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Osaka University
 Essays on growth theory with endogenous technological progress

Kizuku Takao
Essays on growth theory with endogenous technological progress

Kizuku Takao

A Dissertation

Submitted to the Graduate School of Economics, Osaka University,
in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy
in Economics

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4 Growth effect of bubbles in a non-scale endogenous growth model
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Each chapter included in this dissertation is originated from:

- Chapter 1,
  written for this thesis,

- Chapter 2,

- Chapter 3,

- Chapter 4,
Chapter 1
Introduction

1.1 Backgrounds and aims

The sources of economic growth are categorized into human capital and physical capital formation, labor force growth, and technological progress. Among these elements, the last one is considered to be the most fundamental factor to explain cross-country income differences [e.g. Prescott (1998)]. The most important driver of technological progress is R&D activities conducted by private firms to exploit profits. Indeed, in the U.S., about 70% of total national R&D expenditures is funded by private firms.

Starting from the analysis of Romer (1990), many researchers develop canonical models which incorporate R&D investment behaviors by firms to a dynamic general equilibrium setting [e.g. Grossman and Helpman (1991), Aghion and Howitt (1992), Smulders and Klundert (1995), Jones (1995b), Segerstrom (1998), Peretto (1998, 2007, 2011), Howitt (1999), and Dinopoulos and Syropoulos (2007)]. They improve understanding of what factors and how distorts corporate R&D activities. They show that there exist various market failures associated with R&D investments, such as monopoly, imperfect property right, knowledge spillovers, and pecuniary externalities. These distortions make the level of R&D investments in the market economy to be deviated from the social optimum level, affecting aggregate growth and welfare. They seek what policies should be implemented to correct distortions and then improve the level of welfare.

Recently, many empirical studies are also developed, which test important predictions derived from the above-mentioned theoretical analyses, using aggregate and dis-aggregate level data sets around the world [e.g. Jones (1995a), Lainecz and Peretto (2006), Ha and Howitt (2007), Madsen (2008) and Ang and Madsen (2011)]. For example, the first-generation theoretical models have the prediction that equilibrium growth rate is increasing in the market size, which is called as “scale effect
properties” [e.g. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Smulders and Klundert (1995)]. Now it is widely accepted that this scale effect property is empirically refuted. After the criticism, several models which have no scale effect property are developed. These models are roughly classified into the following two prominent types: semi-endogenous type [e.g. Jones (1995b) and Segerstrom (1998)] and fully-endogenous type [e.g. Peretto (1998, 2007, 2011) and Howitt (1999)]. The semi endogenous type assumes diminishing returns to scale in R&D production. In this type, the steady-state growth rate is only pinned down to population growth rate. On the other hand, the fully-endogenous type is the hybrid model where both quality innovation and variety expansion occur. In this type, the steady-state growth rate is dependent on the other parameters and policy variables. A recent growing body of empirical literature tests what type of the second-generation models is well suited to the real economy, although it is still controversial. Ha and Howitt (2007), Madsen (2008), and Ang and Madsen (2011) show that the fully-endogenous type performs well, rather than the semi-endogenous type.

The studies of growth theory considering endogenous technological progress develop not only theoretically but also empirically over the past 25 years. In particular, as mentioned above, the feedback from empirical studies to theoretical studies is now growing. However, many open questions still remain. This thesis tackles some of them. This thesis is composed of three chapters. Each chapter address the following problem.

Chapter 2 contributes to the study of an optimal subsidy problem regarding R&D investments. It is widely accepted that R&D subsidies is needed in order to enable the market equilibrium to replicate the socially optimal allocation [e.g. Grossman and Helpman (1991), Sener (2008), and Grossman, Steger, and Trimborn (2013)]. However, all the analyses are based on the model where technological progress is neutral for consumption goods and capital goods. The literature of growth-accounting addresses that capital goods have advanced dramatically and the relative price of capital goods has fallen. This phenomenon is considered to be derived from sector-specific technological change in the capital goods sector, which is called as “investment-specific technological progress” [e.g. Greenwood, Hercowitz, and Krusell (1997)]. If we focus on the role of investment-specific technological change, what subsidy policy is required to attain the first-best allocation is an open question. In chapter 2, we tackle this question.

The study of macroeconomic implications of tax changes is also an important topic. However, almost analyses are based on the growth model with only physical capital or human capital formation [e.g. Rebelo (1991), Milesi-Ferrettii and Roubini (1998), and Turnovsky (2000)]. Compared with these models, R&D-based growth models potentially have more distortions and thus more various channels derived from tax changes may interact each other. Recently, a few studies [e.g. Zeng
and Zhang (2002), Peretto (2003, 2007, 2011)] examine macroeconomic effects of distortionary tax changes such as a corporate tax cut, on the basis of R&D-based endogenous growth models. In chapter 3, we extend these studies.

Although chapter 2 and chapter 3 examine normative analyses regarding fiscal policies, chapter 4 sheds light on the interactions between financial markets and economic growth. Specifically, chapter 4 conducts a positive analysis about the relationship between the presence of asset bubbles and R&D firms’ investment behaviors. Here asset bubbles are defined as the difference between the fundamental value of an asset and its market value. Empirical evidences suggest that asset bubbles sometimes emerges, and they are accompanied by higher economic growth and a consumption boom. However, existing theoretical studies predict that the presence of asset bubbles increases consumption but retards economic growth [e.g. Grossman and Yanagawa (1993) and Futagami and Shibata (2000)]. Therefore, the theoretical prediction runs contrary to the mentioned above facts. Employing the recent developed growth model with endogenous technological progress, chapter 4 overcomes this conflict. The analysis still remains only in a positive analysis but has wider directions of future normative studies.

1.2 Plan


Chapter 3 examines how changes in various tax rates relevant to corporate activities affect growth and welfare. Here we also focus on their methods of implementation: we examine the effect not only of unanticipated tax changes which have permanent implementation periods, but also anticipated and temporary tax changes. For this purpose, we consider adjustment costs involved in the investment process and allow firms to make a forward looking investment decision in a R&D-based endogenous growth model. Calibrating the model with U.S. data, we find that a dividend tax cut reduces the level of welfare irrespective of implementation method. On the other hand, a capital gains tax cut and a rise in the R&D tax credit rate enhance the level of welfare irrespective of implementation. However, the announcement of these tax changes prior to implementation reduces their effectiveness.

Chapter 4 provides a theoretical explanation for why the presence of asset bub-
bles can lead to higher economic growth in concurrence with high consumption by using a simple endogenous growth model with overlapping generations. In the model economy, long-lived value-maximizing firms continuously improve the quality of their specific products through in-house R&D, while at the same time new firms also enter into the market. Due to an absence of intergenerational altruism, asset bubbles can exist as pyramid schemes whose value is not backed by fundamental value. The presence of asset bubbles then leads to higher interest rates. This requires product proliferation to be impeded, which results in an increase in the demand for differentiated goods at the level of an individual firm. A larger scale of production at the level of an individual firm can encourage in-house R&D of firms and promote economic growth.
Chapter 2

Dynamic analysis of an endogenous growth model with investment-specific technological change

2.1 Introduction

Along with recent technological developments such as information technologies, capital goods have advanced dramatically and the relative price of capital goods has fallen. Krusell (1998) theoretically constructs an endogenous growth model with a sector-specific technological change in the capital goods sector (so-called investment-specific technological change)\(^1\) and explains how R&D performed by firms induces the relative price decline of capital goods, which promotes economic growth. However, since Krusell (1998) focuses only on the steady state, the dynamic properties of the model remain an open question.\(^2\)

In this paper, I examine the transitional process to the steady state and consider a dynamically optimal subsidy policy that enables the market equilibrium to replicate the socially optimal allocation in the Krusell (1998) model. Because the speed of convergence may be slow, it is important to consider how the subsidy policy should change over time as the economy develops. For this purpose, I slightly modify the Krusell (1998) model by reconstructing a continuous-time version model, assuming a

---

\(^1\)According to Greenwood, Hercowitz, and Krusell (1997), investment-specific technological change explains about 60% of US economic growth in the post-war period in contrast to the approximate 40% contribution of sector-neutral technological change.

\(^2\)Boucekkine, del Rio, and Licandro (2005) and Huffman (2007) also analyze an endogenous growth model where investment-specific technological change is induced by R&D in firms. However, their analysis also focuses only on the steady state.
linear technology of R&D production function to simplify analysis of the transitional
dynamics. Otherwise, the environment of the model is the same as in Krusell (1998).

Economic growth is driven by investment in durable capital goods. Long-lived
monopolistic firms repeatedly innovate the firm-specific production technologies of
capital goods. Monopoly power induces each firm to conduct investment and R&D
below an optimal level. Moreover, the presence of social knowledge spillovers further
distorts R&D.

The main contribution of this paper is to show that a combination of the time-
invariant subsidy for investment and the time-variant subsidy for R&D enables the
market equilibrium to replicate the socially optimal allocation. Intuitively, a simple
subsidy policy for investment is required to correct only one distortion, but a complex
subsidy policy for R&D is required to correct the two distortions.

The rest of the paper is organized as follows. Section 2.2 describes the model.
Section 2.3 analyzes the dynamic system of the market equilibrium. Section 2.4
considers an optimal subsidy policy. Finally, section 2.5 summarizes the results.\(^3\)

2.2 Model

In this section, I build the model on a continuous-time version of Krusell (1998).
An economy consists of a final goods sector, a capital goods sector, and households.
The production technology of final goods and capital goods are different. First, I
consider the final goods sector.

The final good, \(Y_t\), is produced by the following technology:

\[
Y_t = \int_0^1 K_{jt}^\alpha \, dj \, L_t^{1-\alpha},
\]

\(\alpha \in (0, 1)\), where \(L_t\) and \(K_{jt}\) respectively represent the inputs of labor and the \(j\)th
capital good at time \(t\). Perfect competition prevails in the final goods market. The
price of final goods is normalized to one. Therefore, I obtain the following optimal
conditions:

\[
w_t = (1 - \alpha) \int_0^1 K_{jt}^\alpha \, dj \, L_t^{-\alpha} \quad \text{and} \quad p_{jt} = \alpha K_{jt}^{\alpha - 1} L_t^{1-\alpha},
\]

where \(w_t\) and \(p_{jt}\) respectively represent the wage rate and the rental price of the \(j\)th capital good at
time \(t\).

Next, I consider the capital goods sector. Monopolistic competition prevails in
the capital goods market and there is a continuum of goods, which is indexed by
.type \(j \in [0, 1]\). From the beginning, the monopoly of each firm is protected by
perfect patent protection. Entry of firms is not considered. Each firm accumulates
its specific capital goods and rents them exclusively to producers of final goods.\(^4\)
One unit of final good can produce \(T_{jt}\) units of capital goods. That is, \(T_{jt}\) represents
the level of the \(j\)th firm’s technology for producing capital goods at time \(t\). Further,

\(^3\)Mathematical calculations and proofs of some of the dynamic properties are omit-
ted. Please refer to the working versions of the paper (http://www.iser.osaka-

\(^4\)This formulation is similar to Smulders and Van de Klundert (1995).
each firm improves this technology by R&D with labor inputs. The law of motion of the capital stocks of the \( j \)th firm is

\[
\dot{K}_{jt} = T_{jt}I_{jt} - \delta K_{jt},
\]

(2.1)

where \( I_{jt} \) and \( \delta > 0 \) respectively represent the inputs of final goods (hereafter referred to as investment) at time \( t \) and the physical depreciation rate of capital stocks. In addition, the law of motion of the production technology of the \( j \)th firm is

\[
\dot{T}_{jt} = \psi T_{jt}^\gamma \bar{T}_t^{1-\gamma} L_{Ajt}, \quad \psi > 0, \quad \gamma \in [0, 1],
\]

(2.2)

where \( L_{Ajt} \) and \( \bar{T}_t \equiv \int_0^1 T_{jt} dj \) respectively represent the labor inputs to R&D at time \( t \) and the average level of production technology across firms.

The profit of the \( j \)th firm at time \( t \) is

\[
\pi_{jt} = p_{jt}K_{jt} - (1 - \tau_I^t)I_{jt} - (1 - \tau_R^t)w_tL_{Ajt},
\]

where \( \tau_I^t \) and \( \tau_R^t \) respectively represent the subsidy rates for investment and for R&D at time \( t \). I assume that the government can impose a lump-sum tax on households to cover these subsidies. In contrast to Krusell (1998), I assume that the R&D technology is specified to be linear in labor inputs, following Grossman and Helpman (1991, ch3). This specification ensures the existence of a balanced growth path in the steady state. \( T_{jt} (\bar{T}_t) \) captures firm-specific (social) knowledge spillovers. \( \gamma \) measures the relative importance of firm-specific dynamic returns to the productivity of R&D. When \( \gamma = 0 \), each firm does not internalize its dynamic returns. When \( \gamma = 1 \), each firm internalizes them completely.

The present value of the \( j \)th firm at time 0 is

\[
V_{j0} = \int_0^\infty (\pi_{jt}) \exp \left(-\int_0^t r_s ds\right) dt,
\]

where \( r_t \) represents the return on safe assets at time \( t \). Symmetry across firms is assumed, so the subscript \( j \) can be dropped. Hence, in equilibrium, \( T_t = \bar{T}_t \). Each firm maximizes the above value function subject to the inverse demand function and the law of motions given \( \bar{T}_t \). To solve the intertemporal maximization problem, I define the following current-value Hamiltonian as

\[
H = p_t K_t - (1 - \tau_I^t)I_t - (1 - \tau_R^t)w_t L_{Ajt} + \mu_t [T_tI_t - \delta K_t] + q_t [\psi T_t^\gamma \bar{T}_t^{1-\gamma} L_{Ajt}] + \mu_t \text{ where the co-state variables } \mu_t \text{ and } q_t \text{ respectively represent the shadow value of investment and R&D at time } t.
\]

I restrict the analysis to an interior solution where both investment and R&D occur. I obtain the following optimal conditions:

\[
\mu_t = \frac{(1 - \tau_I^t)}{T_t},
\]

(2.3)

\[
r_t \mu_t = \alpha^2 K_t^{\alpha-1} L_{yt}^{1-\alpha} - \delta \mu_t + \dot{\mu}_t,
\]

(2.4)

Krusell (1998) assumes that the R&D technology is

\[
T_{jt+1} = T_{jt}^{\gamma} \bar{T}_t^{1-\gamma} H(L_{Ajt}), \quad H'(\cdot) > 0 > H''(\cdot).
\]

5Krusell (1998) assumes that the R&D technology is
\[ q_t = \frac{(1 - \tau_t^R)w_t}{\psi T_t^\gamma T_t^{1-\gamma}}, \]  
\[ r_t q_t = \mu_t I_t + q_t \psi \gamma T_t^{\gamma-1} T_t^{1-\gamma} L_{At} + \dot{q}_t. \]  

In addition, the following transversality conditions must be satisfied:
\[ \lim_{t \to \infty} \mu_t K_t \exp \left( -\int_0^t r_s ds \right) = 0 \]  
and \[ \lim_{t \to \infty} q_t T_t \exp \left( -\int_0^t r_s ds \right) = 0. \]  
In equilibrium, each firm charges a monopoly markup price as \( p_t = \frac{1}{\alpha} u_t \) where \( u_t \equiv r_t + \delta + \frac{\dot{T}_t}{T_t} \) represents the user costs of capital at time \( t \).

Lastly, I consider a representative household’s problem. Population size, \( L \), is constant over time. Each individual supplies one unit of labor inelastically. He maximizes the following lifetime utility function:
\[ U_0 = \int_0^\infty (\log c_t) \exp (-\rho_t) dt, \]  
where \( c_t \) and \( \rho > 0 \) respectively represent the per-capita consumption of final goods at time \( t \) and the individual discount rate.\(^6\)

Solving the intertemporal optimization problem, I obtain the following Euler equation:
\[ \dot{c}_t = r_t - \rho. \]  
Additionally, the following transversality condition must be satisfied: \[ \lim_{t \to \infty} \frac{1}{c_t} a_t \exp (-\rho_t) = 0, \]  
where \( a_t \) represents per-capita assets at time \( t \).

### 2.3 Market equilibrium and dynamics

In this section, I derive a dynamic system of the market equilibrium based on the model in the preceding section. The market equilibrium condition of final goods is \( Y_t = c_t L + I_t \) and the market equilibrium condition of labor is \( L = L_{Y_t} + L_{At} \).

