Innovation, Intellectual Property Rights Protection, and Economic Growth

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1 Introduction

Our country has achieved economic maturity after several decades of rapid economic growth commencing in the 1950s. Our country's population is expected to decline in the future, and there is little possibility of substantial additional capital accumulation. Thus using resources efficiently, which is measured by total factor productivity (TFP), and increasing efficiency, which is driven by research and development (R&D) activities, are very important for attaining sustained growth. In addition, recent years have brought dramatic changes in our economy. The invention of the iPS cell is an impressive development. This breakthrough is expected to lead to substantial changes in the medical, chemical, and food industries, accompanied by major changes in the corporate landscape and a large amount of R&D activity. The discovery of methane hydrate and the growth in shale gas production will also impact the energy industry, and generate innovative and strategic competition between petroleum firms and new energy firms. Regarding relationship with the global economy, the increasing presence of developing countries will create obvious conflicts of interest between developed and developing countries, which affects the choice of technology transfer and R&D activities in developed countries. Therefore, the purpose of this thesis is to illustrate and analyze the complex economic phenomena outlined above, focusing on the role of R&D activity.

In Chapter 2, we analyze the dynamics of macroeconomic variables affected by technological breakthroughs. We develop an R&D-based growth model in which an old sector and a new sector exist simultaneously. We then analyze the transition dynamics from the equilibrium when only the old sector exists to the equilibrium when both the old sector and the new sector exist. We show that following the introduction of a new technology, the growth rate of productivity declines and stock prices fall over several decades , after which the growth rate of productivity and stock prices both begin to rise. These results are consistent with the results of empirical studies such as Jovanovic and Rousseau (2005) and Greenwood and Jovanovic (1999). As this model highlights the trade-off between the growth-enhancing effect of the establishment of a new industry and the growth-impeding effect of the decline in the existing industry, we can use this framework to analyze the dynamic effect of promoting new innovative technology on the growth rate of TFP.

In Chapter 3, we analyze the effect of promoting the entry and R&D activities of new firms. We develop an R&D-based growth model that is different from the basic quality ladder model in the following two ways. First, in our model, the size of the quality increment is determined by a random draw from a given distribution, and thus have different profit flows. Consequently, leader firms differ in terms of their quality lead over their followers, and thus have different profit flows. Second, we assume that the R&D technology of leader firms suffers from diminishing returns, and consequently some leader firms engage in R&D activities. The results show that leaders with larger leads over their rivals have smaller R&D investment. Moreover, we show that subsidizing followers' R&D can promote leaders' aggregate R&D. Subsidies for followers' R&D promote their R&D and impede the leader's R&D. However, promotion of followers' R&D decreases the number of leaders with a larger quality lead and smaller R&D investment and increases that of leaders with a smaller quality lead and larger R&D investment. If this positive effect from a changing distribution outweighs the negative effects on an individual firm's R&D, consequently the promotion of followers' R&D increases leaders' aggregate R&D.

In Chapter 4, we analyze the effect of strengthening intellectual property rights (IPR) protection in developing countries. We develop a product cycle model with costly technology transfer, which requires the resources of both the North and the South. In the basic model, we show that strengthening IPR protection induces a large technology transfer and narrows the North–South wage gap. However, we obtain an ambiguous result regarding the effect on economic growth, which depends crucially on the size of the transfer cost. Although strengthening IPR protection induces a high growth rate when the transfer cost is small, it can induce a low growth rate when the transfer cost is large. In the extended model, in order to examine which factors determine the transfer cost, we consider the situation where the Southern firms may misbehave and the Northern firms incur a cost to monitor them. We show that the degree of investor protection and the degree of morality in developing countries influence the size of the transfer cost, which affects economic growth.

2 Analysis of Transition Dynamics caused by Technological Breakthroughs

2.1 Introduction

Since the beginning of the Industrial Revolution in Britain (1760{1850), we have witnessed a number of dramatic technological breakthroughs, which have had enormous and long-term impacts on economies. Schumpeter refers to these as Kondratieff cycles, caused by technological factors. For example, electrical technologies had a major influence between 1890 and 1930, and the impact of information technology (IT) during the second half of the 20th century was also important. These technologies created new markets that opened up new business opportunities, and induced a sequence of secondary incremental innovation, which Bresnahan and Trajtenberg (1995) characterized as general purpose technology (GPT). For example, household electrical appliance manufacturers and electric power companies both arose as a consequence of electricity, and now continue to improve the quality of their products to sustain consumer demand. Similarly, PC manufacturers, software companies and Internet service providers were set up following the emergence of IT. These companies also continually improve the quality of their products.

The purpose of this paper is to analyze the transition from the emergence of a new technology to its maturity. In particular, we consider the growth rate of productivity, the stock price of incumbents in the preexisting sector and aggregate stock price. Jovanovic and Rousseau (2005) present evidence that, following the emergence of a new and transforming technology, the growth rate of productivity declines for some decades, after which it begins to rise again. Indeed, the productivity slowdown is now recognized as a stylized fact. Our model describes these changes and shows that the slowdown is caused by declining R&D in existing technology. Greenwood and Jovanovic (1999) present evidence that, upon the arrival of a new technology, existing firms' stock prices fall, never to recover. They also show that several years after the emergence of a new technology, aggregate stock prices rise because of the rise of the prices of the stock of new firms exploiting the new technology. Our model describes these facts and shows that the decline in the stock prices of existing firms is caused by the decline in their profits, while the aggregate stock price rises with the new sector's maturity.

The model is the extension of Segerstrom (1998), which is a quality-ladder R&D model, freed from scale effects by assuming that the difficulty of R&D increases as R&D accumulates.[1] His model is a semi-endogenous growth model, where the long-run growth rate is never influenced by political variables or preferences. More recently, the short-run effects in semi-endogenous growth models have received more attention. Steger (2003) found that the speed of convergence in Segerstrom (1998) was only 0.019, which means that it takes about 38 years for half the initial gap to vanish (this is a very long time!). Therefore, analyzing the transition dynamics is very important. In this paper, we examine the technological cycle by analyzing the transition dynamics of an extended Segerstrom (1998) model, which has very long transition dynamics.

In this paper, we expand Segerstrom (1998) by introducing heterogeneity across sectors in four ways: efficiency, difficulty and cost of R&D, and market size.[2] This extension enables the model to simultaneously analyze both the "old sector" and the "new sector", where the old sector has low R&D costs and a high difficulty of R&D , and the new sector has high R&D costs but a low difficulty of R&D. We examine the process caused by a technological breakthrough to analyze the transition dynamics from the existing old-sector equilibrium alone to the equilibrium of both the old and new sectors.

Consider the following situation. At stage 1, there exists an \old sector" with a continuum of products. At stage 2, an exogenous technological breakthrough occurs, leading to emergence of a \new sector" with a continuum of products. The initial cost of R&D in the new sector is high enough that no R&D is conducted in this new sector. Stage 3 is reached later, when another exogenous shock occurs, which drastically reduces the cost of R&D in the new sector, such that it is now conducted. This paper examines how economically important variables such as the growth rate of productivity and stock prices respond to these two enormous exogenous shocks.[3][4]

Some stage 2 results are consistent with empirical facts. First, the growth rate of productivity declines. New technology creates new business opportunities, thus forming the new sector, which captures market share from old-sector firms, thus causing their profits to decrease. This decreases the incentive to enter the old sector, reducing R&D in the old sector. Thus, the instantaneous probability of successful R&D in the old sector decreases. On

[1] Aghion and Howitt (1992),Grossman and Helpman (1991) and Romer (1990) pioneered these R&D models.

[2] Smulders and van de Klundert (1995), Startz (1998), Li (2000), Boucekkine and de la Croix (2003), Doi and Mino (2005) and Chu (2011) have investigated growth models with multiple sectors.

[3] By assuming that R&D cost decreases due to learning by doing, we can extend the model so that the time of initial new-sector R&D investment is determined endogenously; however, including these would complicate the model significantly and so is left for further research.

[4] Kuwahara (2013) studies growth model with the regime change from no R&D regime to with R&D regime.

the other hand, R&D is not being conducted in the new sector because of the very high cost of R&D. Therefore, aggregate R&D decreases, which leads to a decrease in the aggregate probability of successful R&D. Because the growth rate of productivity is calculated as the weighted sum of probabilities of successful R&D, this reduces productivity growth. Second, old-sector incumbents' stock prices decrease because their discounted stream of profits has decreased.

Stage 3 results are also consistent with empirical facts. First, the growth rate of productivity rises. Several years after a new technology is born, the cost of R&D falls. Aggressive R&D activity then begins in the new sector, which dominates the effect of the decline of R&D activity conducted in the old sector. Therefore, aggregate R&D increases, which leads to an increase in the aggregate probability of successful R&D, and the growth rate of productivity rises. Second, old-sector incumbents' stock prices remain low. Because old-sector market share does not change between stages 2 and 3, the discounted profits of old-sector incumbents remain small and hence their stock prices remain low. Third, the aggregate stock price rises when we include the new-sector firms, which were created in response to the new technology. As the new technology matures, the status of new-sector firms stabilizes, raising their discounted profits. Hence, new-sector firms' stock prices rise, dominating the decline of old-sector incumbents' stock prices. Therefore, the aggregate stock price rises.

The contributions of this paper are as follows. First, we explain the transition of economically important variables such as productivity and stock price that is observed empirically using a typical R&D growth model. Second, we construct a quality-ladder R&D model with ex ante heterogeneity (efficiency, difficulty and cost of R&D, and market share differences across sectors evaluated before R&D is conducted). The current literature examines quality-ladder R&D models with ex post heterogeneity (efficiency of product technology, degree of increment quality, shadow price and profits differ across firms evaluated after having conducted R&D).[5] Our model extends these models in another direction. Third, we carefully analyze the transition dynamics of a semi-endogenous growth model. As discussed in Steger (2003), the transition path of a semi-endogenous growth model is so long that analyzing transition dynamics is important for development of this model.

[5]Dinopoulos and Unel (2011) and Minniti, Parello and Segerstrom (2012) examine R&D with ex post heterogeneity.

2.1.1 Related literature

Hornstein and Krusell (1996), Greenwood and Yorukoglu (1997), Helpman and Trajtenberg (1998) and Aghion and Howitt (2008) analyze the relationship between the emergence of a new technology (GPT) and productivity slowdown. Hornstein and Krusell (1996), Greenwood and Yorukoglu (1997), Helpman and Trajtenberg (1998) state that because labor is needed to introduce a new GPT, the amount of labor engaging in manufacturing and in R&D declines, thus reducing output. Aghion and Howitt (2008) use a simple neoclassical model to consider a rate of capital obsolescence that is proportional to the innovation rate. They state that a new GPT raises both the innovation rate and the rate of capital obsolescence, which reduces the growth rate. On the other hand, this paper states that a new technology creates a new sector and diminishes the incentive to carry out R&D in the old sector, which reduces the growth rate of productivity. Besides, this paper shows that if there are some periods in which the cost of R&D in the new sector is high enough, productivity slowdown occurs, although no labor input is needed to introduce a new technology.

Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001) analyze the relationship between the emergence of IT and incumbents' stock prices. They use the Lucas tree model and assume that the emergence of IT causes permanently higher income several years later. The results show that the substitution effect causes current consumption to rise, and thus this reduces demand for assets (the right to future consumption), which reduces asset prices. On the other hand, we show that the initial fall in the stock price is due to the decline in old-sector incumbents' profits and that the subsequent rise is due to the maturation of the new technology sector.

Cheng and Dinopoulos (1996) analyze the endogenous deterministic cycles caused by technological breakthroughs. However, they do not examine the effects of technological breakthroughs on the dynamics of productivity or stock price growth rates. Boucekkine and de la Croix (2003) study the effect on the growth rate of the IT revolution, which changes the productivity of the final goods sector and the efficiency of the R&D sector. They use a simulation model in which efficiency of R&D increases permanently and productivity of the final goods sector decreases temporarily to show that productivity slowdown occurs. They state that the productivity of labor initially decreases because of the negative shock to the final goods sector. However, the positive shock to the R&D sector stimulates R&D activity, which increases productivity. Thus, the productivity of labor eventually increases. This mechanism is clearly different from our model.

The rest of this paper is organized as follows. Section 2 describes the basic setup of the model. Section 3 describes the path of the economy and establishes the balanced-growth equilibrium. Section 4 analyzes the transition from the emergence of the new technology to its maturity. Section 5 concludes.

2.2 The Model

2.2.1 Environment

The economy consists of three agents: households, R&D firms and producing firms (incumbents). Households supply labor inelastically, save by investing in R&D firms as stockholders and consume products. R&D firms hire labor to engage in R&D, which improves products' quality. Producing firms hire labor to produce products, gain profits by selling the products and distribute those profits to households as dividends.

In the main analysis, we consider the following situation. Initially, there exists a sector with a continuum of products, the “old sector”, and subsequently another sector with a continuum of products, the “new sector”, emerges. The two sectors differ in four respects: efficiency, difficulty and cost of R&D, and market size. The old sector, which is denoted sector 0, is so mature that the cost of R&D is low; however, the probability of successful R&D is also low because a large amount of R&D has already been accumulated. This assumption follows Segerstrom's notion that the most obvious ideas are discovered first, making it harder to develop further ideas. The new sector, which is denoted sector 1, is so new that the cost of R&D is high; however, probability of successful R&D is high because little R&D has yet been accumulated.

In each sector, there are continuous industries indexed by $j \in [0, 1]$ producing differentiated goods. Each industry produces goods with different qualities, which are classified into countable quality generation $m = 0, 1, \ldots$. The goods are perfectly substitutes for one another in each industry. Innovation in an industry results in a new generation of goods.

We designate as leaders (producing firms) (incumbents) those firms that use the latest R&D in each industry and are able to produce the state-of-the-art quality product in their industry. At the time of birth of each sector, we assume that the state-of-the-art quality product in each industry is $m = 0$ and that the size of quality increments induced by one successful R&D event is constant in any industry of any sector, which is denoted as $\lambda > 1$. Therefore, in industry j of sector i at time t, state-of-the-art quality is determined by the number of successful R&D events that have occurred in industry j of sector i from the birth of sector i until time t.

We designate as R&D firms those firms that attempt R&D investment in order to win a leadership position through a higher quality product. They may invest in any industry of any sector. If an R&D firm produces successful R&D in industry j of sector i, it can produce a state-of-the-art quality product in industry j of sector i, and thus becomes the leader in industry j of sector i. When the state-of-the-art quality in industry j of sector i is m { 1 generation, denoted as $q^i(j; m-1)$, the successful R&D firm can produce an m generation product, the quality of which is denoted by $q^i(j; m) = \lambda q^i(j; m-1)$. If we normalize the initial quality to unity (i.e., $q^i(j; 0) = 1$), then an m generation of the state-of-the-art quality product in industry j of sector i is expressed as $q^i(j; m) = \lambda^m$.

2.2.2 Households

The economy is populated by a fixed number of identical households. Household members live forever and are endowed with one unit of labor, which is supplied inelastically. The number of members in each household grows exponentially at an exogenous rate $n > 0$. Assuming that the initial population is unity, then the population at time t is $L(t) = e^{nt}$. Each household maximizes its discounted utility:

$$U = \int_0^1 \exp(-(\rho - n)t) \ln u(t) dt; \tag{1}$$

where $\rho > n$ is the subjective discount rate and $\ln u(t)$ is the instantaneous utility per person at time t, which is given by

$$\ln u(t) = \gamma^0 \int_0^1 \ln \left[\sum_m \lambda^m d^0(j; m; t) \right] dj + \gamma^1 \int_0^1 \ln \left[\sum_m \lambda^m d^1(j; m; t) \right] dj; \tag{2}$$

where γ^i is the market share of sector i, $d^i(j;m;t)$ is the quantity demanded of m generation product produced in industry j of sector i at time t.[6] [7] The first term on the right-hand side (RHS) is the instantaneous utility from consuming the products of sector 0; the integration of this term is the instantaneous utility from consuming the products of industry j of sector 0, and the summation of this term is the instantaneous utility from consuming m generation products of industry j of sector 0. The second term on the RHS is the instantaneous utility from consuming products of sector 1, and so on. Because γ^i is the market share of sector i, $\gamma^0 + \gamma^1 = 1$ is always satisfied. If $\gamma^0 = 1$, then this instantaneous utility function is identical to that of Segerstrom (1998).

From (2), the demand function for the product with the lowest quality-adjusted price in industry j of sector i is given by

$$d^i(j;t) = \frac{\gamma^i c(t)}{p^i(j;t)}; \quad \text{for} \quad i = 0;1 \; ; \tag{3}$$

where $p^i(j;t)$ is the price of the product that has the lowest quality-adjusted price in industry j of sector i at time t, $d^i(j;t)$ is the quantity demanded of the product that has the lowest quality-adjusted price in industry j of sector i at time t, and c(t) is households' expenditure at time t.[8] Given (3), intertemporal utility maximization yields

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{4}$$

where r(t) is the interest rate at time t.

2.2.3 Product markets

We assume the product market is monopolistically competitive. In each industry, firms compete in price. Labor is the only input in production. We assume that one unit of labor is required to produce one unit of output regardless of quality level, and that there is no fixed cost. We normalize the wage rate as numeraire. Thus, every firm has a constant marginal cost of production equal to unity.

[6]As seen from (2), we assume that the intersector and interindustry elasticities of substitution are unity, and that the intra-industry elasticity of substitution is infinity.

[7]From this utility function, we interpret that households consume various goods produced by producing firms. However, we can reinterpret that households consume the final good, which is produced by combining the goods produced by producing firms.

[8]Because intra-industry elasticity of substitution is infinity, households demand only the product that has the lowest quality-adjusted price in each industry

In each industry, there is one leader and a number of followers. These followers were formerly able to produce the state-of-the-art quality product but cannot now. Because the quality increment from one successful R&D event is constant, leaders can produce a product of at least λ higher quality compared with the followers. Because the marginal cost of production is unity, followers never charge a price less than unity. Leaders can therefore monopolize demand for their industry's product by charging a price less than λ. Because interindustry elasticity of substitution is unity, leaders have no incentive to lower the price to less than λ.[9] Thus the price of the product is then determined by

$$p(t) = \lambda. \tag{5}$$

Because one unit of labor is required to produce one unit of output and there is no fixed cost, leaders in sector i earn the following profit flow:

$$\pi^i(t) = \gamma^i c(t) L(t)(1 - \frac{1}{\lambda}); \quad \text{for} \quad i = 0; 1; \tag{6}$$

where $\pi^i(t)$ is the leaders' profits in sector i at time t.[10]

2.2.4 R&D races

Labor is the only input to R&D. There is free entry into each R&D race. Any R&D firm that hires $H^i(t) l^i(j; k; t) dt$ units of labor in industry j of sector i at time interval [t,t+ dt] is successful in discovering a state-of-the-art product with instantaneous probability $I^i(j; k; t) = l^i(j; k; t) A^i / X^i(t) dt$, where $H^i(t)$ is the cost of R&D, A^i is the efficiency of R&D and $X^i(t)$ is the difficulty of R&D in industry i. Assuming that the returns to engaging in the R&D race are independently distributed across firms, across industries and over time, the industry-wide instantaneous probability of successful R&D is given by

$$I^i(t) dt = \frac{L^i(t) A^i}{X^i(t)} dt; \quad \text{for} \quad i = 0; 1; \tag{7}$$

[9] We assume that the consumer strictly prefers the higher-quality good when the quality-adjusted price is same.
[10] Note that all industries' profits in the same sector are identical because both the quantity demanded in a sector and their prices are identical.

where $I^i(t)dt$ is industry-wide instantaneous probability of successful R&D in sector i, and $L^i(t)dt$ is the amount of labor engaging in R&D in sector i. Following Segerstrom (1998), the difficulty of R&D in sector i increases as R&D in sector i accumulates. Then we assume

$$\frac{\dot{X}^i(t)}{X^i(t)} = \mu I^i(t); \quad \text{for} \quad i = 0; 1; \tag{8}$$

where $0 < \mu < 1$ is exogenously given. We define s^i as the time of origin in sector i. We also assume that $X^i(s^i) = \bar{X}^i$, that is, the initial difficulty of R&D is identical for all industries in the same sector. Then, the difficulty of R&D in sector i at time t is $X^i(t) = \bar{X}^i \exp\left(\int_{s^i}^{t} \mu I^i_{\nu} d\nu\right)$.

