Concept of inherent strain, inherent stress, inherent deformation and inherent force for prediction of welding distortion and residual stress†

MURAKAWA Hidekazu *, DENG Dean ** and MA Ninshu ***

KEY WORDS: (Inherent strain)(Inherent stress)(Inherent deformation)(Inherent force)(Prediction)(Elastic analysis)(Residual stress)(Distortion)

1. Introduction

There are two methods which can be employed for the prediction of welding distortion and residual stress. One is the thermal elastic plastic analysis in which the welding is treated as a transient nonlinear problem [1]. The other is the inherent strain method in which the distortion and residual stress are computed by elastic analysis using the inherent strain as initial strain [2, 3]. They have both advantages and disadvantages. The latter is advantageous in computational time and disadvantageous because the detail of the welding condition may not be fully considered in some cases. The concept of inherent strain is closely related to those of inherent stress, inherent deformation and inherent force. Since the inherent strain is transformed into the deformation when the restraint is small and it is transformed into the stress when the restraint is strong, the framework of FE code for welding simulation can be naturally and conveniently constructed by combining these concepts. The idea how they are integrated in the FE code is presented in this paper.

2. Inherent Strain Produces Residual Stress and Distortion

The total strain ε can be decomposed into the sum of elastic strain ε^e, plastic strain ε^p, thermal strain ε^T, creep strain ε^c and that produced through phase transformation ε^t.

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^T + \varepsilon^c + \varepsilon^t \qquad (1)$$

Noting that the deformation and the stress are produced by the total strain and elastic strain, Eq. (1) can be rearranged to,

$$\varepsilon - \varepsilon^e = \varepsilon^p + \varepsilon^T + \varepsilon^c + \varepsilon^t = \varepsilon^* \qquad (2)$$

This equation means that the distortion and the residual stress are produced by the inherent strain ε^* which consists of plastic, thermal, creep and that caused by the transformation.

3. Inherent Strain in Thin Plate

Welding deformations in thin plate such as the transverse shrinkage, the longitudinal shrinkage and the angular distortion are mostly produced by the longitudinal and the transverse inherent strains ε_x^* and ε_y^*. The shrinkage of an element can be represented by distributed inherent strain as shown in **Fig. 1**.

Transverse inherent strain ε_x^* (3)

Longitudinal inherent strain ε_y^* (4)

4. Inherent Stress in Thin Plate

Through the constitutive relation, the inherent strains ε_x^*, ε_y^* are transformed into inherent stresses σ_x^*, σ_y^*.

Transverse inherent stress $\sigma_x^* = \frac{E}{1-\nu^2}\left(\varepsilon_x^* + \nu\varepsilon_y^*\right)$ (5)

Longitudinal inherent stress $\sigma_y^* = \frac{E}{1-\nu^2}\left(\nu\varepsilon_x^* + \varepsilon_y^*\right)$ (6)

5. Inherent Deformation in Thin Plate

By integrating the inherent strain over the cross-section normal to the welding line and take the average through the thickness h, the inherent deformations are obtained.

Transverse shrinkage $\delta_T^* = \frac{1}{h}\int \varepsilon_x^* dxdz$ (7)

Longitudinal shrinkage $\delta_L^* = \frac{1}{h}\int \varepsilon_y^* dxdz$ (8)

Transverse bending $\theta_T^* = \frac{1}{I}\int z\varepsilon_x^* dxdz$ (9)

Longitudinal bending $\theta_L^* = \frac{1}{I}\int z\varepsilon_y^* dxdz$ (10)

where, $I = \frac{1}{12}\int z^2 dxdz$

The shrinkage produced by the inherent strain distributed in an element can be replaced by the inherent deformation introduced as the discontinuity of the nodal displacement as shown in **Fig. 2**.

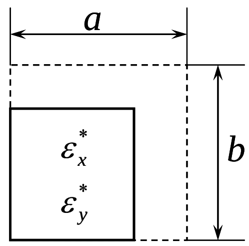


Fig. 1 Shrinkage represented by inherent strain.

† Received on 30 September 2010

* JWRI, Osaka University, Osaka, Japan
** Chongqing University, Chongqing, China
*** JSOL Corp. Osaka, Japan

Transactions of JWRI is published by Joining and Welding Research Institute, Osaka University, Ibaraki, Osaka 567-0047, Japan

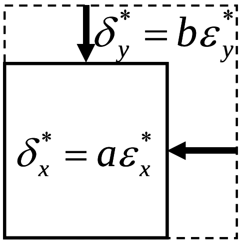


Fig. 2 Shrinkage represented by inherent deformation.

6. Inherent Force in Thin Plate

Further, the inherent forces can be defined by the following equations.

Transverse inherent force $F_T^* = E\int \varepsilon_x^* dxdz = Eh\delta_x^*$ (11)

Longitudinal inherent force $F_L^* = E\int \varepsilon_y^* dxdz = Eh\delta_y^*$ (12)

Transverse inherent moment $M_T^* = E\int z\varepsilon_x^* dxdz = EI\theta_x^*$ (13)

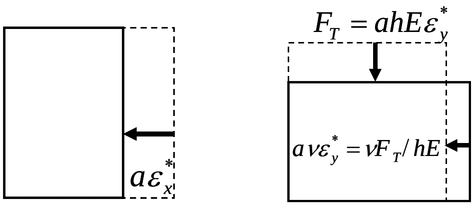
Longitudinal inherent moment $M_L^* = E\int z\varepsilon_y^* dxdz = EI\theta_y^*$ (14)

7. Procedure to Introduce Welding Inherent Strain to FEM

Since the restraint in the welding direction is strong and that in the transverse direction is very weak, the inherent strain in the transverse direction mostly transforms into the deformation and that in the welding direction transforms into the residual stress. Thus it is natural to use the inherent deformation for the transverse shrinkage and the angular distortion. For the longitudinal shrinkage and bending, the inherent force or moment must be used as shown in **Fig. 3**. In this way, the shrinkage in the weld zone can be described by using relatively coarse FE mesh as illustrated in **Fig.4.**

8. Large deformation

To predict the distortion of thin wall welded structures, the large deformation must be taken into account. As shown in **Fig. 5**, the distribution of the stress in an element which is facing the welding line is a combination of that is distributing in an element and that is concentrated at a point in the form of delta function. The effect of large deformation associated with the distributed stress can be taken into account through the standard FE formulation. That associated with the inherent force F_T must be also taken into account in the force vector and the stiffness matrix in FEM.



