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Simulation of Dynamic Crack Propagation in Elastic Plates Using an Interface Element (Report II) †

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Abstract

The stability of the initial crack under static load is analyzed using the proposed interface element. Based on the understanding of the phenomena at the transition from the static and the dynamic behavior, the influence of various factors, such as the pre-loading, the surface energy, the bonding strength and the size of element on the crack speed under steady propagation is investigated.

KEY WORDS: (Crack initiation)(Crack propagation)(Crack speed)(Interface element)(Stability of crack) (Pre-loading)(Interface energy)(Bonding strength)(Elastic problem)(Center crack)

1. Introduction

Failures of structures and machine parts generally involve the formation and the propagation of cracks. It is important to study the behavior of cracks under dynamic as well as static loads for the rational design of structures. The formation and the propagation of the crack are the creation of new surfaces. Thus, to study the behavior of a crack, the mechanical modeling of surface formation is necessary. The authors proposed a combined model which consists of ordinary FEM representing the domain and the interface element which models the formation of crack surfaces in a simple manner. The proposed method was applied to the peeling problem of bonded thin elastic plates and the crack propagation in elastic-plastic materials1), 2). However, the influence of the property of the interface element on the computed crack propagation behavior was examined only for the static peeling problem1). It was shown that the crack extension behavior is governed only by the surface energy and other parameters, such as those describing the interface element and the mesh size have no influence. Thus, the crack propagation problem of a center crack elastic specimen is taken as an example, and the effects of various parameters on the crack propagation are closely examined. The parameters examined are pre-loading, mesh size, surface energy and bonding strength. In addition to these, the criteria for the crack initiation which is defined as the limit of the stability under static loading is also investigated.

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2. Crack Propagation Analysis Using an Interface Element

The interface element represents the interaction between surfaces. The interaction is assumed to be defined through the interface potential per unit area ϕ and the Lennard-Jones type function is selected as the function, i.e.,

$$\phi(\delta) = 2\gamma\left\{\left(\frac{r_0}{r_0+\delta}\right)^{2N} - 2\left(\frac{r_0}{r_0+\delta}\right)^{N}\right\} \quad (1)$$

$$\sigma_0 = \frac{4\gamma N}{r_0}\left\{\left(\frac{N+1}{2N+1}\right)^{\frac{N+1}{N}} - \left(\frac{N+1}{2N+1}\right)^{\frac{2N+1}{N}}\right\} \quad (2)$$

where, δ is the opening displacement. The parameters γ, N and r_0 characterize the properties of the surface. In particular, γ represents the surface energy per unit area and r_0 is the scale parameter. The derivative of the potential ϕ with respect to the opening displacement δ gives the stress acting between the surfaces. As shown in **Fig. 1**, stress σ increases with the increase of the opening displacement and reaches its maximum value σ_0 when $d\phi/d\delta=0$. With a further increase of the opening displacement, the stress rapidly decreases and loses the interaction between the surfaces. The maximum stress

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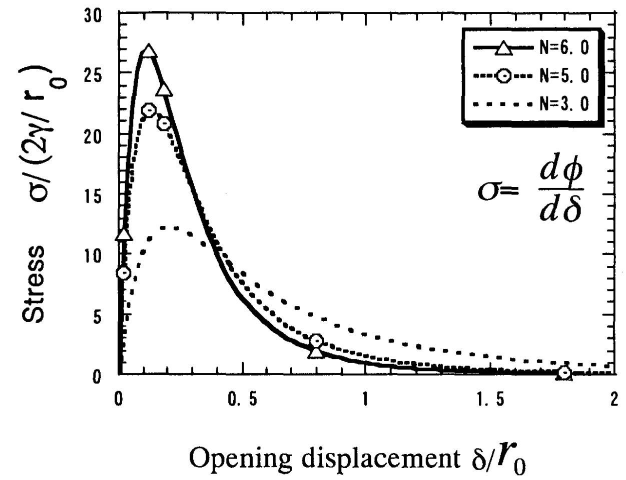

Fig.1 Relation between crack opening displacement and bonding stress.