Let $v^i(t)$ denote the leaders' stock prices in sector i at time t.[11] Each R&D firm attempts to maximize $v^i(t)\frac{A^i l^i(j;k;t)}{X^i(t)}dt - H^i(t)l^i(j;k;t)dt$. If $v^i(t)\frac{A^i}{X^i(t)} > H^i(t)$; then, $l^i = 1$ is profit maximizing, which cannot be the equilibrium. If $v^i(t)\frac{A^i}{X^i(t)} < H^i(t)$, then $l^i = 0$ is profit maximizing. If $v^i(t)\frac{A^i}{X^i(t)} = H^i(t)$, then $l^i \in (0; +\infty)$ is profit maximizing. To sum up,

$$\begin{aligned} v^i(t) &= \frac{X^i(t)H^i(t)}{A^i} \quad \text{when} \quad I^i(t) > 0 \quad \text{for} \quad i = 0; 1 \\ v^i(t) &< \frac{X^i(t)H^i(t)}{A^i} \quad \text{when} \quad I^i(t) = 0 \quad \text{for} \quad i = 0; 1. \end{aligned} \tag{9}$$

2.2.5 No arbitrage condition

Over time interval dt, shareholders receive an income gain $\pi^i(t)dt$ and a capital gain (loss) $\dot{v}^i(t)dt$ in each industry of sector i. However, with instantaneous probability $I^i(t)dt$, a higher-quality product is discovered, the existing leader goes out of business and shareholders suffer a total capital loss $v^i(t)$. Efficient markets require that the expected return to holding the stock of the leader is equal to the risk-free interest rate $r(t)$.[12] Thus $\frac{\pi^i(t)+\dot{v}^i(t)-I^i(t)v^i(t)}{v^i(t)} = r(t)$. From (6), we obtain

[11]As seen below, we interpret $v^i(t)$ as the expected discounted profit for winning an R&D race in sector i at time t. From (6), profits are identical for all industries in the same sector. Thus, the leader's stock price is the same in all industries in the same sector.

[12]Assuming that shareholders are risk neutral and can diversify risk completely.

$$v^i(t) = \frac{\gamma^i\left(\frac{\lambda-1}{\lambda}\right)c(t)L(t)}{r(t) + I^i(t) - \frac{\dot{v}^i(t)}{v^i(t)}}, \quad \text{for} \quad i = 0, 1. \tag{10}$$

In further analysis, let us define indexes of stock prices, $V(t)$ and $\hat{V}(t)$, as follows:

$$V(t) = v^0(t) + v^1(t) \quad \forall I^0(t), \forall I^1(t), \tag{11}$$

where $V(t)$ is defined as the sum of stock prices of leaders across sectors where the leaders are producing goods.

$$\begin{aligned} \hat{V}(t) &= v^0(t) && \text{when} \quad I^0(t) > 0, I^1(t) = 0 \\ \hat{V}(t) &= v^0(t) + v^1(t) && \text{when} \quad I^0(t) > 0, I^1(t) > 0 \end{aligned}, \tag{12}$$

where $\hat{V}(t)$ is defined as the sum of stock prices of leaders across sectors where the leaders are producing goods and R&D firms are carrying on R&D.[13]

2.2.6 The labor market

Because one unit of labor is required to produce one unit of output, $\gamma^i c(t)L(t)/\lambda$ workers are employed by the leader in sector i. $X^i(t)H^i(t)I^i(t)/A^i$ workers are employed by R&D firms in sector i. Because households supply one unit of labor inelastically, we obtain

$$\frac{X^0(t)H^0(t)I^0(t)}{A^0} + \frac{X^1(t)H^1(t)I^1(t)}{A^1} + \frac{c(t)L(t)}{\lambda} = L(t). \tag{13}$$

2.2.7 Growth rate of productivity

Because we assumed that the quality of 0 generation products for all industries of any sector is unity and that the size of an increment of one successful R&D event is λ, the quality of the product manufactured by the leader of industry j of sector i is determined by the number of times that successful R&D occurs in that industry. We define

[13] We define two types of stock price index to deal with the problem of the timing of stock listing (the problem is when we should evaluate new-sector incumbents' stock prices.). Although the above definitions are not complete, they are sufficient for analyzing a rough motion of stock price.

$Q(t)$ as the weighted sum of the quality of products across all industries of all sectors, and express $Q(t)$ as follows:

$$\ln Q(t) = \sum_{i=0}^{1} \gamma^i \int_0^1 \ln \lambda^{m^i(j;t)} dj = \sum_{i=0}^{1} \gamma^i \Phi(i;t) \ln \lambda. \tag{14}$$

where $m^i(j;t)$ is the generation of state-of-the-art quality in industry j of sector i at time t, and $\Phi(i;t) = \int_{s^i}^{t} I(i,v)dv$ is the expected value of successful R&D events that will occur in sector i in time interval $[s^i;t]$.

Define $g(t)$ as the growth rate of the weighted sum of product qualities. From (14), we obtain

$$g(t) \equiv \frac{\dot{Q}(t)}{Q(t)} = \gamma^0 I^0(t) \ln \lambda + \gamma^1 I^1(t) \ln \lambda. \tag{15}$$

Following Grossman and Helpman (1991), we interpret $g(t)$ as the growth rate of productivity.[14]

2.3 Equilibrium

In this section, we derive the equilibrium path in two cases. In one case, R&D is done only in sector 0 (i.e., $I^0(t) > 0$ and $I^1(t) = 0$), and in the other case, R&D is done in both sectors (i.e., $I^0(t) > 0$ and $I^1(t) > 0$).

2.3.1 The case in which $I^0(t) > 0$ and $I^1(t) = 0$

First, derive the equilibrium path in the case where $I^0(t) > 0$ and $I^1(t) = 0$. Define the adjusted difficulty of R&D in sector i: $y^i(t) \equiv X^i(t)H^i(t)/A^i L(t)$ Differentiating this equation with respect to t yields $\dot{y}^i(t)/y^i(t) = I^i(t)\mu - n$ from (8). Because $I^1(t) = 0$, we obtain $y^0(t)I^0(t) + \frac{c(t)}{\lambda} = 1$ from (13). Then

$$\dot{y}^0(t) = \frac{(\lambda - c(t))}{\lambda}\mu - y^0(t)n: \tag{16}$$

Because $I^0(t) > 0$, we obtain $v^0(t) = y^0(t)L(t)$ from (9). We also obtain $v^0(t) = \frac{\gamma^0(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^0(t)}$ from (8) and (10). Combining these two equations and using $y^0(t)I^0(t)+\frac{c(t)}{\lambda} = 1$, we obtain $r(t) = \frac{1}{\lambda y^0(t)}\left[\gamma^0(\lambda-1)c(t) - (1-\mu)(\lambda - c(t))\right]$.

[14] From (2) and (3), we obtain $\ln u(t) = \ln Q(t) + \gamma^0 \ln L^{p0}(t) + \gamma^1 \ln L^{p1}(t)$, where L^{pi} is labor engaging in manufacturing in sector i. Interpreting $u(t)$ as the final output, a higher $Q(t)$ means that more output can be produced with the same input. Therefore, we can take $Q(t)$ as Total Factor Productivity (TFP)and $g(t)$ as the growth rate of TFP.

Substituting this equation into (4),

$$\dot{c}(t) = \frac{1}{\lambda y^0(t)}[(\gamma^0\lambda + \gamma^1 - \mu)(c(t))^2 - (1-\mu)\lambda c(t)] - \rho c(t). \tag{17}$$

This economy's dynamics are described by (16) and (17) in (y^0, c) space, as depicted in figure 1. The vertical axis represents consumption per capita, c, which is a jump variable, and the horizontal axis represents the adjusted difficulty of R&D in sector 0, y^0, which is a state variable. From (16), the $\dot{y}^0(t) = 0$ curve is downward-sloping. When the point (y^0, c) is to the left (right) of the $\dot{y}^0(t) = 0$ curve, then $\dot{y}^0(t) > 0$ ($\dot{y}^0(t) < 0$). From (17), the $\dot{c}(t) = 0$ curve is upward-sloping. When the point (y^0, c) is to the left (right) of the $\dot{c}(t) = 0$ curve, then $\dot{c}(t) > 0$ ($\dot{c}(t) < 0$).

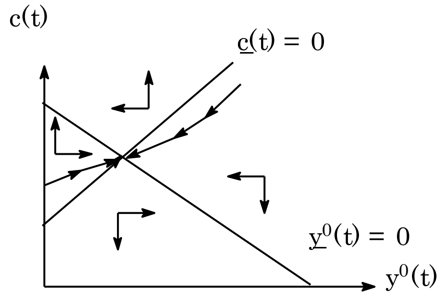

Figure 1. Phase diagram where $I^0(t) > 0$ and $I^1(t) = 0$.

We now derive the value of the balanced-growth equilibrium. Because the balanced-growth equilibrium is determined by the intersection of the $\dot{y}^0(t) = 0$ curve and the $\dot{c}(t) = 0$ curve, we obtain

$$y^{0*} = \frac{\gamma^0\mu(\lambda - 1)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}, \tag{18}$$

and

$$c^* = \frac{\lambda(\mu\rho - \mu n + n)}{\mu\rho - \mu n + n(\gamma^0\lambda + 1 - \gamma^0)}, \tag{19}$$

from (16) and (17), where the steady-state equilibrium level is denoted by "*" in the present case (where $I^0(t) > 0$ and $I^1(t) = 0$).[15] Because c^* is constant in the balanced-growth equilibrium, we obtain $r^* = \rho$ from (4).

[15]The equilibrium is saddle-path stable. Steger (2003) examines the stability conditions carefully.

Substituting (18) and (19) into (13) with $I^1(t) = 0$ and $y^0(t) \equiv X^0(t)H^0(t)/A^0L(t)$, we obtain

$$I^{0*} = \frac{n}{\mu}; \tag{20}$$

and from (15), we obtain

$$g^* = \gamma^0(\ln\lambda)\frac{n}{\mu}: \tag{21}$$

From $v^0(t) = y^0(t)L(t)$ and (18), we obtain

$$v^{0*}(t) = \frac{\mu\gamma^0(\lambda-1)L(t)}{\mu\rho-\mu n+n(\gamma^0\lambda+1-\gamma^0)}; \tag{22}$$

and from (10) and $r^* = \rho$, we obtain

$$v^{1*}(t) = \frac{(\mu\rho-\mu n+n)}{(\mu\rho-\mu n)}\frac{\mu\gamma^1(\lambda-1)L(t)}{\mu\rho-\mu n+n(\gamma^0\lambda+1-\gamma^0)}: \tag{23}$$

Substituting (22) and (23) into (11), we obtain

$$V^*(t) = \frac{(\mu\rho-\mu n+n\gamma^1)(\lambda-1)L(t)}{(\rho-n)\left[\mu\rho-\mu n+n(\gamma^0\lambda+1-\gamma^0)\right]}: \tag{24}$$

From (12), we obtain

$$\hat{V}^*(t) = \frac{\mu\gamma^0(\lambda-1)L(t)}{\mu\rho-\mu n+n(\gamma^0\lambda+1-\gamma^0)}: \tag{25}$$

2.3.2 The case in which $I^0(t) > 0$ and $I^1(t) > 0$

In this subsection, we derive the equilibrium path in the case where $I^0(t) > 0$ and $I^1(t) > 0$. Because $I^0(t) > 0$ and $I^1(t) > 0$, we obtain $v^0(t) = y^0(t)L(t)$ and $v^1(t) = y^1(t)L(t)$ from (9). We also obtain $v^0(t) = \frac{\gamma^0(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^0(t)}$ and $v^1(t) = \frac{\gamma^1(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^1(t)}$ from (8) and (10). Substituting $v^0(t) = y^0(t)L(t)$ into $v^0(t) = \frac{\gamma^0(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^0(t)}$, $v^1(t) =$

$y^1(t)L(t)$ into $v^1(t) = \frac{\gamma^1(\frac{\lambda-1}{\lambda})c(t)L(t)}{r(t)+(1-\mu)I^1(t)}$, eliminating r(t) to combine these two equations, and using (13), we obtain

$$
\begin{aligned}
I^0(t) &= \frac{1}{\lambda(y^0(t)+y^1(t))}\left[\left\{\gamma^0 y^1(t) - \gamma^1 y^0(t)\right\}\frac{(\lambda-1)c(t)}{(1-\mu)y^0(t)} + \lambda - c(t)\right];\\
I^1(t) &= \frac{1}{\lambda(y^0(t)+y^1(t))}\left[\left\{\gamma^1 y^0(t) - \gamma^0 y^1(t)\right\}\frac{(\lambda-1)c(t)}{(1-\mu)y^1(t)} + \lambda - c(t)\right]:
\end{aligned} \tag{26}
$$

Substituting these equations into $\underline{y}^i(t)/y^i(t) = I^i(t)\mu - n$, we obtain

$$
\begin{aligned}
\frac{\underline{y}^0(t)}{y^0(t)} &= \frac{\mu}{\lambda(y^0(t)+y^1(t))}\left[\left\{\gamma^0 y^1(t) - \gamma^1 y^0(t)\right\}\frac{(\lambda-1)c(t)}{(1-\mu)y^0(t)} + \lambda - c(t)\right] - n;\\
\frac{\underline{y}^1(t)}{y^1(t)} &= \frac{\mu}{\lambda(y^0(t)+y^1(t))}\left[\left\{\gamma^1 y^0(t) - \gamma^0 y^1(t)\right\}\frac{(\lambda-1)c(t)}{(1-\mu)y^1(t)} + \lambda - c(t)\right] - n:
\end{aligned} \tag{27}
$$

We define z(t) as the sum of adjusted difficulty of R&D across sectors where R&D is conducted:

$$
\begin{aligned}
z(t) &= y^0(t) + y^1(t) \quad \text{when} \quad I^0(t) > 0; I^1(t) > 0;\\
z(t) &= y^0(t) \quad \text{when} \quad I^0(t) > 0; I^1(t) = 0;
\end{aligned} \tag{28}
$$

then, $\underline{z}(t) = \underline{y}^0(t) + \underline{y}^1(t)$ when R&D is conducted in both sectors.[16] Using (27), we obtain

$$
\underline{z}(t) = \frac{(\lambda - c(t))\mu}{\lambda} - z(t)n: \tag{29}
$$

Substituting $v^i(t) = y^i(t)L(t)$ and (59) into (10), and solving for r(t), we obtain $r(t) = \frac{1}{\lambda z(t)}[(\lambda-\mu)c(t) - (1-\mu)\lambda]$. Substituting this equation into (4), we obtain

$$
\underline{c}(t) = \frac{1}{\lambda z(t)}[(\lambda-\mu)(c(t))^2 - (1-\mu)\lambda c(t)] - \rho c(t): \tag{30}
$$

This economy's dynamics are described by (29) and (30) in (z; c) space, seen in the upper part of figure 2. The

[16]Note that $\underline{z}(t) = \underline{y}^0(t)$ when R&D is conducted only in sector 0. Therefore, we replace $y^0(t)$ with $z(t)$, and $\underline{y}^0(t)$ with $\underline{z}(t)$ in figure 1.

vertical axis represents consumption per capita, c, which is a jump variable, and the horizontal axis represents the sum of adjusted difficulty of R&D across sectors, z, which is a state variable. The upper part of figure 2 is similar to figure 1, but there are two differences between the figures. First, the horizontal axis of figure 1 represents $y^0(t)$, but that of figure 2 represents $z(t)$.[17] Second, the slope and the intercept of the $\dot{c}(t) = 0$ curve of figure 1 are greater than those of figure 2.[18]

From the upper part of figure 2, we can examine the macro variables, including consumption per capita and the sum of stock prices. However, we are also interested in each sector's variables, including the incumbents' stock prices in sector i and the industry-wide instantaneous probability of successful R&D in sector i. Define $Y(t) \equiv y^1(t)/y^0(t)$ as the ratio of adjusted difficulty of R&D in sector 1 to that of sector 0. From (27), we obtain

$$\dot{Y}(t) = \frac{c(t)\mu(\lambda - 1)\gamma^0}{\lambda(1-\mu)y^0(t)}\left\{\frac{\gamma^1}{\gamma^0} - Y(t)\right\}: \tag{31}$$

The dynamics of $Y(t)$, which is a state variable, are illustrated in the lower part of figure 2. The vertical axis represents the adjusted difficulty of R&D in sector 1, y^1, and the horizontal axis represents the adjusted difficulty of R&D in sector 0, y^0. Because $Y(t)$ is the ratio of $y^1(t)$ to $y^0(t)$, the slope of the line connecting the origin and any point in $(y^0(t); y^1(t))$ space represents $Y(t)$. From (31), the $\dot{Y}(t) = 0$ curve is $Y(t) = \frac{\gamma^1}{\gamma^0}$. That is, the ratio of adjusted difficulty of R&D in sector 1 to that of sector 0 is equal to the ratio of market share of sector 1 to that of sector 0. When $Y(t)$ is smaller (greater) than $\frac{\gamma^1}{\gamma^0}$, $\dot{Y}(t) > 0$ ($\dot{Y}(t) < 0$). That is, when the ratio of adjusted difficulty of R&D is smaller (greater) than the ratio of market share at time t, the ratio of adjusted difficulty of R&D increases (decreases) at time t (i.e., industry-wide instantaneous probability of successful R&D in sector 1 is greater (smaller) than that for sector 0). This is because when $Y(t) < \frac{\gamma^1}{\gamma^0}$ ($Y(t) > \frac{\gamma^1}{\gamma^0}$), sector 1 is more (less) attractive than sector 0 because of the relatively high (low) probability of successful R&D or because of the relatively high (low) revenue from becoming the leader, and so more R&D firms are willing to conduct investigations in sector 1 (0). This process continues until the ratio of adjusted difficulty of R&D is equal to the ratio of market share.

[17] However, $y^0(t)$ can be replaced with $z(t)$ and $\dot{y}^0(t)$ with $\dot{z}(t)$ in figure 1 from (28).
[18] Note that $\lambda > 1$.

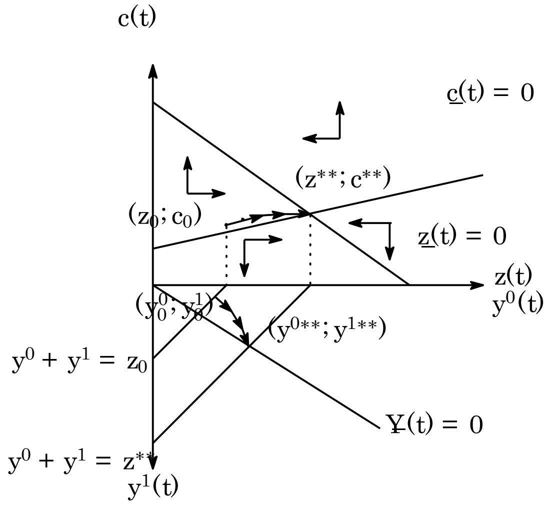

Figure 2. Phase diagram in the case where $I^0(t) > 0$ and $I^1(t) > 0$.

We give an example to promote better understanding of this dynamics. Suppose that the economy is initially at $(z_0; c_0)$ in (z,c) space and at $(y^0_0; y^1_0)$ in $(y^0; y^1)$ space of figure 2. Because $z_0 = y^0_0 + y^1_0$ when $I^0(t) > 0$ and $I^1(t) > 0$, $(y^0_0; y^1_0)$ is on the straight line that passes through the point $(z_0; 0)$ and has a slope of forty-five degrees. Because $(z_0; c_0)$ is to the left of the $\underline{z}(t) = 0$ curve and the $\underline{c}(t) = 0$ curve, the economy moves northeast in (z,c) space. Because $(y^0_0; y^1_0)$ is to the right of the $\underline{Y}(t) = 0$ curve, $Y(t)$ increases. Thus, the economy moves to the $\underline{Y}(t) = 0$ curve satisfying $y^0(t) + y^1(t) = z(t)$ in $(y^0; y^1)$ space, as depicted in figure 2. In the long run, because $z(t)$ converges to z^{**}, $(y^0(t); y^1(t))$ converges to $(y^{0**}; y^{1**})$, which is the intersection of the $\underline{Y}(t) = 0$ curve and the $y^0(t) + y^1(t) = z^{**}$ curve. \"**" denotes the steady-state equilibrium level in which $I^0(t) > 0$ and $I^1(t) > 0$.

Next we find the value of the balanced-growth equilibrium.[19] Because the balanced-growth equilibrium is determined by the intersection of the $\underline{z}(t) = 0$ curve and the $\underline{c}(t) = 0$ curve, from (29) and (30) we obtain

$$z^{**} = \frac{\mu(\lambda - 1)}{\mu\rho - \mu n + n\lambda} \tag{32}$$

and

$$c^{**} = \frac{\lambda(\mu\rho - \mu n + n)}{\mu\rho - \mu n + n\lambda}: \tag{33}$$

[19]The equilibrium is saddle-path stable.