The market equilibrium is characterized by the optimal conditions of the final goods sector, the law of motions, (2.1) and (2.2), the monopolistic firm’s optimal conditions, (2.3), (2.4), (2.5), and (2.6), the household’s Euler equation, and the two market equilibrium conditions. To derive the dynamic system, I define \( Z_t \equiv T_t^{1-\gamma} K_t \), \( S_t \equiv T_t^{1-\gamma} c_t \), and \( Q_t \equiv q_t T_t^{\gamma} T_t^{1-\gamma} \). The following three equations constitute the dynamic system of the market equilibrium:

\[ \frac{\dot{Z}_t}{Z_t} = \frac{1}{1 - \frac{\tau_t}{T_t}} - \psi \frac{L}{1 - \alpha} - \delta, \]  
\[ \frac{\dot{S}_t}{S_t} = \alpha^2 \mu Q_t \left[ \frac{1}{1 - \frac{\tau_t}{T_t}} \right] + \frac{1}{1 - \frac{\tau_t}{T_t}} - \frac{\psi L}{1 - \alpha} - (\delta + \rho) - \frac{\dot{\tau}_t}{1 - T_t}, \]  
\[ \frac{\dot{Q}_t}{Q_t} = \frac{1 - \alpha}{\alpha} \kappa Q_t \left[ -2\alpha + 1 - (1 + \gamma)(1 - \alpha) + \frac{1 - \tau_t}{1 - T_t} \right] - \alpha (1 - \alpha) \kappa Q_t \left[ \frac{1}{1 - T_t} \right]. \]

\(^6\)For simplicity, the utility function is specified to be of the log-utility type although it is specified to be of the CRRA type in Krusell (1998). This simplicity never changes the results qualitatively.
Dynamic analysis of an endogenous growth model with investment-specific technological change

\[
- \frac{1 - \alpha}{\alpha} Q_t^{\frac{\alpha}{1 - \alpha}} S_t L \left( 1 - \tau_I^t \right) + \frac{1 - \alpha}{\alpha} \left[ \left( \frac{\alpha + (1 - \alpha)\gamma}{1 - \alpha} \right) \psi L + \delta + \frac{\dot{\tau}_I^t}{1 - \tau_I^t} - \frac{\dot{\tau}_R^t}{1 - \tau_R^t} \right],
\]

where \( \kappa \equiv (1 - \alpha)^{\frac{1}{\alpha}} \psi^{\frac{a-1}{\alpha}} \). The dynamic system has a unique steady state equilibrium that is locally saddle-point stable.\(^7\) Because the dynamic systems are too complicated, I analyze the transitional dynamics of the market equilibrium growth path (in the absence of subsidy policy) numerically by using the relaxation algorithm.\(^8\) In the numerical simulation, I choose the parameter values as follows. \( \rho \)

\( \text{Figure 2.1: Transitional dynamics of the growth rate of } \{c_t, K_t, T_t, q_t\} \) (dashed line: \( Z_0 = 0.5 \times Z^* \), solid line: \( Z_0 = 2.0 \times Z^* \), \( \gamma = 0.5 \))

\(^7\)When a measure of the productivity of R&D, \( \psi \), or scale of the economy, \( L \), is sufficiently high and the physical depreciation rate of capital stocks, \( \delta \), and the individual discount rate, \( \rho \), is sufficiently low, the uniqueness and existence of the steady state with the positive growth rate is guaranteed.

\(^8\)Trimborn, Koch, and Steger (2008) detail the relaxation algorithm. They also pro-
and $\delta$ respectively are set as 0.05 and 0.2, as is conventional in macroeconomic literature. $\alpha$ is set as 0.8, which implies that the mark-up rate of capital goods is 25%. $L$ is normalized to one. I focus on the case of $\gamma = 0.5$ as a benchmark. $\psi$ is set as 0.025 so that the growth rate of $Y_t$ in the steady state is around 2%. The initial value of the state variable, $Z_0 \equiv T_0^{\frac{1}{\alpha}}K_0$, is set to $Z^* \times 0.5$ or 2.0. This set of parameters and initial conditions also applies to the subsequent numerical exercises.

Figure 2.1 depicts the numerical simulation of the transition path of the growth rate of $\{c_t, K_t, T_t, q_t\}$ in the market equilibrium, starting from the given initial variables $\{K_0, T_0\}$. This figure shows that if an economy initially starts with relatively abundant technologies, then consumption, capital stocks, and the shadow values of R&D grow rapidly, while technologies grow slowly compared to the growth rate in the steady state during an early phase of the transition. The reason is as follows. Since each firm initially has relatively abundant technologies, in the early phase of the transition, the return to investment is higher than the steady-state level, which results in each firm conducting a higher level of investment and a lower level of R&D. Further, this higher return to investment and lower growth rate of technologies derives a higher interest rate in the asset market to satisfy the no-arbitrage condition of investment, (2.4). Therefore, the growth rate of consumption is higher, as households save more and the growth rate of the shadow value of R&D must be also higher to satisfy the no-arbitrage condition of R&D, (2.6). The same logic applies to the case where the economy initially starts out with relatively abundant capital stocks, resulting in the opposite outcome from the case discussed above.

### 2.4 Optimal subsidy policy

In this section, I analyze what subsidy policy enables the market equilibrium to replicate the socially optimal allocation, starting with solving the social planner problem. The inter-temporal optimization problem of the social planner is to maximize the lifetime utility function of households subject to the law of motions, (2.1) and (2.2), and the two market equilibrium conditions. The following three equations constitute the dynamic system of the socially optimal allocation:

\[
\frac{\dot{Z}_t}{Z_t} = \kappa Q_t + \kappa Q_t^{\frac{1}{1-\alpha}} Z_t - S_t Z_t^{-1} L - \frac{\psi L}{1-\alpha} - \delta, \tag{2.10}
\]

\[
\frac{\dot{S}_t}{S_t} = \alpha \kappa Q_t + \kappa Q_t^{\frac{1}{1-\alpha}} Z_t - \frac{\psi L}{1-\alpha} - (\delta + \rho), \tag{2.11}
\]

vide MATLAB programs for the relaxation algorithm, which are downloadable for free at http://www.wiwi.uni-siegen.de/vwli/forschung/relaxation/matlab_applications.html?lang=de.
\[
\frac{\dot{Q}_t}{Q_t} = -(1 - \alpha)\kappa Q_t \frac{1 - \alpha}{\alpha} Q_t^{\frac{1-\gamma}{\alpha}} S_t L + \frac{1}{\alpha} \psi L + \frac{1 - \alpha}{\alpha} \delta. \tag{2.12}
\]

The upper panel of Figure 2.2 depicts the numerical simulation of the transition path of labor inputs to R&D in market equilibrium and socially optimal allocation. This panel shows that during the transition, labor inputs to R&D in the market equilibrium are always less than those of the socially optimal allocation.

Next I examine an optimal transition path of the subsidy rate for investment and R&D. The following two aspects must be considered. First, the market economy in the presence of subsidy policy eventually must attain the steady state of the socially optimal allocation. Second, the market equilibrium growth path to the steady state must also correspond with that of the socially optimal allocation. Therefore, a combination of the optimal subsidy rates must enable (2.7), (2.8), and (2.9) to correspond with (2.10), (2.11), and (2.12), respectively. When \( \gamma = 1 \), a simple combination of the time-invariant subsidy rate, \( \tau_I^t = \tau_R^t = 1 - \alpha \), is required. When \( \gamma \neq 1 \), the optimal subsidy rate for investment is also time-invariant, \( \tau_I^t = 1 - \alpha \), while the optimal subsidy rate for R&D requires a time-variant rate that satisfies the following linear ordinary differential equation:

\[
\dot{\tau}_R^t + A_t \tau_R^t = B_t, \tag{2.13}
\]

where \( A_t \equiv \kappa Q_t^{\text{op}} \frac{1}{\alpha} Z_t^{\text{op}} [-\alpha - \gamma (1 - \alpha)] + Q_t^{\text{op}} \frac{\alpha}{\alpha} S_t^{\text{op}} L + (\gamma - 1) \psi L \) and \( B_t \equiv A_t + \alpha \left[ \kappa Q_t^{\text{op}} \frac{1}{\alpha} Z_t^{\text{op}} - Q_t^{\text{op}} \frac{\alpha}{\alpha} S_t^{\text{op}} L \right]. \)

Setting \( \dot{\tau}_R^t = 0 \) in (2.13), I obtain the following optimal R&D subsidy rate in the steady state:

\[
\tau_R^{**} = 1 - \frac{\alpha \rho}{\alpha \kappa Q^{**} - (\delta + \rho) - [\alpha + \gamma (1 - \alpha)] \left[ \alpha \kappa Q^{**} - (\delta + \rho) \right]},
\]

where \( Q^{**} \) is defined as the steady-state equilibrium value of \( Q_t \) in the socially optimal allocation. Since (2.13) is too complicated for an analytical solution, I provide a numerical simulation of the optimal transition path of the subsidy rate for R&D as shown in the lower panel of Figure 2.2. This panel shows that during the transition to the steady state, the required subsidy rate for R&D is time-variant. In addition, it is shown that the required subsidy rate is high when \( \gamma \) is low and that the movement of the optimal subsidy rate depends on the initial condition.

Why should the optimal subsidy rate for R&D be time-variant while that for investment should be time-invariant in the presence of social knowledge spillovers? Moreover, why should the optimal subsidy rate for R&D and investment be time-invariant if there exist no social knowledge spillovers? The reason is as follows. There

\[\{Z_t^{\text{op}}, S_t^{\text{op}}, Q_t^{\text{op}}\}_{t=0}^\infty \] represents the equilibrium values in the socially optimal allocation.
Figure 2.2: Upper panel: Labor inputs for R&D (dashed line: first-best solution, solid line: market equilibrium solution), Lower panel: the required subsidy rate for R&D ((i) $Z_0 = 0.5 \times Z^*$, (ii) $Z_0 = 2.0 \times Z^*$, Left panel: $\gamma = 0.5$, Right panel: $\gamma = 0.2$)

exist two sources of distortions, monopoly power and social knowledge spillovers.\(^{10}\) Since each firm internalizes only a portion of the social returns, monopoly power induces each firm to conduct investment and R&D below the optimal level. Moreover, the social knowledge spillovers further induce each firm to conduct less R&D because each firm internalizes only firm-specific dynamic returns to the productivity of R&D. Therefore, an R&D decision is dynamically complicated if social knowledge spillovers exist, and hence to correct the two distortions, an optimal subsidy rate for R&D should be time-variant. On the other hand, if there exist no social knowledge spillovers, an optimal subsidy policy for R&D is simply required to be time-invariant.

\(^{10}\)As capital goods are durable, the technological progress makes the capital goods produced in the past obsolete. However, each firm internalizes the obsolescence effect of technological progress. That is, it is internalized not only in the socially optimal allocation but also in the market equilibrium.
because it must only correct distortions of monopoly power. For the same reason, an optimal subsidy rate for investment is simply required to be time-invariant as well, with or without social knowledge spillovers.

2.5 Conclusion

This paper analyzes the transitional dynamics of a continuous-time version of Krusell’s (1998) model and shows that a dynamically complex subsidy policy is required to achieve the socially optimal allocation if socially knowledge spillovers exist. However, whether and how the results change under the discrete-time setting remains to be determined. The optimization problem of each firm is complicated, as each firm controls the levels of capital stocks and technologies dynamically. Hence, an analysis of the transitional dynamics in discrete time is even more difficult. In addition, I assume that the R&D technology is specified to be linear in labor inputs. Relaxing the assumption of linear technology for R&D may allow for the possibility of indeterminacy or a cycle. These topics are left for future research.
Chapter 3

Dynamic effects of anticipated and temporary tax changes in a R&D-based growth model

3.1 Introduction

Technological progress achieved through R&D activities is a major source of economic growth. Firms decide upon the scope of their investment in R&D, considering the costs and benefits of these R&D activities, whose values are dependent upon the applicable statutory tax rate. As Hall and Van Reenen (2000) point out, fiscal incentives for R&D investments differ across countries and change over time. The purpose of this study is to provide the clear policy implications arising from tax changes relevant to R&D activities in the context of a R&D-based endogenous growth model.

The novel feature of our study is its focus on the effect on growth and welfare not only of unanticipated tax changes, which have permanent implementation periods, but also of anticipated and temporary tax changes. Accordingly, we consider an environment where technological progress is driven by in-house R&D by long-lived value-maximizing firms, and these firms make forward-looking investment decisions regarding in-house R&D activities. In the real world, tax changes are usually announced before their implementation and are not permanent but rather only temporary. In such a situation, firms and households have an opportunity to adjust their intertemporal behavior to fit the tax schedule. For better understanding of taxation policy efficacy, it is important to consider what differences arise depending on how tax changes are implemented.

The present analysis is based on a recent endogenous growth model developed by Peretto (2007, 2011). Specifically, the model considers an economy where long-lived
valuemaximizing firms continuously improve upon the quality of their specific product through in-house R&D, while simultaneously new firms also enter the market. The model economy contains two types of investment opportunities, i.e., in-house R&D (quality improving) and the creation of a new firm (product proliferation). The model has the advantage of eliminating the well-known undesirable scale effect [Jones (1995)], while keeping the policy effect property, which is supported by a growing body of recent empirical literature.\footnote{The first generation R&D-based endogenous growth model [e.g. Romer (1990) and Grossman and Helpman (1991)] predicts that the equilibrium growth rate is increasing in the labor endowment. However, Jones (1995a) refutes this assertion using time-series data covering the post-war period. Then, the following two prominent model types are developed. The former type is referred to as the semi-endogenous growth type [e.g. Jones (1995b) and Segerstrom (1998)]. They resolve the undesirable scale effect property by assuming the diminishing returns in R&D production technologies. This specification yields the result that the steady state growth rate is only pinned down to population growth rate. By contrast, the latter type is referred as the fully-endogenous type [e.g. Peretto (1998), Howitt (1999) and Futagami and Ohkusa (2003)]. They assume that both vertical innovation and horizontal innovation occur. This hybrid model yields the conclusion that the steady-state growth rate is also dependent on the other parameters and policy variables. A recent growing body of empirical literature [e.g. Laincz and Peretto (2006), Ha and Howitt (2007), and Ang and Madsen (2011)] report that the latter type performs well, rather than the former type.} Increases in the scale of the aggregate economy are perfectly fragmented by endogenous product proliferation. Aggregate growth is driven by quality growth arising from firm’s in-house R&D activities. The intensity of in-house R&D is dependent on the demand for intermediate goods at the individual firm level, not the aggregate level.\footnote{This prediction is consistent with many empirical studies [e.g. Cohen and Klepper (1996), Adams and Jaffe (1996), and Pagano and Schivardi (2003)].}

However, in the model of Peretto (2007, 2011), a firm’s investment decision regarding in-house R&D turns out to be a static problem. This occurs because the model assumes that the production function of in-house R&D is linear. This implies that the current intensity of in-house R&D reflects only on current market conditions and tax rates, not future variables. As a result, if anticipated and temporary tax changes are incorporated into the model’s setting, the dynamic response of firm’s investment decisions can not be considered, and thus the actual impact of such tax shocks can not be captured.

To overcome this problem, we incorporate the adjustment costs of investment as used in the literature of investment theory.\footnote{See, for example, Hayashi (1982), Abel (1982), and Abel and Blanchard (1983).} More specifically, we assume that firms require the convex adjustment costs associated with in-house R&D investments. This specification is indeed more realistic. Some empirical literature points out the existence of high adjustment costs for R&D investments [e.g., Bernstein and Nadiri (1989), Himmelberg and Petersen (1994), and Brown and Petersen (2011)]. In the presence of adjustment costs, firms’ investment decisions regarding in-house R&D
are a forward-looking problem. The dynamic system of an economy is also characterized by the (tax-adjusted) shadow value of in-house R&D, which determines the intensity of in-house R&D. The shadow value summarizes all informations relevant to in-house R&D investment decisions. The flexibility provided by the shadow value is very useful in analyzing how investment decisions regarding in-house R&D dynamically react to both anticipated and temporary tax changes.

Using this modified model, we study the policy implications of (1) a dividend tax cut, (2) a corporate tax cut, (3) a capital gains tax cut, and (4) a rise in the R&D tax credit rate, taking account of differences arising depending on tax change implementation methods. Calibrating the model with U.S. data, we obtain the following main results. First, a dividend tax cut reduces the level of welfare irrespective of implementation methods. It is detrimental to in-house R&D and aggregate growth after implementation. However, pre-announcement of a dividend tax cut stimulates in-house R&D and aggregate growth up to the point when the tax cut is actually implemented. Second, the policy effect of a corporate tax cut depends upon whether or not in-house R&D expenditures are tax deductible. If in-house R&D expenditures are not deductible, a corporate tax cut leads to higher economic growth and welfare improvements irrespective of implementation. However, if they are fully (or partially) deductible, the policy effect are qualitatively the same as a dividend tax cut. On the other hand, a capital gains tax cut and a rise in the R&D tax credit rate improve the level of welfare irrespective of implementation methods. They stimulate in-house R&D and aggregate growth after implementation. However, pre-announcement of them is detrimental to in-house R&D and aggregate growth up to the point when the tax cut is actually implemented. Therefore, although the overall welfare effect remains positive, the pre-announcement worsens their effectiveness.

Intuitively, the above findings result form the interaction of various channels as follows. First, as firms make forward-looking investment decisions regarding in-house R&D activities, tax changes have a direct effect on incentives to in-house R&D, which dynamically differs depending on how tax changes are implemented. Second, as endogenous firm entry occurs in the present analysis, tax changes also affect firm size at the level of an individual firm, which causes secondary effects on incentives to in-house R&D. In addition, tax changes also affect households’ behavior and the impacts depend on the implementation methods of tax changes. The resulting macroeconomic effects of tax changes depend on what channel dominates through general equilibrium effects.

\(^4\)In Peretto (2007), the dynamic system of the economy is characterized by only one state variable (the number of firms per capita). In Peretto (2011), it is characterized by one state variable (the number of firms per capita) and one jump variable (consumption ratio). On the other hand, the dynamic system of the model used in our analysis is characterized by not only the number of firms per capita and consumption ratio but also one additional jump variable (shadow value of innovation).
Our study is closely related to Peretto (2007, 2011) and the differences between our study and Peretto (2007, 2011) are as follows. First, Peretto (2007, 2011) mainly focuses on the policy effect of a dividend tax cut such as the U.S.’s Jobs Growth and Taxpayer Relief Reconciliation Act of 2003. On the other hand, we also examine the effectiveness of alternative policy instruments rather than the dividend tax cut. Second, although Peretto (2007, 2011) focuses only on unanticipated and permanent tax changes, we also consider anticipated and temporary tax changes in an environment where firms dynamically react to these tax changes.\(^5\)

Our study is also related to the following previous studies. Zeng and Zhang (2002) and Peretto (2003) also study the effects of tax changes on the basis of a non-scale R&D-based growth model. However, both these studies analyze only unanticipated and permanent tax changes and do not consider transitional dynamics and welfare implications. Summers (1981) and Abel (1982) analyze how anticipated and temporary tax changes affect firms’ forward-looking investment decisions by using the framework of adjustment costs for investment. However, their analyses are based on partial equilibrium approaches. As a result, they cannot consider the impacts on aggregate growth and welfare. Strulik and Trimborn (2010) study the effects of anticipated and temporary tax changes in a general equilibrium setting. Their model is based on the neoclassical growth model with endogenous corporate finance, making the steady-state growth rate exogenous in this setting.