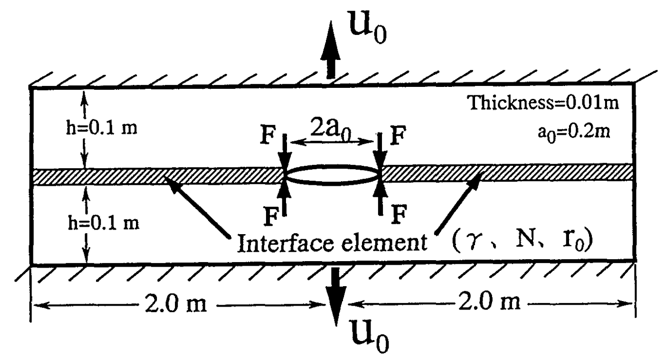

Fig.2 Model for dynamic crack propagation.

σ_0 is referred to as bonding strength in the following discussion.

The interface element, which is a nonlinear spring in nature, can be modeled using the interface potential ϕ which has the property shown in Fig. 1. By arranging such interface elements along the path of the crack, the simulation of free crack propagation under arbitrary loading becomes possible. In this approach, the judgement of crack extension based on the comparison between the crack driving force and the resistance is not necessary, unlike the conventional methods.

3. Crack Propagation Problem in Elastic Plate with Center Crack

3.1 Model for analysis

The model to be analyzed is an elastic plate with a center crack shown in **Fig. 2.** To obtain the steady state of crack propagation, a plate which is wide in the direction of crack propagation, is analyzed. The length, the width and the thickness are 200 mm, 4,000 mm and 10 mm, respectively. The crack is forced to start gently by the following method. A pair of concentrated or distributed loads F are applied on the crack tip to control the starting of the crack propagation. The pre-loading is statically applied as the displacement u_0 prescribed on the top and bottom edges of the specimen. The crack is gently initiated by removing the force F. Then, the crack propagation is analyzed as a dynamic problem. The length of the initial crack is 400 mm and it is assumed that the crack propagates in the direction of the initial crack. Considering the symmetry, 1/8 of the specimen is analyzed using eight-node solid elements.

Table 1 Mechanical properties of elastic plate.

Young's modulus E (GPa)	Poisson's ratio ν	Density ρ (kg/m^3)	Yield stress σ_{Y0} (MPa)
210	0.3	7.85×10^3	∞ (elastic)

Table 2 Parameters for interface potential of elastic plate.

	Surface energy per unit area γ (kN/m)	r_0 (mm)	N	σ_0 (MPa)
Interface element	50 (5 ~ 80)	0.03	6	8955.3
		0.05		5373.4
		0.08		3358.1

The mechanical properties of the plate and the parameters defining the property of the interface elements are given in **Tables 1** and **2.** The time increment used for this analysis is 0.1 μs.

3.2 Stability of crack under static load

Before looking into the dynamic crack propagation problem, the critical displacement under which the initial crack loses its stability is examined. **Fig. 3** shows the effects of the surface energy γ and the size parameter r_0 involved in the interface potential on the value of the critical pre-stretching displacement u_{cr}. The computed results for the cases in which the size of the element is 10 mm are shown. In general, the critical displacement u_{cr} increases when the interface energy γ is large and the