From (18) and (32), we confirm that $z^{**} > y^{0*} = z^{*}$, and from (19) and (33), we confirm that $c^{**} < c^{*}$.[20] That is, the proportion of workers in R&D (manufacturing) is greater (smaller) when $I^0(t) > 0; I^1(t) > 0$ than when $I^0(t) > 0; I^1(t) = 0$ from (13). Because c^{**} is also constant in the balanced-growth equilibrium under $I^0(t) > 0; I^1(t) > 0$, we obtain $r^{**} = \rho$ from (4). From (31), we obtain

$$Y^{**} = \frac{y^{1**}}{y^{0**}} = \frac{\gamma^1}{\gamma^0}: \tag{34}$$

Substituting (34) into (32), we obtain

$$y^{i**} = \frac{\gamma^i \mu(\lambda - 1)}{\mu\rho - \mu n + \lambda n} \quad \text{for} \quad i = 0; 1: \tag{35}$$

From (18) and (35), we confirm $y^{0**} \leq y^{0*}$. Substituting (33) and (35) into (59), we obtain

$$I^{i**} = \frac{n}{\mu} \quad \text{for} \quad i = 0; 1; \tag{36}$$

and from (15), we obtain

$$g^{**} = (\ln \lambda)\frac{n}{\mu}: \tag{37}$$

From (21) and (37), we confirm $g^{**} > g^{*}$.[21] That is, the growth rate of productivity is greater when $I^0(t) > 0; I^1(t) > 0$ than when $I^0(t) > 0; I^1(t) = 0$, because the proportion of workers engaging in R&D is greater when $I^0(t) > 0; I^1(t) > 0$. Because $v^i(t) = y^i(t)L(t); i = 0; 1$ from (9), we obtain

$$v^{i**}(t) = \frac{\gamma^i \mu(\lambda - 1)L(t)}{\mu\rho - \mu n + \lambda n}: \tag{38}$$

From (22) and (38), we confirm $v^{0**}(t) < v^{0*}(t)$. From (23) and (38), we confirm $v^{1**}(t) < v^{1*}(t)$. That is, the

[20] Because $\lambda > 1$, $\gamma^0\lambda + 1 - \gamma^0 < \lambda$.

[21] From (21) and (37), we confirm that the growth rate of productivity is never influenced by the efficiency of R&D, A^i. This result is different from that in Boucekkine and de la Croix (2003), who state that the change in the efficiency of R&D affects the long-run growth rate.

equilibrium value of the incumbents' stock price in both sectors is greater when $I^0(t) > 0; I^1(t) = 0$ than when $I^0(t) > 0; I^1(t) > 0$. From (11), we obtain

$$V^{**}(t) = \hat{V}^{**}(t) = \frac{\mu(\lambda - 1)L(t)}{\mu\rho - \mu n + \lambda n}: \tag{39}$$

From (24) and (39), we obtain $V^{**}(t) < V^{*}(t)$. From (25) and (39), we obtain $\hat{V}^{*}(t) < \hat{V}^{**}(t)$.

2.4 Comparative Dynamics

In this section, we analyze the transition dynamics caused by an exogenous drastic technological breakthrough, which leads to the emergence of the new sector. The process of the emergence of a new technology is classified into three stages. Stage 1 is prior to the emergence of the new technology. In stage 1, there is only the old sector, sector 0 (i.e., $\gamma^0 = 1$). In this stage, households can only consume products of sector 0 and both manufacturing and R&D are carried out only in sector 0. When the economy is in stage 1, an unexpected major technological breakthrough happens and a new sector, sector 1, emerges, which we define as the point at which the economy enters stage 2. Stage 2 follows the emergence of a new technology, but the new technology is so new that R&D firms are not yet willing to carry out R&D in the new sector. In stage 2, $I^0(t) > 0$ and $I^1(t) = 0$ (as discussed in section 3.1). In this stage, households consume the products of both sectors 0 and 1 and manufacturing is carried out in both sectors; however, R&D is conducted only in sector 0. When the economy is in stage 2, unexpected major technological progress happens (cost of R&D falls drastically), which we define as the point at which the economy enters stage 3. By stage 3, the new technology have prevailed. In stage 3, $I^0(t) > 0$ and $I^1(t) > 0$ (as discussed in section 3.2). In this stage, households consume products of both sectors and manufacturing and R&D are carried out in both sectors. This section analyzes each stage in more detail.

2.5 Stage 1: Before the emergence of new technology

Before the emergence of a new technology, the economy is in a balanced-growth equilibrium. [22] The economy of stage 1 can be characterized by the present model with $\gamma^0 = 1$, which is identical with Segerstrom's (1998) model. In

[22]This assumption is based on the fact that major technological breakthroughs are rare.

stage 1, the main endogenous variables are $c(t) = \frac{\lambda(\mu\rho - \mu n + n)}{\mu\rho - \mu n + \lambda n}$, $y^0(t) = z(t) = \frac{\mu(\lambda - 1)}{\mu\rho - \mu n + \lambda n}$, $I^0(t) = \frac{n}{\mu}$, $g(t) = (\ln\lambda)\frac{n}{\mu}$, and $V(t) = \hat{V}(t) = \frac{\mu(\lambda-1)L(t)}{\mu\rho - \mu n + \lambda n}$, from (33), (35), (36), (37), and (39). This is point A in $(z(t); c(t))$ space and point A' in $(y^0(t); y^1(t))$ space of figure 3.[23] Point A is the intersection of the $\underline{z}(t) = 0$ curve: $\frac{(\lambda - c(t))\mu}{\lambda} - z(t)n = 0$ and the $\underline{c}(t) = 0$ curve: $\frac{1}{\lambda z(t)}[(\lambda - \mu)(c(t))^2 - (1-\mu)\lambda c(t)] - \rho c(t) = 0$.[24] Point A' is the intersection of the horizontal axis and the $z(t) = \frac{\mu(\lambda-1)}{(\mu\rho - \mu n + \lambda n)}$ line.

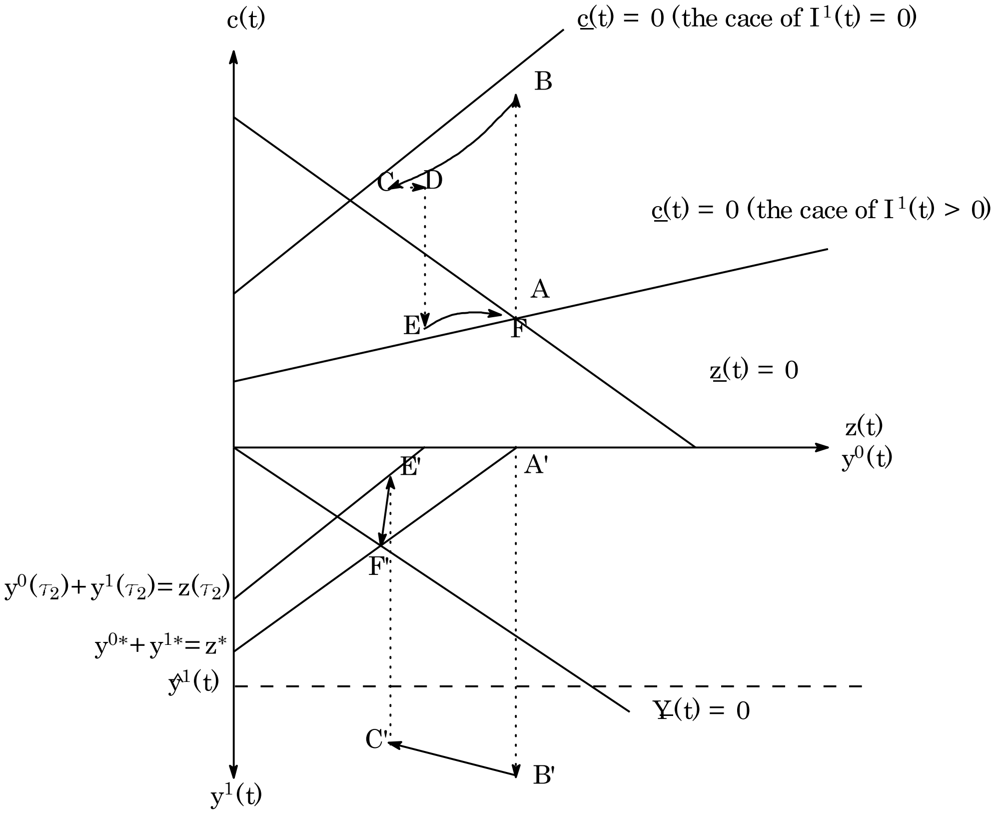

Figure 3. The movement of key variables in a phase diagram.

Figure 4 depicts the movements of the variables. The vertical axis represents the endogenous variables and the horizontal axis represents time, t. The panel on the left depicts the movements of key variables, the middle panel depicts the movements of variables related to the growth rate of productivity, and the panel on the right depicts the movements of variables related to the stock price.

[23] $y^1(t) = 0$ in stage 1 because sector 1 does not yet exist.
[24] Note that when $\gamma^0 = 1$, the $\underline{z}(t) = 0$ curve is identical to the $\underline{y}^0(t) = 0$ curve.

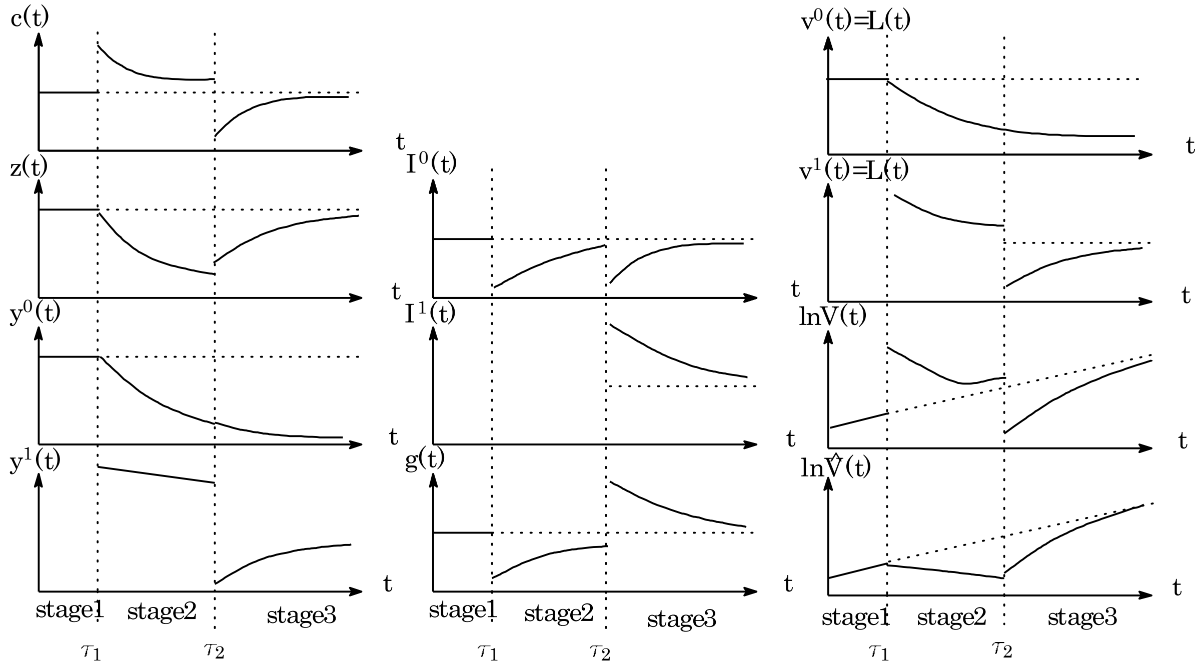

Figure 4. Summary of the movement of key and economically important variables.

2.6 Stage 2: Before the start of R&D in the new sector

Stage 2 covers the period between the time at which a new technology emerges, τ_1, and the beginning of R&D in the new sector. To guarantee the existence of this period, we assume that the cost of R&D in the new sector is too high for R&D firms for several periods after the new technology emerges, τ_2.[25] This assumption is based on the fact that the costs of innovative activity in technologies such as electricity and computing were very high for some time following the initial emergence of these new technologies, which discouraged people from using these new technologies; it is also based on the idea that workers engaging in R&D do not know enough about the new technology to be able to improve the quality of products based on the new technology for that interval of time after the breakthrough.[26] Here, this assumption is reflected in the high value of $H^1(t)$, which means a high value of adjusted difficulty of R&D in sector 1, $y^1(t)$. $\hat{y}^i(t)$ is defined as the critical value for the conduct of R&D sector

[25] If the cost of R&D is not too high, the economy goes directly from stage 1 to stage 3.

[26] Jovanovic and Rousseau (2005) and Greenwood and Yorukoglu (1997) provide evidence that the equipment prices of new technology decline substantially for that interval of time after the breakthrough. Boucekkine, Fernando and Licandro (2003) analyze the role of learning by doing for that interval.

i, for which $\hat{y}^i(t) = \frac{v^i(t)}{L(t)}$ from (9).[27][28] If $y^i(t) \leq \hat{y}^i(t)$, then R&D is conducted in sector i at time t; otherwise, R&D is not conducted in sector i. In stage 2, the inequality $y^1(t) > \hat{y}^1(t)$ is always satisfied because of the high value of $H^1(t)$. Stage 2 is where $I^0(t) > 0$ and $I^1(t) = 0$, as discussed in section 3.1. In stage 2, the $\dot{c}(t) = 0$ curve is expressed as (17), and the $\dot{z}(t) = 0$ curve is expressed as (16) with $z(t) = y^0(t)$.[29] Both the intercept and the slope of the $\dot{c}(t) = 0$ curve are greater in stage 2 than those in stage 1, whereas the $\dot{z}(t) = 0$ curve does not change between stages.

Figure 3 allows us to analyze the transition dynamics. At time τ_1, the economy is at point A in (z,c) space and at point A' in $(y^0; y^1)$ space. As seen in the upper part of figure 3, at time τ_1, the $\dot{c}(t) = 0$ curve shifts counterclockwise, and then the intersection of the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve shifts to the left. Because point A is below the $\dot{c}(t) = 0$ curve in stage 2, point A is no longer the equilibrium. To remain on the saddle path, $c(\tau_1)$ has to shift upward, and then the economy should shift from point A to point B. Because point B is below the $\dot{c}(t) = 0$ curve and above the $\dot{z}(t) = 0$ curve in stage 2, c(t) and z(t) gradually fall to the new equilibrium, which is the intersection of $c(t) = -\frac{\lambda n}{\mu} z(t) + \lambda$ and $c(t) = \frac{\rho\lambda y^0(t) + (1-\mu)\lambda}{(\gamma^0\lambda + \gamma^1 - \mu)}$.[30]

We turn now to the lower part of figure 3. When the new technology emerges, y^1 takes a value such that $y^1(\tau_1) > \hat{y}^1(\tau_1)$. The economy then shifts from point A' to point B'. Because $y^0(t) = z(t)$ throughout stage 2, $y^0(t)$ decreases together with z(t). Because $I^1(t) = 0$ in stage 2, $\dot{y}^1(t)/y^1(t) = -n$. Then $y^0(t)$ and $y^1(t)$ decrease along arrow B'C'.[31] These stage 2 movements can be seen in the left-hand panel of figure 4.

Lemma 2.1. Summary of movement of key variables in stage 2

- Consumption per capita, c, shifts upward at the emergence of the new technology, but decreases throughout stage 2 without ever falling below its level in stage 1.
- *The adjusted difficulty of R&D in sector 0,* $y^0 = z$, remains unchanged at the emergence of the new technology, but decreases throughout stage 2.

[27] The critical value $\hat{y}^i(t)$ depends on time t because leaders' profits, interest rate and economy size vary over time.

[28] If R&D is conducted in sector i, the adjusted difficulty of R&D in sector i is equal to the critical value because of the free entry condition.

[29] Note that $I^1(t) = 0$ in stage 2 and (28).

[30] Because $\dot{c}(t) < 0$ and the size of the decrement of c(t) decreases as it approaches the stage 2 equilibrium, $r(t) < \rho$ is always satisfied and r(t) is increasing throughout stage 2 from (4).

[31] In stage 2, $\frac{\dot{\hat{y}}^1(t)}{\hat{y}^1(t)} = \frac{\dot{v}^1(t)}{v^1(t)} - n$. $\frac{\dot{v}^1(t)}{v^1(t)} < 0$ is also satisfied, which is discussed later. Thus $y^1(t)$ is never less than $\hat{y}^1(t)$ in stage 2 as long as the cost of R&D, $H^1(t)$, remains constant.

In the above analysis, we confirm the movement of key variables in stage 2. Our next task is to analyze the movement of economically important variables such as the growth rate of productivity and stock price of old-sector incumbents in stage 2.

Because the equilibrium value of c in stage 2 is greater than that in stage 1 and c(t) is decreasing throughout stage 2, then consumption per capita in stage 2 is always strictly greater than that in stage 1. Therefore, the proportion of labor in R&D (manufacturing) in stage 2 is always strictly smaller (greater) than in stage 1.[32]

This result is related to the changes in the adjusted difficulty of R&D in sector 0, $y^0(t)$, in stage 2. Because $y^0(t)$ is decreasing and the size of the decrement of $y^0(t)$ decreases as it approaches equilibrium, $I^0(t) < \frac{n}{\mu}$ always holds in stage 2 and $I^0(t)$ approaches $I^{0*} = \frac{n}{\mu}$ from below throughout stage 2. Then, on the only feasible path, $I^0(t)$ shifts downward at the emergence of the new technology and increases throughout stage 2. That is, the instantaneous probability of successful R&D in sector 0 in stage 2 is always strictly less than in stage 1, but increases throughout stage 2. We interpret this result intuitively as follows: at time τ_1, the profits of leaders in sector 0 fall because sector 1 takes market share from sector 0, which means that the rewards for successful R&D in sector 0 decrease. The number of R&D firms trying to enter sector 0 decreases, which means a decrease in the amount of labor engaged in R&D in sector 0. The industry-wide instantaneous probability of successful R&D in sector 0 falls, which is a downshift of $I^0(t)$ at time τ_1. However a lower value of $I^0(t)$ leads to a decrease in the adjusted difficulty of R&D in sector 0, which renders innovative activity more profitable in sector 0.[33] The number of R&D firms trying to enter sector 0 increases throughout stage 2, which means that the industry-wide instantaneous probability of successful R&D in sector 0 increases throughout stage 2.[34] On the other hand, R&D is not conducted in sector 1 in stage 2 because of the high cost of R&D. Thus, from (15), we obtain the important result that the growth rate of productivity is strictly less throughout stage 2 than in stage 1. This result is consistent with the stylized fact known as "the productivity slowdown," as depicted in the middle panel of figure 4.

In this model, the productivity slowdown occurs because of the following. The first cause is the decrease in the industry-wide instantaneous probability of successful R&D in sector 0. Sector 0 leaders' profits decrease because

[32]Dividing (13) by L(t), we obtain $\frac{X^0(t)H^0(t)I^0(t)}{A^0L(t)} + \frac{X^1(t)H^1(t)I^1(t)}{A^1L(t)} + \frac{c(t)}{\lambda} = 1$. Thus, the proportion of labor in manufacturing and R&D is $\frac{c(t)}{\lambda}$ and $1 - \frac{c(t)}{\lambda}$ respectively.

[33]Although the accumulation of R&D in sector 0 continues while $I^0(t) > 0$, the population grows faster than R&D accumulates in stage 2. Thus, the adjusted difficulty of R&D in sector 0 decreases.

[34]Note that because $y^0(t)$ decreases throughout stage 2, $I^0(t)$ never exceeds $\frac{n}{\mu}$.

sector 1 takes market share from sector 0, which decreases the incentive to become the leader in sector 0. This decreases the amount of labor in R&D in sector 0, which in turn decreases the instantaneous probability of successful R&D in sector 0.[35] The second cause is the absence of R&D in sector 1, a consequence of the assumption that the cost of R&D is too high for several periods following the emergence of the new technology. From the evidence in Jovanovic and Rousseau (2005), this assumption seems plausible. However, it may be that the initial cost of R&D in the new sector is not high enough relative to its market share, which means that the economy goes directly from stage 1 to stage 3, with no stage 2. This possibility suggests that whether a technological breakthrough causes a productivity slowdown depends on the initial cost of R&D and the market share of the new sector.

Because $y^0(t)$ decreases over stage 2, $v^0(t)$ also decreases over stage 2 from (9). $v^1(t)$ also decreases over stage 2.[36] Thus, both $V(t)$ and $\hat{V}(t)$ decrease in stage 2 from (11) and (12). Because $y^0(t)$ is not changed by the emergence of a new technology, neither $v^0(t)$ nor $\hat{V}(t)$ is changed by that event. Assuming that sector 1 leaders' stocks are listed at the moment of the technology's emergence, $v^1(t)$ is evaluated at time τ_1. Then $V(t)$ shifts upward to the same degree as $v^1(\tau_1)$ at time τ_1. These movements in stage 2 are depicted in the right-hand panel of figure 4.

What happens to $v^0(t)$ in stage 2? At time τ_1, the new sector emerges, and the old sector loses market share to the new sector, which causes $\pi^0(t)$ to decline. From (10), $v^i(t)$ is an increasing function of $\pi(t)^i$ and a decreasing function of $I^i(t)$. Thus the stock price of incumbents in the old sector decreases unless the replacement risk, $I^0(t)$, changes. However, R&D activity, $I^0(t)$, also decreases to satisfy the free entry condition, then $v^0(t)$ does not shift downward at τ_1.[37] As seen above, $I^0(t) < \frac{n}{\mu}$ always holds in stage 2, in which case $\underline{y}(t)^0 < 0$ also holds.[38] This means that the adjusted difficulty of R&D in sector 0 decreases throughout stage 2, which leads to more R&D activity. Then $I^0(t)$ increases throughout stage 2. Because $\pi^0(t)$ remains small in stage 2, $v^0(t)$ decreases because of an increase in the replacement risk, $I^0(t)$. We note also that because $c(t)$ decreases and $r(t)$ increases throughout

[35] From (7) and (10), we see that $I^0(t)$ must decrease in order to satisfy (7) with equality.

