<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Dynamics of Renminbi Misalignment: A Markov Switching Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Author(s)</strong></td>
<td>Shi, Zongying</td>
</tr>
<tr>
<td><strong>Citation</strong></td>
<td>大阪大学経済学. 61(1) P.91–P.105</td>
</tr>
<tr>
<td><strong>Issue Date</strong></td>
<td>2011-06</td>
</tr>
<tr>
<td><strong>Text Version</strong></td>
<td>publisher</td>
</tr>
<tr>
<td><strong>URL</strong></td>
<td><a href="https://doi.org/10.18910/51772">https://doi.org/10.18910/51772</a></td>
</tr>
<tr>
<td><strong>DOI</strong></td>
<td>10.18910/51772</td>
</tr>
<tr>
<td><strong>rights</strong></td>
<td></td>
</tr>
</tbody>
</table>
Dynamics of Renminbi Misalignment:
A Markov Switching Approach *

Zongying Shi †

Abstract
This paper examines the question of renminbi (RMB) exchange rate misalignment, by applying a Markov switching approach to a standard empirical exchange rate model. The Markov switching model, as applied here, allows us to consider possible state shifts in the evolution of the misalignment process, which may better represent the actual behavior of the RMB exchange rate. Based on the smoothed probabilities of alternative states underlying the misalignment, we find evidence that the quarterly RMB exchange rate alternated between overvaluation and undervaluation (relative to long-run equilibrium) from 1992 to 2009. Moreover, there was asymmetry in the duration of exchange rate misalignment, with overvaluation having greater persistence than undervaluation.

JEL Classification Codes: F31, F37, F41, C32
Key words: Real Exchange Rate; RMB Misalignment; Markov Switching Model

I. Introduction
This paper examines the question of whether the Chinese renminbi (RMB) exchange rate has been misaligned, by applying a Markov switching approach to a standard empirical exchange rate model. The RMB misalignment issue has been a topic of major interest in the international economic policy community over the past several years, notably with the United States government taking a position that the RMB is significantly undervalued and that this has contributed to a large and persistent global payment imbalance involving the United States on the one hand and China on the other.‌ Though a considerable literature has emerged, the debate still continues.

Any empirical basis for taking a position in the debate is inconclusive as long as there is a lack of agreement on what the equilibrium exchange rate is. The standard approach in the literature has so far been to estimate the equilibrium exchange rate from a long-run relationship between the real exchange rate and other variables.

* This is an abridged version of a chapter in the author’s doctoral dissertation submitted to the Graduate School of Economics, Osaka University. The author thanks Professor Shinji Takagi (thesis committee chair), Professor Charles Yuji Horioka, Professor Tsunehiro Otsuki, Professor Jianhuai Shi, Professor Kanemi Ban, Professor Tatsuyoshi Miyakoshi, and Professor Udo Broll for their comments and advice. However, the author alone assumes responsibility for any remaining errors.

† Agricultural Development Bank of China, Beijing

Such a view is behind the series of US congressional bills introduced since 2003 to target the value of the RMB. See Hufbauer and Brunel (2008) for a discussion of the US congressional debate over the Chinese RMB issue.
rate and a set of other macroeconomic variables, and to define misalignment as the difference between the actual exchange rate and the long-run equilibrium exchange rate so estimated. An idea behind this notion of equilibrium is that the actual exchange rate converges to a distribution whose expectation is the equilibrium exchange rate, which can be thought of as an attractor (Funke and Rahn, 2005). Under this methodology, the RMB is considered to be overvalued (undervalued) when the real exchange rate exceeds (falls short of) the estimated equilibrium exchange rate.

There are two conceptual problems with this standard approach. First, the assessment posits the exchange rate to be either overvalued or undervalued, regardless of the magnitude. In practice, what we are more interested in knowing, or the most we can hope to know, is the relative probability of overvaluation versus undervaluation as the true model of exchange rate determination is not known with certainty. Second, the true model of exchange rate determination itself may be changing over time, especially in a dynamic economy like China where significant structural changes must be taking place. Thus, any attempt to capture the RMB’s potential misalignment must also take account of the possibility that the structural relationship between the exchange rate and other macroeconomic variables may be variable.