The remainder of this paper is as follows. Section 3.2 describes the model. Section 3.3 characterizes the dynamic system and the steady-state equilibrium of the market economy. Section 3.4 quantitatively analyzes the transitional adjustment of macroeconomic variables to tax changes and welfare consequences, calibrating the model with U.S. data. Section 3.5 analyzes the sensitivity of the numerical analysis. Finally, Section 3.6 concludes the study.

### 3.2 The model

In this section, we establish our model, which is based on that of Peretto (2011). Time is continuous. The economy is closed and consists of a final goods sector, an intermediate goods sector, households, and government. Perfect competition prevails in the final goods sector, while monopolistic competition prevails in the

---

\(^5\)More specifically, Peretto (2007) analyzes the revenue-neutral tax changes in an environment where the dividend tax rate is endogenously determined to balance the government’s budget constraint and shows that lowering the corporate tax rate and capital gains tax rate or increasing the R&D tax credit rate can lead to higher economic growth and improve welfare levels. Peretto (2011) analyzes the case where the government can finance the outlay required by tax changes via debt, and quantitatively shows that a dividend tax cut leads to the slowdown of in-house R&D and aggregate growth, and thus leads to substantial welfare losses.
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intermediate goods sector. Both the labor and asset markets are competitive. All fiscal variables either change only at discrete events or remain static. Thus, we can treat them parametrically and omit the time index $t$.

### 3.2.1 Final goods sector

The price of final goods is set to be the numeraire. Final goods are consumed by households and used as only one factor of production and investment by the intermediate goods sector. The final goods, $Y_t$ is produced by the following production function:

$$Y_t = \int_0^{N_t} X_{it}^\theta (Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} L_{it})^{1-\theta} di, \quad 0 \leq \alpha, \theta < 1,$$

where $N_t$ is the variety of intermediate goods (the number of intermediate goods firms), $X_{it}$ is the input of intermediate goods $i \in [0, N_t]$ (produced by firm $i$), and $L_{it}$ is the input of labor that uses intermediate goods $i$. The productivity of $L_{it}$ depends not only on the quality of intermediate good $i$, $Z_{it}$, but also on the average quality level across all intermediate goods, $\bar{Z}_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_{jt} dj$. Therefore, we obtain the following optimal conditions:

$$X_{it} = \left( \frac{\theta}{P_{it}} \right)^{\frac{1}{1-\theta}} (Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha} L_{it}),$$

$$L_{it} = \left( \frac{1-\theta}{W_t} \right)^{\frac{1}{\theta}} X_{it}^{\alpha} (Z_{it}^{\alpha} \bar{Z}_t^{1-\alpha})^{1-\alpha},$$

where $P_{it}$ and $W_t$ represent the price of intermediate good $i$ and the wage rate, respectively.

### 3.2.2 Intermediate goods sector

Firm $i$ can exclusively produces its differentiated good at quality, $Z_{it}$, because each firm’s monopoly is permanently protected by perfect patent protection. Producing one unit of intermediate goods requires one unit of final goods. Firms improve the quality of their specific product through their in-house R&D. In contrast to Peretto (2007, 2011), however, we assume that given increases in firm-specific quality level, $R_{it} \geq 0$, involve adjustment costs associated with innovation, following Hayashi (1982). Specifically, the law of motion pertaining to firm-specific quality is

$$\dot{Z}_{it} = R_{it},$$
and the amount of R&D expenditure is given by

$$\Phi(R_{it}, Z_{it}) = R_{it} + \frac{h}{2} \frac{R^2_{it}}{Z_{it}},$$

(3.5)

where $h > 0$ reflects the extent of adjustment costs associated with in-house R&D and the case of $h = 0$ corresponds to the specification of Peretto (2007, 2011).\(^6\)

At each point in time, fixed operating costs, $\phi Z_t (\phi > 0)$, are imposed. Accordingly, the gross cash flow is $F_{it} = (P_{it} - 1)X_{it} - \phi Z_t$, where the first term represents revenue minus variable production costs and the second term represents fixed operating costs. Let $\sigma$ represent the rate of R&D tax credits (the fraction of R&D expenditure that firms are allowed to deduct from their corporate taxable amount).\(^7\)

The total amount of corporate tax is $\tau \Pi [F_{it} - \sigma \Phi(R_{it}, Z_{it})]$, where $\tau \Pi$ represents the corporate tax rate. The gross cash flow must therefore be distributed as follows:

$$F_{it} = \tau \Pi [F_{it} - \sigma \Phi(R_{it}, Z_{it})] + E_{it}d_{it} + J_{it},$$

where $E_{it}$ is the number of equities, $d_{it}$ is the pre-tax dividends on a per-share basis, and $J_{it}$ is the retained earnings. A firm’s financial constraint is written by $J_{it} + \dot{E}_{it}v_{it} = \Phi(R_{it}, Z_{it})$, where $\dot{E}_{it}$ and $v_{it}$ represent the number of newly issued equities and the equity price on a per-share basis, respectively. Since we do not consider here the case where in-house R&D is financed by a bond issue, the above identity indicates that in-house R&D investments must be financed by retaining earnings, newly issued equities, or both.\(^8\)

Along the lines of Peretto (2011), we focus only on the scenario where the marginal source of in-house R&D is limited only to retaining earnings. The scenario is called “New view” in the corporate finance literature. In this scenario, $\Phi(R_{it}, Z_{it}) = J_{it}$ because $\dot{E}_{it} = 0$.

Let $V_{it} \equiv E_{it}v_{it}$ and $D_{it} \equiv E_{it}d_{it}$. Without loss of generality, $E_{it}$ is normalized to one. Dividends is given by

$$D_{it} = (1 - \tau \Pi)F_{it} - (1 - \sigma \tau \Pi)\Phi(R_{it}, Z_{it}).$$

(3.6)

The return on equity can be rewritten by

$$r_t = (1 - \tau_D) \frac{D_{it}}{V_{it}} + (1 - \tau_V) \frac{\dot{V}_{it}}{V_{it}},$$

(3.7)

---

\(^6\)This functional form is based on Turnovsky (2000).

\(^7\)Although $\sigma$ is assumed to be zero for simplification in Peretto (2011), we follow the specification of Peretto (2007) so that we can consider the effects of the tax credit policy for R&D investment as well.

\(^8\)See Turnovsky (1990) for a detailed discussion.
where \( \tau_D \) is the dividend tax rate and \( \tau_V \) is the capital gains tax rate.

Integrating (3.7) yields the value of firm \( i \) as follows:

\[
V_{it} = \int_t^\infty \exp \left( \int_t^s \frac{1}{1 - \tau_V} r_v dv \right) \left( \frac{1 - \tau_D}{1 - \tau_V} \right) \left[ (1 - \tau_H) F_{is} - (1 - \sigma \tau_H) \Phi(R_{is}, Z_{is}) \right] ds.
\]

Throughout this analysis, we consider a symmetric equilibrium by assuming that any new firm starts with the same technology level as incumbents so that the subscript \( i \) can be dropped. In the equilibrium, \( Z_t = \bar{Z} \) holds. Each firm maximizes its value, subject to (3.2) and (3.4), given \( \bar{Z} \). To solve the inter-temporal maximization problem, we define the following current-value Hamiltonian as

\[
H \equiv \frac{1 - \tau_D}{1 - \tau_V} \left[ (1 - \tau_H) F_t - (1 - \sigma \tau_H) \Phi(R_t, Z_t) \right] + q_t \left[ R_t \right],
\]

where the co-state variable, \( q_t \), represents a shadow value for in-house R&D. We obtain the following optimal conditions:

\[
P_t = \frac{1}{\hat{q}}, \quad (3.8)
\]

\[
q_t = \frac{(1 - \tau_D)(1 - \sigma \tau_H)}{(1 - \tau_V)} \left[ 1 + h \frac{R_t}{Z_t} \right], \quad (3.9)
\]

\[
r_t = (1 - \tau_D)(1 - \tau_H) \frac{\partial F_t}{\partial Z_t} \frac{1}{q_t} + (1 - \tau_D)(1 - \sigma \tau_H) \frac{h}{2} \left( \frac{R_t}{Z_t} \right)^2 \frac{1}{q_t} + (1 - \tau_V) \frac{\dot{q}_t}{q_t}. \quad (3.10)
\]

The transversality condition is \( \lim_{s \to \infty} \exp \left( -\frac{1}{1 - \tau_V} \int_t^s r_v dv \right) Z_s q_s = 0 \). From (3.4) and (3.9), the quality growth rate is given by

\[
\dot{z}_t \equiv \frac{\dot{Z}_t}{Z_t} = \begin{cases} 
\frac{1}{h} \left[ \frac{(1 - \tau_V)}{(1 - \tau_D)(1 - \sigma \tau_H)} q_t - 1 \right] & \text{if } \dot{q}_t > 1, \\
0 & \text{if } \dot{q}_t \leq 1.
\end{cases} \quad (3.11)
\]

(3.8) represents the pricing rule with constant mark-up. (3.9) indicates that firms invest in-house R&D up to the point where the shadow value of in-house R&D (RHS) equals the cost of in-house R&D (LHS). Since in-house R&D is funded only by retaining earnings, the outlay of one dollar for in-house R&D decreases dividend payments for shareholders by \( \frac{(1 - \tau_D)(1 - \sigma \tau_H)}{(1 - \tau_V)} \). Thus, reductions in the dividend tax rate and corporate tax rate increase the cost of in-house R&D, whereas reductions in the capital gains tax rate and a higher R&D tax credit rate lowers the cost of in-house
R&D.\footnote{If \( \sigma = 0 \), a decrease in the corporate tax does not change the cost of in-house R&D.} (3.10) represents the no-arbitrage condition between the return on equity and that on in-house R&D. Hereafter, we call \( \tilde{q}_t \equiv \frac{(1-\tau_D)(1-\sigma\tau_V)}{(1-\tau_D)(1-\sigma\tau_V)} q_t \) as modified \( q_t \) along the lines of Hayashi (1982).\footnote{Modified \( q_t \) corresponds to Tobin’s marginal \( q \).} If there are no adjustment costs \( (h = 0) \), modified \( q_t \) always pins down to one.\footnote{See Peretto (2007, 2011).} By contrast, in our setting, modified \( q_t \) is endogenously determined and has a transitional process in equilibrium. (3.11) shows that the rate of quality growth is an increasing function of modified \( q_t \). Since modified \( q_t \) is derived from firms’ intertemporal optimization problem, all informations relevant to in-house R&D decisions are summarized by modified \( q_t \).

Developing new products requires entering costs, \( \beta Z_t (\beta > 1) \). New entry firms are financed by issuing equity. Free-entry conditions yields

\[
V_t = \beta Z_t \quad \Leftrightarrow \quad \dot{N}_t > 0. \tag{3.12}
\]

From (3.6) and (3.12), the return on equity, (3.7), can be rewritten by

\[
r_t = (1 - \tau_D) \left[ \frac{F_t}{\beta Z_t} - (1 - \sigma\tau_V) \frac{\Phi(R_t, Z_t)}{\beta Z_t} \right] + (1 - \tau_V) \frac{\dot{Z}_t}{Z_t}. \tag{3.13}
\]

### 3.2.3 Households

The model’s economy has identical households. Each individual household member is identically endowed with one unit of time and provides labor supply elastically. The population grows at a constant rate, \( \lambda > 0 \). Without loss of generality, the population size at time 0 is normalized to one. Hence, the number of population at time \( t \) is given by \( e^{\lambda t} \). Households maximize the following utility function:

\[
U_t = \int_t^\infty e^{-(\rho-\lambda)(s-t)} \left[ \log C_s e^{-\lambda s} + \zeta \log (1 - l_s) \right] \, ds,
\]

where \( C_t \) is the aggregate consumption, \( l_t \) is the fraction of time allocated to work per capita, \( \zeta > 0 \) is the measure of preference for leisure, and \( \rho (> \lambda) \) is the rate of the time preference. The household budget constraint is given by

\[
\dot{N}_t V_t = N_t \left[ (1 - \tau_D) D_t - \tau_V \dot{V}_t \right] + (1 - \tau_L) W_t l_t e^{\lambda t} - (1 + \tau_C) C_t - T_t,
\]

where \( \tau_L \) is the labor income tax rate, \( \tau_C \) is the consumption tax rate, and \( T_t \) is the lump-sum tax. Solving the inter-temporal optimization problem yields the following
optimal conditions:

\[
\frac{\dot{C}_t}{C_t} = r_t - \rho + \lambda, \quad (3.14)
\]

\[
l_t = 1 - \frac{(1 + \tau_C)\zeta C_t}{(1 - \tau_L)W_t e^\lambda}. \quad (3.15)
\]

The transversality condition is \( \lim_{s \to \infty} e^{-(\rho - \lambda)(s-t)}a_s \mu_s = 0 \), where \( \mu_t \) represents the shadow value of holdings assets.

### 3.2.4 Government

Government spending is given by \( G_t = gY_t \) \((0 < g < 1)\), where the share of the government spending to outputs is assumed to be exogenously given. Along the lines of Peretto (2007, 2011), it is assumed that government spending does not affect a household’s utility or the efficiency of production activities. This allows the effects of distortionary taxes to be isolated from the effects of government expenditure. The government’s budget constraint is given by

\[
G_t = \tau_L W_t e^\lambda t + \tau_C C_t + \tau_N N_t [F_t - \sigma \Phi(Z_t, R_t)] + \tau_D N_t D_t + \tau_V N_t \dot{V}_t + T_t.
\]

Since the Ricardian equivalence holds, the same equilibrium dynamics occurs as in the economy with public debt.

### 3.3 Market equilibrium

#### 3.3.1 Equilibrium dynamics

In this section, we derive the dynamic system of market equilibrium. The market equilibrium condition of final goods is given by

\[
Y_t = G_t + C_t + N_t [X_t + \phi Z_t + \Phi(Z_t, R_t)] + \beta Z_t \dot{N}_t. \quad (3.16)
\]

Define the number of firms per capita as \( n_t \equiv N_t / e^\lambda t \) and the ratio of the aggregate consumption to output as \( c_t \equiv C_t / Y_t \). With full proof presented in Appendix 1, the labor supply per capita is given by

\[
l(c_t) = \frac{1}{1 + \Gamma c_t}, \quad \Gamma \equiv \frac{(1 + \tau_C)\zeta}{(1 - \tau_L)(1 - \theta)} > 0. \quad (3.17)
\]

The reduced-form aggregate production function of final goods is given by

\[
Y_t = \Omega l(c_t) e^\lambda t Z_t, \quad \Omega \equiv \theta^{2\theta} \cdot (3.18)
\]
For simplifying the notation, we hereafter define

\[ S = \frac{(1 - \tau_V)}{(1 - \tau_D)(1 - \sigma \tau)} \quad \text{and} \quad \eta = \frac{1 - \sigma \tau}{1 - \tau} \].

In Appendix 2, we provide the deviation of the following simultaneous differential equation which constitutes the economy’s dynamical system (in the case where \( \tilde{q}_t > 1 \)):

\[
\begin{align*}
\dot{n}_t &= \left[ 1 - \theta^2 - g - c_t \right] \frac{\Omega l(c_t)}{\beta} - \left[ \phi + \frac{(S q_t)^2 - 1}{2h} + \beta \lambda \right] \frac{n_t}{\beta}, \\
\dot{c}_t &= c_t [1 + \Gamma c_t] \left[ r_t - \rho - \frac{S q_t - 1}{h} \right], \\
\dot{q}_t &= \frac{1}{1 - \tau_V} r_t q_t - \frac{\alpha \theta (1 - \theta) \Omega l(c_t)}{S \eta} \frac{n_t}{\beta} - \frac{(S q_t - 1)^2}{2Sh},
\end{align*}
\]

where the interest rate (return on equity) is given by

\[ r_t = \frac{(1 - \tau_V)}{\beta S \eta} \left[ \theta (1 - \theta) \frac{\Omega l(c_t)}{n_t} - \phi - \eta \left( \frac{(S q_t)^2 - 1}{2h} \right) \right] + (1 - \tau_V) \frac{S q_t - 1}{h} \].

See Appendix 3 for proof of the dynamic system in the case where \( \tilde{q}_t \leq 1 \).