[36] From (10) and $I^1(t) = 0$, we obtain $\frac{\underline{v}^1(t)}{v^1(t)} = r(t) - \gamma^1(\frac{\lambda-1}{\lambda})\frac{c(t)}{v^1(t)}L(t)$. This equation shows that $\underline{v}^1(t)$ increases with $v^1(t)$, $r(t)$ and decreases with $c(t)$. In stage 2, $r(t)$ increases and $c(t)$ decreases. Thus, $\frac{\underline{v}^1(t)}{v^1(t)}$ is increases throughout stage 2 as long as $v^1(t)$ does not decrease. Then, if $\frac{\underline{v}^1(t)}{v^1(t)} > 0$, $v^1(t)$ goes to infinity and never converges to the equilibrium. Therefore, $\frac{\underline{v}^1(t)}{v^1(t)} < 0$ must always hold in stage 2 if convergence is to occur. Therefore, the only feasible path of $v^1(t)$ is that $v^1(t)$ shifts upward at time τ_1 and decreases throughout stage 2.

[37] From (9), $v^i(t)$ is a function of the state variable, $X(t)^i$, and exogenous variables, $H^i(t)$ and A^i, when $I^i(t) > 0$. Thus $v^i(t)$ does not shift unless the exogenous variables change.

[38] Note that a differential equation of $y(t)^i$ is $\underline{y}^i(t)/y^i(t) = I^i(t)\mu - n$

stage 2, the decline in $v^0(t)$ and $v^1(t)$ will be exacerbated.[39] As shown in the last paragraph, both $V(t)$ and $\hat{V}(t)$ decrease throughout stage 2, which is consistent with the empirical data. This shows that the aggregate stock price decreases regardless of the timing of new firms' stock listing. Thus the decline of $v^0(t)$, which is caused by the decline of the market share of sector 0, plays an important role in the decline of the aggregate stock price.

Proposition 2.1. Summary of the movements of economically important variables in stage 2, in which R&D is not conducted in sector 1 after the emergence of a new technology.

- The growth rate of productivity, g, shifts downward at the moment of the emergence of the new technology, and increases throughout stage 2; however, it never goes over the level of stage 1.
- The stock price of incumbents in the old sector, v^0, decreases throughout stage 2.
- The sum of the stock price indexes, V and $\hat{V}$, decrease throughout stage 2.

2.7 Stage 3: After R&D in the new sector begins

Third, we analyze stage 3, which is the stage after R&D begins in the new sector. We refer to the moment when R&D firms in new sector begin to carry out R&D as τ_2. When the economy is in stage 2, unexpected drastic technological progress occurs (cost of R&D falls drastically), which forces the economy to move from stage 2 to stage 3. In our model, this phenomenon reflects an unexpected sufficient drop in $H^1(t)$, which causes an unexpected drop in $y^1(t)$ satisfying $y^1(t) \leq \hat{y}^1(t)$ and $y^1(t)/y^0(t) < \gamma^1/\gamma^0$. Although this assumption seems to be restrictive, we obtain similar results by extending this model to the model where the cost of R&D decreases due to learning by doing.[40] However, as this expansion is somewhat complex and the results do not change dramatically, for now we regard the cost of R&D as an exogenous variable.

Because of the unexpected exogenous shock, $y^1(t)$ does not become greater than $\hat{y}^1(t)$; hence, we express the economy of stage 3 as the case where $I^0(t) > 0$ and $I^1(t) > 0$, which is discussed in section 3.2. Then, in stage 3, the $\dot{c}(t) = 0$ curve is expressed as (30), and the $\dot{z}(t) = 0$ curve is expressed as (29). Both the intercept and the

[39] From equation (10), $v^i(t)$ is an increasing function of $c(t)$ and a decreasing function of $r(t)$.

[40] We define the differential equation of $H^1(t)$ as $\begin{matrix} \dot{H}^1(t) = -\nu^1 \frac{A^1}{X^1(t)} L^{p1} & \text{when} & H^1(t) > \bar{H}(t) \\ \dot{H}^1(t) = 0 & \text{when} & H^1(t) = \bar{H}(t) \end{matrix}$, where $\bar{H}(t)$ and ν^1 are parameters and L^{p1} is the amount of labor engaging in manufacturing in sector 1. Then, the cost of R&D, $H^1(t)$, becomes an endogenous variable and the regime switch from stage 2 to stage 3 is determined endogenously.

slope of $\dot{c}(t) = 0$ curve are smaller in stage 3 than those in stage 2, whereas the $\dot{z}(t) = 0$ curve is identical in both stages. On the other hand, the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve of stage 3 and those of stage 1 are completely identical.

We then analyze the economic movement by using figure 3 again. At time τ_2, we consider that the economy is at point C in (z,c) space and at point C' in $(y^0; y^1)$ space (i.e., the unexpected shock occurs when economy is at point C and point C'). First, we examine the upper part of figure 3. At time τ_2, $\dot{c}(t) = 0$ curve shifts clockwise, and the intersection of the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve shifts to the right. Because $I^0(t) > 0$ and $I^1(t) > 0$ by the drastic technological progress, z(t) should be evaluated as $z(t) = y^0(t) + y^1(t)$ (c.f., z(t) is evaluated as $z(t) = y^0(t)$ in stage 2). Then z(t) shifts to the right to the same degree as $y^1(\tau_2)$, and the economy shifts from point C to point D. Because point D is above the $\dot{c}(t) = 0$ curve and above the intersection of the $\dot{c}(t) = 0$ curve and the $\dot{z}(t) = 0$ curve of stage 3, then point D is no longer on the saddle path. To return to the saddle path, $c(\tau_2)$ has to shift downward, and then the economy should shift from point D to point E. Because point E is above the $\dot{c}(t) = 0$ curve and below the $\dot{z}(t) = 0$ curve, c(t) and z(t) increase gradually to the new equilibrium point F, which is the intersection of $c(t) = -\frac{\lambda n}{\mu} z(t) + \lambda$ and $c(t) = \frac{\rho\lambda z(t) + (1-\mu)\lambda}{\lambda-\mu}$.[41]

We next examine the lower part of figure 3. At the moment that the unexpected exogenous shock occurs, the adjusted difficulty of sector 1 drops dramatically, which satisfies $y^1(\tau_2) \leq \hat{y}^1(\tau_2)$ and $y^1(\tau_2)/y^0(\tau_2) < \gamma^1/\gamma^0$. The economy then shifts from point C' to point E'. In stage 3, the dynamics of y^0 and y^1 are determined by (31). Because we assumed that $y^1(t)/y^0(t) < \gamma^1/\gamma^0$, the ratio of adjusted difficulty of R&D of sector 1 to that of sector 0, Y(t), increases in stage 3. Note that from (18) and (35), $y^{0**} < y^{0*}$ and y^0 is a state variable; therefore, y^0 decreases in stage 3. Because z(t) increases and $y^0(t)$ decreases in stage 3, $y^1(t)$ must increase in stage 3. Then $y^0(t)$ and $y^1(t)$ move to F", which is on the $\dot{Y}(t) = 0$ curve satisfying $y^{0**} + y^{1**} = z^{**}$. These movements are depicted in stage 3 of the left-hand panel of figure 4.

Lemma 2.2. Summary of movement of key variables in stage 3

- Consumption per capita, c, shifts downward the moment R&D in the new sector begins, but increases throughout stage 3. However, it is never above the level in stage 1.

[41] Because $\dot{c}(t) > 0$ and the size of increment of c(t) decreases when approaching the equilibrium in stage 3, $r(t) > \rho$ is always satisfied and r(t) decreases throughout stage 3 from (4).

- *The sum of the adjusted difficulty of R&D across sectors, z, shifts upward the moment R&D in the new sector* begins, and increases throughout stage 3. However, it is never above the level in stage 1.
- *The adjusted difficulty of R&D in sector 0,* y^0, remains unchanged the moment R&D in the new sector begins, but decreases throughout stage 3.
- *The adjusted difficulty of R&D in sector 1,* y^1, shifts downward the moment R&D in the new sector begins, but increases throughout stage 3.

In the above analysis, we confirm the movement of key variables in stage 3. Next, we analyze the movement of economically important variables such as the growth rate of productivity, the value of old-sector leaders' stock and the sum of stock prices in stage 3.

Because the equilibrium value of consumption per capita in stage 3 is smaller than that in stage 2 and c(t) increases throughout stage 3, consumption per capita in stage 3 is always strictly smaller than that in stage 2. Therefore, the share of labor engaging in R&D (manufacturing) in stage 3 is always strictly greater (smaller) than in stage 2 from (13). Although the total share of labor engaging in R&D is greater than that in stage 2, because $\underline{y}^0(t) < 0$, the industry-wide instantaneous probability of successful R&D in sector 0 is always less than the equilibrium value, $I^{0**} = n/\mu$, but increases throughout stage 3 to $I^{0**} = n/\mu$. Although the impact of $I^0(t)$ at time τ_2 is ambiguous, if stage 2 is long enough (i.e., $I^{0*} \simeq n/\mu$), then $I^0(\tau_2)$ shifts downward at the moment that R&D begins in sector 1. On the other hand, because $\underline{y}^1(t) > 0$ and the size of the increment to $y^1(t) > 0$ decreases throughout stage 3, the industry-wide instantaneous probability of successful R&D in sector 1 is always greater than the equilibrium value, $I^{1**} = n/\mu$, and decreases throughout stage 3 to $I^{1**} = n/\mu$. Then the only feasible path, $I^1(t)$, shifts upward precisely at τ_2 and decreases throughout stage 3. Therefore, $I^0(t) \leq I^1(t)$ is always satisfied throughout stage 3, which is consistent with $\underline{Y}(t) > 0$ throughout stage 3. Because $\underline{z}(t) > 0$, $\underline{Y}(t) > 0$ and $I^0(t) < I^1(t)$ throughout stage 3, $g(t) = \gamma^0 I^0(t) + \gamma^1 I^1(t) > n/\mu$ throughout stage 3 as depicted in the middle panel of figure 4 (see Appendix). This result is consistent with the empirical fact that the growth rate of productivity returns to a high level after a lapse of some years from the emergence of new technology.

This result has a simple explanation. When the economy arrives at stage 3, the cost of R&D in sector 1 falls dramatically. R&D firms in the new sector can then carry out R&D with a high probability of success (i.e., a low value of $X^1(t)$) and with a low cost of R&D, (i.e. a low value of $H^1(t)$), which induces a large volume of R&D activity by R&D firms in sector 1. Thus, the number of workers engaging in R&D activity in the new sector increases, which leads to a high industry-wide instantaneous probability of successful R&D in the new sector. By contrast, the number of workers engaging in R&D in sector 0 decreases because R&D firms in sector 1 take workers from R&D firms in sector 0, which reduces the industry-wide instantaneous probability of successful R&D in sector 0. However, the analysis in our appendix shows that the former effect dominates latter, and thus the growth rate of productivity is high throughout stage 3.

Because $y^0(t)$ ($y^1(t)$) decreases (increases) in stage 3, $v^0(t)$ ($v^1(t)$) also decreases (increases) in stage 3 from (7). Furthermore, because $z(t)$ increases in stage 3, the stock price indexes, $V(t)$ and $\hat{V}(t)$, also increase in stage 3 from (11), (12) and (28). Because $y^0(t)$ does not change at τ_2, $v^0(t)$ does not change at that point either. $v^1(t)$ in stage 2 is always higher than in stage 3 because new-sector leaders face no threat of replacement. In stage 3, these leaders face replacement because R&D commences in sector 1. Thus, the stock price of sector 1 leaders will fall as soon as R&D begins in sector 1. Therefore, $V(t)$ shifts downward at time τ_2. Consider instead the index $\hat{V}(t)$; because sector 1 leaders' stocks are listed at R&D in sector 1 begins (i.e., $v^1(t)$ is evaluated at time τ_2), then $\hat{V}(t)$ shifts upward to the same degree as $v^1(\tau_2)$ at time τ_2. These movements are shown in the right-hand panel of figure 4.

The reason $v^1(t)$ remains small and never returns to its stage 1 level is very simple: it is that $\pi^1(t)$ also remains small and never returns to its stage 1 level because of the emergence of the new sector. Then the cause of the increase in $V(t)$ and $\hat{V}(t)$ throughout stage 3 is the increase in $v^1(t)$ in stage 3. This comes mainly from the decrease in $I^1(t)$ throughout stage 3. As the new sector matures, that is, as product quality in the new sector improves, subsequent new-sector R&D becomes more difficult. This reduces the probability of successful R&D in the new sector (decreases the risk of new-sector leaders being replaced), which increases $v^1(t)$ throughout stage 3. In addition, the value of the increment of $v^1(t)$ relies in part on the results that $c(t)$ increases and $r(t)$ decreases throughout stage 3. Because the effect of this increment dominates the effect of the decline in $v^0(t)$, $V(t)$ and $\hat{V}(t)$ increase throughout stage 3.

Proposition 2.2. Summary of movements of economically important variables in stage 3, where R&D is conducted in sector 1

- The growth rate of productivity, g, shifts upward when R&D in sector 1 begins, and decreases throughout stage 3. However, it never falls below the level in stage 1.
- The stock price of old-sector incumbents, v^1, remains small and never returns to the level in stage 1.
- The sum of the stock prices, V and $\hat{V}$, increases throughout stage 3.

2.8 Conclusion

Using a typical R&D growth model, we examined the transition dynamics of a technological breakthrough. We showed that a technological breakthrough causes a productivity slowdown and a fall in the stock prices of existing firms. Both these results are consistent with events of the 1970s. Our model provides an explanation for these effects of technological breakthroughs and shows that productivity slowdown occurs if and only if the initial cost of R&D activity is high enough.

Our model is enough simple that there is room to expand more complex but realistic model. For example, one can expand utility function having any elasticity of substitution between sectors. In present model, market share is exogenous and constant throughout the transition, and it may make someone wonder that all firms in new sector emerge simultaneously and instantaneously. This problem is caused by our utility function having a unit elasticity of substitution between sectors. Expanding a utility function having any constant elasticity of substitution between sectors, we can confirm that the market share of a new sector is small initially and grows throughout the transition. Another example is to make cost of R&D endogenous variable. This extension enable us to confirm that regime switch from stage 2 to stage 3 is determined endogenously. These are important direction for future work.

2.9 Appendix

2.9.1 A1

We show that the growth rate of productivity in stage 3 is greater than the equilibrium value. Because $\underline{z}(t) > 0$ in stage 3, and note that $\underline{y}^i(t)/y^i(t) = I^i(t)\mu - n$, the following holds:

$$
\begin{aligned}
& \underline{z}(t) > 0 \\
, \quad & I^0(t)y^0(t)\mu - ny^0(t) + I^1(t)y^1(t)\mu - ny^1(t) > 0 \\
, \quad & I^0(t)\frac{y^0(t)}{y^0(t)+y^1(t)} + I^1(t)\frac{y^1(t)}{y^0(t)+y^1(t)} > \frac{n}{\mu} \\
, \quad & I^0(t)\frac{1}{1+Y(t)} + I^1(t)\frac{Y(t)}{1+Y(t)} > \frac{n}{\mu}:
\end{aligned}
\tag{40}
$$

Because $I^0(t) < I^1(t)$, then $I^0(t)\frac{1}{1+Y(t)} + I^1(t)\frac{Y(t)}{1+Y(t)}$ is an increasing function of $Y(t)$. From $\underline{Y}(t) > 0$, we obtain $Y(t) < \frac{\gamma^1}{\gamma^0}$. Then the following holds:

$$
\begin{aligned}
& I^0(t)\frac{1}{1+Y(t)} + I^1(t)\frac{Y(t)}{1+Y(t)} \\
< \; & I^0(t)\frac{1}{1+\frac{\gamma^1}{\gamma^0}} + I^1(t)\frac{\frac{\gamma^1}{\gamma^0}}{1+\frac{\gamma^1}{\gamma^0}} \\
= \; & I^0(t)\frac{1}{\frac{\gamma^0+\gamma^1}{\gamma^0}} + I^1(t)\frac{\frac{\gamma^1}{\gamma^0}}{\frac{\gamma^0+\gamma^1}{\gamma^0}} \\
= \; & I^0(t)\gamma^0 + I^1(t)\gamma^1:
\end{aligned}
\tag{41}
$$

From (40) and (41), we obtain $g(t) = \gamma^0 I^0(t) + \gamma^1 I^1(t) > n/\mu$

3 Innovation by Heterogeneous Leaders

3.1 Introduction

In developed economies, total factor productivity (hereafter, TFP) growth from research and development (hereafter, R&D) activities is the most important source of sustained economic growth. Since the seminal contributions by Grossman and Helpman (1991) and Aghion and Howitt (1992), many studies have developed R&D-based growth models and examined how aggregate R&D activities and the growth rate of output are determined. However, most of the earlier studies have two features that are not consistent with the evidence. First, in most of the earlier studies, innovation is conducted by new entrants or follower firms, not by the leader that has the state-of-the-art technology in each industry. In reality, in many industries, the leader firm conducts R&D activities and contributes to innovation. For instance, Bartelsman and Doms (2000) report that 75 percent of TFP growth results from R&D activities by the incumbent firms. Second, in most of the earlier studies, profits from innovation are symmetric among firms. However, in reality, the profits from innovation differ among firms. Indeed, commencing with Scherer (1965), many empirical studies have examined the distribution of profits among firms.

In order to construct an R&D-based growth model that is consistent with this evidence, we assume that (i) R&D technology of leader firms exhibits decreasing returns and (ii) the size of the quality increment is determined by a random draw from a given distribution. As a result, we develop a Schumpeterian growth model where leader firms have different profits and some of them conduct R&D activities. Examining this heterogeneous-leaders model, first, we find the following interesting tendency in the leader firms' behaviors. The leader that has higher quality compared with its followers and earns larger profits makes smaller R&D investments. The reason is as follows. The leader loses the value of their current position when they succeed in the next innovation. Therefore, the net benefit of R&D for the leader that has a larger quality lead over their followers is higher, and thus a higher-quality leader has lower R&D expenditure. Consequently, a higher-quality leader is more likely to be leapfrogged by the other firms. This theoretical result is consistent with actual firm behavior. Sony succeeded in developing the high-quality flat cathode-ray tube TV and did not invest in the development of a liquid crystal TV. Consequently, Sony did not obtain a large share of the market for liquid crystal TVs. Eastman{ Kodak, which was the earliest developer of film cameras, went bankrupt because of the diffusion of digital cameras. Second, we examine the effects of subsidizing followers' R&D and obtain a counter-intuitive result. R&D subsidies for followers promote R&D by followers naturally. However, this promotion of followers' R&D can also promote aggregate R&D by leaders. The reason is as follows. The promotion of followers' R&D impedes R&D by leaders. However, the promotion of followers' R&D also shifts the distribution of leaders; it reduces the number of leaders with larger leads and smaller R&D investment and increases that of leaders with smaller leads and larger R&D investment. If this positive effect associated with the changing distribution outweighs the negative effects on an individual firm's R&D, the promotion of followers' R&D increases leaders' aggregate R&D. This result implies that R&D subsidies for new entry firms are effective in raising aggregate productivity growth, and provides a rationale for actual promotion policies for R&D activities by new entry firms.[42]

Some earlier studies including Thompson and Waldo (1994), Segerstrom and Zornierek (1999), Etro (2004), and Segerstrom (2007) developed R&D-based growth models where the leader firms conduct R&D activities and reexamined how aggregate R&D is determined. However, in the models, there is no factor that brings about heterogeneity among the leaders, and thus the leader firms are symmetric among industries. In contrast, Minniti, Parello, and Segerstrom (2013) incorporate uncertainty of innovation size (quality increment) into Segerstrom (1998) and construct a Schumpeterian growth model where the innovation size and profits are different among industry leaders. However, this study assumes that R&D technology has constant returns, and thus the leader firms do not conduct R&D activities in equilibrium. Therefore, constructing a realistic model where the R&D activities of leaders are different is one of the contributions of the present study.

The rest of the paper is structured as follows. Section 2 develops a heterogeneous-leaders Schumpeterian growth model and derives the equilibrium of the model. Section 3 conducts comparative statics. Section 4 concludes.

3.2 The Model

As stated in the Introduction, we extend the quality-ladder model of Grossman and Helpman (1991) so that (i) the R&D technology of the leader firms exhibits decreasing returns and (ii) innovation size, namely the quality increment, is stochastic. By doing so, we develop a Schumpeterian growth model where the leader firms have

[42] For instance, one of the innovation-promoting policies that the European Commission decided to implement in "Innovation Union" was to promote entry and R&D activities by small and medium-sized enterprises (European Commission, 2010).

different quality increments and profits, and further conduct different volumes of R&D activities.