In order to address these problems we make use of a Markov switching model in this paper. The Markov model, as applied here, allows us to consider possible shifts in the evolution of the misalignment process, which may better represent the actual behavior of the RMB exchange rate. Although we adopt the widely used exchange rate modeling technique of Clark and MacDonald (1998), unlike the previous applications of this technique to the RMB (e.g., Zhang, 2001; Zhang, 2002; Chen, 2007), we are not just interested in the simple question of whether the RMB is overvalued or undervalued. Instead, we ask whether any identified misalignment is large enough to be “meaningful” in a probabilistic sense, by calculating the probabilities of alternative states. We do this in a model that explicitly considers a state switching factor, which we believe is fundamentally important for many economic and financial series that are subject to a shift from one type of behavior to another and back again (Hamilton, 1989; Bergman and Hansson, 2005; Lee and Chen, 2006).

In particular, we follow Terra and Valladares (2010) to employ a two-state Markov switching model, which our preliminary examination of the data suggested was a reasonable approximation to the evolution of quarterly RMB real effective exchange rate misalignment during the period Q1: 1992-Q4:2009. Because the sample period is rather long, we use the real exchange rate (as opposed to the nominal exchange rate) in order to reflect relative price level developments between China and its major trading partners. As a measure of the RMB real exchange rate, we use the real effective exchange rate (REER) as calculated by the International Monetary Fund (IMF).

It turns out that the application of the Markov switching approach to a standard empirical exchange rate model of the RMB indeed yields evidence of a pronounced Markov property, with the mean value of the RMB’s REER switching between alternate states. Over the sample period, the RMB was not always in one state or the other, but was undervalued in some cases and overvalued in others. More importantly, it becomes evident that the duration of overvaluation was longer on average (nine
quarters) than that of undervaluation (only two quarters). The RMB was more often overvalued than undervalued in a probabilistic sense.

The rest of the paper is organized as follows. Section II presents a brief review of existing work on empirical equilibrium exchange rate models and the application of a Markov switching model to exchange rate data. Section III considers the conceptual basis of an equilibrium exchange rate (Behavioral Equilibrium Exchange Rate, or BEER) model we use to estimate the quarterly equilibrium RMB exchange rate, as well as the theoretical underpinnings of a Markov switching model we employ to identify the property of RMB misalignment. Section IV explains the choice of variables for the equilibrium exchange rate equation, the sources of the data, and how we specify the BEER and Markov switching models for estimation purposes; it then presents the empirical results. Finally, section V presents concluding remarks.

II. A Brief Review of Previous Work

A number of approaches have been used in the literature to estimate the equilibrium exchange rate (EER) of a currency. Among the more popular concepts of equilibrium exchange rate are: Fundamental Equilibrium Exchange Rate (FEER), proposed by Williamson (1994); Desired Equilibrium Exchange Rate (DEER) by Bayoumi et al. (1994); Natural Equilibrium Exchange Rate (NATREX) by Stein (1994); and Behavioral Equilibrium Exchange Rate (BEER) by Clark and MacDonald (1998). Each concept differs from the others in terms of time horizon and the way of modeling the dynamics, but they all share the basic feature of defining equilibrium in terms of a historical relationship between the exchange rate and a set of other macroeconomic variables (for a review, see Driver and Westaway, 2004).

The BEER approach has been particularly popular as a tool of policy analysis mainly because of its simplicity. The empirical application of BEER essentially involves estimating a single reduced-form equation by such econometric procedures as cointegration, and is especially suitable for developing countries for which the estimation of a large and complex model is not feasible because of data limitation even when it may be desirable (Zhang, 2001). In contrast, for example, FEER would be quite sensitive to a slight modification of parameters when assumptions change. BEER’s typical use of cointegration is also an appealing way of identifying a long-run relationship between the real exchange rate and a set of underlying fundamentals, as is BEER’s definition of equilibrium as an attractor towards which the actual real exchange rate gravitates (Zhang, 2001). This explains why many studies, including Chen (2007), adopted the BEER approach to address the issue of RMB misalignment. This is the approach we also take in this paper.

As noted in the introduction, the weaknesses of the existing applications of the BEER model are that (i) they do not take an explicitly probabilistic approach to the issue of overvaluation versus undervaluation and that (ii) they do not take account of the possibility of a structural shift in the long-run relationship between the real exchange rate and other fundamentals. At most, what the existing literature offers in this context is the use of a Chow test to see if there was a structural break in the sample. But this presupposes that the breakpoints in the sample are known a priori. In reality, while
a discrete policy shift (say from a fixed to a flexible exchange rate system) may well be known, a potential structural shift in the behavior of exchange rates could involve more subtle factors, such as productivity growth and per capital income changes that may alter the rate of time preference. The benefit of the Markov switching technique we employ is to treat any regime shift as endogenous to the model.