### 3.3.2 Steady-state equilibrium

Let \( y_t \equiv Y_t/(l_t e^{lt}) \), which represents the output per worker. From (3.18), the growth rate of output per worker is given by \( \dot{y}_t \equiv \dot{y}_t/y_t = \dot{z}_t = (\tilde{q}_t - 1)/h \). In what follows, we characterize the steady-state equilibrium, \( \{ n^*, c^*, \tilde{q}^* (\equiv S q^*), l^*, r^*, \tilde{y}^* \} \). From (3.20), \( \dot{c}_t = 0 \) and \( c^* > 0 \) implies (if \( \tilde{q}^* > 1 \))

\[ r^* = \rho + \frac{\tilde{q}^* - 1}{h}. \]  

From (3.21), \( \dot{q}_t = 0 \) and \( \tilde{q}^* > 1 \) implies

\[ r^* = (1 - \tau_V) \frac{\alpha \theta (1 - \theta) \Omega l(c^*)}{\eta} \frac{1}{n^*} \frac{1}{\tilde{q}^*} + (1 - \tau_V) \frac{(\tilde{q}^* - 1)^2}{2h} \frac{1}{\tilde{q}^*}. \]

This equation represents the no-arbitrage condition between the return on in-house R&D and that on equity in the steady-state equilibrium. Other things being equal, a dividend tax cut has no direct impact on the return from in-house R&D. A dividend tax cut boosts a firm’s after-tax gross cash flow and thus enhances the gross benefit
Dynamic effects of anticipated and temporary tax changes in a R&D-based growth model

derived from quality growth through in-house R&D. But the tax cut also increases the cost of in-house R&D, as previously discussed. As described in public finance literature [e.g. Summers (1981) and Hassett and Hubbard (2002)], the effects of the dividend tax cut cancel each other out. On the other hand, a corporate tax cut, a capital gains tax cut, and an increase in the R&D tax credit rate directly enhance the return on in-house R&D. From (3.23) and (3.24), we can determine that eliminating \( r^* \) yields (if \( \tilde{q}^* > 1 \))

\[
\frac{\Omega l(c^*)}{n^*} = \frac{\eta}{\alpha \theta (1 - \theta)} \left\{ \frac{1}{1 - \tau_V} \left( \rho + \frac{\tilde{q}^* - 1}{h} \right) \tilde{q}^* - \frac{(\tilde{q}^* - 1)^2}{2h} \right\}.
\]  

(3.25)

Substituting (3.23) and (3.25) into (3.22), we find that \( \tilde{q}^* \) is derived by solving \( f(\tilde{q}) = 0 \) with respect to \( \tilde{q} \) where

\[
f(\tilde{q}) \equiv \begin{cases} 
\frac{1}{1 - \tau_V} \left[ \rho + \frac{\tilde{q} - 1}{h} \right] (S\alpha\beta - \tilde{q}) + \frac{(\tilde{q} - 1)^2}{2h} + \alpha \frac{\tilde{q}^2 - 1}{2h} - S\alpha\beta \frac{\tilde{q} - 1}{h} + \frac{\alpha \phi}{\eta}, & \text{if } \tilde{q} > 1, \\
\frac{\rho}{1 - \tau_V} (S\alpha\beta - \tilde{q}) + \frac{\alpha \phi}{\eta}, & \text{if } \tilde{q} \leq 1.
\end{cases}
\]

(3.26)

If \( S\alpha\beta \leq 1 - \frac{(1 - \tau_V)\alpha \phi}{\eta \rho} < 1 \), \( f(1) \leq 0 \) and \( f'(\tilde{q}) < 0 \). In such a case, no steady-state equilibrium exists with a positive quality growth rate. On the other hand, if \( 1 - \frac{(1 - \tau_V)\alpha \phi}{\eta \rho} < S\alpha\beta \), \( f(1) > 0 \). In such a case, two types of configurations of \( f(\tilde{q}) \) can be considered, as shown in Figure 3.1-(a, b). Figure 3.1-(a) represents the case where \( f(\tilde{q}) \) is monotonically decreasing in \( \tilde{q} \) for \( \tilde{q} > 1 \). Figure 3.1-(b) represents the case where \( f(\tilde{q}) \) is inverted U-shapes for \( \tilde{q} > 1 \). The both figures assure that if \( 1 - \frac{(1 - \tau_V)\alpha \phi}{\eta \rho} < S\alpha\beta \), then \( \tilde{q}^* \) is uniquely determined at the point where \( \tilde{q}^* \) is higher than 1. In what follows, we focus on the case where \( 1 - \frac{(1 - \tau_V)\alpha \phi}{\eta \rho} < S\alpha\beta \). In such a case, there exists a unique steady-state equilibrium with a positive rate of quality growth. See Appendix 4 for proof.

Since the Jacobian matrix derived from the linear approximation of (3.19)-(3.21) in the neighborhood of the steady-state equilibrium is complicated, we cannot analytically examine the stability of the dynamic system. However, our numerical simulations confirm that the unique steady state is locally saddle-point stable in the benchmark setting and in the subsequent sensitivity analysis, as shown below.\(^{12}\)

\(^{12}\)Since the dynamic system has one state variable \( (n_t) \) and two jump variables \( (c_t) \) and \( (q_t) \), it must have two positive characteristic roots and one negative characteristic root to assure that the unique steady state is saddle-point stable. Our numerical simulation reports that the value of three characteristic root corresponding to the dynamic system are −0.4129, 0.2240, and 0.1478 in the benchmark parameter setting.
From (3.19) and (3.25), \( \dot{n}_t = 0 \) and \( n^* > 0 \) implies

\[
c^* = \left[ 1 - \theta^2 - g \right] - \frac{\alpha \theta(1-\theta)}{\varphi(\tilde{q}^*)} \left[ \phi + \frac{\tilde{q}^2 - 1}{2h} + \beta \lambda \right],
\]

where

\[
\varphi(\tilde{q}^*) \equiv \frac{\eta}{1 - \tau_V} \left[ \rho + \frac{\tilde{q}^* - 1}{h} \right] \tilde{q}^* - \eta \frac{(\tilde{q}^* - 1)^2}{2h}.
\]

Rewriting (3.25) yields

\[
n^* = \frac{\alpha \theta(1-\theta)\Omega(c^*)}{\varphi(\tilde{q}^*)}.
\]

The mechanism that eliminates the scale effect on the steady-state growth rate of output is consistent with the case where adjustment costs are absent [Peretto (2007, 2011)]. In the steady-state equilibrium, modified \( q \) is independent of the scale factor for the economy, \( l(c^*) \) [see (3.26)]. Increases in the economy’s scale factor lead to higher aggregate demand for intermediate goods at the individual firm level. This larger scale of production at the individual firm level allows in-house R&D expenditures to be spread over a greater number of units of goods, thus having a direct positive effect on incentives for a firm to engage in R&D in-house. This effect is called the cost-spreading effect. However, higher aggregate demand for intermediate goods also attracts new firms to enter the market as firm values rise. Thus, the per-firm market share of intermediate goods demand shrinks. This reduces the scale of production at the individual firm level, which in turn lowers incentives to conduct in-house R&D activities. This effect is called the market share effect. In the steady-state equilibrium, the market share effect derived from higher aggregate demand for intermediate goods perfectly cancels out the cost-spreading effect [see discussion in Peretto (2007)].

3.3.3 Steady-state growth effect of tax changes

The manner in which a permanent change of tax variables affects steady-state growth rate of output per workers is also consistent with Peretto (2007, 2011). We summarize those findings as follows:

Furthermore, we confirm that the comparative statics of the parameters in the steady-state equilibrium obtain similar results to those in Peretto (2007, 2011). Increases in \( \alpha, \beta, \) and \( \phi \) enhance the steady-state growth rate of outputs, respectively. Increases in \( \alpha \) allow each firm to more intensely internalize positive returns derived from its own in-house R&D activities. Increases in \( \beta \) and \( \phi \) make it more difficult for potential new firms to enter the market, thus reallocating resources from product proliferation to quality improving. An increase in \( h \) reduce the steady-state growth rate of output because it directly increases the cost of in-house R&D. On the other hand, the effect of \( \rho \) upon the steady-state growth rate of outputs is ambiguous.
Table 3.1: Steady-state growth effect of tax changes

<table>
<thead>
<tr>
<th>( \frac{\partial \hat{y}^*}{\partial \tau_D} )</th>
<th>( \frac{\partial \hat{y}^*}{\partial \tau_{II}} )</th>
<th>( \frac{\partial \hat{y}^*}{\partial \tau_V} )</th>
<th>( \frac{\partial \hat{y}^*}{\partial \sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0</td>
<td>&lt; 0 (if ( \sigma = 0 ))</td>
<td>&lt; 0 (if ( S\alpha\beta &lt; \frac{1 + h\rho}{1 - \tau_V} ))</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>&gt; 0 (if ( \sigma \geq 1 ))</td>
<td>( \leq 0 ) (if ( \sigma \in (0, 1) ))</td>
<td>( \leq 0 ) (otherwise)</td>
<td>( \leq 0 ) (otherwise)</td>
</tr>
</tbody>
</table>

Steady-state growth rate of output per workers is increasing in the rates of dividend tax and corporate tax (if \( \sigma \geq 1 \)) and R&D tax credit rate. On the other hand, it is decreasing in the rate of the corporate tax (if \( \sigma = 0 \)) and the rate of capital gains tax (if \( S\alpha\beta < \frac{1 + h\rho}{1 - \tau_V} \)). Increases in the corporate tax rate (if \( \sigma \in (0, 1) \)) and capital gains tax rate (if \( S\alpha\beta \geq \frac{1 + h\rho}{1 - \tau_V} \)) have ambiguous effects upon the steady-state growth rate of output per workers.

Proof can be found in Appendix 5. A dividend tax cut has no direct impact on a firms’ incentive to pursue in-house R&D, as previously discussed. On the other hand, a dividend tax cut directly enhances the returns on equity. Given the aggregate market demand for intermediate goods, the number of firms per capita increases. The resulting product proliferation lowers incentives to conduct in-house R&D through the market share effect. Thus, a dividend tax cut unambiguously has an negative effect on quality growth.\(^\text{14} \)

On the other hand, a higher R&D tax credit rate unambiguously has an positive effect on quality growth. It reduces the cost of in-house R&D, which dominates the other effect so that it functions like a direct subsidy for in-house R&D.

Both a corporate tax cut (if \( \sigma \in (0, 1) \)) and a capital gains tax cut generally have ambiguous effects on quality growth. These tax cuts directly enhance both the returns on in-house R&D and on equity. However, if \( \sigma = 0 \), it is shown that a corporate tax cut unambiguously enhances quality growth in the steady state. Furthermore, if \( \alpha \) and \( \beta \) are sufficiently low, a capital gains tax cut enhances quality growth in the steady state.

\(^{14}\text{If } \sigma = 1, \text{ a corporate tax cut also has the same qualitative effect as a dividend tax cut. When in-house R&D expenditures are fully deductible against corporate tax, no qualitative difference exists between the dividend tax and corporate tax.} \)
3.4 Numerical analysis

3.4.1 Data and methodology

Since analytically examining the transitional adjustment of aggregate economy in response to various tax changes is complicated, we calibrate the model with U.S. data by using relaxation algorithm method developed by Trimborn, Koch, and Steger (2008).\(^\text{15}\)

As the benchmark, we use the value of all tax variables, following the methodology in Peretto (2011).\(^\text{16}\) The values of $\theta$ and $\rho$ are set to 0.30 and 0.03, respectively, which are conventional values in the macroeconomic literature. The value of $\lambda$ is set to 0.01, which is consistent with the average annual population growth rate in the U.S. economy. The parameter choice associated with adjustment costs, $h$, is less clear. According to Schubert and Turnovsky (2011), the parameter of adjustment costs for physical capital investment is generally assumed to fall within 10-15 in the literature [e.g., Auerbach and Kotlikoff (1987) and Ortigueira and Santos (1997)]. Bernstein and Nadiri (1989) and Himmelberg and Petersen (1994) report the extent to which adjustment costs associated with R&D investment equals or surpasses that associated with physical capital investment. Therefore $h = 15.0$ is employed as the benchmark. The parameter associated with entry costs, $\beta$, is also less clear. Following Peretto (2011), we employ $\beta = 6.55$ as the benchmark.\(^\text{17}\) The values of $\alpha$ and $\phi$ are set to 0.1616 and 0.1785, respectively, so that the consumption ratio and growth rate of output in the steady state are 0.69 and 0.02, respectively. $\zeta$ is set to 1.459 so that the fraction of time devoted to labor supply is 0.33.

Table 3.2 summarizes the benchmark parameter values. Table 3.3 reports the values of key endogenous variables in the steady-state equilibrium, $\{n^*, c^*, q^*, l^*, r^*, y^*\}$, which are characterized under the benchmark parameter setting.

In what follows, we investigate the specific transitional adjustments in key macro variables and welfare induced by the following specific tax changes: (a) a 5% point reduction in the dividend tax rate, (b) a 5% point reduction in the corporate tax rate, (c) a 5% point reduction in the capital gains tax rate, and (d) a 10% point reduction in the capital gains tax rate. For details, see Peretto (2011) for a detailed explanation of this estimation.

---

\(^{15}\)Trimborn, Koch, and Steger (2008) details the relaxation algorithm. They also provide MATLAB programs for the relaxation algorithm, which are downloadable for free at http://www.wiwi.uni-siegen.de/vwli/forschung/relaxation/matlab_applications.html?lang=de. Using this method, Strulik and Trimborn (2010) examine how both anticipated and temporary tax reforms affect the aggregate economy within the framework of the neoclassical (exogenous) growth model.

\(^{16}\)R&D costs are in fact fully deductible against corporate tax liability in the U.S. tax code. However, setting $\sigma = 0$ allows us to clearly see the fundamental distinction between corporate and dividend taxes. If R&D costs are assumed to be fully deductible ($\sigma = 1.0$), then a corporate tax cut has the same qualitative effects upon the economy as a dividend tax cut.

\(^{17}\)See Peretto (2011) for a detailed explanation of this estimation.
Dynamic effects of anticipated and temporary tax changes in a R&D-based growth model

Table 3.2: Benchmark parameter setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>0.143</td>
<td>Government expenditure share</td>
</tr>
<tr>
<td>( \tau_D )</td>
<td>0.35</td>
<td>Dividend tax rate</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.0</td>
<td>R&amp;D tax credit rate</td>
</tr>
<tr>
<td>( \tau_H )</td>
<td>0.335</td>
<td>Corporate tax rate</td>
</tr>
<tr>
<td>( \tau_V )</td>
<td>0.20</td>
<td>Capital gains tax rate</td>
</tr>
<tr>
<td>( \tau_C )</td>
<td>0.05</td>
<td>Consumption tax rate</td>
</tr>
<tr>
<td>( \tau_L )</td>
<td>0.256</td>
<td>Labor income tax rate</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.1616</td>
<td>Appropriable quality</td>
</tr>
<tr>
<td>( 1 - \theta )</td>
<td>0.7</td>
<td>Labor share</td>
</tr>
<tr>
<td>( h )</td>
<td>15.0</td>
<td>The extent of adjustment costs</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.1785</td>
<td>The extent of fixed operating costs</td>
</tr>
<tr>
<td>( \beta )</td>
<td>6.55</td>
<td>The extent of entry costs</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>1.459</td>
<td>Preference for leisure</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.03</td>
<td>Time discount rate</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.01</td>
<td>Population growth rate</td>
</tr>
</tbody>
</table>

Table 3.3: Steady-state equilibrium values (benchmark)

<table>
<thead>
<tr>
<th>( n^* )</th>
<th>( c^* )</th>
<th>( \hat{q}^* )</th>
<th>( l^* )</th>
<th>( r^* )</th>
<th>( \hat{y}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0339</td>
<td>0.69</td>
<td>1.30</td>
<td>0.33</td>
<td>0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

rise in the R&D tax credit rate. And we consider the following three different implementation scenarios: (1) an unanticipated and permanent tax change, (2) an anticipated and permanent tax change, and (3) an unanticipated and temporary tax change. In every scenario, the economy initially (at \( t = 0 \)) remains in the steady-state equilibrium before the tax change. In implementation scenario (1), each tax change suddenly comes into effect at \( t = 5 \) and lasts forever from that point forward. In implementation scenario (2), all economic agents expect at \( t = 0 \) that each tax change will be implemented at \( t = 5 \) and last forever from that point on. In implementation scenario (3), each tax change comes into effect unexpectedly at \( t = 0 \) and but reverts to its initial level after \( t = 5 \). This reversion is expected by all economic agents at \( t = 0 \).

Figures 3.2-3.5 show the transitional path of key macro variables in response to each tax change within the different implementation scenarios as given above. Specifically, each panel of these figures represents the transitional path of: (1) the number of firms per capita (\( n_t \)), (2) the consumption ratio (\( c_t \)), (3) modified q (\( \hat{q}_t \)),...
(4) hours worked per capita \((l_t)\), (5) the interest rate \((r_t)\), (6) the growth rate of output per workers \((\dot{y}_t)\), (7) the growth rate of the number of firms \((\dot{N}_t)\), and (8) the ratio of distortionary tax revenue to output.\(^{18}\) The horizontal axes in each panel measure years. In the vertical axes, \(r_t\), \(\dot{y}_t\), and \(\dot{N}_t\) are measured by their actual values, whereas all the other variables are measured by their percentage deviation from pre-reform levels.

Table 3.4 reports welfare consequences arising from the tax changes. Welfare level is measured as a consumption equivalent: what constant relative increases in annual consumption per capita must be induced so that households’ pre-reform utility levels equal the household utility levels in the case where the economy moves to a new steady-state equilibrium due to the tax change.\(^{19}\)

<table>
<thead>
<tr>
<th>Policy change</th>
<th>Unanticipated (Permanent)</th>
<th>Anticipated (Permanent)</th>
<th>Temporary</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta t_D = -0.05)</td>
<td>-10.49</td>
<td>-9.19</td>
<td>-2.49</td>
</tr>
<tr>
<td>(\Delta t_{II} = -0.05)</td>
<td>3.20</td>
<td>3.46</td>
<td>0.0741</td>
</tr>
<tr>
<td>(\Delta t_{IV} = -0.05)</td>
<td>8.68</td>
<td>7.37</td>
<td>2.18</td>
</tr>
<tr>
<td>(\Delta \sigma = 0.1)</td>
<td>7.03</td>
<td>6.47</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Note: Welfare gains are measured in consumption equivalent and expressed in percentage points.