3.2.1 Consumers and workers

The economy has a fixed measure L of consumers. Intertemporal utility is given by:

$$U = \int_0^{\infty} e^{-\rho t} \left[\log u_t + (\bar{\ell} - \ell_t)\right] dt,$$

where ρ is a subjective discount rate, ℓ_t denotes supplied labor at time t, $\bar{\ell}$ denotes the maximum labor supply, and u_t represents utility from consumption at time t.[43] We specify utility per capita from consumption as follows:

$$\log u_t = \int_0^1 \log \sum_j q_t(j,\omega) d_t(j,\omega) d\omega, \tag{42}$$

where $q_t(j,\omega)$ and $d_t(j,\omega)$ denote the jth-highest quality in industry ω and the consumption volume of the good, respectively.

Solving the utility maximization problem of the household, we now derive the demand for each differentiated good $\omega \in [0,1]$. First, in each differentiated goods industry, households consume only the good with the lowest quality-adjusted price, which is provided by the firm having the highest quality in equilibrium. Letting $p_t(\omega)$ denote the price of the highest quality in industry ω, the demand for the good with the highest quality in industry ω is given by:

$$d_t(\omega) = \frac{E_t}{p_t(\omega)}, \tag{43}$$

where E_t denotes per capita consumption. Substituting the demand into the utility, we can derive the indirect utility as:

$$\log u_t = \log E_t - \left\{ \int_0^1 \ln \left[\frac{p_t(\omega)}{q_t(\omega)} \right] dj \right\}.$$

[43]Labor supply is assumed to be perfectly elastic as in Aghion, Harris, and Vickers (1997). As a result, utility is quasilinear.

Second, we must solve the dynamic optimization problem as follows:

$$\max \int_0^\infty e^{-\rho t} \left[\log E_t - \left\{ \int_0^1 \ln \left[\frac{p_t(\omega)}{q_t(\omega)} \right] d\omega \right\} + (\bar{\ell} - \ell_t) \right] dt;$$

subject to

$$\dot{a}_t = r_t a_t + w_t \ell_t - E_t;$$

where a_t denotes the asset holding per capita and w_t denotes the wage rate. We take the wage rate as a numeraire, therefore $w_t = 1$. As long as $\ell_t < \bar{\ell}$, $1 = \frac{w_t}{E_t}$ must hold. This optimality condition can be reduced to $E_t = 1$, and thus from the Euler equation, $\frac{\dot{E}_t}{E_t} = r_t - \rho$, we get $r_t = \rho$; that is, the interest rate is constant over time. Hereafter, we let r denote the constant interest rate.

3.2.2 Firms

In each industry, there are different qualities that consumers can buy. However, as mentioned before, consumers buy only the good with the lowest quality-adjusted price. Hereafter, we refer to the firm producing the highest quality in each industry as the leader and refer to the other firms conducting R&D activities so as to leapfrog the leader as followers. If a firm succeeds in achieving quality improvement, the quality increases by λ, which we assume is determined by a random draw from a given distribution. If the highest quality in industry ω is $q(\omega)$ before innovation, the quality of the new leader is given by $\lambda q(\omega)$ while the quality of its followers is $q_{\omega,t}$. In each industry, the leader charges a price so that the followers cannot earn a positive profit. Therefore, the leader charges a price equal to the quality lead λ. Thus the profit per consumer is given by $\pi = (p_t(\omega) - 1)d_t(\omega) = 1 - \frac{1}{\lambda}$.

Originally, we identify leaders with quality increments λ that are obtained by their innovations. However, for analytical simplicity, we assume that profit per consumer π is distributed among $[0, \bar{\pi}]$, and hereafter we identify leaders with profit sizes of $\pi \in [0, \bar{\pi}]$ and refer to the leader with profit π as π-leader.

We let $V(\pi)$ and $I(\pi)$ denote the value and R&D intensity of π-leader respectively. The R&D technology of leaders is assumed to exhibit diminishing returns. By devoting $\frac{C}{2} I(\pi)^2 dt$ units of labor in infinitesimal time interval dt, the leaders will succeed in the invention of new quality with probability $I(\pi)dt$. Therefore, the Hamilton–Jacobi–

Bellman equation (hereafter, HJB equation) of the π-leader is given by:

$$\mathrm{rV}(\pi) = \max_{\mathrm{I}(\pi)} \left\{ \pi L - \frac{\mathrm{C}}{2} [\mathrm{I}(\pi)]^2 + \mathrm{I}(\pi) \left[\mathrm{EV} - \mathrm{V}(\pi)\right] - \mathrm{I_F}\,\mathrm{V}(\pi) \right\}; \tag{44}$$

where EV is the expected value of the firm that succeeds in innovating and $\mathrm{I_F}$ is the probability of innovation by followers. Letting $\mathrm{g}(\pi)$ denote the distribution of π, we can express EV as follows:

$$\mathrm{EV} \equiv \mathrm{E}[\mathrm{V}(\pi)] = \int_0^{\bar{\pi}} \mathrm{V}(\pi)\mathrm{g}(\pi) d\pi.$$

From (44), the first-order condition for leaders is:

$$\mathrm{EV} - \mathrm{V}(\pi) = \mathrm{CI}(\pi): \tag{45}$$

if $\mathrm{EV} \geq \mathrm{V}(\pi)$, otherwise $\mathrm{I}(\pi) = 0$.

The firms other than the leaders, that is, the followers, also conduct R&D activities. Next, we consider the behaviors of the followers. We let $\mathrm{V_F}$ denote the value of a follower. The HJB equation of the follower is: $\mathrm{rV_F} = \max_{\mathrm{I_F}} \{0 - (1 - \mathrm{s_F})\bar{\mathrm{C}}_\mathrm{F}\mathrm{I_F} + \mathrm{I_F}\,[\mathrm{EV} - 0]\}$, where $\bar{\mathrm{C}}_\mathrm{F}$ denotes the R&D cost for followers and $\mathrm{s_F}$ denotes the R&D subsidy rate for followers. The R&D equilibrium condition for followers is:

$$\mathrm{EV} = (1 - \mathrm{s_F})\bar{\mathrm{C}}_\mathrm{F} \equiv \mathrm{C_F} \tag{46}$$

and thus we get $\mathrm{V_F} = 0$.

3.3 Equilibrium

We derive the equilibrium in the present model and effects of subsidies for followers' R&D on π-leader's R&D, followers' R&D, and aggregate R&D by leaders, and the sum of leaders' R&D and followers' R&D.

3.3.1 R&D by leaders

First, we derive the equilibrium R&D of π-leader $I(\pi)$ given I_F.

From the first-order condition of R&D for leaders (45) and the equilibrium condition of R&D for followers (46), we get:

$$CI(\pi) = \begin{cases} C_F - V(\pi) & \text{if} \quad 0 \leq V(\pi) \leq C_F; \\ \\ 0 & \text{if} \quad V(\pi) > C_F: \end{cases} \tag{47}$$

For the leaders with a stock value that satisfies $0 \leq V(\pi) \leq C_F$, substituting the first-order condition into the HJB equation (44) yields the quadratic equation with respect to $V(\pi)$ as follows:

$$V(\pi) = \frac{\pi L + \frac{1}{2C}\left[C_F - V(\pi)\right]^2}{r + I_F}: \tag{48}$$

For the leaders with a stock value that satisfies $V(\pi) > C_F$, they conduct no R&D activity and the stock value consists of only profit flow as follows:

$$V(\pi) = \frac{\pi L}{r + I_F}:$$

For the leader with the stock value that satisfies $V(\pi) = C_F$,

$$C_F = \frac{\pi L}{r + I_F}:$$

We define the critical level of profit in terms of whether the leader conducts R&D activities or not as π^*, and π^* is given by:

$$\pi^* = (r + I_F)\frac{C_F}{L}: \tag{49}$$

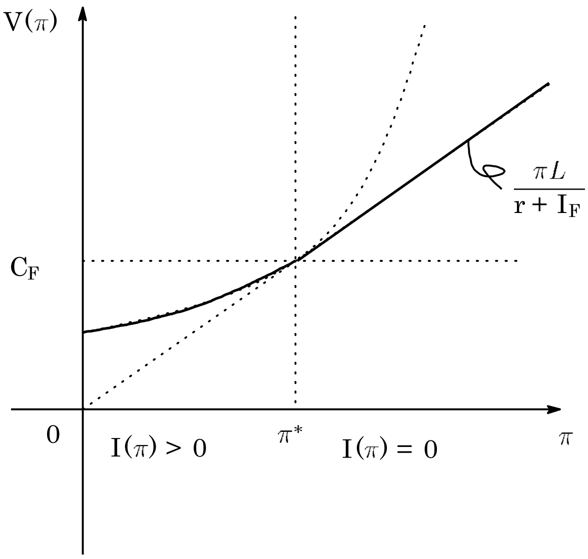

Figure 1: The value of π-leader

As a result, if the profit flow π exceeds the critical level, the leader with profit flow π conducts no R&D activity, and the stock value is given by $\pi L/(r+I_F)$, which satisfies $V(\pi) > C_F$. If the profit flow π is lower than the critical level, the leader conducts R&D activities, and the stock value is given by the smaller solution of the quadratic equation (48), which satisfies $V(\pi) < C_F$, as shown in Fig. 1. The stock value is given by:

$$V(\pi) = \begin{cases} C_F + (r + I_F)C - C\sqrt{(2\frac{C_F}{C} + r + I_F)(r + I_F) - 2\pi\frac{L}{C}} & \text{if} \quad 0 \leq \pi \leq \pi^*; \\ \\ \frac{\pi L}{r + I_F} & \text{if} \quad \pi^* < \pi \leq \bar{\pi}. \end{cases} \tag{50}$$

Substituting the stock value into the optimality condition for R&D (47), given I_F, we obtain π-leader's R&D as follows:

$$I(\pi) = \begin{cases} \sqrt{(2\frac{C_F}{C} + r + I_F)(r + I_F) - 2\frac{L}{C}\pi} - (r + I_F); & \text{if} \quad 0 \leq \pi \leq \pi^*; \\ \\ 0 & \text{if} \quad \pi^* < \pi \leq \bar{\pi}. \end{cases} \tag{51}$$

(51) shows that the innovation intensity of the leader is a decreasing function of π. Therefore, we can summarize the result in the following proposition:

Proposition 3.1. *The leader that has higher quality compared with followers and earns higher profit makes a* smaller R&D investment.

This result implies that leaders that obtain larger leads over follower firms have lower R&D expenditure. This theoretical result is consistent with some actual firms' behaviors. For instance, Sony succeeded in developing the high-quality flat cathode-ray tube TV and did not invest in the development of a liquid crystal TV. Consequently, Sony did not obtain a large share of the market for liquid crystal TVs.

In addition, some empirical studies show that R&D investment by smaller firms accounts for a large proportion of important innovations. [44] This paper's result is consistent with this tendency.

3.3.2 R&D by followers

Next, we derive the equilibrium R&D of followers I_F.

In the rest of the analysis, we assume that the distribution of profit is a uniform distribution over $[0; \bar{\pi}]$ for analytical simplicity. Hereafter, we use $R \equiv r + I_F$ instead of I_F, and we use $\tilde{C}_F \equiv C_F{=}C$ and $A \equiv L{=}C$ for notational simplicity. We can calculate the expected value of $V(\pi)$ as follows:

$$
\begin{aligned}
E[V(\pi)] &= \int_0^{\pi^*} V(\pi)g(\pi)d\pi + \int_{\pi^*}^{\bar{\pi}} V(\pi)g(\pi)d\pi \\
&= \frac{C}{A\bar{\pi}}\left((\tilde{C}_F + R)\tilde{C}_F R + \frac{1}{3}\left\{R^3 - \left[(2\tilde{C}_F + R)R\right]^{\frac{3}{2}}\right\}\right) + \frac{C}{2A\bar{\pi}}\left(\frac{(A\bar{\pi})^2}{R} - \tilde{C}_F^2 R\right):
\end{aligned}
$$

We can rewrite this as follows:

$$
E[V(\pi)] = \frac{C}{A\bar{\pi}}G(R)
$$

[44] According to Scherer (1984), companies with fewer than 1,000 employees were responsible for 47.3 percent of important innovations, while large companies with over 10,000 employees were responsible for only 34.5 percent of important innovations.

where

$$G(R; c_F) \equiv \left((C_F + R) C_F R + \frac{1}{3} \left\{ R^3 - \left[(2C_F + R) R \right]^{\frac{3}{2}} \right\} \right) + \frac{1}{2} \left(\frac{(A\pi)^2}{R} - C_F^2 R \right):$$

This expected value must satisfy the R&D equilibrium condition for followers (46), and thus we get:

$$G(R; C_F) = A\pi \; C_F: \tag{52}$$

This equation determines the equilibrium value of R.

As shown in Appendix 3.5.1, we can show that $\frac{dR}{dC_F} < 0$. Thus, we can summarize the result as follows:

Proposition 3.2. A decrease in R&D costs for followers from, for example, subsidies promotes followers' R&D.

3.3.3 Effects of R&D subsidies for followers on leaders' R&D

We have shown that subsidies for followers' R&D promote followers' R&D. Now we examine the effects of subsidies for followers' R&D on leaders' R&D.

Differentiating (51), we get:

$$\frac{dI(\pi)}{dC_F} = \frac{1}{I(\pi) + R} \left\{ \left[C_F - I(\pi) \right] \frac{dR}{dC_F} + R \right\}: \tag{53}$$

As shown in Appendix 3.5.2, if the elasticity of followers' R&D with respect to R&D costs is sufficiently high to satisfy the condition

$$\text{Condition (R)} \qquad - \frac{C_F}{R} \frac{dR}{dC_F} > \frac{C_F}{R + C_F};$$

the effect of an increase in C_F on $I(\pi)$ is an increasing function of $I(\pi)$, as depicted in Fig. 2. We can summarize the result as follows:

Proposition 3.3. *If the elasticity of followers' R&D with respect to R&D subsidies is sufficiently high to satisfy Condition (R), the effect of subsidies for followers' R&D on* π-leader's R&D decreases as π increases.

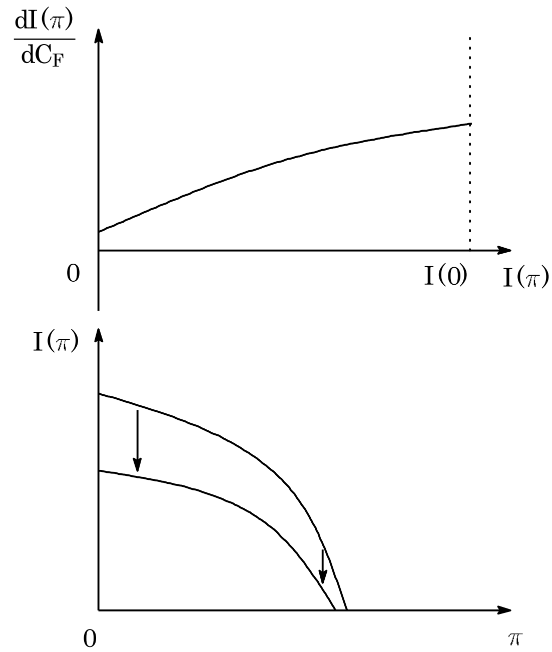

Figure 2: The effects of subsidies for followers' R&D on leaders' R&D

Next, we examine the effect of an increase in $\tilde{C}_F$ on the R&D of the π^*-leader, the leader with the minimum profit who conducts no R&D activity. When $I(\pi^*) = 0$, we can rewrite (53) as follows:

$$\frac{dI(\pi^*)}{d\tilde{C}_F} = \frac{\tilde{C}_F}{R}\frac{dR}{dC_F} + 1: \tag{54}$$

Therefore, if the elasticity of followers' R&D is sufficiently high to satisfy the following condition,

$$\text{Condition (R')} \qquad -\frac{\tilde{C}_F}{R}\frac{dR}{d\tilde{C}_F} > 1;$$

R&D subsidies for followers promote R&D by the leader who conducts no R&D investment. On the assumption that profit is uniformly distributed, this case does not hold. However, we can show that this case holds if we generalize the distribution of profit.

3.3.4 Effects on aggregate R&D by leaders

In this section, we examine how aggregate R&D by leaders is determined and examine the effects of subsidies for followers' R&D on aggregate R&D by leaders.

We let $\mu_t(\pi)$ denote the distribution of π-leaders, and thus we can express the aggregate innovation of leaders as follows:

$$I_{L;t} = \int_0^{\bar{\pi}} I(\pi)\mu_t(\pi)d\pi. \tag{55}$$

We derive the stationary distribution of π-leaders. The inflow of π-leaders in time interval dt is a proportion $g(\pi)$ of the number of leaders and followers that succeed in innovation at the interval, that is, $(I_{L;t}dt + I_F dt)g(\pi)$. The outflow of π-leaders is the sum of the number of π-leaders that succeed in innovation in the interval and that of followers that target the industries of present π-leaders and succeed in innovation in the interval, that is, $[I(\pi)dt + I_F dt]\mu_t(\pi)$. Therefore, the change in the distribution of π-leaders is given by:

$$\frac{\partial \mu_t(\pi)}{@t} = (I_{L;t} + I_F)g(\pi) - [I(\pi) + I_F]\mu_t(\pi): \tag{56}$$

We focus our analysis on the stationary distribution for simplicity. Then, from (56), we obtain the stationary distribution of the leaders as follows:

$$\mu(\pi) = \frac{I_L + I_F}{I(\pi) + I_F}\frac{1}{\bar{\pi}}: \tag{57}$$

Substituting (57) into (55) yields the equation that determines the equilibrium leaders' aggregate R&D I_L as follows:

$$I_L = \int_0^{\bar{\pi}} \frac{I_L + I_F}{I(\pi) + I_F}\frac{1}{\bar{\pi}} I(\pi)d\pi. \tag{58}$$

Now, we examine effects of subsidies for followers' R&D on aggregate R&D by leaders I_L. As mentioned above,

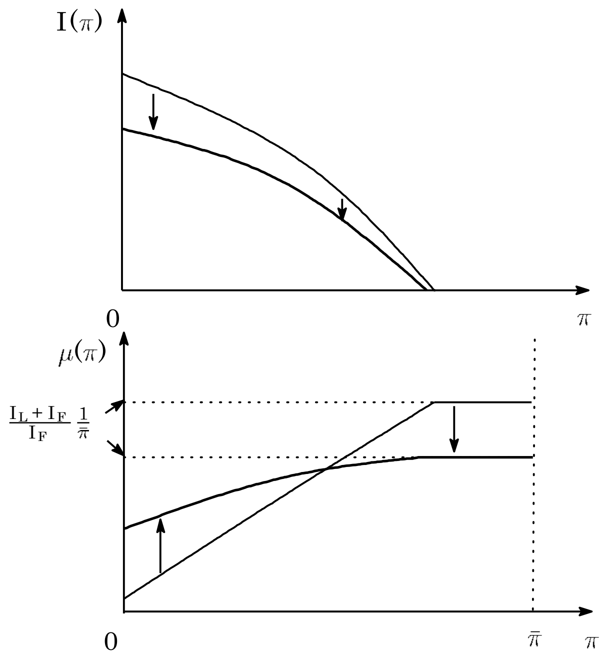

Figure 3: Change in the stationary distribution of leaders

if the elasticity of followers' R&D is lower, subsidies for followers' R&D tend to decrease leaders' R&D irrespective of the profit level of the leader. Regardless, subsidies for followers' R&D may increase aggregate R&D by leaders. Subsidies for followers' R&D affect the aggregate R&D by leaders through the following two channels. First, subsidies for followers' R&D affect the R&D level of each leader. We refer to this effect as the individual effect. Through this effect, subsidies necessarily reduce the aggregate R&D by leaders, as depicted in the upper panel of Fig. 3. Second, they change the stationary distribution of π-leader. Subsidies for followers' R&D promote followers' R&D I_F. From the stationary distribution (57), an increase in I_F increases (decreases) the distribution of the π-leader that conducts higher (lower) levels of R&D than the average level of R&D by leaders, that is, $I(\pi) > (<) I_L$, as depicted in the lower panel of Fig. 3. From Proposition 3.1, leaders with lower profits conduct higher levels of R&D. Therefore, through the change in the distribution, followers' R&D increased by subsidies raises the aggregate R&D by leaders. We refer to this effect as the distribution effect. If the distribution effect overwhelms the individual effect, subsidies for followers' R&D raise the aggregate R&D by leaders.