Kim and Nelson (1999) show that a Markov switching model can be an important tool to capture occasional but recurrent and endogenous state shifts in time series. Although the state variable underlying such shifts is unobservable, its probability in each period can be estimated by the maximum likelihood method (Hamilton, 1989; Bergman and Hansson, 2005; and Lee and Chen, 2006). Because the time at which a structural break occurs is endogenously determined within the model, the Markov switching model eliminates the need to make an arbitrary decision as to where the breakpoint may be. It attempts to identify a state shift in the real exchange rate and helps make inferences about the probability of a particular state at each point in time.

For example, Engel and Hamilton (1990) model exchange rate dynamics as characterized by a sequence of stochastic, segmented time trends, and show that the predictive performance of a Markov switching model dominates that of a simple random walk model. Likewise, Lee and Chen (2006), using a Markov switching model to estimate an exchange rate process, show that the model fits the data well, possibly reflecting the behavior of a central bank that alternates between intervention and non-intervention under a managed floating regime. In fact, they clearly show that some parameters, such as the constant term (corresponding to the rate of exchange rate depreciation), are indeed state-dependent. Terra and Valladares (2010) present another application of a Markov switching model to exchange rate data, in which they use the estimated probabilities to identify overvaluation from undervaluation episodes for 85 countries (though not including China) during 1960-98.

III. Conceptual Framework

BEER model

According to Clark and MacDonald (1998), the BEER approach starts with uncovered interest rate parity (UIP):

\[
E_t(e_{t+1}) - e_t = i_t - i_t^*
\]

where \( e_t \) is the nominal exchange rate in period \( t \), defined in terms of domestic currency per unit of foreign currency (i.e., a rise in \( e \) denotes a depreciation of home currency); \( E_t(e_{t+1}) \) denotes the expected nominal exchange rate for period \( t+1 \) formed in period \( t \); and \( i_t \) and \( i_t^* \) are, respectively, domestic and foreign interest rates in period \( t \).

Subtracting the expected inflation differential from both sides of equation (1) we obtain the following real interest parity equation:

\[
E_t(q_{t+1}) - q_t = r_t - r_t^*,
\]

where \( q_t \) is the expected rate of inflation in period \( t \), and \( r_t \) and \( r_t^* \) are, respectively, domestic and foreign real interest rates in period \( t \).
where \( q_t \) is the realized real exchange rate in period \( t \); \( r_t \) and \( r_t^* \) refer, respectively, to domestic and foreign real interest rates in period \( t \); and \( E_t(q_{t+1}) \) denotes the expected real exchange rate for period \( t+1 \) formed in period \( t \). Rearranging equation (2) yields:

\[
q_t = E_t(q_{t+1}) - (r_t - r_t^*)
\]

This means that the realized real exchange rate \( q_t \) can be represented as a function of the expected real exchange rate for period \( t+1 \) and the current real interest rate differential.

Now, let us assume that the (unobservable) expected real exchange rate \( E_t(q_{t+1}) \) can be fully determined by a vector of long-run economic fundamentals \( Z_t \). Thus, the BEER approach produces an estimate of the equilibrium real exchange rate \( q_{BEER} \), which incorporates both long-run economic fundamentals \( Z_t \) and the short-run interest rate differential:

\[
q_{BEER} = f(Z_t, (r_t - r_t^*))
\]

Equation (4) is used as the basis for estimating the equilibrium exchange rate.

**Markov switching model**

Consider a model with structural breaks in its parameters as follows:

\[
y_t = x_t \beta_{S_t} + \nu_t, \ t=1,2, \ldots, T \quad \nu_t \sim N(0, \sigma^2_{\nu}) \quad S_t = 0 \text{ or } 1
\]

where \( y \) is a dependent variable (e.g., the real exchange rate), \( x \) is a vector of independent variables, \( \beta \) is a vector of coefficients, subscript \( S_t \) is a state that prevails in period \( t \), and \( \nu \) is an error term. Unlike a more conventional linear model, both the set of parameters \( \beta \) and the variance of the error term \( \nu \) in the model are permitted to assume different values, according to an unobservable state variable \( S_t \).

Although there could theoretically be many states, most empirical work in the past has considered two alternative states for tractability, which may correspond, in the context of exchange rate dynamics, to such factors as recession or boom, intervention or nonintervention, appreciation or depreciation, and the like. Thus, \( S_t \) can be made to take discrete values such as 0 or 1 (e.g., \( S_t = 0 \) for overvaluation; \( S_t = 1 \) for undervaluation).

