Buckling Behavior of Plates under Idealized Inherent Strain†

Xiao-Min ZHONG*, Hidekazu MURAKAWA** and Yukio UEDA***

Abstract

The buckling behavior of plate due to welding is investigated using the inherent strain as an equivalent load. The analysis is based on the elastic large deflection theory of plate. A finite element approach is employed to predict the buckling deformation. Numerical results are examined to understand the buckling behavior and the effects of factors, such as the plate thickness, the ratio of breadth to the length of plate, and the distribution of the inherent strain which varies with the heat input.

KEY WORDS: (Welding Deformation) (Buckling) (Inherent Strain) (Thin Plate) (Finite Element Method)

1. Introduction

When a plate or a thin walled structure is thermally processed with a non uniform temperature field, transient deformation occurs inevitably during the process and residual deformation may remain. From the engineering point of view, these residual deformations of structures might significantly affect their performance and reliability. Especially when the plate thickness is less than a critical value, deflection due to instability or buckling occurs during the thermal process. The same thermo-mechanical phenomenon can be utilized to form curved plate for practical purposes. Therefore, it is important to make investigations on the relationship between deformation and the process conditions.

Over the past decades, a number of investigations have been done[1-17]. The form of the investigation and the sophistication of the analysis have hinged on the mathematical modeling and the available analytical or numerical means. In 1955, Masubuchi investigated the buckling distortion of a bead-welded plate, and obtained the buckling pattern along the weld line[1),2)]. Watanabe and Satoh made a theoretical study to calculate the values of the critical thickness for buckling distortion of plate with butt welds[1),3)]. In 1978, Terai et al. conducted an extensive experimental and analytical investigations on out-of-plane distortion during the welding of thin panel structures[1),4)].

With the development of computational methods and powerful computers, the numerical simulation becomes a widely used tool. From the end of the 1970's till now, a considerable number of detailed calculations of deformations under thermal process have been made by Ueda[5)], Friedman[6)], Karlsson[7)] and many others. Recently, a computer aided process planning system for plate bending by line heating was developed by Ueda, Murakawa and et al.[8-11)], and the relationships between the final form of the plate and the inherent strain or the heating condition have been discussed.

Most of these simulations were done using the Finite Element Method. After these early successes, more and more advanced Finite Element Models have been developed to compute the transient temperatures, displacements and stress fields. Material constitutive models have been developed from the early rate independent thermo-elasto-plastic models to the present various thermo-elasto-viscoplastic models, unified constitutive models and so on. For examples, in 1989, Oddy introduced the transformation plasticity into the thermo-mechanical analysis[12)]. In 1991, Mahin presented a study based on the rate dependent elasto-plastic

† Received on Nobember 24, 1995

* Lecturer, Shanghai Jiao Tong University
** Associate Professor
*** Professor

Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan.

model proposed by Bammann[13]. In 1992, Sheng and Chen proposed a method in which both liquid and solid phases of the weld metal are modeled[14].

Although, it is necessary to develop material models for a complete understanding of the thermo-mechanical process, the difficulty is greatly increased when these constitutive equations are used for practical applications. It is obviously unreasonable and meaningless to take into account all the factors in the model at the same time. It is not necessary to carry out the complete thermo-mechanical computation when the interest is not to trace the transient behavior. In the past decade, a rather simple method was developed[15]. The elastic analysis using the inherent strain as an equivalent load which depends on the process conditions was proposed[17].

In this paper, the buckling behavior of plate is investigated. The analysis is based on the elastic large deflection theory of plates. A Finite Element approach is employed to predict the buckling under given inherent strain. Numerical results are examined to understand the buckling behavior of plates and the effects of various factors, such as the plate thickness and the heat input.

2. FEM Model and Assumed Inherent Strain

Considering the primary objective of the investigation, geometrically nonlinear large deflection Finite Element analysis should be employed. In the Mindlin plate theory which is employed in the present study, the displacements are assumed in the following form,

$$U_k = u_k - x_3\theta_k \quad (k=1,2) \tag{1}$$

$$U_3 = u_3 \tag{2}$$

where, u_1, u_2 and u_3 are the in-plane and out-of-plane displacements on the midsurface, respectively. θ_1 and θ_2 are the rotations. The Green strain tensors defined in the Total Lagrangian description are given as,

$$\varepsilon_{ij} = (1/2)[u_{i,j} + u_{j,i} - x_3(\theta_{i,j} + \theta_{i,j}) + u_{3,i}u_{3,j}] \tag{3}$$

$$\varepsilon_{i3} = (1/2)[u_{3,i} - \theta_i] \tag{4}$$

$$(i=1,2,3),\ (j=1,2)$$

The 2nd Piola-Kirchhoff stress which is coupled with Green strain in energy is employed. Their relationship can be described in the following form,

$$\sigma_{ij} = D_{ijrs}\varepsilon_{rs} \tag{5}$$

When the material is isotropic and elastic, the constitutive equation is given as;