By providing numerical examples, we show that subsidies for followers' R&D raise the aggregate R&D by leaders under certain parameter values. In the numerical example, we use the following parameters: $\rho = 0{:}07; \bar{\pi} = 0{:}4; A(\equiv$

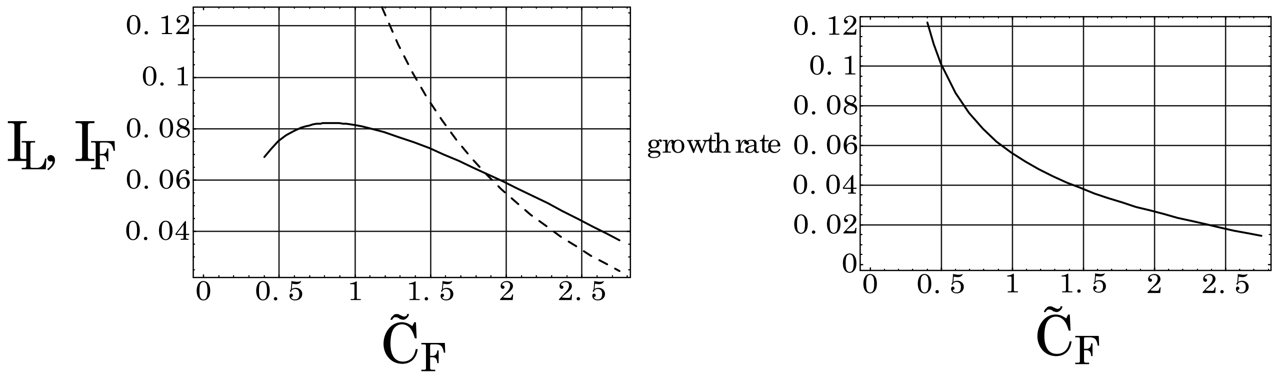

Figure 4: Aggregate R&D by leaders, followers' R&D, and growth rate of u_t
The solid line represents the aggregate R&D by leaders and the broken line represents followers' R&D I_F.

$L/C) = 1$, and $\tilde{C}_F \in [0.4, 2.75]$. First, Mehra and Prescott (1985) estimated the average real return on the stock market for the past century as 7%, and thus we set the interest rate at this level. In this model, $r_t = \rho$, and we set the subjective discount rate at 0.07. Second, empirical studies such as Norrbin (1993) and Basu (1996) estimate the range of markup of price over marginal cost as [1.05, 1.4]. Thus we set the upper value of the profit $\bar{\pi}$ at 0.4 so that the expected profit equals 0.2, which is obtained when the markup is 1.25, that is, in the middle of the estimated range. Finally, we choose the value of A and the range of $\tilde{C}_F$ so that the range of the growth rate of the utility is realistic as the range of the GDP per capita growth rate.[45]

As shown in the left panel of Fig. 4, under larger followers' R&D costs, a decrease in followers' R&D costs increases the aggregate R&D by leaders, while under smaller followers' R&D costs, it decreases the aggregate R&D by leaders. We can surmise the reason for this result as follows. When followers' R&D costs are smaller, followers' R&D is larger, as shown in the left panel of Fig. 4. This makes the distribution of leaders flatter and the increase in aggregate R&D by leaders through the distribution effect tends to be smaller, and thus overwhelmed by the decrease in aggregate R&D by leaders through the individual effect.

[45]We derive the growth rate of the utility in this model later.

3.3.5 Effects on aggregate R&D

Next, we examine how the aggregate R&D, that is, the sum of R&D by followers and R&D by leaders, is determined. Rewriting the definition of I_L (58), we get the equation of the aggregate R&D $I(\equiv I_L + I_F)$ as follows: [46]

$$\frac{1}{I_L + I_F} = \int_0^{\pi^*} \frac{1}{I(\pi) + I_F} \frac{1}{\bar{\pi}} d\pi + \int_{\pi^*}^{\bar{\pi}} \frac{1}{I_F} \frac{1}{\bar{\pi}} d\pi. \tag{59}$$

This equation determines the equilibrium aggregate R&D I. Total differentiation of (59) yields:

$$\frac{1}{I^2} \frac{dI}{d\tilde{C}_F} = \int_0^{\pi^*} \frac{1}{[I(\pi) + I_F]^2} \left[\frac{dI(\pi)}{d\tilde{C}_F} + \frac{dI_F}{d\tilde{C}_F} \right] \frac{1}{\bar{\pi}} d\pi + \int_{\pi^*}^{\bar{\pi}} \frac{1}{{I_F}^2} \frac{dI_F}{d\tilde{C}_F} \frac{1}{\bar{\pi}} d\pi.$$

By using (53), we can rewrite $\frac{dI(\pi)}{d\tilde{C}_F} + \frac{dI_F}{d\tilde{C}_F}$ as follows:

$$\begin{aligned} \frac{dI(\pi)}{d\tilde{C}_F} + \frac{dI_F}{d\tilde{C}_F} &= \frac{1}{I(\pi) + R} \left\{ \left[\tilde{C}_F - I(\pi) \right] \frac{dR}{d\tilde{C}_F} + R \right\} + \frac{dR}{d\tilde{C}_F} \\ &= \frac{1}{I(\pi) + R} \left\{ \left(\tilde{C}_F + R \right) \frac{dR}{d\tilde{C}_F} + R \right\}: \end{aligned}$$

If Condition (R) holds, subsidies for followers' R&D increase the sum of leaders' R&D and followers' R&D in each industry where leaders' R&D is positive irrespective of the profit of the industry leader. From Proposition 3.2, the second term of the RHS is necessarily negative, and thus we can get the result of the effects on the aggregate R&D as in the following proposition:

Proposition 3.4. *If the elasticity of followers' R&D with respect to R&D costs is sufficiently high to satisfy* Condition (R), subsidies for followers' R&D increase the sum of leaders' R&D and followers' R&D in each industry, and consequently increase the aggregate R&D.

This result implies that if the elasticity of followers' R&D with respect to R&D subsidies is higher, R&D subsidies for new entry firms tend to be effective in raising aggregate productivity growth.

[46] Subtracting both sides of the following equation from one,

$$\frac{I_L}{I_L + I_F} = \int_0^{\bar{\pi}} \frac{I(\pi)}{I(\pi) + I_F} \frac{1}{\bar{\pi}} d\pi,$$

we get (59).

3.3.6 Effects on the growth rate of utility

Finally, we derive the growth rate of utility from consumption, u_t. We can calculate $\log u_t$ as:

$$\begin{aligned}\log u_t &= \int_0^1 \log \sum_j q_t(j,\omega)\, d_t(j,\omega)\, d\omega \\ &= \int_0^1 \log q_t(j_t(\omega),\omega)(1/\lambda_t(\omega))\, d\omega \\ &= \int_0^1 \log q_t(j_t(\omega),\omega)\, d\omega - \int_0^1 \lambda_t(\omega)\, d\omega\,,\end{aligned}$$

where $\lambda_t(\omega)$ denotes the quality increment of the state-of-the-art quality in industry ω. We define the average quality as $\log Q_t \equiv \int_0^1 \log q_t(j_t(\omega),\omega)\, d\omega$. The growth rate of u_t equals that of Q_t, which is given by:

$$\frac{d \log Q_t}{dt} = E[\log \lambda]\,(I_L + I_F)\,,$$

where we can derive

$$E[\log \lambda] = \frac{1 - [1 - \log(1-\bar{\pi})]\,(1-\bar{\pi})}{\bar{\pi}}.$$

Under the above parameter values, the growth rate is as in the right panel of Fig. 4.

3.4 Conclusion

Incorporating stochastic quality increments and diminishing returns of R&D of leaders into the standard quality-ladder model, we constructed a new Schumpeterian growth model where leader firms' quality leads over their followers are different among industries and leaders' R&D investment depends on the size of the quality lead. In this model, we obtained two main results. First, leaders with larger quality leads make smaller R&D investments. This tendency is consistent with actual behaviors of previous leader firms such as Sony and Eastman–Kodak. Second, subsidies for followers' R&D can increase the aggregate R&D of leaders. This theoretical result provides a rationale for the actual promotion policies for R&D activities of new entry firms, such as Innovation Union, which is the innovation policy in the EU.

3.5 Appendix

3.5.1 A1

In this appendix, we show that $\frac{dR}{dC_F} < 0$.

Totally differentiating (52) yields:

$$\frac{\partial G}{\partial R}\frac{dR}{dC_F} = A\bar{\pi} - \frac{\partial G}{\partial C_F}. \tag{60}$$

First, from (52) we get $\frac{\partial G(R)}{\partial R}$ as follows:

$$\frac{\partial G(R)}{\partial R} = \frac{1}{2}\left(C_F^2 - \left(\frac{A\bar{\pi}}{R}\right)^2\right) + \left[\left(2C_F + R\right)R\right]^{\frac{1}{2}}\left\{\left[\left(2C_F + R\right)R\right]^{\frac{1}{2}} - \left(C_F + R\right)\right\}.$$

The first term of the RHS is negative because $\pi^* = \frac{RC_F}{A} < \bar{\pi}$ is satisfied at the equilibrium and the second term of the RHS is also negative because

$$\begin{aligned}
&\left[\left(2C_F + R\right)R\right]^{\frac{1}{2}} - \left(C_F + R\right) < 0 \\
&\Leftrightarrow\ 2C_F R + R^2 - C_F^2 - 2C_F R - R^2 < 0 \\
&\Leftrightarrow -C_F^2 < 0.
\end{aligned}$$

Thus, we can show that $\frac{\partial G(R)}{\partial R} < 0$.

Second, we examine the sign of the term $(A\bar{\pi} - \frac{dG}{dC_F})$. From (52) we get $\frac{\partial G(R)}{\partial C_F}$ as follows:

$$\frac{\partial G(R)}{\partial C_F} = R\left(C_F + R\right) - R\left[\left(2C_F + R\right)R\right]^{\frac{1}{2}}.$$

Substituting this into $(A\bar{\pi} - \frac{dG}{dC_F})$, we get:

$$\begin{aligned}
A\bar{\pi} - \frac{dG}{dC_F} &= A\bar{\pi} - R\left(C_F + R\right) + R\left[\left(2C_F + R\right)R\right]^{\frac{1}{2}} \\
&= A\left\{\bar{\pi} - \frac{RC_F}{A}\right\} + R\left\{\left[\left(2C_F + R\right)R\right]^{\frac{1}{2}} - R\right\}.
\end{aligned}$$

The first term of the RHS is positive because $\pi^* = \frac{R\tilde{C}_F}{A} < \bar{\pi}$ is satisfied in the equilibrium. The second term is also positive. Thus, we get $(A\bar{\pi} - \frac{dG}{d\tilde{C}_F}) > 0$. As a result, we can show that $\frac{dR}{d\tilde{C}_F} < 0$.

3.5.2 A2

We show that $\frac{dI(\pi)}{dC_F}$ is an increasing function of $I(\pi)$ if Condition (R) holds.

$$
\begin{aligned}
\frac{\partial}{\partial I(\pi)} \frac{dI(\pi)}{d\tilde{C}_F} &= \frac{1}{[I(\pi) + R]^2} \left(-[I(\pi) + R] \frac{dR}{d\tilde{C}_F} - \left\{ \left[\tilde{C}_F - I(\pi) \right] \frac{dR}{d\tilde{C}_F} + R \right\} \right) \\
&= \frac{1}{[I(\pi) + r + I_F]^2} \left[\left(-\frac{dR}{d\tilde{C}_F} \right) \left(\tilde{C}_F + R \right) - R \right] :
\end{aligned}
$$

Therefore, we get $\frac{\partial}{\partial I(\pi)} \frac{dI(\pi)}{d\tilde{C}_F} > 0$ if $\frac{C_F}{R} \left(-\frac{dR}{d\tilde{C}_F} \right) > \frac{R}{C_F + R}$.

4 International intellectual property rights protection and economic growth with costly transfer

4.1 Introduction

Since a range of Trade-Related Aspects of Intellectual Property Rights (TRIPS) agreement, which requires developing countries that are members of the World Trade Organization (WTO) to establish minimum standards of intellectual property right (IPR) protection, were signed in the Uruguay Round, many studies have analyzed the effects of strengthening IPR protection in developing countries. These studies typically construct a model consistent with the product cycle, as highlighted in Vernon (1966). New goods are invented and production takes place initially in high-wage developed countries. Subsequently, production shifts to low-wage developing countries, accompanied by technology transfer. Technology diffuses from developed countries to developing countries through many channels. Regardless of the channels, technology transfer involves substantial resource cost in both countries. For example, when technology transfer takes place through foreign direct investment (FDI), affiliates receiving technology will typically need to conduct R&D to absorb the technology and to modify it for the local market [47], and multinational firms will need to train workers in developing countries and to acquire knowledge about foreign customs and regulations. When technology transfer takes place through licensing, there is a negotiation cost incurred to establish an agreement. However, many theoretical papers ignore the cost of technology transfer. In particular, no study has considered that technology transfer involves a resource cost in both developed countries and developing countries. This paper, therefore, develops a product-cycle model with costly technology transfer, which requires resource use in both developed and developing countries.

We assume that there are two countries in the world, the North and the South. New goods are invented in the North, and inventors of the new good can earn a profit flow because they are protected by perfect IPR protection in the North. They have an incentive to transfer technology to the South in order to produce goods using Southern labor, whose wage is relative low. However they suffer from a one-off imitation risk and transfer cost. These trade-offs determine the amount of technology transfer in the equilibrium.

[47] Fors (1997) presented evidence that multinationals spend substantial amounts on R&D performed abroad.

Then we analyze the effect of strengthening IPR protection. Strengthening IPR protection (decreasing imitation risk) induces a large technology transfer and narrows the North{South wage gap. Strengthening IPR protection reduces imitation risk, which decreases the disadvantage of transferring technology. Then, more inventors are willing to transfer technology, and more production shifts to the South, which induces increased demand for Southern labor. This reduces the North{South wage gap. These results are consistent with models in related studies in which technology diffuses through FDI or licensing. However, we obtain ambiguous results regarding the effect on economic growth. Although strengthening IPR protection induces a high growth rate when the transfer cost is small, strengthening IPR protection can induce a low growth rate when the transfer cost is large. Strengthening IPR protection induces a production shift to the South, which is accompanied by a technology transfer. On the one hand, more production takes place in the South, which makes Northern resources shift from production to R&D activities, which promotes economic growth. On the other hand, more technology transfer takes place, which requires Northern resources. The resources used in R&D will shift to the transfer activities, which impedes economic growth. If the former (latter) effect dominates the latter (former) effect, strengthening IPR protection promotes (harms) economic growth.

Then what factors determine the transfer cost, which plays a key role in our model? In the real world, it depends not only on the characteristics of the firms in the developed country such as the ability to conduct adaptive R&D and to accumulate knowledge, but also on the characteristics of the host countries such as the legal system, education level and morals of Southern workers. Specifically, entrepreneurs may misbehave and the Northern firms must monitor their behaviors so that they do not misbehave in countries with weaker investor protection or lower morality. Then we extend the basic model by introducing the framework of contract theory in order to examine how weaker investor protection or lower morality affect transfer cost and consequently R&D activities.

We assume that for firms in developed countries to shift production to developing countries, it is necessary to cooperate with entrepreneurs in the developing countries, and that entrepreneurs have an incentive to enjoy private benefits by misbehaving. These private benefits depend on the degree of investor protection, the morals of workers in developing countries, and the monitoring by the firms transferring the technology. When the degree of investor protection and the morals of laborers are low, private benefits are high. However firms can reduce the private benefits of entrepreneurs by monitoring, which involves a resource cost. Although the characteristics of host countries are not so important when the degree of investor protection and the morals of laborers are sufficiently high, they have a crucial effect on the transfer cost when the degree of investor protection and the morals of laborers are low because firms must monitor entrepreneurs to ensure that they do not misbehave. The transfer cost is high if the degree of investor protection and the morals of laborers in the host country are low. In this situation, strengthening IPR protection impedes economic growth. This result implies that strengthening IPR protection does not promote R&D in all developed countries but rather depends on the characteristics of the host countries.

This paper develops a product-cycle model where the channel of technology transfer is FDI or licensing. Lai (1998), Yang and Maskus (2001), Glass and Wu (2007), Tanaka, Iwaisako and Futagami (2007), Dinopoulos and Segerstrom (2010), Branstetter and Saggi (2011), and Tanaka and Iwaisako (2014) constructed such a model. Most of these papers showed that strengthening IPR protection induces a large technology transfer, narrows the North{South wage gap and produces a high growth rate.[48] Our paper shows that the effect of strengthening IPR protection on the growth rate is ambiguous when we consider the role of costly transfer activities. Some papers have considered costly technology transfer. In the context of illegal imitation models, Grossman and Helpman (1991) and Mondal and Gupta (2007, 2009) assumed that illegal imitation as a channel of technology transfer is costly and requires Southern labor. In the context of licensing, Yang and Maskus (2001) and Tanaka, Iwaisako and Futagami (2007) assumed that firms incur negotiation cost to obtain licensing agreements, which is reduced by strengthening IPR protection. However, Yang and Maskus (2001) assumed that only Northern labor is needed for negotiation, while Tanaka, Iwaisako and Futagami (2007) assumed that only Southern labor is needed in negotiation. In the context of FDI models, Dinopoulos, and Segerstrom (2010) assumed that firms should conduct adoptive R&D to transfer technology, which requires only Southern labor. Our model generalizes these papers such that transfer activities require both Northern and Southern labor.

The rest of this paper is structured as follows. Section 2 describes the basic model and derives the equilibrium. Section 3 describes the extended model. Section 4 provides concluding comments.

[48]There is another category of product-cycle model in which technology transfer occurs through illegal imitation. Grossman and Helpman (1991), Helpman (1993), Arnold (2002) and Mondal and Gupta (2007, 2009) constructed such a model. Most of these papers showed that strengthening IPR protection induces small technology transfers, widens the North{South wage gap and reduces the economic growth rate. Glass and Saggi (2002) and Parello (2008) developed hybrid models in which technology transfers occur through both FDI and illegal imitation.

4.2 Basic model

4.2.1 Setting

There are two countries in the world, the North and the South. Both country's populations are exogenous given M^N and M^S, respectively, and do not grow. They are linked by free international trade in differentiated goods. The wage rate of the North is higher than that of the South. Only the North has a research sector; therefore, new differentiated goods are invented in the North. Inventors of the new goods are protected by perfect IPR protection in the North but are protected by imperfect IPR protection in the South.

4.2.2 Households

We construct an overlapping-generations model.[49] Individuals live for two periods, young and old. When they are young, they supply one unit of labor inelastically and save all of their wage. When they are old, they do not leave a bequest, as they use all of their savings to consume differentiated goods and obtain utility as follows:

$$U_{t-1} = \left\{ \int_0^{n_t} [x_t(z)]^{\frac{\sigma-1}{\sigma}} dz \right\}^{\frac{\sigma}{\sigma-1}}; \tag{61}$$

where U_{t-1} is the utility of an individual of generation $t-1$, $x_t(z)$ is consumption of differentiated goods in industry z in period t, $\sigma > 1$ is the elasticity of substitution of differentiated goods, and n_t is the available variety of differentiated goods in period t, which increases through R&D activity. Solving the utility-maximization problem yields the following demand function:

$$x_t(z) = \frac{p_t(z)^{-\sigma}}{\int_0^{n_t} p_t(u)^{1-\sigma} du} E_t; \tag{62}$$

where $p_t(z)$ is the price of differentiated goods in industry z in period t, and E_t is aggregate world consumption in period t. As young individuals do not consume, and old individuals do not work, aggregate savings of generation

[49] Tanaka and Iwaisako (2009) and Chou and Shy (1991,1993) have also constructed overlapping-generations models with a variety-expansion type of innovation.

t, S_t, equals discounted aggregate consumption in period $t+1$ E_{t+1}:

$$S_t = \frac{E_{t+1}}{1+r_t}; \tag{63}$$

where r_t is the interest rate in period t.