In the most basic form of Hamilton’s filter (Hamilton, 1989), the state variable \( S_t \) is postulated to be evaluated according to a first order Markov process, such that:

\[
\text{prob}[S_t = 0 | S_{t-1} = 0] = p_{00} \quad \text{prob}[S_t = 1 | S_{t-1} = 0] = p_{10} \\
\text{prob}[S_t = 0 | S_{t-1} = 1] = p_{01} \quad \text{prob}[S_t = 1 | S_{t-1} = 1] = p_{11}
\]

where \( p_{00} \) denotes the probability of being in state 0 in period \( t \) when the system was also in state 0 in the previous period. Likewise, \( p_{10} \) denotes the probability of being in state 1 when the system was also in state 1 in the previous period. Analogously, \( p_{01} \) and \( p_{11} \) define the probability of switching from one state to the other. Needless to say, \( p_{00} \) and \( p_{10} \) would sum to unity, as would \( p_{01} \) and \( p_{11} \). Although \( S_t \) is not observable, the probability of each state at each point in time can be inferred by the Hamilton filter based on the information available up to period \( t (\Omega_t) \).
According to Kim and Nelson (1999), for a nonlinear model such as equation (5), the maximum likelihood estimator can be obtained by maximizing the following log likelihood function to generate the current sample \((y_T, y_{T-1}, \ldots, y_2, y_1)\)

\[
\ln L = \sum_{t=1}^T \ln(f(y_t | S_t))
\]  

(7)

where \(y\) is the dependent variable. The marginal density of \(y\) in period \(t\) is calculated by the following formula:

\[
f(y_t | \Omega_{t-1}) = \sum_{S_t=0}^1 f(y_t | S_t, \Omega_{t-1})P_r(S_t | \Omega_{t-1})
\]

(8)

That is, the marginal density can be interpreted as a weighted average of the conditional densities, given \(S_t = 0\) and \(S_t = 1\). This requires us to calculate the weighting factors \(\Pr[S_t = 0 | \Omega_{t-1}]\) and \(\Pr[S_t = 1 | \Omega_{t-1}]\) in order to obtain the log likelihood function.

Given \(\Pr[S_t = i | \Omega_{t-1}]\), the weighting factors \(\Pr[S_t = j | \Omega_{t-1}]\) can be calculated by the Hamilton filter as:

\[
\Pr[S_t = j | \Omega_{t-1}] = \frac{\sum_{i=0}^1 \Pr[S_t = i | S_{t-1} = i] \Pr[S_{t-1} = i | \Omega_{t-1}]}{\sum_{j=0}^1 \sum_{i=0}^1 \Pr[S_t = j | S_{t-1} = i] \Pr[S_{t-1} = i | \Omega_{t-1}]}
\]

(9)

where \(i=0,1; j=0,1; \) and \(\Pr[S_t = j | S_{t-1} = i]\) are the transition probabilities in equation (6). Once \(y_t\) is observed at the end of period \(t\), the probability term can be updated as:

\[
\Pr[S_t = j | \Omega_t] = \frac{\sum_{i=0}^1 \Pr[S_t = i | S_{t-1} = i] \Pr[S_{t-1} = i | \Omega_{t-1}]}{\sum_{j=0}^1 \sum_{i=0}^1 \Pr[S_t = j | S_{t-1} = i] \Pr[S_{t-1} = i | \Omega_{t-1}]}
\]

(10)

where \(\Omega_t = \{ \Omega_{t-1}, y_t \}\). These two steps, given by equations (9) and (10), may be iterated to obtain \(\Pr[S_t = j | \Omega_t], t=1,2,\ldots,T\). The log likelihood function is therefore a function of the vector of parameters \((\beta_0, \beta_1, \sigma^2_0, \sigma^2_1, p_{00}, p_{11})\) in equations (5) and (6).

To sum up, the parameters of the model can be estimated by maximizing the log likelihood function. Inferences about the state in each period can then be made, conditional on the parameter estimates of the model. The explicit form of the log likelihood function is given by:

\[
\ln L = \sum_{t=1}^T \ln \left( \sum_{i=0}^1 \sum_{j=0}^1 \Pr[S_t = i | \Omega_{t-1}] f(y_t | S_t, \Omega_{t-1}) \Pr[S_t = j | \Omega_{t-1}] \Pr[S_{t-1} = i | \Omega_{t-1}] \right)
\]

We can also obtain the smoothed probability of each state by the Kim smoother, which is an algorithm similar to the Hamilton filter but differs in the information horizon, as follows:

\[
\Pr[S_t = i | \Omega_t] = \sum_{j=1}^T \Pr[S_t = i, S_{t+1} = j | \Omega_t]
\]

(11)

where \(\Pr[S_t = i, S_{t+1} = j | \Omega_t] = \Pr[S_{t+1} = j | \Omega_t] \Pr[S_t = i | \Omega_t] \Pr[S_{t+1} = j | S_t = i] / \Pr[S_{t+1} = j | \Omega_t] \)
Specifically, the smoothed probability is inferred based on the whole sample information \( \Omega_T \) (available up to the ending point \( T \) of the sample), whereas the filtered probability is inferred based on the partial sample information \( \Omega_t \) (available up to period \( t \) of the sample; see Kim and Nelson, 1999 for details).

