}(\sigma - i\varepsilon) P(\sigma - i\varepsilon)^{-1} \\ &\qquad - \dot{P}(\sigma + i\varepsilon) P(\sigma + i\varepsilon)^{-1} \Big) \hat{\phi}(\lambda - \sigma) d\sigma. \end{aligned} \tag{16}$$

As in [2], one can deduce from (16) that

$$\frac{d}{d\lambda}(\widetilde{N} * \hat{\phi})(\lambda) = \sum_{j=0}^{\infty} \gamma_j \lambda^{n-2-j}, \tag{17}$$

which clearly implies (15). □

The analytic case is treated similarly using that in this case, according to the results in [20], the resonances in Λ_1 satisfy

$$0 < \mathrm{Im}\, \lambda_j \le C e^{-\gamma |\mathrm{Re}\, \lambda_j|}$$

for some constants $C, \gamma > 0$.

Addentum

The purpose of this addentum is to derive (17) from the semi-classical asymptotics in [2]. Let $\varphi \in C_0^\infty(\mathbf{R})$, $\varphi(t) = 1$ for $|t| \le 1$, and set $\varphi_\lambda(t) = \varphi(\lambda^\delta t)$, $0 < \delta \ll 1$. Denoting the LHS of (16) by $\zeta(\lambda)$, we have

$$\begin{aligned}(\text{A.1}) \qquad \zeta(\lambda) = \lambda \,\mathrm{tr}(2\pi i)^{-1} \lim_{\varepsilon \to 0+} \int (&\dot{P}(\lambda(1 - z - i\varepsilon))P(\lambda(1 - z - i\varepsilon))^{-1} \\ -\dot{P}(\lambda(1 + z + i\varepsilon))&P(\lambda(1 + z + i\varepsilon))^{-1})\varphi_\lambda(z)\hat{\phi}(\lambda z)dz + O(\lambda^{-\infty}).\end{aligned}$$

On the other hand, given any integer $m \gg 1$, for $z \in \mathbf{C}$, $|z| \le C\lambda^{-\delta}$, we have

$$(\text{A.2}) \qquad P(\lambda(1 - z)) = P(\lambda) - \lambda z\dot{P}(\lambda) + \sum_{k=2}^{m-1} z^k T_k(\lambda) + \widetilde{O}(\lambda^{-\delta m}),$$

where $T_k(\lambda)$ are $\lambda - \Psi DO$'s of class $L_{cl}^{0,0}(\Gamma)$, independent of z. Here and in what follows $\widetilde{O}(\lambda^{-\delta m})$ denotes a $\lambda - \Psi DO$ of class $L_{0,0}^{0,-\delta m}(\Gamma)$. Since $\dot{P}(\lambda)$ is elliptic (of class $L_{cl}^{0,-1}(\Gamma)$), (A.2) can be rewritten in the form

$$(\text{A.3}) \qquad P(\lambda(1 - z)) = (Q_m(\lambda, z) - z)R_m(\lambda, z),$$

where $Q_m(\lambda, z)$ is a $\lambda - \Psi DO$, depending analyticaly on a parameter z, of the form

$$(\text{A.4}) \qquad Q_m(\lambda, z) = G_m(\lambda) + \widetilde{O}(\lambda^{-\delta m})$$

with a $\lambda - \Psi DO$, $G_m(\lambda)$, of class $L_{cl}^{0,0}(\Gamma)$, independent of z. $R_m(\lambda, z)$ is elliptic of class $L_{cl}^{0,0}(\Gamma)$ uniformly in z. It follows from the analysis in [13] that $(Q_m(\lambda, z) - z)^{-1}$ is analytic in z for $\mathrm{Im}\, z \ne 0$, and by Lemma 5.1 of [13], we have

$$(\text{A.5}) \qquad \|(Q_m(\lambda, z) - z)^{-1}\|_{\mathcal{L}(H^s(\Gamma), H^s(\Gamma))} \le \frac{C_s}{|\mathrm{Im}\, z|},$$

for $\mathrm{Im}\, z \ne 0$, with a constant C_s independent of z and λ. We also have, in view of (A.3),

$$\begin{aligned}(\text{A.6}) \qquad -\lambda\dot{P}(\lambda(1 - z)) &= \frac{dP(\lambda(1 - z))}{dz} \\ &= (-1 + \widetilde{O}(\lambda^{-\delta m}))R_m(\lambda, z) + (Q_m(\lambda, z) - z)\frac{dR_m(\lambda, z)}{dz}.\end{aligned}$$

Thus, by (A.1), (A.3) and (A.6), we get

$$\zeta(\lambda) = \mathrm{tr}(2\pi i)^{-1} \lim_{\varepsilon \to 0+} \int ((Q_m(\lambda, z + i\varepsilon) - z - i\varepsilon)^{-1}$$

$$-(Q_m(\lambda, z - i\varepsilon) - z + i\varepsilon)^{-1})\varphi_\lambda(z)\hat{\phi}(\lambda z)dz + O(\lambda^{-\delta m}). \tag{A.7}$$