### 3.4.2 Dividend tax cut

Figure 3.2-(a) shows the impulse responses of key macro variables to the 5% point permanent dividend tax cut under the benchmark parameter setting. The dashed lines in the panels of Figure 3.2-(a) plot the impulse responses in the case where the permanent tax cut is unanticipated. When the tax cut is implemented (at \(t = 5\)), the consumption ratio and modified \(q\) instantaneously fall, whereas the number of firms per capita starts to rise. These variables gradually converge to the new steady-state level. Hours worked reacts in a contrary manner against the consumption ratio. The growth rate of output per workers falls from \(0.391\) to \(0.3901766\) at \(t = 5\).

\[^{18}\]More formally, welfare differences are evaluated as follows. \(U_0(c^O,I^O,\dot{y}^O,n^O)\) is defined as a household’s level of the utility in the case where the economy remains in the initial steady-state equilibrium before a tax change. We define \(U_0^N(c^N_t,I^N_t,\dot{y}^N_t,n^N_t)\) as in the case when the economy moves to the new steady-state equilibrium due to the tax changes. Here, we measure the consumption equivalent by \(\delta\), which is defined as the value that satisfies \(U_0^O(c^O(1+\delta),I^O,\dot{y}^O,n^O) = U_0^N(c^N_t,I^N_t,\dot{y}^N_t,n^N_t)\). See Appendix 6 for details on how to calculate household utility levels.
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before it converges to 0.01771. The tax cut is detrimental to quality growth during all transition phases. Since the tax cut proportionally and permanently increases both after-tax gross cash flows and the cost of in-house R&D at the same time, it therefore has no direct effect on a firm’s incentive to conduct in-house R&D. It only directly increases dividend payments and returns on equity, which lead to product proliferation. This negatively impacts the incentives to in-house R&D through the market share effect.

Table 3.4 shows that the tax cut carries welfare costs of around 10.49% points of per capita annual consumption. The negative welfare consequence results from the slowdown of quality growth as well as the decline of consumption and household leisure times.

If the permanent tax cut is anticipated, then the impulse responses, which are plotted by the solid lines in Figure 3.2-(a), become quite different. When the news arrives (at $t = 0$), households and firms can incorporate the future tax cut and change their inter-temporal behavior. At $t = 0$, all variables rather than the state variable (the number of firms per capita) instantaneously changes. The consumption ratio falls, whereas modified $q$ jumps up and the growth rate of per worker output rises from 0.02 to 0.02234. Up to the point when the tax cut is actually implemented, the consumption ratio further decreases, whereas modified $q$ increases and the growth rate of per worker output continues to rise. The number of firms per capita gradually rises. After the tax cut is implemented (at $t = 5$), both the consumption ratio and the number of firms per capita gradually increase to converge to a new steady-state level. On the other hand, at $t = 5$, modified $q$ and the growth rate of output per worker drastically drops lower than they were pre-reform values. Then they also converge to a new steady-state level.

Why does pre-announcement of the tax cut have a positive impact on quality growth temporarily? Recall that the tax cut proportionally increases both after-tax gross cash flows and the cost of in-house R&D after implementation. Other things being equal, the news that the tax cut will be implemented in future makes firms to incorporate that gross benefits derived from conducting in-house R&D today increase, whereas the cost of conducting in-house R&D today remains in the same level as that before announcement, up to the point when the tax cut is actually implemented. Hence, the anticipated tax cut temporarily has a direct positive effect on a firm’s incentive to conduct in-house R&D activities. On the other hand, it also leads to product proliferation, which results in a indirect negative effect on incentives to conduct in-house R&D. The former positive direct effect outweighs the latter negative effect.

Table 3.4 shows that an anticipated dividend tax cut is estimated to impose the loss of around 9.19% points of annual consumption per capita, indicating that the welfare costs are mitigated compared to the welfare effect of an unanticipated
tax cut. This outcome reflects the fact that the rate of quality growth temporarily increases up to the point when the tax cut is actually implemented, whereas consumption and hours worked adjust more smoothly.

Figure 3.2-(b) shows the impulse responses to a temporary 5% point dividend tax cut under the benchmark setting. When the tax cut is implemented (at $t = 0$), all variables other than the state variable instantaneously change. After the tax cut is terminated (at $t = 5$), all variables gradually revert to their pre-reform levels. Remarkably, during its implementation, modified $q$ declines more sharply compared to the case of the permanent tax cut. At $t = 0$, the growth rate of output per worker falls to 0.01546 and then decrease further until the tax cut is terminated, in reaction to the temporary increase in the cost of in-house R&D. As Table 3.4 shows, the temporary dividend tax cut yields welfare costs of an estimated 2.49% points of per capita annual consumption.

### 3.4.3 Corporate tax cut

Figures 3.3-(a, b) show the impulse responses to a 5% point corporate tax cut under the benchmark parameter setting. Except for modified $q$ and the growth rate of output per worker, the impulse responses are qualitatively similar to those found in the case of the dividend tax cut. If the tax cut is unanticipated, then at $t = 5$, modified $q$ jumps up and the growth rate of output per worker rises. These variables then further increase to the new steady-state level. If the tax cut is anticipated, then at $t = 0$, modified $q$ jumps up and the growth rate of output per workers rises. Again, these variables then further increase to the new steady-state level. If the tax cut is temporary, the effect on quality growth is also positive during all transitional phases.

Why does the tax cut unambiguously exert a positive impact on the quality growth during all phases of the transition irrespective of implementation? Recall that under the benchmark parameter setting, in-house R&D expenditures are not deductible against corporate tax ($\sigma = 0$). Other things being equal, pre-announcement of the future tax cut directly increases gross benefits derived from conducting in-house R&D today, whereas it does not change the cost of conducting in-house R&D today, up to the point when the tax cut is actually implemented. Hence, the anticipated tax cut temporarily has a direct positive effect on incentives to conduct in-house R&D. On the other hand, it also leads to product proliferation, which results in a indirect negative effect on incentives to conduct in-house R&D. The former positive direct effect outweighs the latter indirect negative effect. Table 3.4 shows that the welfare effect is positive irrespective of implementation method.
3.4.4 Capital gains tax cut

Figure 3.4-(a) shows the impulse responses to the unanticipated (or anticipated) and permanent 5% point capital gains tax cut under the benchmark parameter setting. Although the steady-state effect of a capital gains tax cut on quality growth is qualitatively ambiguous, our calibration shows that the tax cut increases the quality growth rate in the new steady state.\(^{20}\) If the permanent tax cut is unanticipated, modified \(q\) initially rises before converging to the new steady-state level. During all transition phases, the quality growth rate is higher than its pre-reform level. Table 4 shows that the tax cut yields welfare gains of around 8.68% points of per capita annual consumption.

If the permanent tax cut is anticipated, however, quality growth slows up to the point when the tax cut is actually implemented. The future capital gains tax cut reduces the future cost of conducting in-house R\&D so that firms have an opportunity to dynamically adjust their investment plans and thus delay in-house R\&D investments. This effect dominates the other effects. Although the overall effect of the capital gains tax cut on welfare remains positive, the temporary slowdown of quality growth has a negative effect on welfare. On the other hand, the anticipated tax cut makes household behavior more smoothly, resulting in a positive effect on welfare. However, the latter positive effect cannot outweigh the former negative effect. As a result, the anticipated tax cut reduces welfare gains by 1.31% points compared to the welfare effect of the unanticipated tax cut.

Figure 3.4-(b) shows the impulse response to a temporary 5% point capital gains tax cut under the benchmark parameter setting. Remarkably, quality growth accelerates during its implementation. This temporary acceleration of quality growth is more significant compared to the steady-state effect of the permanent tax cut. Mainly, this occurs because the temporary tax cut reduces the cost of conducting in-house R\&D during its implementation. As Table 3.4 shows, the temporary tax cut also yields welfare gains of an estimated 2.18% points of per capita annual consumption.

3.4.5 Increases in the R\&D tax credit rate

Figure 3.5-(a) shows impulse responses to the unanticipated (or anticipated) 10% point permanent increase in the R\&D tax credit rate under the benchmark parameter setting. The tax change increases the steady-state rate of quality growth. Remarkably, in the steady state, the tax changes is shown to be self-financing: the ratio of distortionary tax revenues to output is higher than pre-reform levels. If the permanent tax change is unanticipated, then during all transition phases, both the

\(^{20}\)The subsequent robustness checks show that the steady-state effect on quality growth is positive.
growth rate of outputs per workers and the consumption ratio are higher than their pre-reform levels. As Table 3.4 shows, the tax change yields welfare gains estimated to be around 7.03% points of per capita annual consumption.

On the other hand, if the permanent tax change is anticipated, modified q and the growth rate of output per worker are lower than their pre-reform levels up to the point when the tax cut is actually implemented. This findings parallel that in the case of the anticipated capital gains tax cut. Future rises of the R&D tax credit rate directly reduce the future cost of conducting in-house R&D, leading firms to delay in-house R&D investments until after the tax change is actually implemented. This negative effect dominates the other effects. As a result, although the tax cut has an overall positive effect on welfare, the implementation lags from the tax change reduce these welfare gains by 0.56% points of per capita annual consumption, compared to the welfare effect of the unanticipated tax change.

Figure 3.5-(b) shows the impulse responses to a temporary 10% point increase in the R&D tax credit rate under the benchmark parameter setting. The effect on quality growth parallels that found in the case of the temporary capital gains tax cut. Temporary increases in the R&D tax credit rate reduce the cost of conducting in-house R&D during its implementation. Furthermore, the temporary acceleration of quality growth is more significant compared to the steady-state effect of the permanent tax change. As Table 3.4 shows, the temporary tax change also yields welfare gains estimated to be around 1.26% points of per capita annual consumption.

3.5 Sensitivity analysis

3.5.1 Parameter changes

We now conduct robustness checks for identified tax changes effects by changing certain parameters. First, we increase or decrease the values of $\beta$ and $h$ by 50% points. In all these cases, we re-estimate $\alpha$ and $\phi$ so that the consumption ratio and the growth rate of output in the pre-reform steady-state equilibrium remain the same as in the benchmark parameter setting. We find that the impulse responses are qualitatively the same as in the benchmark analysis. As Table 3.5 reports, the welfare consequences of tax changes are quantitatively modified but our main findings in the benchmark analysis qualitatively hold.

Second, we consider the case of $\sigma = 1.0$. In fact, the U.S. tax code sets $\sigma = 1.0$ even though in the benchmark analysis, we set to $\sigma = 0.0$. We find that except for the case of the corporate tax cut, the impulse responses and welfare consequences remain qualitatively the same as in the benchmark analysis.

We then consider the case in which labor supply is inelastic (i.e., $\zeta = 0$). We find that except for the case of the corporate tax cut, the impulse responses and welfare
3.5.2 Social returns to product variety

In the model as described thus far, and as (3.18) shows, the number of firms (product variety) per capita does not directly contribute to the production of final goods. Given the aggregate market demand for intermediate goods, changes in the number of firms per capita merely affect the market structure for intermediate goods firms. The policy that leads to a higher number of firms per capita indirectly distorts incentives for a firm to conduct in-house R&D. In this section, we relax this somewhat extreme feature. Along the lines of Peretto (2007, 2011), we consider the case where socially positive returns to product variety exist for the production of final goods as follows:

\[ Y_t = n_t^v \int_0^{N_t} X_{it}^\theta \left( Z_{it}^{1-\alpha} L_{it} \right)^{1-\theta} \, di, \quad v > 0, \]

where the contribution of product variety on final goods output is assumed to be external to all agents. In this case, the reduced-form production function of final goods can be rewritten by

\[ Y_t = n_t^v \Omega(l(c_t)) e^{\lambda_t} Z_t, \quad \kappa \equiv \frac{v}{1-\theta}. \]

The dynamic system of the economy is modified as follows:

\[ \dot{n}_t = \left[ 1 - \theta^2 - g - c \right] \Omega(l(c_t)) \beta n_t^{-\kappa} - \left[ \phi + \frac{(Sq_t)^2 - 1}{2h} + \beta \lambda \right] \frac{n_t}{\beta}, \]

\[ \dot{c}_t = c_t \left[ 1 + \Gamma c_t \right] \left[ r_t - \rho - \frac{Sq_t - 1}{h} - \frac{\kappa \dot{n}_t}{n_t} \right], \]

\[ \dot{q}_t = \frac{1}{1 - \tau V} \tau V \dot{c}_t - \frac{\alpha \theta (1-\theta) \Omega(l(c_t))}{S \eta} \frac{n_t^{1-\kappa} - (Sq_t - 1)^2}{2Sh}, \]

where

\[ r_t = \frac{(1 - \tau V)}{\beta S \eta} \left[ \theta (1-\theta) \frac{\Omega(l(c_t))}{n_t^{1-\kappa}} - \phi - \eta \frac{(Sq_t)^2 - 1}{2h} \right] + (1 - \tau V) \frac{Sq_t - 1}{h}. \]

The growth rate of output per worker is given by \( \dot{q}_t + \frac{\kappa \dot{n}_t}{n_t} \). Since the steady-state number of firms per capita is constant, the steady-state growth rate of output is only dependent of modified \( q_t \), as is also the case for \( \kappa = 0 \). If \( \kappa > 0 \), then the steady-state number of firms per capita is given by \( \left( n^* \right)^{1-\kappa} \), where \( n^* \) is consistent with the steady-state value in the case of \( \kappa = 0 \). The other steady-state values...
coincide with those in the case of $\kappa = 0$. That is, social returns to product variety ($\kappa > 0$) simply add to the direct positive effect on the production of final goods, and they only change the steady-state value of the number of firms per capita; thus, the steady-state effect from tax changes upon macroeconomic variables is consistent with the case of $\kappa = 0$.

The impulse responses of key macro variables not involving the growth rate of output per workers are qualitatively the same as in the case of $\kappa = 0$. The growth rate of output per worker is also dependent on the transition growth rate of the number of firms per capita. If the intensity of the growth rate for the number of firms per capita dominates that for quality growth, then the impulse response of the growth rate of output per worker is modified. As an example, Figures 3.6-(a, b) depict the impulse responses to a 5% point dividend tax cut in the case of $\kappa = 0$. The figure shows that even if the tax cut is unanticipated and permanent (or temporary), the growth rate of output per worker initially shows a sharp increase. That is, the positive growth rate in the number of firms per capita initially outweighs the slowdown in quality growth.

In the case of $\kappa > 0$, household welfare is also dependent of the number of firms per capita. Higher product variety directly increases household welfare. Table 3.6 reports the welfare consequences of tax changes in the cases of $\eta = 0.1, 0.3, 0.5$, and $0.7$. As the intensity of social return to product variety, $\eta$, increases, welfare losses arising from the dividend tax cut are mitigated, while the welfare gains resulting from the corporate tax cut increase significantly. On the other hand, the welfare gains resulting from the capital gains tax cut increase, while the welfare gains resulting from increases in the R&D tax credit rate are reduced, but these variations are not significant compared to the impacts from cuts in the rate of dividend and corporate taxes. In any tax change, however, the sign of the welfare effect does not change, and the effect of implementation lags holds qualitatively, as in the case of the benchmark analysis.

3.6 Conclusion

We first summarize our results and then discuss their implications. A dividend tax cut reduces the level of welfare irrespective of implementation methods. The tax cut is detrimental to both in-house R&D and aggregate growth after implementation. Consumption and household leisure time also decrease. Therefore, the tax cut yields overall welfare losses. However, pre-announcement of the tax cut stimulates in-house R&D and aggregate growth up to the point when the tax cut is actually implemented. Households also can adjust the timing of their consumption and leisure more smoothly. Both these effects arising from the foreknowledge of the tax cut have a positive effect on welfare. Therefore, the pre-announcement mitigates
the welfare losses compared to the case of an unanticipated tax cut, although the overall welfare effect still remains negative. On the other hand, the policy effect of a corporate tax cut is dependent on the specific R&D tax credit rate.

A capital gains tax cut and increases in the R&D tax credit rate lead to welfare gains irrespective of implementation methods. These tax changes stimulate in-house R&D and aggregate growth after implementation. The acceleration of quality growth yields welfare gains. However, pre-announcement of these tax changes are detrimental to in-house R&D and aggregate growth up to the point when the tax cut is actually implemented. On the other hand, the pre-announcement lead households to smooth their behavior, which yields positive effects on welfare. However, the former negative effects surmount the latter positive effects. As a result, the pre-announcement of these tax changes worsens their effectiveness, although the overall welfare effect still remains positive.

Our analysis suggests that a capital gains tax cut and increases in the R&D tax credit rate are effective policy instruments. However, when considering their implementation in terms of scope and timing, policy makers should be careful to ensure that their effectiveness is maximized.

3.7 Appendices

3.7.1 Appendix 1

The perfect distribution in the final goods sector (letting $L_t = L_t$) yields:

\begin{align*}
\theta^2 Y_t &= N_t X_t, \\
(1 - \theta) Y_t &= W_t N_t L_t.
\end{align*}

Using the definition of $c_t$, (3.28), and the market equilibrium condition of labor, $N_t L_t = e^{N_l} L_t$, then (3.15) can be rewritten as (3.17). Substituting (3.2) and the market equilibrium condition of labor to (3.1) yields (3.18).

3.7.2 Appendix 2

Dividing both sides of (3.16) by $Y_t$ and using the definition of $n_t$ and $c_t$, (3.27), and (3.18), we obtain

\begin{equation}
1 - \theta^2 - g - c_t = \frac{n_t}{\Omega(c_t)} \left[ \phi + \frac{\Phi(R_t, Z_t)}{Z_t} + \beta \left( \frac{\dot{n}_t}{n_t} + \lambda \right) \right].
\end{equation}
Dividing (3.5) by $Z_t$ and using (3.11), we obtain
\[
\frac{\Phi(R_t, Z_t)}{Z_t} = \frac{(Sq_t)^2 - 1}{2h}. \tag{3.30}
\]
Substituting (3.30) to (3.29), we obtain (3.19).

From (3.27) together with the definition of $n_t$, and (3.18), we obtain
\[
\frac{F_t}{Z_t} = \left(1 - \theta \right) \frac{X_t}{Z_t} - \phi,
= \theta(1 - \theta) \frac{\Omega(c_t)}{n_t} - \phi. \tag{3.31}
\]
Using (3.30), (3.31), and (3.11), we can rewrite (3.13) as
\[
r_t = \frac{(1 - \tau_D)(1 - \tau_{II})}{\beta} \left[ \theta(1 - \theta) \frac{\Omega(c_t)}{n_t} - \phi \right] - \frac{(1 - \tau_D)(1 - \sigma_{II})}{\beta} \left[ \frac{(Sq_t)^2 - 1}{2h} \right] \\
+ (1 - \tau_V) \frac{Sq_t - 1}{h}.
\]
Then, from the definition of $S$ and $\eta$, rearranging the above equation yields (3.22).