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \tag{6}$$

where, μ and λ are Lame elastic constants and δ_{ij} is Kronecker delta. If the inherent strain ε'_{ij} is considered as the initial strain, the stress strain relationship can be rewritten as follows.

$$\sigma_{ij} = 2\mu(\varepsilon_{ij} - \varepsilon'_{ij}) + \lambda(\varepsilon_{kk} - \varepsilon'_{kk})\delta_{ij} \tag{7}$$

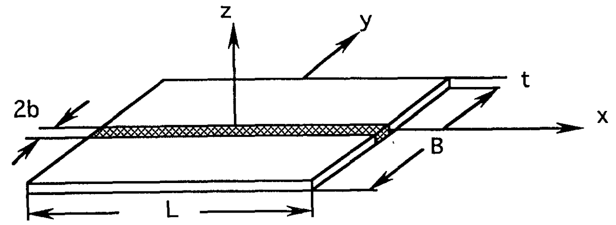


Fig.1 The model for analysis.

The four-node quadrilateral element proposed by Hughes is adapted and selective reduced integration to remove the transverse shear locking is used. Discretized equilibrium equations are derived via the virtual work theorem which is described as;

$$\psi(U) = \int\delta\varepsilon^T\sigma dv - \int\delta U^T b dv - \int\delta U^T t ds = 0 \tag{8}$$

where σ is the 2nd Piola-Kirchhoff stress. $\delta\varepsilon$ and δU are the virtual strain and the virtual displacement. b and t are the body force and the surface force, respectively. A direct linearization based on the Newton-Raphson method is employed to derive the Finite Element equations.

To study the buckling behavior of thin plates, the following assumptions are made to model the inherent strain due to welding.

(1) When the plate is thin, the temperature distribution across the thickness is almost uniform and it can be estimated by using appropriate analytical formulae.

(2) The inherent strain is produced in the area where the thermal strain exceeds the yield strain.

(3) Welding residual stress distribution through the thickness is almost uniform in thin plate. The longitudinal stress component is the most important in relation to buckling. The magnitude of this component is equal to or slightly bigger than the yield stress.

(4) The transverse shrinkage and the angular distortion are mainly related to the transverse inherent strain component.

Based on the discussions above, the inherent strain distribution can be assumed as follows. It is assumed that only the components ε'_x and ε'_y are non-zero. These two components are distributed in a zone with dimensions ***L*, *2b*** and ***t*** as shown in **Fig. 1.**

$$b = \alpha Q/(2tc\varrho\varepsilon_y\sqrt{2\pi e}) \tag{9}$$

$$\varepsilon'_x = -\sigma_y/E \tag{10}$$

$$\varepsilon'_y = -S/2r - z\delta/b \tag{11}$$

$$(-t/2 \le z \le t/2)$$

where, Q is the heat input (J/cm). For the given heat input Q, transverse shrinkage S and angular distortion δ can be determined using Satoh's experimental curve. The other data necessary in the computation are:

thermal expansion ratio

$$\alpha = 1.3*10^{-5} \cdots\cdots 1/°c$$

heat conductivity

$$\lambda = 0.675 \cdots\cdots J/cm \bullet s \bullet °c$$

thermal capacity

$$c = 0.475 \cdots\cdots J/g \bullet °c$$

density

$$\varrho = 7.82 \cdots\cdots g/cm^3$$

Young's modulus

$$E = 200.0 \cdots\cdots Gpa$$

Poison's ratio

$$\upsilon = 0.30$$

yield stress

$$\sigma_y = 300 \cdots\cdots Mpa$$

A rectangular plate subjected to an inherent strain is employed as the model to be examined. The dimensions of the plate are ***L, B*** and ***t*** as shown in Fig.1.