Let $\widetilde{\varphi}$ be an almost analytic extension of φ such that $\widetilde{\varphi} \in C_0^\infty(\mathbf{C}), \widetilde{\varphi}(z) = \varphi(z), \forall z \in \mathbf{R}$, and $\partial_{\bar{z}}\widetilde{\varphi}(z) = O(|\mathrm{Im}\, z|^N), \forall N \in \mathbf{N}$. Set $\widetilde{\varphi}_\lambda(z) = \widetilde{\varphi}(\lambda^\delta z)$. By Stokes' theorem, we can rewrite (A.7) in the form

$$\zeta(\lambda) = \mathrm{tr}\, \pi^{-1} \int_{\mathbf{C}} (Q_m(\lambda, z) - z)^{-1} \partial_{\bar{z}}\widetilde{\varphi}_\lambda(z)\hat{\phi}(\lambda z)L(dz) + O(\lambda^{-\delta m}), \tag{A.8}$$

where $L(dz)$ denotes the Lebesgue measure on $\mathbf{C}$. Now, in view of (A.4) and (A.5), the trace in (A.8) can be treated in precisely the same way as the trace in Theorem 2 of [2] (with $h = \lambda^{-1}$) giving asymptotics with error terms $O(\lambda^{-\delta m})$ with possibly a new, smaller $\delta > 0$. Since m is arbitrary, this gives an asymptotics of $\zeta(\lambda)$ modulo $O(\lambda^{-\infty})$.

References

[1] F. Cardoso and G. Vodev: *Asymptotic behaviour of the scattering phase in linear elasticity for a strictly convex body*, Comm. in Partial Diff. Equations, **22** (1997), 2025–2049.

[2] M. Dimassi and J. Sjöstrand: *Trace asymptotics via almost analytic extensions, in: Partial Differential Equations and Mathematical Physics*, The Danish – Swedish Analysis Seminar, 1995, L. Hörmander and A. Melin, eds. Birkhäuser, Boston, 1996, 126–142.

[3] V. Ivrii: *On the second term in the spectral asymptotic for the Laplace-Beltrami operator on a manifold with boundary*, Funct. Anal. Appl. **4** (1980), 98–106.

[4] M. Kawashita: *On a region free from the poles of the resolvent and decay rate of the local energy for the elastic wave equation*, Indiana Univ. Math. J. **43** (1994), 1013–1043.

[5] A. Majda and J.V. Ralston: *An analogue of Weyl's theorem for unbounded domains I, II, III*, Duke Math. J. **45** (1978), 183–196; Duke Math. J. **45** (1978), 513–536; Duke Math. J. **46** (1979), 725–731.

[6] R.B. Melrose: *Scattering theory and the trace of the wave group*, J. Funct. Anal. **45** (1982), 29–40.

[7] R.B. Melrose: *The trace of the wave group*, Contemporary Math. **27** (1984), 127–167.

[8] R.B. Melrose: *Weyl asymptotic for the phase in obstacle scattering*, Comm. in Partial Diff. Equations, **13** (1988), 1431–1439.

[9] V. Petkov and G. Popov: *Asymptotic behaviour of the scattering phase for nontrapping obstacles*, Ann. Inst. Fourier Grenoble, **32** (1982), 111–149.

[10] D. Robert: *On the Weyl formula for obstacles, in: Partial Differential Equations and Mathematical Physics*, The Danish – Swedish Analysis Seminar, 1995, L. Hörmander and A. Melin, eds., Birkhäuser, Boston, 1996, 262–285.

[11] Yu. Safarov and D. Vassiliev: *Branching Hamiltonian billiards*, Soviet Math. Dokl. **38** (1989), 64–68.

[12] Yu. Safarov and D. Vassiliev: *The asymptotic distribution of eigenvalues of partial differential operators*, Translations of Math. Monographs, American Math. Soc. **155** (1996).

[13] J. Sjöstrand and G. Vodev: *Asymptotics of the number of Rayleigh resonances*, Math. Ann. **309** (1997), 287–306.

[14] J. Sjöstrand and M. Zworski: *Lower bounds on the number of scattering poles. II*, J. Funct. Anal. **123** (1994), 336–367.

[15] P. Stefanov and G. Vodev: *Distribution of resonances for the Neumann problem in linear elasticity outside a strictly convex body*, Duke Math. J. **78** (1995), 677–714.

[16] P. Stefanov and G. Vodev: *Neumann resonances in linear elasticity for an arbitrary body*, Commun. Math. Phys. **176** (1996), 645–659.

[17] D. Vassiliev: *Two-term asymptotics of the spectrum of a boundary value problem under an interior reflection of general form*, Funct. Anal. Appl. **18** (1984), 267–277.

[18] D. Vassiliev: *Two-term asymptotics of the spectrum of a boundary value problem in the case of a piecewise smooth boundary*, Soviet Math. Dokl. **33** (1986), 227–230.

[19] G. Vodev: *Sharp bounds on the number of scattering poles for perturbations of the Laplacian*, Commun. Math. Phys. **146** (1992), 205–216.

[20] G. Vodev: *Existence of Rayleigh resonances exponentially close to the real axis*, Ann. Inst. H. Poincaré (Physique Théorique), **67** (1997), 41–57.

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