From logarithmic differentiation of $c_t$ with respect to time yields
\[
\frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = r_t - \rho + \frac{\dot{i}(c_t)}{i(c_t)} + \frac{\dot{Z}_t}{Z_t}.
\]
Using (3.17) and (3.11), the above equation can be rewritten as
\[
\frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{\Gamma \dot{c}_t}{1 + \Gamma c_t} - \frac{Sq_t - 1}{h}. \tag{3.32}
\]
Rearranging (3.32) yields (3.20).

From (3.2), (3.8), and the market equilibrium condition of labor, we obtain
\[
\frac{\partial F_t}{\partial Z_t} = \alpha \theta (1 - \theta) \frac{\Omega(c_t)}{n_t}. \tag{3.33}
\]
Using the definition of $S$ and $\eta$, (3.11), and (3.33), we can rewrite (3.10) as
\[
\frac{\alpha \theta (1 - \theta) \Omega(c_t)}{S\eta} \frac{1}{n_t} + \frac{(Sq_t - 1)^2}{2Sh} = \frac{1}{1 - \tau_V} r_t q_t - \dot{q}_t.
\]
Then, using (3.17), the above equation is rewritten as (3.21).

### 3.7.3 Appendix 3

The dynamical system of the economy where \( \tilde{q}_t \leq 1 \) is constituted by

\[
\dot{n}_t = \left[ 1 - \theta^2 - g - c_t \right] \frac{\Omega l(c_t)}{\beta} - \left[ \phi + \beta \lambda \right] \frac{n_t}{\beta},
\]

\[
\dot{c}_t = c_t \left[ 1 + \Gamma c_t \right] \left[ r_t - \rho \right],
\]

\[
\dot{q}_t = \frac{1}{1 - \tau_V} r_t q_t - \frac{\alpha \theta (1 - \theta) \Omega l(c_t)}{S \eta} \frac{n_t}{\beta},
\]

where the interest rate is given by

\[
r_t = \frac{(1 - \tau_V)}{\beta S \eta} \left[ \theta (1 - \theta) \frac{\Omega l(c_t)}{n_t} - \phi \right].
\]

### 3.7.4 Appendix 4

Differentiating (3.26) with respect to \( \tilde{q} \) yields

\[
f'(\tilde{q}) = \begin{cases} 
1 + \alpha - \frac{2}{1 - \tau_V} \frac{\tilde{q}}{h} + \left[ \frac{1}{1 - \tau_V} - 1 \right] \frac{S \alpha \beta + 1}{h} - \frac{\rho}{1 - \tau_V}, & \text{if } \tilde{q} > 1, \\
-\frac{\rho}{1 - \tau_V}, & \text{if } \tilde{q} \leq 1.
\end{cases}
\]

Moreover, second order differentiation of (3.26) with respect to \( \tilde{q} \) yields

\[
f''(\tilde{q}) = \begin{cases} 
[1 + \alpha - \frac{2}{1 - \tau_V}] \frac{1}{h} < 0, & \text{if } \tilde{q} > 1, \\
0, & \text{if } \tilde{q} \leq 1.
\end{cases}
\]

Here,

\[
f(1) = \frac{\rho}{1 - \tau_V} \left[ S \alpha \beta - 1 \right] + \frac{\alpha \phi}{\eta},
\]

\[
\lim_{\tilde{q} \to 1_+} f'(\tilde{q}) = \frac{1}{h(1 - \tau_V)} \left[ \tau_V (S \alpha \beta - 1) - (1 - \alpha) - h \rho \right].
\]

If \( S \alpha \beta \leq 1 - \frac{(1 - \tau_V) \alpha \phi}{\eta} \) \((< 1)\), \( f(1) \leq 0 \) and \( \lim_{\tilde{q} \to 1_+} f'(\tilde{q}) < 0 \). Then, \( f'(\tilde{q}) < 0 \) for \( \tilde{q} > 1 \) as \( f''(\tilde{q}) < 0 \) for \( \tilde{q} > 1 \). Therefore, in this case, \( f(\tilde{q}) \) has only one solution...
of $\tilde{q}$ with a value less than one. That is, no steady-state equilibrium exists with a positive growth rate of output. On the other hand, if $1 - \frac{(1 - \tau_V)\alpha\eta\rho}{\eta} < S\alpha\beta$, then $f(1) > 0$. No matter whether $\lim_{\tilde{q} \to 1+0} f'(\tilde{q}) < 0$ is positive or negative, $f(\tilde{q})$ has unique solution of $\tilde{q}$ with a value higher than one, as depicted in Figure 3.1-(a, b).

### 3.7.5 Appendix 5

Differentiating the RHS of (3.26) with respect to $\tau_D$, $\tau_\Pi$, $\tau_V$, and $\sigma$ yields

\[
\frac{\partial f(\tilde{q})}{\partial \tau_D} = \left\{ \frac{\alpha\beta}{1 - \tau_V} \rho + \left[ \frac{1}{1 - \tau_V} - 1 \right] \alpha\beta \frac{\hat{q} - 1}{h} \right\} \frac{1}{(1 - \tau_D)} S > 0, \\
\frac{\partial f(\tilde{q})}{\partial \sigma} = \left\{ \frac{\alpha\beta}{1 - \tau_V} \rho + \left[ \frac{1}{1 - \tau_V} - 1 \right] \alpha\beta \frac{\hat{q} - 1}{h} \right\} \frac{\tau_\Pi}{(1 - \sigma\tau_\Pi)} S + \frac{\alpha\phi}{\eta^2} \frac{\sigma}{(1 - \tau_\Pi)} > 0, \\
\frac{\partial f(\tilde{q})}{\partial \tau_\Pi} = \left\{ \frac{\alpha\beta}{1 - \tau_V} \rho + \left[ \frac{1}{1 - \tau_V} - 1 \right] \alpha\beta \frac{\hat{q} - 1}{h} \right\} \frac{\sigma}{(1 - \sigma\tau_\Pi)} S - \frac{\alpha\phi(1 - \sigma)}{(1 - \sigma\tau_\Pi)^2} < 0, \\
\frac{\partial f(\tilde{q})}{\partial \tau_V} = -\rho \left( \frac{\tilde{q}}{(1 - \tau_V)^2} \right) + \frac{\hat{q} - 1}{h(1 - \tau_V)} \left[ S\alpha\beta - \frac{\hat{q}}{1 - \tau_V} \right] = \Gamma(\tilde{q}) \geq 0.
\]

Since $f(\tilde{q})$ is a decreasing function of $\tilde{q}$ in the neighborhood around the steady-state solution, the above derivations imply that $\tilde{q}^*$ is increasing in $\tau_D$, $\tau_\Pi$ (if $\sigma = 1$) and is decreasing in $\sigma$, $\tau_\Pi$ (if $\sigma = 0$) and the effects of tax changes in $\tau_\Pi$ (if $\sigma \in (0, 1)$) and $\tau_V$ are ambiguous.

In addition, we also find

\[
\Gamma(1) = -\frac{\rho}{(1 - \tau_V)^2} < 0, \\
\Gamma'(\tilde{q}) = \frac{1}{(1 - \tau_V)^2 h} \left[ -2\tilde{q} - h\rho + 1 + S\alpha\beta(1 - \tau_V) \right] \geq 0, \\
\Gamma''(\tilde{q}) = -\frac{2}{(1 - \tau_V)^2 h} < 0.
\]

Then, if $S\alpha\beta < (1 + h\rho)/(1 - \tau_V)$, $\Gamma(\tilde{q}) < 0$ for $\tilde{q} > 1$. Therefore, it is shown that if $S\alpha\beta < (1 + h\rho)/(1 - \tau_V)$, then $\tilde{q}^*$ is a decreasing function of $\tau_V$. 

3.7.6 Appendix 6

We define $\Psi_t \equiv U_t - \frac{1}{\rho - \lambda} \log Z_t$. From the definition of $c_t$, (3.11), and (3.18), differentiating $\Psi_t$ with respect to time yields

$$\dot{\Psi}_t = (\rho - \lambda)\Psi_t - \log \Omega - \log c_t - \log l_t - \zeta \log (1 - l_t) - \frac{1}{\rho - \lambda} \frac{S q_t - 1}{h}.$$

In the steady state, $\Psi_t$ is constant over time. Calculating the dynamic path of $\Psi_t$ numerically using the relaxation algorithm, we can obtain the initial value of $\Psi_t$, $\Psi_0 = U_0 - \frac{1}{\rho - \lambda} \log Z_0$. Without loss of generality, $Z_0$ is normalized to one. Hence, we obtain $U_0 = \Psi_0$. 
Figure 3.1: The steady-state equilibrium
Dynamic effects of anticipated and temporary tax changes in a R&D-based growth model

(a) Anticipated vs. unanticipated permanent reduction in the dividend tax rate by 5 percentage points in the benchmark setting. Solid (Dashed) lines plots the impulse response of each variable to the anticipated (unanticipated) tax cut. The circle marks on the left (right) vertical axis indicates the steady-state level before (after) the tax cut.

(b) Temporary reduction in the dividend tax rate by 5 percentage points in the benchmark setting. The circle mark on the vertical axis indicates the initial level.

Figure 3.2
(a) Anticipated vs. unanticipated permanent reduction in the corporate tax rate by 5 percentage points in the benchmark setting. Solid (Dashed) lines plots the impulse response of each variable to the anticipated (unanticipated) tax cut. The circle marks on the left (right) vertical axis indicates the steady-state level before (after) the tax cut.

(b) Temporary reduction in the corporate tax rate by 5 percentage points in the benchmark setting. The circle mark on the vertical axis indicates the initial level.

Figure 3.3
Dynamic effects of anticipated and temporary tax changes in a R&D-based growth model

(a) Anticipated vs. unanticipated permanent reduction in the capital gains tax rate by 5 percentage points in the benchmark setting. Solid (Dashed) lines plots the impulse response of each variable to the anticipated (unanticipated) tax cut. The circle marks on the left (right) vertical axis indicates the steady-state level before (after) the tax cut.

(b) Temporary reduction in the capital gains tax rate by 5 percentage points in the benchmark setting. The circle mark on the vertical axis indicates the initial level.

Figure 3.4
(a) Anticipated vs. unanticipated permanent rise in the tax credit rate by 10 percentage points in the benchmark setting. Solid (Dashed) lines plots the impulse response of each variable to the anticipated (unanticipated) change. The circle marks on the left (right) vertical axis indicates the steady-state level before (after) rise of the tax credit.

(b) Temporary rise in the tax credit rate by 10 percentage points in the benchmark setting. The circle mark on the vertical axis indicates the initial level.

Figure 3.5
Dynamic effects of anticipated and temporary tax changes in a R&D-based growth model

Table 3.5: Welfare gains of tax changes (parameter changes)

<table>
<thead>
<tr>
<th>Tax change</th>
<th>Unanticipated</th>
<th>Anticipated</th>
<th>Temporary</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(Permanen-</td>
<td>(Permanen-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>t)</td>
<td>t)</td>
<td></td>
</tr>
<tr>
<td>$\beta = 9.825$ ($with \alpha = 0.1086$ and $\phi = 0.2760$)</td>
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<tr>
<td>$\Delta t_D = -0.05$</td>
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<td>-8.92</td>
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</tr>
<tr>
<td>$\Delta t_{II} = -0.05$</td>
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<td>3.48</td>
<td>0.0785</td>
</tr>
<tr>
<td>$\Delta t_V = -0.05$</td>
<td>8.29</td>
<td>6.97</td>
<td>2.16</td>
</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
<td>6.88</td>
<td>6.32</td>
<td>1.25</td>
</tr>
<tr>
<td>$\beta = 3.275$ ($with \alpha = 0.3152$ and $\phi = 0.0812$)</td>
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</tr>
<tr>
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<tr>
<td>$\Delta t_{II} = -0.05$</td>
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<tr>
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<td>8.77</td>
<td>2.24</td>
</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
<td>7.51</td>
<td>6.98</td>
<td>1.28</td>
</tr>
<tr>
<td>$h = 22.5$ ($with \alpha = 0.1775$ and $\phi = 0.1775$)</td>
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<tr>
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<td>6.91</td>
<td>1.69</td>
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<td>$\Delta \sigma = 0.1$</td>
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<td>5.79</td>
<td>0.97</td>
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<td>$h = 7.5$ ($with \alpha = 0.1455$ and $\phi = 0.1796$)</td>
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<td>$\Delta \sigma = 0.1$</td>
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<td>7.38</td>
<td>1.97</td>
</tr>
<tr>
<td>$\sigma = 1.0$ ($with \alpha = 0.1102$ and $\phi = 0.1719$)</td>
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<td>2.15</td>
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<td>$\zeta = 0$</td>
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</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
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<td>6.48</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Note: Welfare gains are measured in consumption equivalent and expressed in percentage points. Other values of the tax variables and parameters are same as the benchmark setting.
(a) Anticipated vs. unanticipated permanent reduction in the dividend tax rate by 5 percentage points in the case of $\kappa = 0.3$. Solid (Dashed) lines plots the impulse response of each variable to the anticipated (unanticipated) tax cut. The circle marks on the left (right) vertical axis indicates the steady-state level before (after) the tax cut.

(b) Temporary reduction in the dividend tax cut by 5 percentage points in the case of $\kappa = 0.3$. The circle mark on the vertical axis indicates the initial level.

Figure 3.6
Table 3.6: Welfare gains of tax changes (positive social spillover of product variety)

<table>
<thead>
<tr>
<th>Tax change</th>
<th>Unanticipated (Permanent)</th>
<th>Anticipated (Permanent)</th>
<th>Temporary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0.1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t_D = -0.05$</td>
<td>$-9.91$</td>
<td>$-8.57$</td>
<td>$-2.46$</td>
</tr>
<tr>
<td>$\Delta t_{\Pi} = -0.05$</td>
<td>$3.69$</td>
<td>$3.99$</td>
<td>$0.0906$</td>
</tr>
<tr>
<td>$\Delta t_V = -0.05$</td>
<td>$8.79$</td>
<td>$7.50$</td>
<td>$2.17$</td>
</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
<td>$6.94$</td>
<td>$6.39$</td>
<td>$1.25$</td>
</tr>
<tr>
<td>$\kappa = 0.3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t_D = -0.05$</td>
<td>$-8.40$</td>
<td>$-7.01$</td>
<td>$-2.33$</td>
</tr>
<tr>
<td>$\Delta t_{\Pi} = -0.05$</td>
<td>$4.98$</td>
<td>$5.35$</td>
<td>$0.17$</td>
</tr>
<tr>
<td>$\Delta t_V = -0.05$</td>
<td>$9.09$</td>
<td>$7.84$</td>
<td>$2.16$</td>
</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
<td>$6.73$</td>
<td>$6.17$</td>
<td>$1.22$</td>
</tr>
<tr>
<td>$\kappa = 0.5$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t_D = -0.05$</td>
<td>$-6.29$</td>
<td>$-4.88$</td>
<td>$-2.08$</td>
</tr>
<tr>
<td>$\Delta t_{\Pi} = -0.05$</td>
<td>$6.80$</td>
<td>$7.21$</td>
<td>$0.32$</td>
</tr>
<tr>
<td>$\Delta t_V = -0.05$</td>
<td>$9.50$</td>
<td>$8.31$</td>
<td>$2.14$</td>
</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
<td>$6.43$</td>
<td>$5.89$</td>
<td>$1.18$</td>
</tr>
<tr>
<td>$\kappa = 0.7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t_D = -0.05$</td>
<td>$-3.23$</td>
<td>$-1.83$</td>
<td>$-1.69$</td>
</tr>
<tr>
<td>$\Delta t_{\Pi} = -0.05$</td>
<td>$9.43$</td>
<td>$9.87$</td>
<td>$0.56$</td>
</tr>
<tr>
<td>$\Delta t_V = -0.05$</td>
<td>$10.07$</td>
<td>$8.99$</td>
<td>$2.10$</td>
</tr>
<tr>
<td>$\Delta \sigma = 0.1$</td>
<td>$6.01$</td>
<td>$5.52$</td>
<td>$1.10$</td>
</tr>
</tbody>
</table>

Note: Welfare gains are measured in consumption equivalent and expressed in percentage points. Other values of tax variables and parameters are same as the benchmark setting.
Chapter 4

Growth effect of bubbles in a non-scale endogenous growth model with in-house R&D

4.1 Introduction

Asset bubbles sometimes emerge, and they are accompanied by higher economic growth and a consumption boom. A seminal study by Tirole (1985) shows the existence of a rational deterministic bubble on an asset in an economy with overlapping generations. It is found that the presence of asset bubbles increases consumption but retards economic growth [e.g., Saint-Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000)]. The reason is that the higher consumption absorbs resources and crowds out investment. As a result, the theoretical prediction runs contrary to the mentioned above fact. The purpose of this paper is to overcome this conflict and provide a theoretical explanation for why the presence of asset bubbles can lead to higher economic growth in concurrence with high consumption by using a simple endogenous growth model.

The present analysis is based on a recent endogenous growth model developed by Peretto (2007, 2011). Specifically, the model considers the economy where long-lived value-maximizing firms continuously improve upon the quality of their specific product through in-house R&D, while simultaneously new firms also enter into the market. That is, in the model economy, there are two dimensional investment oppor-

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1. Asset bubbles are defined as the difference between the fundamental value of an asset and its market value. For example, if an intrinsically useless asset whose fundamental value is zero has a positive value, we say there exist asset bubbles.
2. The introduction of Martin and Ventura (2012) provides excellent surveys about the fact about asset bubbles.
tunities: in-house R&D (quality improving) and the creation of a new firm (product proliferation). The main source of economic growth is obtained from technological progress, endogenously derived from the in-house R&D of firms. The advantage of the model is elimination of the well-known undesirable scale effect [e.g. Jones (1995a, b)], while keeping the policy effect property supported by recent growing empirical literature. Increases in the scale of the aggregate economy are perfectly fragmented by the endogenous product proliferation.