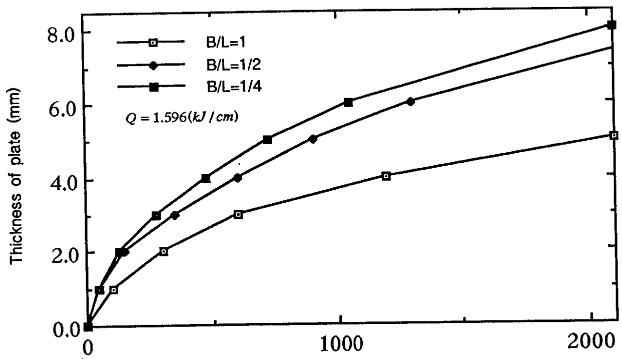


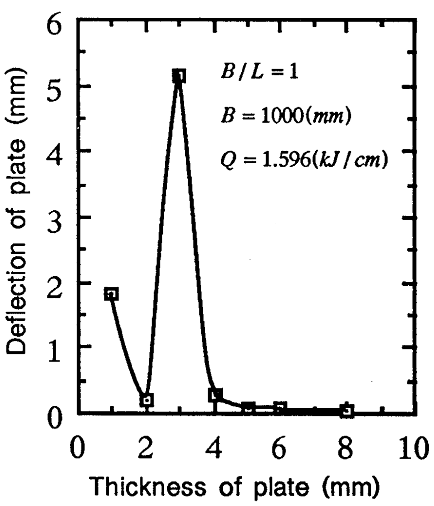
Fig.2 Effect of the size and the thickness of plate on the critical plate thickness for buckling.

3. Numerical Results

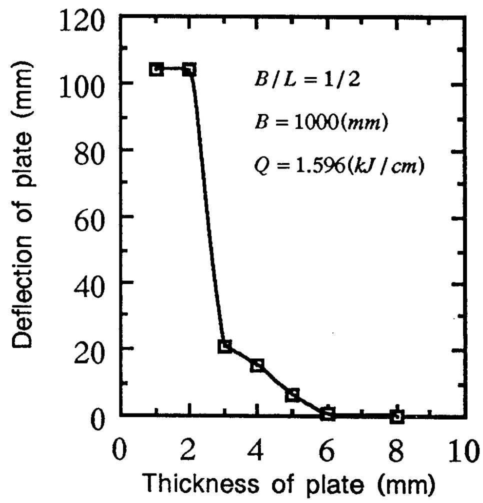
The types of FEM used for the present study are divided into two. The first type is to clarify the possibility of buckling by counting the number of negative eigen values of the stiffness matrix. If the stiffness matrix has negative eigen values, the plate can buckle. The number of the negative eigen values is counted using Sturm's theorem. In this computation, the plate is constrained to flat form. The second type is the ordinary deformation analysis to determine the magnitude and the mode of deformation.

The rectangular plates employed in the analysis are subjected to no constraint. Only six components of the displacements on the plate are fixed to prevent rigid body motion. Due to the strong non-linearity, the inherent strain is given incrementally in the computation. The program can automatically adjust the loading steps if necessary for the convergence. The differences between the maximum and the minimum deflection on the plate ***w*** is used to compare the magnitude of the buckling deformation.

Through the first type of analysis, the relation between the critical thickness of the plate and the geometry of the plate is investigated. The heat input per unit length is kept at Q=1.596 (kJ/cm). The number of negative eigen values is counted for the plate with different thickness, width and length. In **Fig.2**, the cases which locate on the boundary between the regions with and without negative eigen values are connected as lines. These lines give the critical thicknesses of the plate with



(1) square plate



(2) retanglar plate

Fig.3 Relationship between the deflection and the thickness.

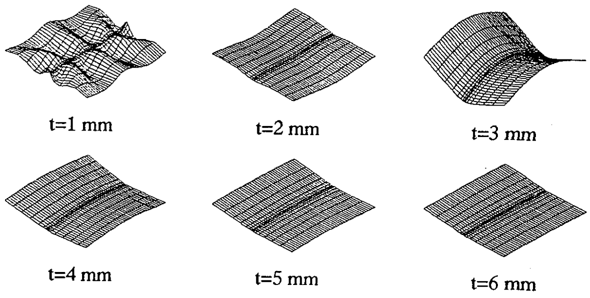


Fig.4 Deformed shape of square plate when the heat input Q is 1.596 kJ/cm.

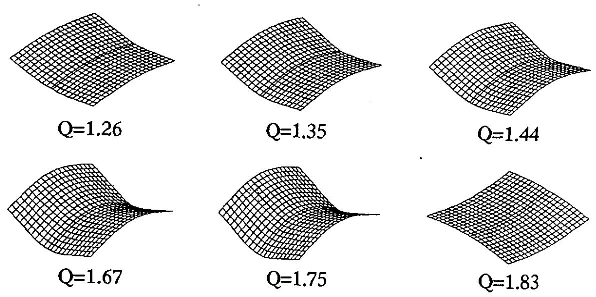


Fig.7 Deformed shape of square plate under the defferent heat input Q when the plate thickness is 3 mm.