IV. Empirical Results

Choice of variables

The empirical model consists of the RMB’s real effective exchange rate and five other variables, as follows:

**The real effective exchange rate** (REER), the dependent variable in the BEER equation, is the RMB’s trade-weighted, consumer price index (CPI)-based real exchange rate against the currencies of China’s major trading partners, as obtained from the IMF’s *International Financial Statistics* (IFS). China’s REER declined sharply during the first quarter of 1994 (Figure 1), in connection with an exchange rate system reform when the official and parallel market exchange rates were unified and the rate was pegged to the dollar at a more depreciated level. The REER then began to appreciate until it reached a peak during the first quarter of 1998 when the central government committed to maintaining the RMB’s nominal rate against the US dollar in the immediate aftermath of the Asian financial crisis. The REER subsequently displayed swings before beginning a new round of appreciation from the third quarter of 2007 (when another exchange rate system reform revalued the currency and increased its flexibility). During this latter period, the RMB was officially managed in relation to a basket of currencies.

![Figure 1. China’s Quarterly Real Effective Exchange Rate (2005 = 100)](image)

Note: An increase indicates a real appreciation of the RMB.

**Terms of trade** (TOT) is a main channel through which macroeconomic shocks are transmitted from one country to another (Obstfeld and Rogoff, 1996). The impact of TOT on REER works through the adjustment of nontradable goods prices arising from demand shifts, with the sign depending on the relative sizes of the income and substitution effects. If the income effect dominates the substitution effect, the sign is expected to be positive, namely, an improvement in TOT leads to a real currency
appreciation. Because no unit export and import price indices are available for China, they are constructed from the relevant import and export price indices of China’s 26 major trading partners, appropriately weighted by the share of China in the respective countries’ imports and exports, as follows:

\[ \text{TOT} = \frac{\sum_{i=1}^{26} \alpha_i \cdot IMP_i}{\sum_{i=1}^{26} \beta_i \cdot EXP_i} \times 100 \]

where subscript \( i \) refers to China’s \( i \)th trading partner; \( EXP_i \) and \( IMP_i \) are the \( i \)th partner’s export and import price indices; \( \alpha_i \) is the share of the \( i \)th trading partner in China’s exports; and \( \beta_i \) is its share in China’s imports. These weights \( \alpha_i \) and \( \beta_i \) are calculated from the trade statistics for 1992-2008.

**Openness** (OPEN) measures the extent to which a country is exposed to international trade and is given by:

\[ \text{OPEN} = \frac{(EX + IM)}{GDP} \times 100 \]

where \( EX \) is the value of China’s exports, \( IM \) is the value of China’s imports, and \( GDP \) is China’s GDP. A rise in openness is expected to cause a shift of demand away from nontradable goods to importables, implying that an increase in OPEN would lead to real exchange rate depreciation. The quarterly data on Chinese exports and imports come from the IMF’s *Direction of Trade Statistics* (DOT), while the data on Chinese GDP come from the *People’s Bank of China Quarterly Bulletin*, converted into US dollars at the period average exchange rate obtained from the IFS.

**Government expenditure** (GOV) as a share of GDP is used as a proxy for government size. An increase in government size would trigger a shift of demand between tradables and nontradables, the direction of which depends on whether government expenditure is more or less intensive in the use of tradable versus nontradable goods. The data come from the *People’s Bank of China Quarterly Bulletin*.

**Net foreign assets** (NFA) play a key role in inter-temporal real exchange rate determination models (e.g., Obstfeld and Rogoff, 1996). Because net foreign debt must be repaid out of future trade surpluses, a fall in NFA is expected to lead to real exchange rate depreciation. Conversely, an increase in NFA would lead to currency appreciation over the medium to long term. We use the balance of foreign assets and liabilities in the monetary authorities’ balance sheet as a proxy for China’s net foreign assets position. The data come from the IFS.