We introduce the above-mentioned structure into a continuous-time overlapping generations model developed by Weil (1989). Specifically, we assume that economic agents live forever but have no intergenerational altruism. We also assume that new generations are born at a constant rate. Additionally, we assume that there initially exists an intrinsically useless asset (fiat money). The reason there exist asset bubbles in equilibrium is the same as the reason spelled out in Tirole (1985). Due to an absence of intergenerational altruism, one generation can transfer the intrinsically useless asset to the other generation as a pyramid scheme only if the price appreciation of the intrinsically useless asset is equal to the return on a real asset (equity) whose value is backed by the fundamental value.

Our analysis stipulates the theoretical mechanism by which the presence of asset bubbles can lead to higher economic growth with high consumption. Here the endogeneity of the market structure plays a key role. If asset bubbles emerge, households think that they are wealthier and thus want to consume more. This leads to a higher reservation interest rate of households in the asset market. To satisfy the higher reservation interest rate, the return on a real asset (equity) must increase. This requires product proliferation to be impeded so that demand for differentiated goods, at the level of an individual firm, increases. Consequently, if the positive effect on in-house R&D of firms due to the larger scale of production at the level of an individual firm exceeds the negative effect derived from the higher interest rate, the intensity of in-house R&D increases, thus enhancing economic growth. In addition, the lower product proliferation provides available economic resources for a higher consumption by households.

This paper is related to several recent studies that examine the relationship between asset bubbles and economic growth [e.g. Hirano and Yanagawa (2010),

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3See, for example, Laincz and Peretto (2006), Ha and Howitt (2007), and Ang and Madsen (2011).

4If we employ the alternative setup that there exists a positive death rate following Blanchard (1985), our main claims do not change.

5For simplicity, as a theoretical devise, we distinguish the pure bubbles from real assets whose values are backed by the fundamental values along with many other studies. As one notable exception, Olivier (2000) analyzes the case where bubbles attach directly on real assets such as the equity of firms, and Olivier (2000) also demonstrates that bubbles on equity may enhance economic growth in the framework of the endogenous growth model developed by Romer (1990).
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Kunieda and Shibata (2012), and Martin and Ventura (2012)]. Their analyses consider an environment where the credit market is imperfect and the efficiency of investment among firms is heterogeneous. In this environment, the presence of asset bubbles can reallocate resources from inefficient to efficient investments. As a result, the reallocation may raise the productivity of aggregate outputs and promote economic growth. By contrast, our model economy is characterized by an environment, where the credit market is perfect, and the efficiency of investment among firms is homogeneous. The key feature of our analysis is that the endogeneity of market structure affects the scale of production at the level of an individual firm, which in turn determines the intensity of in-house R&D.

The rest of the paper is organized as follows. Section 4.2 describes the model. Section 4.3 analyzes the dynamic system of the market equilibrium. Section 4.4 analyzes the steady state equilibrium as well as compares the steady state equilibrium with and without asset bubbles. We conclude in Section 4.5.

4.2 The model

In this section, we establish a continuous-time overlapping-generations version of the model developed by Peretto (2011).\(^6\) We also assume that there exists an initial and intrinsically useless asset, that is, fiat money. The economy is closed and consists of a final goods sector, an intermediate goods sector, and households.

4.2.1 The final goods sector

Perfect competition prevails in the final goods sector. The (real) price of final goods is set to be numeraire. Final goods are consumed by households and used as only one factor of the production and investment of intermediate goods sector. The final goods, \(Y_t\) is produced by the following production function:

\[
Y_t = \int_0^{N_t} X_{it}^\theta (Z_{it}^{1-\alpha} L_{it})^{1-\theta} di, \quad 0 < \alpha, \theta < 1,
\]

where \(N_t\) is the variety of intermediate goods (the number of intermediate goods firms), \(X_{it}\) is the input of intermediate good \(i \in [0, N_t]\) (produced by firm \(i\)), and \(L_{it}\) is the input of labor which uses intermediate goods \(i\). The productivity of \(L_{it}\) depends not only on the quality of intermediate good \(i\), \(Z_{it}\), but also on the average level of the quality across intermediate goods, \(\bar{Z}_t \equiv \int_0^{N_t} \frac{1}{N_t} Z_{jt} dj\). Therefore, we

\(^6\)The model of Peretto (2011) is a lab-equipment style versions of the model developed by Peretto (2007).
obtain the following optimal conditions:

\[
X_{it} = \left( \frac{\theta}{p'_{it}} \right) \frac{1}{\sigma} \left( Z^n_{it} \bar{Z}^{1-\alpha} L_{it} \right), \tag{4.2}
\]

\[
L_{it} = \left( \frac{1 - \theta}{w_t} \right) \frac{1}{\sigma} X_{it} \left( Z^n_{it} \bar{Z}^{1-\alpha} \right) \frac{1}{\sigma}, \tag{4.3}
\]

where \( p'_{it} \) and \( w_t \) represent the price of intermediate good \( i \) and the wage rate, respectively.

### 4.2.2 The intermediate goods sector

Monopolistic competition prevails in the intermediate goods sector. Firm \( i \) exclusively produces its differentiated good with its quality \( Z_{it} \). The monopoly of each firm is permanently protected by perfect patent protection. Producing one unit of intermediate goods requires one unit of final goods. Firms improve upon the quality of their specific product through their in-house R&D. The law of motion of the firm-specific quality is as follows:

\[
\dot{Z}_{it} = R_{it}, \tag{4.4}
\]

where \( R_{it} \) is the inputs for in-house R&D. Fixed operating costs, \( \phi \bar{Z}_t \) (\( \phi > 0 \)), are also required at each point in time. Then, the profit flow of firm \( i \) is \( \Pi_{it} = (p'_{it} - 1) X_{it} - \phi \bar{Z}_t - R_{it} \). Without loss of generality, the number of equity per firm is normalized to one. The return on equity of firm \( i \) is given by

\[
r_t = \frac{\Pi_{it}}{V_{it}} + \frac{\dot{V}_{it}}{V_{it}}. \tag{4.5}
\]

Integrating (4.5) forward yields the value of firm \( i \) as

\[
V_{it} = \int_{t}^{\infty} \Pi_{is} \left( e^{-\int_{s}^{t} r_v dv} \right) ds.
\]

Throughout this analyses we consider a symmetric equilibrium in which \( Z_{it} = \bar{Z}_t \) holds. Each firm maximizes its value, subject to (4.2) and (4.4), given \( \bar{Z} \). Solving the inter-temporal optimization problem, we obtain the following optimal conditions:

\[
p'_{it} = \frac{1}{\theta}. \tag{4.6}
\]

\(^7\)Since here we assume that perfect capital market prevails and that there are no distortionary taxes, it does not matter whether firms finance their in-house R&D by issuing equity or by using retained earnings.
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\[ q_t = 1 \iff R_t > 0, \quad (4.7) \]

\[ r_t = \alpha - \theta Z_t^{\alpha-1} \dot{L}_t + \frac{\dot{q}_t}{q_t}, \quad (4.8) \]

and the transversality condition is given by \( \lim_{s \to \infty} q_s Z_s e^{-\int_s^t r_v dv} = 0 \), where \( q_t \) is a co-state variable (a shadow value of in-house R&D) associated with the current-value Hamiltonian for this optimization problem, where (4.6) represents pricing rule with constant mark-up, and where (4.8) represents the no-arbitrage relationship between the return on equity and the return on in-house R&D.

### 4.2.3 Firm entry

Development of new product requires \( \beta Z_t (\beta > 1) \). A new firm is set up by issuing equities. Free-entry condition yields

\[ V_t = \beta Z_t \iff \dot{N}_t > 0. \quad (4.9) \]

This equation implies entry is positive until the value of a firm is equal to the set up cost. In our model economy, as we will see later, population grows perpetually and then the aggregate demand for intermediate goods also continues to grow. Therefore, at each point in time, new entry of firms occurs.

### 4.2.4 Households

The specification of household behavior is based on that in Weil (1989). At each point in time, a new generation is born at a constant rate \( \lambda > 0 \). Since the probability of death is assumed to be zero, each generation lives forever. And there is no intergenerational altruism. Without loss of generality, the total population at time 0 is normalized to 1 so that the total population at time \( t \) is \( e^{\lambda t} \).

At the beginning of the time 0, there is an initial generation endowed with initial wealth. Hereafter we call this generation as generation 0⁻. The initial wealth consists of a real asset (equity) and an intrinsically useless asset (fiat money). To decide upon a fixed terminology, we label the intrinsically useless asset a “bubbly asset”. Specifically, the generation 0⁻ initially possesses the equity of firms (the initial number of firms is \( N_0 \)), whose value is given by \( N_0 V_0 \) in real terms and...
$B_0$ pieces of the bubbly asset in nominal terms.\footnote{That is, the total value of the initial asset is equal to $N_0V_0 + B_0/p_0^M$ in real terms, where $p_0^M$ is the price of the bubbly asset at time 0.} By contrast, the subsequent generation does not have any initial wealth. Hereafter we label the generation born at time $t \geq 0$ as “generation $t$”.

Each household supplies one inelastic unit of labor at each point in time. The representative household in generation $s \leq t$ maximizes the following utility function:

$$U(s,t) = \int_t^\infty \log[c(s,x)]e^{-\rho(x-t)}dx, \quad \rho > 0,$$

subject to the following budget constraint of generation $s$:

$$\frac{db(s,t)}{dt} \frac{1}{p_t^M} + \frac{dk(s,t)}{dt} = r_t k(s,t) + w_t - c(s,t),$$

where $b(s,t)$ is holdings of pieces of a bubbly asset (in nominal terms), $1/p_t^M$ is the price of the bubbly asset, $k(s,t)$ is holdings of a real asset, and $c(s,t)$ is consumption.

Since the bubbly asset has no intrinsic or fundamental value, households possess both the real asset and the bubbly asset only if the price appreciation of the bubbly asset is equal to the return on the real asset, that is,

$$-\frac{p_t^M}{p_t^M} = r_t. \quad (4.11)$$

Let $m(s,t) \equiv b(s,t)/p_t^M$ and $a(s,t) \equiv k(s,t) + m(s,t)$. Using (4.11) and the definitions, the flow budget constraint of generation $s$ can be rewritten as

$$\frac{da(s,t)}{dt} = r_t a(s,t) + w_t - c(s,t). \quad (4.12)$$

Maximizing (4.10) subject to (4.12) yields the following Euler equation of generation $s$:

$$\frac{dc(s,t)}{dt} = (r_t - \rho) c(s,t). \quad (4.13)$$

And the following No Ponzi game condition must be satisfied:

$$\lim_{x \to \infty} a(s,x)e^{\int_0^x r_v dv} = 0.$$
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\[
A_t \equiv a(0^-, t) + \lambda \int_0^t a(s, t)e^{\lambda s}ds,
\]
\[
W_t \equiv w_t + \lambda \int_0^t w_s e^{\lambda s}ds.
\]

In Appendix 1, we show that the dynamic path of aggregate consumption and aggregate wealth is respectively given by

\[
\dot{C}_t = (r_t - \rho + \lambda) C_t - \rho \lambda A_t,
\]
(4.14)
\[
\dot{A}_t = r_t A_t + W_t - C_t.
\]
(4.15)

### 4.3 Market equilibrium and Dynamics

In this section, we derive the dynamics system of the market equilibrium based on the model in the preceding section. The market equilibrium condition of final goods is given by \(Y_t = C_t + N_t [X_t + \phi Z_t + R_t] + \beta Z_t \dot{N}_t\). The market equilibrium condition of assets is given by \(A_t = M_t + N_t V_t\), where \(M_t\) is defined as the aggregate value of the bubbly asset, that is \(B_0/p_t M_t\). This condition holds that in equilibrium the aggregate wealth must be equal to the aggregate value of the bubbly asset and the aggregate value of equity \((N_t V_t)\). Here we remark that the real asset in the economy is only the equity of firms. The market equilibrium condition of labor is given by \(e^{-\lambda t} = N_t L_t\).

Define the number of firms per capita as \(n_t \equiv N_t/e^{-\lambda t}\). From (4.7) and the market equilibrium condition of labor, if \(R_t > 0\), (4.8) is rewritten by

\[
r_t = \alpha \theta (1 - \theta) \Omega \frac{1}{n_t},
\]
(4.16)

where \(\Omega \equiv \theta \frac{\alpha}{1 - \theta} \frac{\phi}{e^{-\lambda t}}\). From (4.9) and the market equilibrium condition of labor, if \(R_t > 0\), (4.5) is rewritten by

\[
r_t = \frac{1}{\beta} \left[ \theta (1 - \theta) \Omega \frac{1}{n_t} - \phi - 1 \right] \frac{\beta - 1}{\lambda} \frac{\dot{Z}_t}{Z_t}.
\]
(4.17)

From (4.16) and (4.17), the rate of quality growth is given by

\[
\dot{z}_t(n_t) \equiv \frac{\dot{Z}_t}{Z_t} = \begin{cases} 
\frac{\alpha \beta - 1}{\beta - 1} \theta (1 - \theta) \Omega \frac{1}{n_t} + \frac{\phi}{\beta - 1}, & \text{if } n_t > \tilde{n}, \\
0, & \text{if } \alpha \beta < 1 \text{ and } 0 < n_t \leq \tilde{n},
\end{cases}
\]
(4.18)
and the interest rate (the return on equity) is given by

\[ r_t = \begin{cases} 
\alpha \theta (1 - \theta) \frac{\Omega}{n_t}, & \text{if } n_t > \tilde{n}, \\
\frac{1}{\beta} \left[ \theta (1 - \theta) \frac{\Omega}{n_t} - \phi \right], & \text{if } \alpha \beta < 1 \text{ and } 0 < n_t \leq \tilde{n},
\end{cases} \]  

(4.19)

where \( \tilde{n} = \max \left\{ 0, \frac{(1 - \alpha \beta \theta (1 - \theta) \Omega)}{\phi} \right\} \). More specifically, the intensity of in-house R&D is determined at the point where the return on in-house R&D (the RHS of (4.16)) is equal to the return on equity (the RHS of (4.17)). (4.18) indicates if \( \alpha \beta > 1 \) (< 1), the lower number of firms per capita leads to the higher (lower) rate of quality growth. If \( \alpha \) and \( \beta \) are sufficiently high (low), the return on in-house R&D is more (less) sensitive to changes in the number of firms per capita. Hence, the lower number of firms per capita raises the return on in-house R&D than the return on equity, resulting in the higher (lower) rate of quality growth. On the other hand, (4.19) shows that the interest rate is unambiguously decreasing in the number of firms per capita. The lower number of firms per capita leads to higher demand for intermediate goods at the level of an individual firm.

More intuitively, whether the rate of quality growth is increasing or decreasing in the number of firms per capita is dependent on the extent of the following two contradictory forces. Other things being equal, the lower number of firms per capita increases demand for intermediate goods at the level of an individual firm. The larger scale of production at the level of an individual firm allow in-house R&D expenditures to be spread over more units of goods, thus having a positive effect on incentives to in-house R&D (the cost spreading effect). On the other hand, a higher interest rate associated with a lower number of firms, per set of firms, lowers incentives to in-house R&D (the interest rate effect). If \( \alpha \beta > 1 \), the positive cost spreading effect exceeds the negative interest rate effect. The same logic applies to the case where \( \alpha \beta < 1 \).\(^{11}\)

The perfect distribution in the final goods sector implies \( N_t X_t = \theta^2 Y_t \) and \( w_t N_t L_t = (1 - \theta) Y_t \). Applying these relationships to (4.1), we obtain the following reduced-form aggregate production function of final goods, which is given as

\[ Y_t = \Omega e^{\lambda t} Z_t. \]  

(4.20)

The growth rate of outputs per capita is given as \( \hat{z}_t(n_t) \).

Define the ratio of aggregate consumption to outputs as \( c_t \equiv C_t / Y_t \) and the ratio of aggregate value of the bubbly asset to outputs as \( m_t \equiv M_t / Y_t \). In Appendix 2,

\(^{11}\)If \( \phi = 0 \), \( \alpha \beta \) is restricted to be higher than 1 so that the steady state equilibrium with a positive rate of quality growth exists.
we show that the following three equations constitute the dynamic system of the economy:

\[ \dot{n}_t = \frac{\Omega}{\beta} \left[ 1 - \theta^2 - c_t \right] - \frac{n_t}{\beta} \left[ \phi + \hat{z}_t(n_t) + \beta \lambda \right], \quad (4.21) \]

\[ \dot{c}_t = \left[ \phi(n_t) - \rho \right] c_t - \lambda \rho \left[ m_t + \beta \frac{n_t}{\Omega} \right], \quad (4.22) \]

\[ \dot{m}_t = \left[ \phi(n_t) - \lambda \right] m_t, \quad (4.23) \]

where the difference between the interest rate and the rate of quality growth is given by

\[ \phi(n_t) \equiv r_t - \hat{z}_t = \begin{cases} 
(1 - \alpha) \theta (1 - \theta) \frac{\Omega}{n_t} - \frac{\phi}{\beta - 1}, & \text{if } n_t > \tilde{n}, \\
\frac{1}{\beta} \left[ \theta (1 - \theta) \frac{\Omega}{n_t} - \phi \right], & \text{if } \alpha \beta < 1 \text{ and } 0 < n_t \leq \tilde{n}. 
\end{cases} \quad (4.24) \]

\[ (4.25) \]

\[ (4.25) \text{ shows that the } \dot{n}_t = 0\text{-locus is independent of the value of } m \text{ and on the } \dot{n}_t = 0\text{-locus } c \text{ is negatively related to } n. \text{ This locus represents the resource constraint of the economy. The downward-sloping shape of this locus implies that the higher number of firms per capita absorbs available resources for consumption of households. From} \]

\[ 4.4 \text{ The steady state equilibrium} \]

This section characterizes the steady state equilibrium. As we will see later, there may exist multiple steady state equilibrium: the steady state equilibrium without asset bubbles and the steady state equilibrium with asset bubbles. In Appendix 3, we examine the local stability of the steady state equilibrium. We prove that the equilibrium path toward the steady state equilibrium with asset bubbles is locally saddle-point stable, whereas the equilibrium path toward the steady state equilibrium without asset bubbles is locally indeterminate. Thus, a global indeterminacy arises along with previous studies, such as in Grossman and Yanagawa (1993) and Futagami and Shibata (2000).