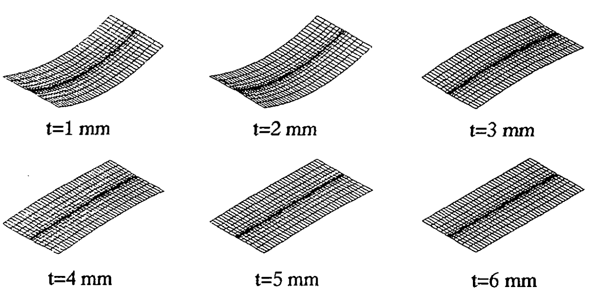


Fig.5 Deformed shape of rectangular plate when the heat input Q is 1.596 kJ/cm.

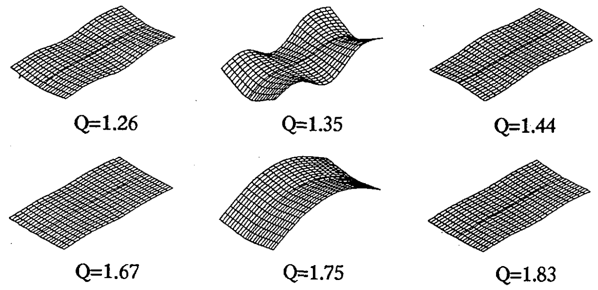


Fig.8 Deformed shape of rectangular plate under the defferent heat input Q when the plate thickness is 3 mm.

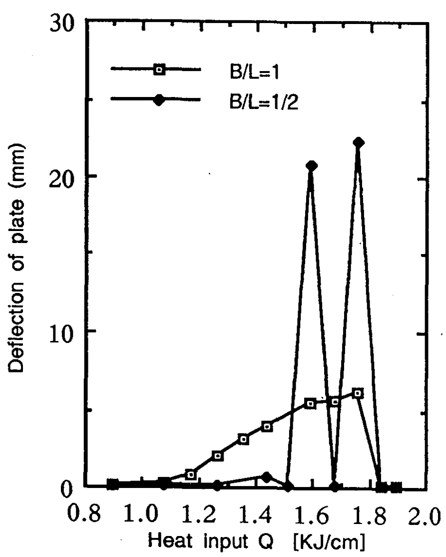


Fig.6 Effect of the heat input on the deflection of the plate when plate thickness is 3 mm.

respect to the buckling. If the thickness of the plate is smaller than the curve, the plate buckles. Thus, it is seen that the plate is more likely to buckle when the width and the length of the plate are large.

The deformations of the plates when the heat input is Q=1.596(kJ/cm) are computed through the second type analysis. As shown by **Fig.3-(1)** and **Fig.3-(2)**, the square plate buckles when the thickness is less than 4 mm and this critical thickness becomes 5 mm when the plate is rectangular (***B/L***=1/2). These agree with the prediction based on the eigen value which is shown in Fig. 2. The sudden change of the deflection observed in Fig.3 is due to the difference in the buckling mode. For both the square and rectangular plates, the deformation modes are shown in **Figs.4** and **5**, respectively. It is seen that the modes when t=1 (mm) and t=2 (mm) are different from others which show the lowest buckling mode.

The effect of the heat input on the buckling deformation when the thickness and the size of the plate are the same is shown in **Fig.6**. The deformation modes for both the square and the rectangular plates are shown in **Figs.7** and **8**. In case of the square plate, the deflection increases monotonically with the heat input up to Q=1.75 (kJ/cm). When Q=1.83 (kJ/cm), the magnitude of deflection decreases as the computed buckling mode changes from other cases. Such sudden variation of the deflection is observed more frequently in the rectangular plate. This may be due to the fact that there are more than one possible buckling modes when the heat input is large. Thus, the numerically obtained buckling

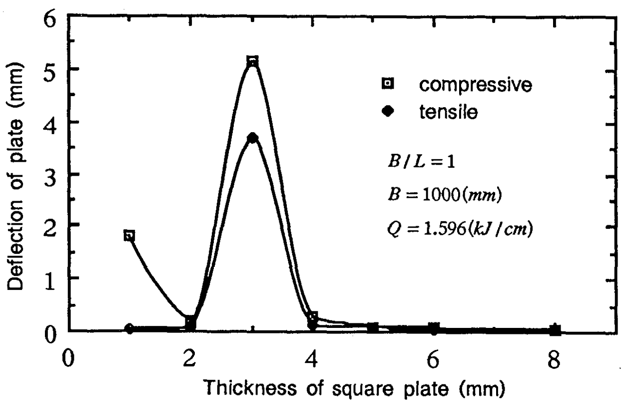


Fig.9 Effect of changing the sign of inherent strain.

deformation is not necessarily the mechanically most favorably mode.