The **interest rate differential** (INT) between domestic and foreign interest rates affects the exchange rate through its impact on capital flows. As capital inflows respond positively to a greater profit opportunity, a larger interest rate differential (favoring domestic assets) should lead to real

---

2 Australia, Brazil, Canada, Colombia, Denmark, Finland, Germany, Greece, Hong Kong SAR, Hungary, Ireland, Israel, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Poland, Singapore, Spain, Sweden, Thailand, Turkey, the United Kingdom, and the United States. The share of these countries in China’s trade is about 76 percent for exports and 58 percent for imports.
exchange rate appreciation. We use the RMB’s one-year lending rate as the domestic interest rate and the Federal fund rate in the United States as the foreign interest rate. The data come from the IFS.

**Model specification**

The estimation procedure involves three steps. First, we estimate the equilibrium value of the RMB by the BEER approach:

\[
\text{reer}_t = \beta_0 + \beta_1 \text{tot}_t + \beta_2 \text{open}_t + \beta_3 \text{gov}_t + \beta_4 \text{nfa}_t + \beta_5 \text{int}_t + \epsilon_t
\]

(12)

where \( \text{reer, tot, nfa, open, gov and int} \) (all in lower-case letters) refer to the natural logarithms of REER, TOT, OPEN, GOV, NFA, and INT, respectively. Second, we calculate the size of misalignment (\( \text{mis} \)) for each period as:

\[
\text{mis}_t = \text{reer}_t - \hat{\text{reer}}_t
\]

(13)

where \( \hat{\text{reer}} \) is the estimated long-run equilibrium real effective exchange rate. Finally, the estimated size of misalignment is used as an input in the following two-state, fourth-order autoregression (AR) Markov Switching model, along the lines of Hamilton (1989) and Terra and Valladares (2010):

\[
\text{mis}_t - \mu_{n_s} = \sum_{i=1}^{4} \phi_i (\text{mis}_{t-i} - \mu_{n_s}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \quad S_t = 0 \text{ or } 1
\]

(14)

\[
\mu_{n_s} = \mu_1 S_t + \mu_2 S_{t-1}
\]

where \( \mu \) is the mean, \( \sigma^2 \) is the variance, \( \phi \) is the AR coefficient and \( \epsilon \) is a Gaussian innovation.

Let us define the state in which the RMB is overvalued as \( S_t = 0 \) and the state in which it is undervalued as \( S_t = 1 \). Let us also allow the dynamics of overvaluation to be qualitatively different from that of undervaluation, but assume that the variances are invariant. The transition matrix can then be rewritten in a more compact form as:

\[
P = \begin{bmatrix}
P_{oo} & P_{ou} \\
P_{uo} & P_{uu}
\end{bmatrix}
\]

(15)

where \( P_{oo} \) means the probability of remaining in the state of overvaluation for two consecutive periods; \( P_{uu} \) is the probability of remaining in the state of undervaluation for two consecutive periods; \( P_{uo} \) is the probability of transition from the state of overvaluation to the state of undervaluation; and \( P_{ou} \) is the probability of transition from the state of undervaluation to the state of overvaluation.

**Regression analysis**

Because a unit root was detected for each variable in level but not when first differenced, we proceed to test for a cointegration relationship between \( \text{reer} \) and the independent variables. By Johansen’ maximum eigen-value and rank tests, we find at least one cointegration vector (Table 1). This means that a long-run relationship exists between the RMB’s real effective exchange rate and its fundamentals, allowing the equilibrium value of the RMB to be estimated by a normalized cointegration equation (Table 2).
The estimated long-run relationship gives the equilibrium value of the RMB (Figure 2, where the BEER's trend line is obtained by a Hodric-Prescott filter), and the difference between the actual REER and the filtered BEER represents misalignment (Figure 3, where it is expressed in percentage terms). It is evident from Figure 2 that, for the period Q1:1992-Q3:2009, the REER and BEER hardly intersected. Thus, the RMB was almost constantly misaligned in one direction or the other. Some have used a pre-established threshold value (say 15 percent or 25 percent) to judge whether the size of misalignment is large enough to be an overvaluation or undervaluation (Goldfajn and Valdés, 1999). The choice of a threshold value, of course, is arbitrary.