(a) transverse shrinkage (b) longitudinal shrinkage

Fig. 3 Shrinkage represented by inherent deformation and force.

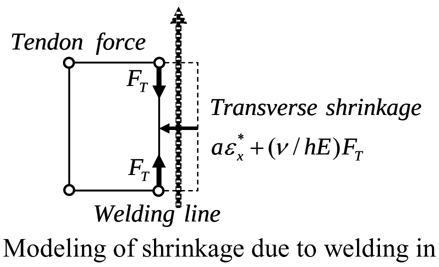


Fig. 4 Modeling of shrinkage due to welding in FEM.

In case of large deformation problem, the strain and the total potential energy of the structure at time $t + \Delta t$ can be symbolically described as the following equations.

$$\varepsilon(u + \Delta u) = \varepsilon(u) + \Delta^1\varepsilon + \Delta^2\varepsilon \quad (15)$$

$$\pi(u + \Delta u) = \pi(u) + \Delta^1\pi + \Delta^2\pi \quad (16)$$

where, u and Δu are the displacement and its increment. $\Delta^1\varepsilon, \Delta^2\varepsilon, \Delta^1\pi, \Delta^2\pi$ are the first and second order terms of Δu in the strain and the total potential energy, respectively. From the first order term $\Delta^1\pi$, the force vector is derived while the stiffness matrix is derived from the second order term $\Delta^2\pi$. Noting that the inherent force (Tendon force) is the concentrated stress described as a delta function, the contributions of the tendon force to the large deformation through the force vector and the stiffness matrix are given by the following equations.

$$\Delta^1\pi = ...+\int \sigma(u)\Delta^1\varepsilon\, dv = ..+\int \sigma(u)\theta(u)\Delta\theta(u)dv$$
$$= ..+\int F_T\theta(u)\Delta\theta(u)ds \quad (17)$$

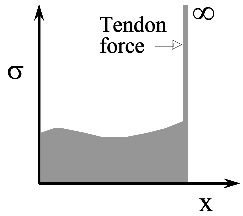


Fig. 5 Stress distribution in element.

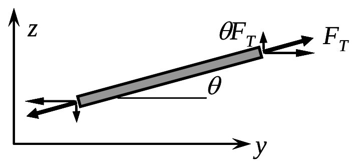


Fig. 6 Force produced by large deformation.

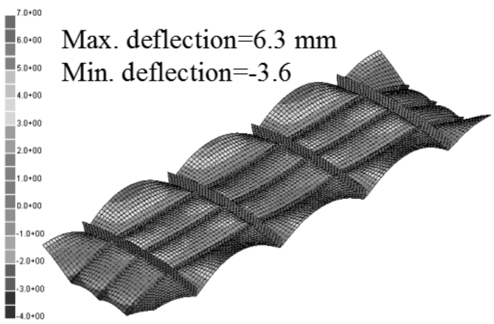


Fig. 7 Deflection computed under small deformation.

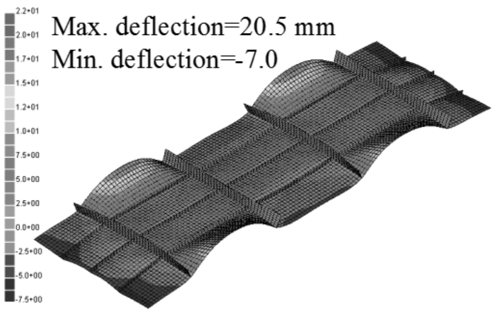


Fig. 8 Deflection computed under large deformation.

$$\Delta^2 \pi = ... + \int \sigma(u) \Delta^2 \varepsilon \, dv = ... + \int \sigma(u) \{\Delta\theta(u)\}^2 dv$$
$$= .. + \int F_T \{\Delta\theta(u)\}^2 ds \qquad (18)$$

where, θ symbolically represents the out of plane rotation due to the deflection. **Figure 6** illustrates the lateral force produced by the out of plane deformation on the welding line.

9. Example of Computed Results

Welding distortion of stiffened panels with 12 m length and 4 m width is computed as both small deformation and large deformation problems. The computed deformations are shown in **Figs. 7 and 8** for small and large deformation.

10. Conclusions

The idea of combined use of inherent deformation and inherent force is presented and its effectiveness for large deformation problems is demonstrated.

References

[1] Y. Ueda and T. Yamakawa: Thermal Stress Analysis of Metals with Temperature Dependent Mechanical Properties, Trans. JWS, 2-2 (1971), pp.90-100.

[2] Y. Ueda, K. Fukuda, K. Nakacho and S. ENDO: Fundamental Concept in Measurement of Residual Stresses Based on Finite Element Method and Reliability of Estimated Values, Theoretical and Applies Mechanics, 25 (1977), pp. 539-554.

[3] H. Murakawa, Y. Luo and Y. Ueda: Inherent Strain as an Interface between Computational Welding Mechanics and Its Industrial Application, Mathematical Modelling of Weld Phenomena 4, Ed. H. Cerjak (1998), 597-619.