4.2.3 Production

There are three types of firms producing goods: Northern firms, Multinational firms and Imitation firms. Northern firms and Multinational firms supply goods monopolistically, and Imitation firms supply goods competitively. Northern firms hire one Northern laborer to produce one good. Multinational firms and Imitation firms hire one Southern laborer to produce one good. There are no fixed costs or transport costs. The instantaneous profits of Northern firms, Multinational firms and Imitation firms are expressed as $\pi_t^N = x_t^N\left(p_t^N - w_t^N\right)$, $\pi_t^M = x_t^M\left(p_t^M - w_t^S\right)$ and $\pi_t^I = x_t^I\left(p_t^I - w_t^S\right)$ respectively, where x_t^N, x_t^M and x_t^I are the outputs and p_t^N, p_t^M and p_t^I are the prices of Northern firms, Multinational firms and Imitation firms respectively, w_t^N is the wage rate of the North, and w_t^S is the wage rate of the South. As Northern firms and Multinational firms supply goods monopolistically, they choose the optimal price $p_t^N = \frac{\varepsilon}{\varepsilon-1}w_t^N$, $p_t^M = \frac{\varepsilon}{\varepsilon-1}w_t^S$ respectively. As Imitation firms supply goods competitively, the price of Imitation firms is $p_t^I = w_t^S$. Using (62), we obtain:

$$x_t^N = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}}{n_t^N\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} + n_t^M\left(\frac{\varepsilon}{\varepsilon-1}\omega_t\right)^{1-\varepsilon} + n_t^I\left(\omega_t\right)^{1-\varepsilon}}E_t; \tag{64}$$

$$x_t^M = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\omega_t\right)^{-\varepsilon}}{n_t^N\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} + n_t^M\left(\frac{\varepsilon}{\varepsilon-1}\omega_t\right)^{1-\varepsilon} + n_t^I\left(\omega_t\right)^{1-\varepsilon}}E_t; \tag{65}$$

$$x_t^I = \frac{\left(\omega_t\right)^{-\varepsilon}}{n_t^N\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} + n_t^M\left(\frac{\varepsilon}{\varepsilon-1}\omega_t\right)^{1-\varepsilon} + n_t^I\left(\omega_t\right)^{1-\varepsilon}}E_t; \tag{66}$$

where n_t^N, n_t^M and n_t^I are the available variety of differentiated goods produced by Northern firms, Multinational firms and Imitation firms respectively in period t, $!_t \equiv \frac{w_t^S}{w_t^N}$ is the relative wage and $E_t \equiv \frac{E_t}{w_t^N}$. Substituting (64) and (65) into the profit function, we obtain relative instantaneous profit as follows:

$$\frac{\pi_t^N}{\pi_t^S} = (!_t)^{"-1}: \tag{67}$$

4.2.4 R&D activity

As inventors of new goods are protected by perfect IPR protection in the North, they can become Northern monopolistic firms and earn profit flow. Then new firms undertake R&D activity. We assume that a new firm must hire $1{=}n_t$ units of Northern labor to invent a new product.[50] Then the return from the R&D activity is $v_t^N - \frac{1}{n_t} w_t^N$, where v_t^N is the value of Northern firms in period t.[51] Assuming free entry into each R&D race, we obtain:2009

$$v_t^N = \frac{w_t^N}{n_t}: \tag{68}$$

Aggregating labor engaged in R&D yields the following dynamic equation of variety:

$$g_t^N \equiv \frac{n_{t+1} - n_t}{n_t} = H_t^R; \tag{69}$$

where H_t^R is aggregate labor engaged in R&D.

4.2.5 Technology transfer

Northern firms have an incentive to transfer technology to the South in order to produce goods using Southern labor, whose wage is relative low. Producing goods at low cost allows these firms to earn large profits, which is an advantage of technology transfer. However, they suffer from a one-off imitation risk and transfer cost, which are the disadvantages of technology transfer. As IPR protection in the South is imperfect, the Northern firms

[50]We assume a simple setting in which the efficiency of R&D is unity, a spillover occurs from the aggregate available variable n_t and there is no uncertainty.

[51]We assume that the stock value v_t^N is the ex dividend stock value. Tanaka and Iwaisako (2009) describe in detail the sequence in each period.

can lose their profit flow with probability δ because of imitation when transferring their technology.[52] That is, Northern firms succeed in transferring technology and become Multinational firms with probability $1-\delta$, but they lose profit flow, and goods are produced by the Imitation firms in their industry with probability δ. Then $n_{t+1}^M - n_t^M = (1-\delta)\left(n_{t+1}^S - n_t^S\right)$ and $n_{t+1}^I - n_t^I = \delta\left(n_{t+1}^S - n_t^S\right)$ are satisfied, where $n_t^S \equiv n_t^M + n_t^I$ is the variety produced by Southern labor. In the long run, n_t^M and n_t^I approximate $n_t^M = (1-\delta)\, n_t^S$ and $n_t^I = \delta n_t^S$ respectively, regardless of the initial conditions. We additively assume that transferring technology needs both Northern labor and Southern labor, and should satisfy the following CES restriction:[53]

$$n_t \left[b\left(Bh_t^{FN}\right)^{\frac{\eta-1}{\eta}} + (1-b)\left(h_t^{FS}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \geq f_F; \tag{70}$$

where h_t^{FN} is Northern labor engaged in technology transfer, h_t^{FS} is Southern labor engaged in technology transfer, $\eta \geq 0$ is the elasticity of substitution between Northern labor and Southern labor, $1/f_F$ is the efficiency of technology transfer, $b \in [0, 1]$ is the degree of contribution of Northern labor and B is the efficiency of Northern labor. They choose optimal h_t^{FN} and h_t^{FS} to minimize transfer cost $c_t^F \equiv w_t^N h_t^{FN} + w_t^S h_t^{FS}$ subject to (70). Then we obtain:

$$h_t^{FN} = \frac{1}{n_t} F^{FN}(\omega_t); F^{FN}(\omega_t) \equiv \frac{f_F}{B\,(b)^{\frac{\eta-1}{\eta}}} \left[\frac{(B\omega_t)^{\eta-1}(b)^{\eta}}{(B\omega_t)^{\eta-1}(b)^{\eta} + (1-b)^{\eta}} \right]^{\frac{\eta}{\eta-1}}; \tag{71}$$

$$h_t^{FS} = \frac{1}{n_t} F^{FS}(\omega_t); F^{FS}(\omega_t) \equiv \frac{f_F}{(1-b)^{\frac{\eta-1}{\eta}}} \left[\frac{(1-b)^{\eta}}{(B\omega_t)^{\eta-1}(b)^{\eta} + (1-b)^{\eta}} \right]^{\frac{\eta}{\eta-1}}; \tag{72}$$

$$c_t^F = v_t^N F^{FC}(\omega_t); F^{FC}(\omega_t) \equiv \frac{f_F}{(b)^{\frac{\eta-1}{\eta}} B} \left[\frac{(B\omega_t)^{\eta-1}(b)^{\eta} + (B\omega_t)^{\frac{\eta-1}{\eta}} b(1-b)^{\eta-1}}{(B\omega_t)^{\eta-1}(b)^{\eta} + (1-b)^{\eta}} \right]^{\frac{\eta}{\eta-1}}; \tag{73}$$

[52] Related studies assumed that Multinational firms suffer from Poisson imitation risk instead of a one-off imitation risk. Although these studies use settings that are more realistic than ours, both settings play a similar role in the model, and our setting makes analysis of the model easier. Then we adopt a one-off imitation risk.

[53] When the Northern firm transfers technology, on the one hand, they send their staff in order to provide the necessary knowledge, such as how to manage Southern workers, etc. Then Northern labor is required. On the other hand, Southern laborers must learn how to produce new goods, which requires time. Then Southern labor is required.

which are functions of relative wage rate ω. For the latter analysis, we check the properties of the above function. First, $\frac{dF^{FN}(\omega)}{df_F} > 0, \frac{dF^{FS}(\omega)}{df_F} > 0, \frac{dF^{FC}(\omega)}{df_F} > 0$ are satisfied, which means that low efficiency of technology transfer induces many Northern laborers and Southern laborers to engage in technology transfer and makes the technology transfer costly. Second, $\frac{dF^{FN}(\omega)}{d\omega} > 0, \frac{dF^{FS}(\omega)}{d\omega} < 0$ and $\frac{dF^{FC}(\omega)}{d\omega} > 0$ are satisfied, which means that the high Southern wage, keeping the Northern wage constant, induces many Northern laborers and few Southern laborers to engage in technology transfer and incurs a large technology transfer cost. Third, $\frac{dF^{FC}(\omega)}{d\eta} < 0$ is satisfied, which means that the low elasticity of substitution between Northern labor and Southern labor induces a high transfer cost. Finally, F^{FC} is U-shaped with respect to the degree of contribution b.[54]

The expected gain from technology transfer is $\left((1-\delta)\,\phi v_t^S - v_t^N\right) - c_t^F$ where v_t^S is the value of Multinational firms and $\phi \in [0,1]$ is the profit share of Northern firms.[55] If the expected gain from technology transfer is strictly negative, Northern firms prefer not to transfer technology. If the expected gain from technology transfer is strictly positive, transfer takes place, and the demand for Southern labor increases. Then the North–South wage gap falls, which decreases the incentive to transfer. This mechanism continues until the expected gain reaches zero.[56] Therefore, when technology transfer takes place, using (73):

$$\frac{v_t^N}{v_t^S} = \frac{(1-\delta)\phi}{F^{FC}(\omega_t) + 1} \tag{74}$$

is satisfied. Our assumption about the technological transfer cost is a generalization of that in related studies. When $f_F = 0$ and $\phi = 1$, our model reflects the economy of Lai (1998). When $f_F > 0$ and $b = 1$, our model reflects the economy of Yang and Maskus (2001). When $f_F > 0$ and $b = 0$ and we interpret ϕ as the profit share of Southern licensed firms, our model reflects the economy of Tanaka, Iwaisako and Futagami (2007). When $f_F > 0$, $b = 0$ and $\phi = 1$, our model reflects the economy of Dinopoulos and Segerstrom (2010).

[54] A high (low) b induces many Northern (Southern) laborers to engage in technology transfer. An increase in b means an increase (decrease) in the efficiency of Northern (Southern) labor. Thus when few (many) Northern laborers are engaged in technology transfer, an increase in b induces an increase (decrease) in the transfer cost.

[55] We assume that v_t^S is the ex dividend stock value.

[56] We only focus on the case in which production takes place in both the North and South.

4.2.6 Labor market

Northern laborers are engaged in the production of Northern firms, R&D activity and technological transfer. Northern labor supply is exogenously given. The labor market clearing condition in the North is:

$$x_t^N n_t^N + H_t^R + (n_{t+1}^S - n_t^S) h_t^{NF} = M^N: \tag{75}$$

Southern laborers are engaged in the production of Multinational firms and Imitation firms, and technological transfer. Southern labor supply is exogenously given. The labor market clearing condition in the South is:

$$x_t^S n_t^S (1-\delta) + x_t^I n_t^S \delta + (n_{t+1}^S - n_t^S) h_t^{SF} = M^S: \tag{76}$$

4.2.7 Asset market

We assume that the international asset market between two countries is complete, and thus their asset markets are integrated.[57] Young individuals save their asset in the form of stock in the new firms, existing Northern firms and existing Multinational firms. The asset market clearing condition is:

$$\frac{E_{t+1}}{1+r_t} = w_t^N H_t^R + n_t^N v_t^N + (1-\delta) n_t^S v_t^S: \tag{77}$$

When both stock of existing Northern firms and stock of existing Multinational firms are held, the return on these stocks is equal. The no-arbitrage condition satisfies:

$$1 + r_t = \frac{\pi_{t+1}^N + v_{t+1}^N}{v_t^N} = \frac{\pi_{t+1}^S + v_{t+1}^S}{v_t^S}; \tag{78}$$

where r is the net return from holding riskless assets in period t.

[57]Alternatively, one could assume that the international asset market between two countries is incomplete, and thus that their asset markets are isolated. Tanaka and Iwaisako (2009) used this assumption.

4.3 Equilibrium

From (69), (71) and (75), the difference equation of the ratio of goods produced by Southern labor to all produced goods $\zeta_t \equiv \frac{n_t^S}{n_t}$ is as follows:

$$\zeta_{t+1} = \frac{\frac{1}{F^{FN}(!_t)}\frac{1}{\zeta_t}\left[M^N - x_t^N n_t^N - g_t^N\right] + 1}{g_t^N + 1}\zeta_t; \tag{79}$$

In the following, we focus only on the balanced growth path (BGP) equilibrium where g^N, r, ζ, ! , and E are constants. We normalize the Northern wage so that the it grows at the interest rate r and $w_0^N = 1$. [58]

First we describe the equilibrium North{South relative wage. From (67), (74), (78) and $!_{t+1} = !_t$, we obtain:

$$!^{"-1}\left(F^{FC}(!) + 1\right) = (1-\delta)\,\phi. \tag{80}$$

The North{South relative wage $! = \frac{w^S}{w^N}$ is determined by this equation. The relative wage is an increasing function of ", η, ϕ and a decreasing function of δ and f_F, which means that enhancing the transfer incentive increases the relative wage. [59]

Lemma 4.1. Strengthening IPR protection (decreasing δ) narrows the North{South wage gap.

This lemma is consistent with the results of related studies such as Lai (1998), Tanaka, Iwaisako and Futagami (2007), Dinopoulos and Segerstrom (2010), Branstetter and Saggi (2011) and Tanaka and Iwaisako (2014).

Next we describe the equilibrium value of the growth rate g^N and production ratio ζ. As $\zeta_{t+1} = \zeta_t$ is satisfied on the BGP equilibrium, from (75) and (79), we obtain:

$$\zeta = \frac{FDI}{g^N}; \tag{81}$$

[58] Many papers standardize some endogenous variable z as the numeraire $z_0 = 1; z_1 = 1; z_2 = 1; \ldots$. Our standardization is $z_0 = 1; z_1 = 1 + r; z_2 = (1 + r)^2; \ldots$ instead of a more common standardization, which produces a simple expression for the asset market clearing condition.

[59] As the high elasticity of substitution of differentiated good " leads to a low price and high output (labor demand), the incentive to hire lower wage labor is high. A high elasticity of substitution of the labor-hiring transfer activity η and a high efficiency of technology transfer result in a low transfer cost, which enhances the transfer incentive. Low imitation risk and a high share of profit of Northern firms lead to a high expected return of technology transfer, which increases the transfer incentive. As $F^{FC}(!)$ is inverse U-shaped with respect to the degree of contribution b, the relative wage is U-shaped with respect to b.

where $\mathrm{FDI} \equiv \frac{n^S_{t+1} - n^S_t}{n_t}$ is the transfer rate.

As $E \equiv \frac{E_t}{w^N_t}$ is constant on the BGP and w^N grows at the rate r, $\frac{E_{t+1}}{E_t} = 1 + g^{w^N} = 1 + r$ is satisfied. Then from (68), (74) and (77), we obtain:

$$E = g^N + 1 - \zeta + \frac{\zeta}{\phi}\left(1 + F^{FC}(\omega)\right). \tag{82}$$

From (75), (81) and (82), we obtain:

$$g^N = \frac{M^N - X^N\left(1 - \zeta + \frac{\zeta}{\phi}\left(1 + F^{FC}(\omega)\right)\right)}{1 + F^{FN}(\omega)\zeta + X^N}; \tag{83}$$

where $X^N \equiv \frac{x^N n^N}{E} = \frac{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}}{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} + \zeta(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\omega\right)^{1-\varepsilon} + \zeta\delta(\omega)^{1-\varepsilon}}$, which satisfies $\frac{dX^N}{d\zeta} < 0$ and $\frac{dX^N}{d\delta} < 0$.[60] This equation depicts a two-dimensional N-curve whose vertical axis measures the growth rate g^N and horizontal axis measures the production ratio ζ respectively. A rising ζ induces two effects on the N-curve. The first effect is through $\frac{dX^N}{d\zeta} < 0$. This effect means that increasing the ratio of goods produced by Southern labor decreases the ratio of goods produced by Northern labor, which increases labor engaged in R&D and g^N. The second effect is through $\frac{d\zeta F^{FN}(\omega)}{d\zeta} > 0$ and $\frac{d\zeta F^{FC}(\omega)}{d\zeta} > 0$. This effect means that increasing the ratio of goods produced by Southern labor increases technology transfer, which increases labor engaged in the transfer, which then decreases labor engaged in R&D and g^N. Therefore, the slope of the N-curve is ambiguous. If F^{FN} and F^{FC} are large, this slope is negative because the second effect is strong. If F^{FN} and F^{FC} are small, this slope is positive because the second effect is weak. Decreasing δ shifts the N-curve downward because $\frac{dX^N}{d\delta} < 0$, $\frac{dF^{FC}(\omega)}{d\omega}\frac{d\omega}{d\delta} < 0$ and $\frac{dF^{FN}(\omega)}{d\omega}\frac{d\omega}{d\delta} < 0$ are satisfied.

From (76), (81) and (82), we obtain:

$$g^N = \frac{M^S - X^S\left(1 - \zeta + \frac{\zeta}{\phi}\left(1 + F^{FC}(\omega)\right)\right)}{\zeta F^{FS}(\omega) + X^S}; \tag{84}$$

where $X^S \equiv \frac{x^M_t n^M_t + x^I_t n^I_t}{E} = \frac{\left[\left(\frac{\varepsilon}{\varepsilon-1}\omega\right)^{-\varepsilon}(1-\delta) + (\omega)^{-\varepsilon}\delta\right]\zeta}{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} + \zeta\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\omega\right)^{1-\varepsilon} + \delta(\omega)^{1-\varepsilon}\right]}$, which satisfies $\frac{dX^S}{d\zeta} > 0$ and $\frac{dX^S}{d\delta} > 0$.[61]

[60] This is confirmed in the appendix.
[61] This is confirmed in the appendix.

This equation depicts an S-curve. Increasing ζ decreases g^N in the S-curve because $\frac{dX^S}{d\zeta} > 0$, $\frac{d\zeta F^{FS}(!)}{d\zeta} > 0$ and $\frac{d\zeta F^{FN}(!)}{d\zeta} > 0$ are satisfied. The S-curve is always downward sloping. Decreasing δ shifts the S-curve upward (with small ζ) because $\frac{dX^S}{d\delta} > 0$, $\frac{dF^{FS}(!)}{d!}\frac{d!}{d\delta} > 0$ are satisfied.[62]

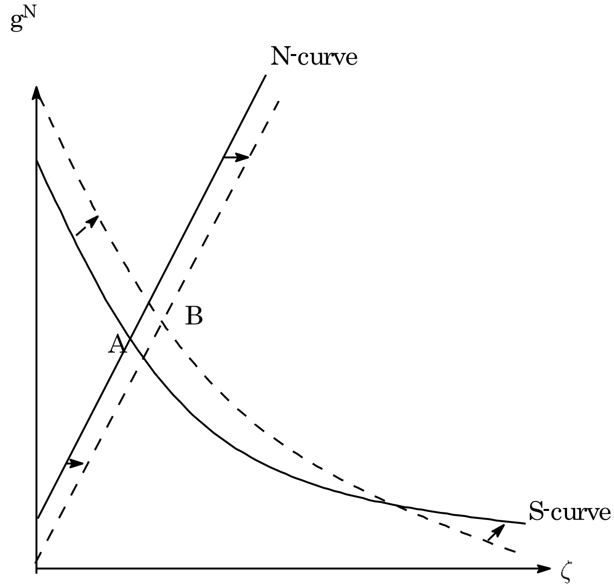

Figure 1. Upward-sloping N-curve.

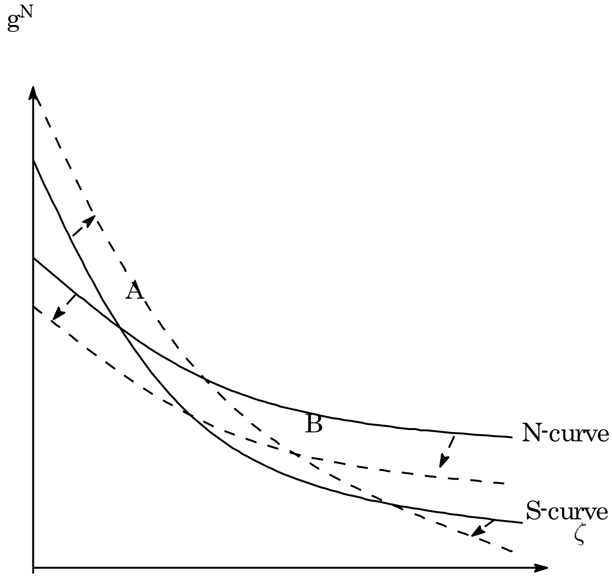

Figure 2. Downward-sloping N-curve.

[62]With large ζ, decreasing δ shifts the S-curve downward because the effect of $\frac{dF^{FC}(!)}{d!}\frac{d!}{d\delta} < 0$ outweighs the two other effects. However, when b is not close to unity, decreasing δ shifts the S-curve upward in the range of ζ 2 [0; 1]. Therefore, we focus only on this case.

Then we can analyze the effect of strengthening IPR protection (decreasing imitation probability δ). First we consider the case of a low transfer cost F^{FC} (low f_F and high η). In this case, the N-curve is upward sloping, which is depicted in Figure 1. Strengthening IPR protection shifts the N-curve downward and the S-curve upward from the solid lines to the broken lines, and the equilibrium shifts to point B from point A. Then the ratio of goods produced by Southern labor ζ increases. The effect of growth rate g^N is ambiguous.[63] Next, we consider the case of a high transfer cost F^{FC} (high f_F and low η). In this case, the N-curve is downward sloping, which is depicted in Figure 2. In Figure 2, solid lines shift broken lines, and the equilibrium shifts to point B from point A. Then the ratio of goods produced by Southern labor ζ increases and growth rate g^N decreases.

Proposition 4.1. Strengthening IPR protection (decreasing δ) induces a large technology transfer, and the ratio of goods produced by Southern labor ζ *increases. However, the effect on the growth rate is ambiguous. Although* strengthening IPR protection induces a high growth rate when the transfer cost is low, strengthening IPR protection induces a low growth rate when the transfer cost is high.