In the case of the real welding process, the plate also buckles due to the tensile thermal strain in the heating process, and in the final state it buckles with the compressive residual plastic strain. **Figure 9** shows the deflection of the plates which have the compressive and the tensile inherent strain with the same magnitude. As it is shown by the figure, the plate can buckle under both the compressive and the tensile inherent strain and the critical plate thickness is almost the same in this case.

4. Conclusions

From the FEM analysis conducted in the present study, the following conclusions can be drawn.

1) The nonlinear large deflection elastic FEM using the inherent strain as the equivalent load can be employed to predict the possibility of buckling and deflection of plates.
2) The investigations show that the buckling response of rectangular plate is closely related to the plate thickness, the aspect ratio of the plate, and the distribution and the magnitude of the inherent strain.
3) Different buckling modes can occur depending on the conditions. The deflection of the plate becomes small when the higher buckling mode appears.
4) The tensile inherent strain can also cause buckling of the plate as well as the compressive inherent strain.

References

1) K. Masubuchi, "Analysis of welded structures" Pergamon Press. 1980, pp.300-312.
2) K. Masubuchi, "Buckling-type deformation of thin plate due to welding" Proceedings of Third National Congress for Applied Mechanics of Japan 1955, pp.107-111.
3) M. Watanabe, and K. Satoh, "Fundamental studies on buckling of thin steel plate due to bead-welding" Journal of the Japan Welding Society 27(6) 1958, pp.313-320 (in Japanese).
4) K. Terai, et al., "Study on prevention of welding deformation in thin-skin plate structures" Kawasaki Technical Review 61 Aug. 1978, pp.61-66.
5) Y. Ueda, and T. Yamkawa, "Thermal nonlinear behavior of structures", Advance in Computer Methods in Structure and Design, UAH Press 1972, pp.375-392.
6) E. Friedman, "Thermomechanical analysis of the welding process using the finite element method" Journal Press. Vessel Tech. 97(3) 1975, pp.206-213.
7) M. Jonsson, L. Karlsson. and L. E. Lindgren, "Deformations and stresses in the butt-welding of large plate" 'Num. Methods in Heat Transfer' Wiley & Sons Ltd. 1985, pp.35-38.
8) Y Ueda, H. Murakawa, A. M. Rashwan, and et al., "Development of computer aided process planning system for bending by line-heating (report 1) -Relation between the final form of plate and the inherent strain-", Transactions of JWRI 20(2) 1991, pp.129-139.
9) Y. Ueda, H. Murakawa, A. M. Rashwan and et al., "Development of computer aided process planning system for bending by line-heating (report 2) -Practice for plate bending in shipyard viewed from aspect of inherent strain-", Transactions of JWRI 21(1) 1992, pp.123-133.
10) Y. Ueda, H. Murakawa, A. M. Rashwan and et al., "Development of computer aided process planning system for bending by line-heating (report 3) -Relation between heating condition and deformation-" Transactions of JWRI 22(1) 1993, pp.145-156.
11) Y. Ueda, H. Murakawa, A. M. Rashwan and et al., "Development of computer aided process planning system for bending by line-heating (report 4) -Decision making of heating condition, location and direction-" Transactions of JWRI 22(2) 1993, pp.305-313.
12) A. Oddy, M. McDill, and J. Goldak, "A general transformation plasticity relation for 3d finite element analysis of welds" submitted to Euro. J. Mechanics.
13) K. W. Mahin, W. Winters, T. M. Holden, R. R. Hosbons, and S. R. MacEwen, "Prediction and measurement of residual elastic strain distributions in Gas Tungsten Arc welds" Welding Journal 70((9) 1991, pp.245-s--260-s.
14) C. Sheng and Y. Chen, "Modeling welding by surface heating" Journal of Engineering Materials and Technology Vol.(114) Oct., 1992, pp.439-449.
15) Y. Ueda, K. Nakacho and M. Yuan, "Application of FEM to theoretical analysis, measurement and prediction of welding residual stress" Transactions of JWRI 20(2) 1991, pp.97-107.
16) Y. Ueda, T. Yao, K. Nakacho and M. Yuan, "Prediction of welding residual stress, deformation and ultimate strength of plate panels" Transactions of JWRI 21(2) 1992 pp.137-143.
17) Y. Ueda, and M. Yuan, "A predicting method of welding residual stress using source of residual stress (report 3) " Transactions of JWRI 22(2) 1993, pp.157-168.