The approach we take to determine whether the size of misalignment is large enough to be relevant is the Markov switching model (Terra and Valladares, 2010). In particular we use the model to derive a probability criterion, by using the misalignment series as the dependent variable in the Markov

## Table 1. Cointegration Tests of the RMB Real Exchange Rate and Other Fundamental Macroeconomic Variables

<table>
<thead>
<tr>
<th>Hypothesized no. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace statistic</th>
<th>0.05 critical value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.491076</td>
<td>136.2221</td>
<td>103.8473</td>
<td>0.0001</td>
</tr>
<tr>
<td>At most 1 *</td>
<td>0.398916</td>
<td>90.29106</td>
<td>76.97277</td>
<td>0.0034</td>
</tr>
<tr>
<td>At most 2 *</td>
<td>0.360126</td>
<td>55.67770</td>
<td>54.07904</td>
<td>0.0357</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.192723</td>
<td>25.31680</td>
<td>35.19275</td>
<td>0.3813</td>
</tr>
<tr>
<td>At most 4</td>
<td>0.097347</td>
<td>10.75878</td>
<td>20.26184</td>
<td>0.5655</td>
</tr>
<tr>
<td>At most 5</td>
<td>0.054272</td>
<td>3.794430</td>
<td>9.164546</td>
<td>0.4436</td>
</tr>
</tbody>
</table>

Unrestricted cointegration rank (maximum eigenvalue) test

<table>
<thead>
<tr>
<th>Hypothesized no. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-eigen statistic</th>
<th>0.05 critical value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.491076</td>
<td>45.93106</td>
<td>40.95680</td>
<td>0.0127</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.398916</td>
<td>34.61336</td>
<td>34.80587</td>
<td>0.0527</td>
</tr>
<tr>
<td>At most 2 *</td>
<td>0.360126</td>
<td>30.36089</td>
<td>28.58808</td>
<td>0.0294</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.192723</td>
<td>14.55802</td>
<td>22.29962</td>
<td>0.4123</td>
</tr>
<tr>
<td>At most 4</td>
<td>0.097347</td>
<td>6.964353</td>
<td>15.89210</td>
<td>0.6738</td>
</tr>
<tr>
<td>At most 5</td>
<td>0.054272</td>
<td>3.794430</td>
<td>9.164546</td>
<td>0.4436</td>
</tr>
</tbody>
</table>

Notes: * denotes that the hypothesis of no cointegration is rejected at the 0.05 level; MacKinnon-Haug-Michelis (1999) p-values.

## Table 2. Normalized Cointegrating Vector

<table>
<thead>
<tr>
<th>reer</th>
<th>tot</th>
<th>gov</th>
<th>nfa</th>
<th>open</th>
<th>int</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>-2.019</td>
<td>0.057</td>
<td>-0.068</td>
<td>0.317</td>
<td>-0.013</td>
<td>2.867</td>
</tr>
<tr>
<td>(0.187)</td>
<td>(0.025)</td>
<td>(0.019)</td>
<td>(0.023)</td>
<td>(0.007)</td>
<td>(0.736)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors are in parentheses.
switching model. The key parameters of the model are estimated by a numerical optimization algorithm and are presented in Table 3. In the table, $\mu_O$ and $\mu_U$ are, respectively, the average value of REER misalignment under the states of overvaluation and undervaluation; $\phi_1$ through $\phi_4$ are the coefficients of the first through fourth order autoregressions; $\sigma^2$ is the variance of the error term; $P_{OO}$ and $P_{UU}$ are the diagonal elements of the transitional matrix that indicate the probability of remaining in the same state as the previous period, with the first term referring to overvaluation and the second term undervaluation. Accordingly, the switching probability from overvaluation to undervaluation
or from undervaluation to overvaluation can be calculated as \((1-P_{OO})\) or \((1-P_{UU})\). The log likelihood function is the maximum log likelihood function of generating the existing sample of the real effective exchange rate misalignment series.

It is obvious from these parameters that the mean values of misalignment under the alternative states are significantly different: \(\mu_O\) is positive (the state of overvaluation) while \(\mu_U\) is negative (the state of undervaluation). The coefficient of the fourth-order autoregression is significant. \(P_{OO}\) is larger than \(P_{UU}\), so that the diagonal elements of the matrix of the transition probabilities contain important information about the expected duration of a state (Kim and Nelson, 1999). Note that the expected duration is given by \(D_{ii} = 1/(1-P_{ii})\), where subscript \(ii\) can be either \(OO\) or \(UU\). On average, the state of overvaluation seems to have a longer duration (nine quarters, indicated by \(Dur_O\) in Table 3) than the state of undervaluation (two quarters, indicated by \(Dur_U\)). Using the Kim filter, we can estimate the smoothed probability of the state variable at each point in time (Figure 4). In the figure, the solid line is the probability of overvaluation, while the dots are the complementary probability of undervaluation.