4.4 Extended model

In the previous section, we show that the effect of strengthening IPR protection on the growth rate depends crucially on the transfer cost, which is a function of the efficiency of technology transfer f_F. This parameter summarizes many factors such as the experience of Northern firms, the degree of globalization, the environments of host countries, etc. In this section, we focus on the characteristics of the host countries. Then we introduce a moral hazard framework following Holmstrom and Tirole (1997) and Antras, Desai and Foley (2009). In this setting, when Northern firms transfer their technology to the South, they need to cooperate with Southern entrepreneurs, who have no profitable opportunity except for cooperation with Northern firms. However, they cannot fully control Southern entrepreneurs; therefore, they have to contract with Southern entrepreneurs. After technology is transferred, Southern entrepreneurs choose to behave or to misbehave. When Southern entrepreneurs choose to behave, the transfer succeeds with probability $p^H(1-\delta)$, and Southern entrepreneurs enjoy no private benefits. When Southern entrepreneurs choose to misbehave, the transfer succeeds with probability $p^L(1-\delta)$, and the

[63] However, in the extreme case that $f_F = 0$ is satisfied (Lai (1998)'s economy), we can easily show that g^N increases with decreasing δ (see the appendix).

Southern entrepreneurs enjoy private benefits.[64] For simplicity, we assume that $p^H = 1$ and $p^L = 0$.[65] Following Antras, Desai and Foley (2009), we also assume that the private benefits of Southern entrepreneurs are proportional to the value of Multinational firms and are influenced by the investor protection in the South and the monitoring by Northern firms. If the investor protection is strong, it is difficult for Southern entrepreneurs to misbehave and enjoy private benefits. Even if the investor protection is not sufficiently strong, Northern firms can reduce the private benefits of Southern entrepreneurs by monitoring, which needs Northern labor. Severe monitoring by Northern firms also makes it difficult to obtain private benefits. Then, following Antras, Desai and Foley (2009), we specify the private benefits of Southern entrepreneurs as $(1-\mu)(1-\delta) v_t^M C\left(h_t^{FN}\right)$, where μ is the degree of investor protection or morality in the host country, and $C\left(h_t^{FN}\right)$ is the monitoring function, which satisfies $C^0\left(h_t^{FN}\right) < 0$. For simplicity, we specify the monitoring function as $C\left(h_t^{FN}\right) = \frac{1}{a^M n_t h_t^{FN}}$, where n_t is a spillover and a^M is the efficiency of monitoring.

Then Northern firms choose the volume of employment engaged in technology transfer h_t^{FN}, h_t^{FS} and the share of the transfer cost κ such that they maximize the transfer gain and Southern entrepreneurs choose to behave. This optimal contract is given as follows:

$$\begin{aligned}
\max_{h_t^{FN}; h_t^{FS}, \kappa} \quad & (1-\delta)\phi v_t^S - \kappa c_t^F \\
\text{s:t:} \quad & n_t \left[b\left(h_t^{FN}\right)^{\frac{\eta-1}{\eta}} + (1-b)\left(h_t^{FS}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \geq f_F \\
& (1-\delta)(1-\phi) v_t^S - (1-\kappa) c_t^F \geq 0 \\
& (1-\delta)(1-\phi) v_t^S - (1-\kappa) c_t^F \geq (1-\mu)(1-\delta) v_t^S \frac{1}{a^M n_t h_t^{FN}} - (1-\kappa) c_t^F
\end{aligned} \tag{85}$$

The objective function is the transfer gain of Northern firms, which is the same as in the previous section except for the presence of κ.[66] The first constraint is a resource constraint on technology transfer, which was used in the previous section. The second constraint is the participation constraint of Southern entrepreneurs, given that

[64] Private benefits are interpreted as perquisites for the Southern entrepreneurs associated with leaking technology, diverting funds, doing careless work, making less effort, etc.

[65] This assumption enables us to focus only on the case in which Northern firms ensure that Southern entrepreneurs do not misbehave; then, Southern entrepreneurs do not choose to misbehave in equilibrium.

[66] In this section, we allow the share of transfer cost κ to be selected to obtain a simple solution for the optimal contract. In our model, ϕ is exogenously given, which is an endogenous variable in Antras, Desai and Foley (2009). They analyzed how the share of profit and the share of cost are affected by the Southern investor protection. They showed that both are high in the low protection case, and vice versa. Although their setting is more realistic, our setting is rich enough to analyze the effect of Southern investor protection and the level of IPR protection on the growth rate.

they have no other option for earning a profit. Southern entrepreneurs gain an exogenous $(1-\phi)$ share of profit and pay an endogenous κ share of cost. This constraint says that the expected gain of Southern entrepreneurs must be nonnegative. The third constraint is an incentive compatibility constraint of Southern entrepreneurs. The left-hand-side of this inequality is the benefit of Southern entrepreneurs when they choose to behave, and the right-hand-side is the benefit of Southern entrepreneurs when they choose to misbehave. This inequality must hold so that Southern entrepreneurs choose not to misbehave.

The first constraint is always binding because excess investment leads to a high transfer cost, which reduces the transfer gain. The second constraint is also always binding in our setting because the Northern firm has an incentive that continues to decrease κ if this inequality is strictly positive. Then $\kappa = 1-(1-\delta)(1-\phi)\frac{v_t^S}{c_t^F}$ is satisfied. The third constraint is not binding when μ is high enough, which yields $h_t^{FN} = \frac{1}{n_t}F^{FN}(\omega_t)$, $h_t^{FS} = \frac{1}{n_t}F^{FS}(\omega_t)$ and $c_t^F = v_t^N F^{FC}(\omega_t)$. These results are the same as in the previous section. However, when μ is low, the third constraint is binding, which yields:

$$h_t^{FN} = \frac{1}{n_t}\hat{F}^{FN}; \hat{F}^{FN} \equiv \frac{1}{a^M}\frac{1-\mu}{1-\phi}; \tag{86}$$

$$h_t^{FS} = \frac{1}{n_t}\hat{F}^{FS}; \hat{F}^{FS} \equiv \left[\left(\frac{1}{1-b}\right)(f_F)^{\frac{1-\eta}{\eta}} - \left(\frac{b}{1-b}\right)\left(\frac{1}{a^M}\frac{1-\mu}{1-\phi}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}; \tag{87}$$

$$c_t^F = v_t^N \hat{F}^{FC}(\omega_t); \hat{F}^{FC}(\omega_t) \equiv \left\{\frac{1}{a^M}\frac{1-\mu}{1-\phi} + \omega_t\left[\left(\frac{1}{1-b}\right)(f_F)^{\frac{1-\eta}{\eta}} - \left(\frac{b}{1-b}\right)\left(\frac{1}{a^M}\frac{1-\mu}{1-\phi^N}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right\}; \tag{88}$$

where $\hat{F}^{FN} > F^{FN}(\omega_t)$, $\frac{d\hat{F}^{FN}}{d\mu} > 0$, $\hat{F}^{FC}(\omega_t) > F^{FC}(\omega_t)$ and $\frac{d\hat{F}^{FC}(\omega)}{d\mu} > 0$ are satisfied. Therefore, when Southern investor protection is weak, more Northern laborers must be engaged in technology transfer than the optimal hiring level that the third constraint is not binding in order for the Southern entrepreneur not to misbehave, and therefore the cost of transfer is also higher than the optimal level. We can easily verify that relaxing Southern investor protection (decreasing μ) induces increasing h_t^{FN} and c_t^F. The other setting, except for the cost of transfer, is the same as in the previous section, which gives us the following equations:

$$! ^{"-1}\left(\hat{F}^{FC}(!)+1\right) = (1-\delta); \tag{89}$$

$$g^N = \frac{M^N - X^N\left(1+\zeta\hat{F}^{FC}(!)\right)}{1+\zeta\hat{F}^{FN}(!)+X^N}; \tag{90}$$

$$g^N = \frac{M^S - X^S\left(1+\zeta\hat{F}^{FC}(!)\right)}{\zeta\hat{F}^{FS}(!)+X^S}; \tag{91}$$

instead of (80), (83) and (84), which determines the equilibrium value of $!$, ζ and g^N. As discussed in the previous section, strengthening IPR protection reduces the growth rate when $\hat{F}^{FC}(!)$ is high. Then we get following proposition.

Proposition 4.2. Strengthening IPR protection (decreasing δ) induces a low growth rate when Southern investor protection and the level of morality in the host country are weak.

4.5 Conclusion

We developed a product-cycle model with costly technology transfer that requires resources from the North and South. We showed that whether or not strengthening IPR protection enhances growth or impedes growth depends heavily on the size of the transfer cost. This result is not obtained in related studies using a product-cycle model where the channel of technology transfer is FDI or licensing, because they do not take into account the resource cost of technology transfer in both countries. If only Southern labor is used in the transfer, shifts in production to the South always increase R&D activity in the North. If only Northern labor is used in the transfer, the interior point equilibrium may tend to be unstable when the efficiency of transfer is low. In this situation, many Northern laborers are used in the transfer, which increases the demand for Northern labor. Then when production shifts to the South, the North{South wage gap increases, which increases the incentive to transfer technology. This mechanism continues until all production shifts to the South, which generates extreme point equilibrium. By reason of the above, our generalization of technology transfer is meaningful.

In the extended model, we introduced a contract theory framework, which is usually analyzed in a partial equilibrium model. Our general equilibrium model identified the effect of investor protection and the level of morality in developing countries on the growth rate through the labor market; weaker investor protection raises the monitoring cost, which wastes labor resource in the North and thus impedes R&D activities. This effect cannot be caught in partial equilibrium models.

We showed that characteristics of developing countries affect transfer cost. However, another factor affecting transfer cost is possible. Analyzing the various aspects of the transfer cost may yield interesting interactions between strengthening IPR protection and economic growth. This is an important direction for future research.

4.6 Appendix

4.6.1 A1

In this appendix, we confirm the differential calculus of $X^N = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\zeta)}{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\zeta(\omega)^{1-\varepsilon}\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]}$ and $X^S = \frac{(\omega)^{-\varepsilon}\zeta\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\zeta(\omega)^{1-\varepsilon}\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]}$. As $\varepsilon > 1$, $\omega < 1$ and $\frac{\partial\omega}{\partial\delta} < 0$ are satisfied, we can easily show $\frac{\partial X^N}{\partial\zeta} = \frac{-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(\omega)^{1-\varepsilon}\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]}{\left\{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\zeta(\omega)^{1-\varepsilon}\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]\right\}^2} < 0$, $\frac{\partial X^N}{\partial\delta} = \frac{-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\zeta)\left\{\zeta(\omega)^{1-\varepsilon}\left[1-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\right]-(\varepsilon-1)(\omega)^{-\varepsilon}\zeta\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]\frac{\partial\omega}{\partial\delta}\right\}}{\left\{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\zeta(\omega)^{1-\varepsilon}\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]\right\}^2} < 0$ and $\frac{\partial X^S}{\partial\zeta} = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}(\omega)^{-\varepsilon}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}{\left\{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\zeta(\omega)^{1-\varepsilon}\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]\right\}^2} > 0$. Because we can easily show $\frac{\partial}{\partial\delta}\left\{\frac{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}}{\zeta(\omega)^{-\varepsilon}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}\right\} = \frac{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\zeta\varepsilon(\omega)^{-\varepsilon-1}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]\frac{\partial\omega}{\partial\delta}}{\left\{\zeta(\omega)^{-\varepsilon}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]\right\}^2} < 0$ and $\frac{\partial}{\partial\delta}\left\{\frac{\omega\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]}{\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}\right\} = \frac{-\omega\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}-\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}\right]+\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}(1-\delta)+\delta\right]\frac{\partial\omega}{\partial\delta}}{\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta} < 0$, $\frac{1}{X^S} = \frac{(1-\zeta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}}{\zeta(\omega)^{-\varepsilon}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]} + \frac{\omega\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]}{\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}$ is a decreasing function of δ. Then $\frac{dX^S}{d\delta} > 0$ is satisfied.

4.6.2 A2

Here we confirm that strengthening IPR protection always induces high growth when $f_F = 0$ (Lai (1998)'s economy). In this case, (83) and (84) are expressed as $g^N = \frac{M^N - X^N}{1+X^N}$ and $g^N = \frac{M^S - X^S}{X^S}$. Then we get the equilibrium value of ζ explicitly as $\zeta = \frac{\frac{M^S}{M^N+1}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}+\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\right]}{(\omega)^{-\varepsilon}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]-\frac{M^S}{M^N+1}(\omega)^{1-\varepsilon}\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}(1-\delta)+\delta\right]+\frac{M^S}{M^N+1}\left\{\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}+\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\right\}}$. Substituting this into $g^N = \frac{M^S - X^S}{X^S}$ yields $1 + g^N = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}}{\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}+\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\right]}\left\{\left(M^N + 1\right)\left(\frac{\varepsilon}{\varepsilon-1}\right) + M^S\omega\frac{\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}(1-\delta)+\delta\right]}{\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}\right\}$. As $\frac{\partial}{\partial\delta}\left\{\frac{\omega\left[(1-\delta)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}+\delta\right]}{\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}(1-\delta)+\delta\right]}\right\} < 0$, $\frac{dg^N}{d\delta} < 0$ is satisfied.

References

[1] Aghion, P., Harris, C., Vickers, J., 1997. Competition and growth with step-by-step innovation: An example, European Economic Review 41(3-5), 771{782

[2] Aghion, P., Howitt, P., 1992, A model of growth through creative destruction, Econometrica, 110, 323{351.

[3] Aghion, P., Howitt, P., 2008, The Economics of Growth, MIT Press.

[4] Antras, P., Desai, M., Foley,C.F., 2009, Multinational firms, FDI flows and imperfect capital markets, Quarterly Journal of Economics, 124, 1171{1219

[5] Arnold, L.G., 2002, On the growth effects of North–South trade: the role of labor market flexibility, Journal of International Economics, 58, 451{466

[6] Bartelsman, E.J., Doms, M., 2000. Understanding productivity: lessons from longitudinal microdata, Journal of Economic Literature 38(3), 569{594.

[7] Basu, S., 1996. Procyclical productivity: Increasing returns or cyclical utilization, Quarterly Journal of Economics 111, 709{751.

[8] Branstetter, L., Saggi,K., 2011, Intellectual property right, foreign direct investment and industrial development, Economic Journal, 121, 1161{1191

[9] Chou, C.F., Shy, O., 1991, An overlapping generations model of self-propelled growth, Journal of Macroeconomics, 13, 511{521

[10] Chou, C.F., Shy, O., 1993, The Crowding-out effects of long duration of patents, RAND Journal of Economics, 24, 304{312

[11] Chu, Angus, 2011, The welfare cost of one-size-fits-all patent protection, Journal of Economic Dynamics and Control, 35, 876{890.

[12] Boucekkine, R., de la Croix, D., 2003, Information technologies, embodiment and growth, Journal of Economic Dynamics and Control, 27, 2007{2034.

[13] Boucekkine, R., Del Rio, F., Licandro, O., 2003, Embodied Technological Change, Learning-by-doing and the Productivity Slowdown, The Scandinavian Journal of Economics, 105, 87{97.

[14] Bresnahan, T., Trajtenberg, M., 1995, General purpose technologies: \Engines of growth"?, Journal of Econometrics, 65, 83{108.

[15] Cheng, L., Dinopoulos, E., 1996, A multisectoral general equilibrium model of Schumpeterian growth and fluctuations, Journal of Economic Dynamics and Control, 20, 905–923.

[16] Doi, J., Mino, K., 2005, Technological spillovers and patterns of growth with sector-specific R&D, Journal of Macroeconomics, 27, 557{578.

[17] Dinopoulos, E., Segerstrom, P., 2010, Intellectual property rights, multinational firms and economic growth, Journal of Development Economics, 92, 13{27

[18] Dinopoulos, E., Unel, B., 2011, Quality Heterogeneity and Global Economic Growth, European Economic Review, 55, 595-612.

[19] Etro, F., 2004. Innovation by leaders. Economic Journal 114(495), 281{303.

[20] European Commission, 2010. Europe 2020 Flagship Initiative { Innovation Union. COM(2010) 546 final.

[21] Fors, G., 1997, Utilization of R&D results in the home and foreign plants of multinationals, Journal of International Economics, 45, 342{358

[22] Glass, A.J., Saggi, K., 2002, Intellectual property rights and foreign direct investment, Journal of International Economics, 56, 387{410

[23] Glass, A.J., Wu,X., 2007, Intellectual property rights and quality improvement, Journal of Development Economics, 82, 393{415

[24] Greenwood, J., Yorukoglu, M., 1997, 1974, Carnegie-Rochester Conference Series on Public Policy 46, 49{95.

[25] Grossman, G., Helpman, E., 1991, Innovation and Growth in the Global Economy, MIT Press.

[26] Helpman, E., 1993, Innovation, imitation, and intellectual property rights, Econometrica, 61, 1247{1280

[27] Helpman, E. and Trajtenberg, M., 1998, A time to sow and a time to reap: General purpose technologies, General Purpose Technologies and Economic Growth, 55{84.

[28] Hbijin, B., Jovanovic, B., 1999, The information-technology revolution and the stock market, American Economic Review, 89, 116{122.

[29] Hbijin, B., Jovanovic, B., 2001, The IT revolution and the stock market: Evidence, American Economic Review, 91, 1203{1220.

[30] Holmstrom, B., Torple, J., 1997, Financial intermediation, loanable funds, and the real sector, Quarterly Journal of Economics, 112, 663{691

[31] Hornstein, A., Krusell, P., 1996, Can technology improvements cause productivity slowdowns? In NBER Macroeconomics Annual 11, NBER Chapters, 209{276. National Bureau of Economic Research, Inc.

[32] Jovanovic, B., Rousseau, P., 2005, General purpose technologies, Handbook of Economic Growth, volume 1, Part B, 1181{1224.

[33] Kuwahara, S., 2013, Dynamical analysis of the R&D-based growth model with a regime switch, Journal of Economics, 108, 35{57

[34] Lai, E. L.-C., 1998, International intellectual property rights protection and the rate of product innovation, Journal of Development Economics, 55, 133{153

[35] Li, C.-W., 2000, Endogenous vs. semi-endogenous growth in a two-R&D-sector model, The Economic Journal, 110, 109{122.

[36] Mehra, R., Prescott, E., 1985. The equity premium: A puzzle, Journal of Monetary Economics 15, 145{161.

[37] Minniti, A., Parello, C., Segerstrom, P., (Forthcoming), A Schumpeterian growth model with random quality improvements, Economic Theory.

[38] Mondal, D., Gupta, M.R., 2007, Intellectual property rights protection and unemployment in a North{South model: A theoretical analysis, Economic Modeling, 25, 463{484

[39] Mondal, D., Gupta, M.R., 2009, Endogenous imitation and endogenous growth in a North{South model: A theoretical analysis, Journal of Macroeconomics, 31, 668{684

[40] Norrbin, S. C., 1993. The relationship between price and marginal cost in US industry: A contradiction, Journal of Political Economy 101, 1149{1164.

[41] Parello, C.P., 2008, A north{south model of intellectual property rights protection and skill accumulation, Journal of Development Economics, 85, 253{281

[42] Romer, P., 1990, Endogenous technological change, Journal of Political Economy, 98, 71{102.

[43] Scherer F. M., 1965. Firm size, market structure, opportunity, and the output of patented inventions, American Economic Review 55, 1097{1123.

[44] Scherer F. M., 1983. R&D and declining productivity growth, American Economic Review , 73(2), 215{218.

[45] Segerstrom, P., 1998, Endogenous growth without scale effects, American Economic Review, 88, 1290–1310.

[46] Segerstrom, P. S., 2007. Intel economics. International Economic Review 48(1), 247{280.

[47] Segerstrom, P. S., Zornierek, ., 1999. The R&D incentives of industry leaders. International Economic Review 40(3), 745{766.

[48] Smulders, S., van de Klundert, T., 1995, Imperfect competition, concentration and growth with firm-specific R&D, European Economic Review, 39, 139{160.

[49] Startz, R., 1998, Growth states and shocks, Journal of Economic Growth, 3, 203{216.

[50] Steger, T., 2003, The Segerstrom model: Stability, speed of convergence and policy implications, Economic Bulletin, 15, 1{8.

[51] Tanaka, H., Iwaisako, T., Futagami, K., 2007, Dynamic analysis of innovation and international transfer of technology through licensing, Journal of International Economics, 73, 189{212

[52] Tanaka, H., Iwaisako, T., 2009, Product cycles, endogenous skill acquisition, and wage inequality, Canadian Economics Association, 42, 300{331

[53] Tanaka, H., Iwaisako, T., 2014, Intellectual property rights and foreign direct investment: A welfare analysis, European Economic Review, 67, 107{124

[54] Thompson, P., Waldo, D., 1994. Growth and trustified capitalism. Journal of Monetary Economics 34(3), 445{462.

[55] Vernon, R., 1966, International investment and international trade in the product cycle, Quarterly Journal of Economics, 80, 190{207

[56] Yang, G., Maskus, K.H., 2001, Intellectual property rights, licensing, and innovation in an endogenous product-cycle model, Journal of International Economics, 53, 169{187