From (4.21), \( \dot{n}_t = 0\)-locus is given by

\[ c = \begin{cases} 
(1 - \theta) \left[ 1 + \frac{\beta \theta (1 - \alpha)}{\beta - 1} \right] - \frac{\beta n}{\Omega} \left[ \frac{\phi}{\beta - 1} + \lambda \right], & \text{if } n > \tilde{n}, \\
(1 - \theta^2) - \frac{\beta n}{\Omega} \left[ \frac{\phi}{\beta} + \lambda \right], & \text{if } \alpha \beta < 1 \text{ and } 0 < n \leq \tilde{n}. 
\end{cases} \quad (4.25) \]
(4.22), \( \dot{c}_t = 0 \)-locus is given by

\[
c = \begin{cases} 
\frac{\lambda \rho}{(1 - \alpha) \theta (1 - \theta) \Omega n - \beta - 1 - \rho} \left[ m + \frac{\beta n}{\Omega} \right], & \text{if } n > \tilde{n}, \\
\frac{\lambda \rho}{\theta (1 - \theta) \Omega n - \beta - 1 - \rho} \left[ m + \frac{\beta n}{\Omega} \right], & \text{if } \alpha \beta < 1 \text{ and } 0 < n \leq \tilde{n}.
\end{cases}
\]  

(4.26)

(4.26) shows that on the \( \dot{c}_t = 0 \)-locus, given the value of \( m \), \( c \) is positively related to \( n \) starting from the origin in the region of \( 0 < n < \tilde{n} \) where \( \tilde{n} \equiv \text{argsolve } \{ \varphi(n) = \rho \}. \)

This locus is derived from the aggregation of the Euler equation of households. The upward-sloping shape of this locus implies that the higher number of firms per capita leads to a lower interest rate, and thus households want to consume more.\(^{13}\) First, we consider a steady state equilibrium without asset bubbles, \( \{ n^*, c^*, m^* \} \) where \( m^* = 0 \). In Figure 4.1 and Figure 4.2, we plot the \( \dot{n}_t = 0 \)-locus and \( \dot{c}_t = 0 \)-locus with \( m^* = 0 \) by solid lines.\(^{14}\) These figures show that \( \{ n^*, c^* \} \) is uniquely determined.

Second, we consider the possibility of the existence for the steady state equilibrium with asset bubbles, \( \{ n^{**}, c^{**}, m^{**} \} \), where \( m^{**} > 0 \). From (4.23), if \( m^{**} > 0 \), \( \dot{m}_t = 0 \) implies \( \varphi(n^{**}) = \lambda \). Therefore, from (4.18) and (4.24), we obtain

\[
n^{**} = \begin{cases} 
\frac{(1 - \alpha) \theta (1 - \theta) \Omega}{(\beta - 1) \lambda + \phi}, & \text{if } \alpha \beta > 1 \text{ or } \alpha \beta < 1 \text{ and } \phi > \tilde{\phi}, \\
\frac{\theta (1 - \theta) \Omega}{\beta \lambda + \phi}, & \text{if } \alpha \beta < 1 \text{ and } 0 < \phi \leq \tilde{\phi},
\end{cases}
\]  

(4.27)

---

\(^{12}\)If the steady state number of firms per capita is higher than \( \tilde{n} \), the steady state consumption ratio becomes negative.

\(^{13}\)The higher number of firms per capita also yields a higher ratio of the aggregate value of equity to output, that is, \( NV/Y \). Hence, the higher wealth ratio induces households to consume more, given an interest rate. This reinforces the extent of the upward-sloping shape of this locus. However, this is dependent on the specification of the entry cost. For example, if we consider the alternative assumption in which the entry cost is related to the production volumes of incumbents, that is, \( X_t \) along with Peretto (2007), then the ratio of the aggregate value of equity to output becomes constant.

\(^{14}\)Specifically, Figure 4.1 shows the configuration of \( \dot{n}_t = 0 \)-locus and \( \dot{c}_t = 0 \)-locus when \( \alpha \beta > 1 \), while Figure 4.2 shows that when \( \alpha \beta < 1 \) and \( \phi > \phi \) (see the definition of \( \phi \) for the latter main text). If \( \alpha \beta < 1 \) and \( \phi > \phi \) is satisfied, the steady state quality growth is positive both in the steady state equilibrium without asset bubbles and in that with asset bubbles.
and
\[
\dot{z}(n^{**}) = \begin{cases} 
\frac{(\alpha \beta - 1)\lambda + \alpha \phi}{1 - \alpha}, & \text{if } \alpha \beta > 1 \text{ or } \alpha \beta < 1 \text{ and } \phi > \tilde{\phi}, \\
0, & \text{if } \alpha \beta < 1 \text{ and } 0 < \phi \leq \tilde{\phi},
\end{cases} 
\tag{4.28}
\]
where \(\tilde{\phi} \equiv \max \left\{0, \frac{1-\alpha \beta}{\alpha}\right\}\). From (4.25), (4.27), and (4.28), \(c^{**}\) is given by
\[c^{**} = (1 - \theta).\tag{4.29}\]

From (4.26), (4.27), and (4.29), \(m^{**}\) is given by
\[
m^{**} = \begin{cases} 
\frac{(1 - \theta)}{\lambda \rho} \left[\lambda - \rho - \frac{(1 - \alpha)\beta \theta \lambda \rho}{(\beta - 1)\lambda + \phi}\right], & \text{if } \alpha \beta > 1 \text{ or } \alpha \beta < 1 \text{ and } \phi > \tilde{\phi}, \\
\frac{(1 - \theta)}{\lambda \rho} \left[\lambda - \rho - \frac{\beta \theta \lambda \rho}{\beta \lambda + \phi}\right], & \text{if } \alpha \beta < 1 \text{ and } 0 < \phi \leq \tilde{\phi}.
\end{cases}
\tag{4.30}
\]

Therefore, the necessary and sufficient condition for the existence of the steady state equilibrium with an asset bubble is given by
\[
\begin{cases} 
\lambda - \rho > \frac{(1-\alpha)\beta \theta \lambda \rho}{(\beta - 1)\lambda + \phi} > 0, & \text{if } \alpha \beta > 1 \text{ or } \alpha \beta < 1 \text{ and } \phi > \tilde{\phi}, \\
\lambda - \rho > \frac{\beta \theta \lambda \rho}{\beta \lambda + \phi} > 0, & \text{if } \alpha \beta < 1 \text{ and } 0 < \phi \leq \tilde{\phi}.
\end{cases}
\tag{4.31}
\]

If this condition is satisfied, we find that in the steady state equilibrium without asset bubbles the growth rate of final output exceeds the interest rate. That is, the necessary condition for the presence of asset bubbles is same as previous studies such as Grossman and Yanagawa (1993) and Futagami and Shibata (2000). Hereafter we consider the case where (4.31) holds.

If there exists the steady state equilibrium with asset bubbles, \(\dot{c}_t = 0\)-locus moves up in a counterclockwise direction as opposed to the case in which no asset bubbles exist. In Figure 4.1 and Figure 4.2, we plot \(\dot{c}_t = 0\)-locus with \(m^{**} > 0\) by a dotted line. These figures show that the number of firms per capita in the steady state with asset bubbles is unambiguously lower than that in the steady state equilibrium without asset bubbles. Moreover, it is shown that the consumption ratio in the steady state equilibrium with asset bubbles is unambiguously higher than that in the steady state equilibrium without asset bubbles.

The key mechanism is as follows: If asset bubbles emerge, households think that they are wealthier, and thus want to consume more. This leads to higher reservation interest rate of households in the asset market. In order to satisfy the

\[15\] See Appendix 4 for the proof.
higher reservation interest rate, the return on equity must increases. This requires the number of firms per capita to be lower given the market size of the economy because the return on equity is positively related to demand for intermediate goods at the level of an individual firm. As a result, the presence of asset bubbles yields a larger scale of production at the level of an individual firm.

Consequently, if the cost spreading effect exceeds the interest rate effect (if $\alpha \beta > 1$), as previously noted, the presence of asset bubbles encourages in-house R&D of firms and promotes economic growth. The same logic applies to the case in which the interest rate effect surmounts the cost spreading effect (if $\alpha \beta < 1$). In addition, the lower product proliferation provides available economic resources for a higher household consumption, regardless of the effect on economic growth.

4.5 Conclusion

This paper provides a theoretical explanation for why the presence of asset bubbles can lead to higher economic growth in concurrence with high consumption by using a continuous-time overlapping generation model with in-house R&D investment and firm entry. The theoretical mechanism is simple and the endogeneity of the market structure plays a key role. The presence of asset bubbles leads to a higher interest rate in the asset market. This requires product proliferation to be impeded, which would result in an increase in the demand for differentiated goods, at the level of an individual firm. A larger scale of production at the level of an individual firm can encourage incentives to in-house R&D of firms and promote economic growth. In addition, the lower product proliferation provides available economic resources for a higher consumption by households.

4.6 Appendices

4.6.1 Appendix 1

Integrating (4.12) yields

$$
\lim_{x \to \infty} a(s, x) e^{(-f_r^s r dv)} - a(s, t) + \int_t^\infty c(s, x) e^{(-f_r^s r dv)} dx = \int_t^\infty (w_x) e^{(-f_r^s r dv)} dx = h_t.
$$

Applying (4.13) and the No Ponzi game condition to the above equation, we obtain $c(s, t) = \rho [a(s, t) + h_t]$. Aggregating this yields

$$
C_t = \rho \left[ a(0^-, t) + h_t \right] + \rho \lambda \int_0^t a(s, t) e^{\lambda s} ds + \rho \lambda \int_0^t h_t e^{\lambda s} ds,
$$
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Figure 4.1: Steady state equilibrium (in the case where $\alpha \beta > 1$)

Figure 4.2: Steady state equilibrium (in the case where $\alpha \beta < 1$ and $\phi > \tilde{\phi}$)
\[ C_t = \rho (A_t + H_t), \quad \text{where} \quad H_t \equiv h_t + \lambda \int_0^t h_t e^{\lambda s} ds. \quad (4.32) \]

From (4.13) and (4.32), differentiating the aggregate consumption with respect to time yields

\[ \dot{C}_t = (r_t - \rho) c(0^-, t) + c(t, t) \lambda e^{\lambda t} + (r_t - \rho) \lambda \int_0^t c(s, t) e^{\lambda s} ds, \]

\[ \Leftrightarrow \dot{C}_t = \rho \lambda e^{\lambda t} [a(t, t) + h_t] + (r_t - \rho) C_t, \]

\[ \Leftrightarrow \dot{C}_t = \rho \lambda H_t + (r_t - \rho) C_t, \quad (\because \ a(t, t) = 0), \]

\[ \Leftrightarrow \dot{C}_t = (r_t - \rho + \lambda) C_t - \rho \lambda A_t. \]

And aggregating (4.12) yields (4.15).

### 4.6.2 Appendix 2

Let \( a_t \equiv A_t / Y_t \). From (4.14) and (4.20),

\[ \frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} - \dot{z}_t - \lambda \Leftrightarrow \dot{c}_t = [\phi(n_t) - \rho] c_t - \lambda \rho a_t. \quad (4.33) \]

From (4.20), the market equilibrium condition of assets is rewritten by

\[ a_t = m_t + \beta \frac{n_t}{\Omega}. \]

Then, substituting this condition into (4.33) yields (4.22).

From (4.20) and \( N_t X_t = \theta^2 Y_t \), the market equilibrium condition of final goods is rewritten by

\[ 1 = c_t + \theta^2 + \frac{n_t}{\Omega} \left[ \phi + \dot{z}_t + \beta \frac{\dot{N}_t}{N_t} \right], \]

Then, from this condition and the definition of \( n_t \), we obtain (4.21).

From the market equilibrium condition of labor and \( N_t w_t L_t = (1 - \theta) Y_t \), (4.15) is rewritten by

\[ \frac{\dot{A}_t}{A_t} = r_t + (1 - \theta) \frac{1}{a_t} - \frac{c_t}{a_t}. \]

Using the above equation and (4.20), we obtain

\[ \dot{a}_t = [\phi(n_t) - \lambda] a_t + (1 - \theta) - c_t. \]

Differentiating the market equilibrium condition of assets with respect to time and
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using (4.21) and the law of motion of the asset ratio, we obtain (4.23).

4.6.3 Appendix 3

From (4.21), (4.22), and (4.23), the system of the linearized differential equation around the steady state is given by

$$
\begin{bmatrix}
\dot{n}_t \\
\dot{c}_t \\
\dot{m}_t
\end{bmatrix} =
\begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{bmatrix}
\begin{bmatrix}
n_t - n^{st} \\
{c}_t - c^{st} \\
m_t - m^{st}
\end{bmatrix},
$$

(4.34)

where \(\{n^{st}, c^{st}, m^{st}\}\) represents \(\{n^*, c^*, m^*\}\) or \(\{n^{**}, c^{**}, m^{**}\}\) and

- \(J_{11} = -\frac{1}{\beta} \left[ \phi + \hat{z}(n^{st}) + \beta \lambda \right] - \frac{n^{st}}{\beta} \left. \frac{\partial \hat{z}(n)}{\partial n} \right|_{n=n^{st}} < 0\),
- \(J_{12} = -\frac{\Omega}{\beta} < 0\),
- \(J_{13} = 0\),
- \(J_{21} = c^{st} \left. \frac{\partial \phi(n)}{\partial n} \right|_{n=n^{st}} - \lambda \rho \frac{\beta}{\Omega} < 0\),
- \(J_{22} = \varphi(n^*) - \rho > 0\) if \(m^{st} = m^*\) (\(\therefore n^* > n^{**}\)),
- \(\begin{cases} J_{22} = \lambda - \rho > 0 & \text{if } m^{st} = m^{**} \\
\end{cases}\),
- \(J_{23} = -\lambda \rho < 0\),
- \(J_{31} = 0\) if \(m^{st} = m^*\),
- \(\begin{cases} J_{31} = m^{**} \left. \frac{\partial \phi(n)}{\partial n} \right|_{n=n^{**}} < 0 & \text{if } m^{st} = m^{**} \\
\end{cases}\),
- \(J_{32} = 0\),
- \(J_{33} = \varphi(n^*) - \lambda < 0\) if \(m^{st} = m^*\) (\(\therefore n^* > n^{**}\)),
- \(\begin{cases} J_{33} = 0 & \text{if } m^{st} = m^{**} \\
\end{cases}\).

We define matrix \(J\) as \(J \equiv \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}\). We obtain the following characteristic equation of the coefficient matrix on the right-hand side of (4.34):

\(0 = |J - qI| = -q^3 + \text{Tr}JJq^2 - BJq + \text{Det}J \equiv f(q)\).
where $I$ is the identity matrix, $\text{Tr} J$ is the trace of $J$, $\text{Det} J$ is the determinant of $J$, and $BJ ≡ \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} + \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}$.

The dynamic system has one state variable, $n_t$, and two jump variables, $c_t$, and $m_t$. Hence, if the number of negative eigenvalues around the steady state is one, there exists a unique equilibrium saddle path toward the steady state. If the number of negative eigenvalues around the steady state is more than one, there exists a continuum of equilibrium path toward the steady state.

Following Benhabib and Perli (1994), we now check the number of negative eigenvalues around the steady state by applying the Routh-Hurwitz theorem:

**Theorem.** The number of roots of the polynomial in (4.34) with positive real parts is equal to the number of variations of sign in the scheme

\[-1 \ \text{Tr} J \ - BJ + \frac{\text{Det} J}{\text{Tr} J} \ \text{Det} J. \tag{4.35}\]

Around the steady state without asset bubbles, we obtain $\text{Det} J = J_{12}J_{23}J_{31} - J_{12}J_{21}J_{33} > 0$ and $\text{Tr} J = J_{11} + J_{22} + J_{33} < 0 \ (\therefore \ \varphi(n^*) < \lambda)$. Then, we can conclude that around the steady state without asset bubbles there are one positive eigenvalue and two eigenvalues with negative real part because there is only one change of the sign in the scheme (4.35) regardless of the value of $BJ$. Therefore, there exists a continuum of equilibrium path toward the steady state without asset bubbles.

Around the steady state with asset bubbles, we obtain $\text{Det} J = J_{12}J_{23}J_{31} < 0$, $\text{Tr} J = J_{11} + J_{22} < 0$, and $BJ = J_{11}J_{22} - J_{12}J_{21} < 0$. Hence, we also obtain $-BJ + \text{Det} J/BJ > 0$. Then, we can conclude that around the steady state with asset bubbles there are two eigenvalues with one positive real part and one negative eigenvalue, because there are two changes of the sign in the scheme (4.35). Therefore, the equilibrium path toward the steady state with asset bubbles is locally determinate.

### 4.6.4 Appendix 4

From (4.24), we find that the difference between the interest rate and the growth rate of final output, $\varphi(n) - \lambda$, is decreasing in $n$. In the steady state equilibrium with asset bubbles, we obtain $\varphi(n^{**}) - \lambda = 0$. Since it is shown that $n^*$ is higher than $n^{**}$, we find that $\varphi(n^*) - \lambda < 0$. 
Bibliography