Table 3. Estimated Parameters of the Markov Switching Model

<table>
<thead>
<tr>
<th>(\mu_O)</th>
<th>(\mu_U)</th>
<th>(\phi_1)</th>
<th>(\phi_2)</th>
<th>(\phi_3)</th>
<th>(\phi_4)</th>
<th>(\sigma^2)</th>
<th>(P_{OO})</th>
<th>(P_{UU})</th>
<th>(Dur_O)</th>
<th>(Dur_U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.669</td>
<td>-7.652</td>
<td>-0.063</td>
<td>-0.149</td>
<td>0.051</td>
<td>0.556</td>
<td>9.352</td>
<td>0.887</td>
<td>0.477</td>
<td>8.8</td>
<td>1.9</td>
</tr>
<tr>
<td>(0.648)</td>
<td>(1.717)</td>
<td>(0.126)</td>
<td>(0.108)</td>
<td>(0.118)</td>
<td>(0.147)</td>
<td>(2.328)</td>
<td>(0.053)</td>
<td>(0.165)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: \(Dur_O\) and \(Dur_U\) denote the duration of overvalued and undervalued states, respectively, in terms of average number of quarters; standard errors are in parentheses.

Figure 4. Smoothed Probabilities of Alternative States of RMB Misalignment

[Graph showing smoothed probabilities of overvaluation and undervaluation]
Figure 4 shows that the state of overvaluation dominates the state of undervaluation throughout the sample period, as the probability of overvaluation stays closer to one while the probability of undervaluation remains close to zero. For example, if we use 0.5 as the threshold, 57 (10) out of the 67 observations represent overvaluation (undervaluation) in a probabilistic sense. A histogram of the overvaluation and undervaluation probabilities gives an even clearer indication of this, namely, the quarterly RMB exchange rate was more often overvalued than undervalued (Figure 5).

V. Concluding Remarks

This paper has explored the question of whether the RMB exchange rate was misaligned during 1992-2009, by applying a Markov switching approach to a standard empirical exchange rate model. The application of the Markov switching model to quarterly data has allowed us to consider possible state shifts in the evolution of the misalignment process, which may better represent the actual behavior of the RMB exchange rate in view of likely structural shifts in the relationship between the exchange rate and a set of economic fundamentals in a dynamic economy such as China. Unlike the existing literature on the RMB misalignment issue, our interest went beyond simply asking whether the RMB was overvalued or undervalued. Rather, we have approached the issue from a probabilistic point of view, by asking whether any identified misalignment was large enough to be “meaningful” in a probabilistic sense.

Specifically, we have applied a Behavioral Equilibrium Exchange Rate (BEER) model to the quarterly real effective exchange rate of the RMB to obtain the misalignment series. We have then
used a two-state Markov switching model to derive the smoothed probabilities of alternative states
(overvaluation and undervaluation) underlying the misalignment, which gave clear evidence that the
quarterly RMB exchange rate alternated between overvaluation and undervaluation. Moreover, there
was asymmetry in the duration of exchange rate misalignment, with overvaluation tending to have
greater persistence than undervaluation.

The principal finding of this paper represents a significant departure from the existing empirical
literature on the RMB misalignment issue (see Chen, 2007 for a survey). Existing studies, based on
an equilibrium exchange rate model of one type or another, typically conclude that the RMB was
undervalued in some but overvalued in other periods. But this is not a surprising result, given the
fact that empirical equilibrium exchange rate modeling is an attempt to find a long-run relationship
between the exchange rate and a set of fundamentals on the basis of historical data. One would
naturally expect the actual exchange rate to fall on each side of the estimated equilibrium rate, with
nearly equal probability.

Our approach to the RMB misalignment issue has yielded a totally different insight: the RMB
exchange rate was more often overvalued than undervalued. A qualification may be necessary,
however. By following the conventional exchange rate modeling technique of the literature on the
RMB misalignment, we have not explicitly taken account of the potential impact of China’s long-run
sustainable current account balance, much less global factors, on the equilibrium exchange rate. When
such factors were fully incorporated, the conclusion that the RMB was more often overvalued than
undervalued could change. Even so, we have demonstrated the fundamental importance of taking an
explicitly probabilistic approach to any sensible discussion of exchange rate misalignment issues that
have far-reaching political implications.

References
International Money and Finance, 24, 121-138.
Rate Calculations to Alternative Assumptions and Methodologies,” in Williamson, J. (ed.),
Monetary Fund.
Empirical Study,” paper presented at the conference on “Dynamics, Economic Growth, and
International Trade (DEGIT),” Melbourne.