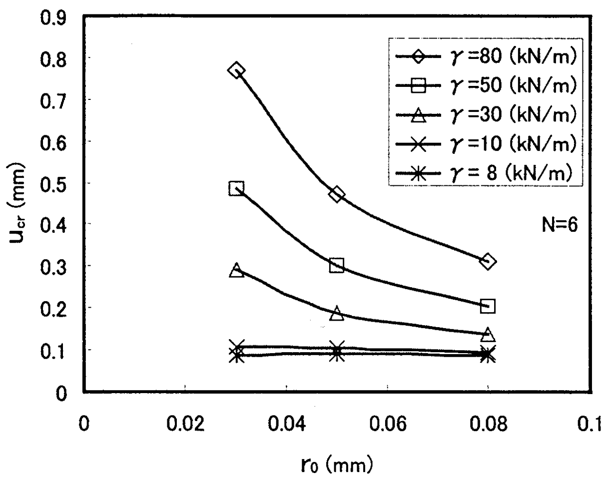

Fig.3 Influence of γ and r_0 on critical displacement under static load.

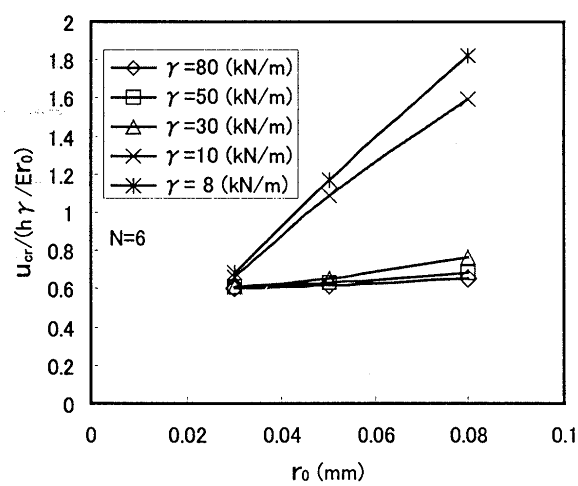

Fig.4 Influence of γ and r_0 on normalized critical displacement under static load based on stress criterion.

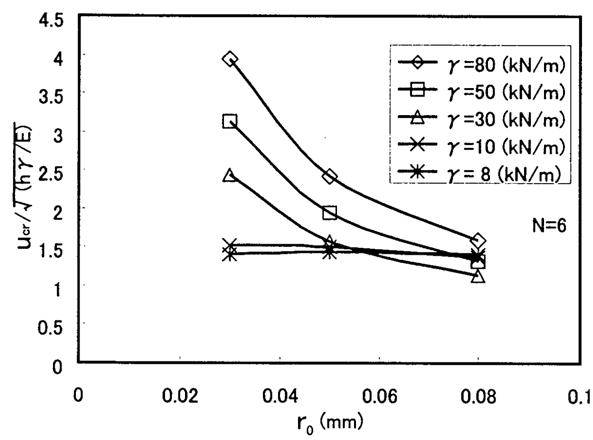

Fig.5 Influence of γ and r_0 on normalized critical displacement under static load based on energy criterion.

scale parameter r_0 is small. Further, the critical displacements are normalized in **Figs. 4** and **5**. In Fig. 4, the critical displacement is normalized by γ/Er_0 which is proportional to the bonding strength σ_0. It is observed that the normalized critical displacements converge to almost the same value when the interface energy γ is greater than 30 kN/m. In these cases, the statically loaded crack becomes unstable when the stress at the crack tip reaches the critical value. The critical stress is the bonding strength σ_0 which is proportional to γ and inversely proportional to r_0. In Fig. 5, the critical displacement is normalized based on the idea that the crack propagation is controlled by the interface energy. The critical stress is normalized by $(h\gamma/E)^{1/2}$. In this case, the normalized critical displacements tend to converge to a single value with the increase of the scale parameter r_0 when the interface energy is smaller than 10 kN/m. Figures 4 and 5 tell us that the instability of statically loaded cracks can be divided into two groups, namely those controlled by stress and energy. The detail will be discussed later.

Figure 6 shows the influence of mesh size on the critical displacement u_{cr} when the scale parameter r_o is 0.05 mm. When the interface energy g is larger than 30 kN/m, the critical displacement becomes small with the decrease of element size. This can be explained since the stress at the crack tip becomes large when the element size becomes small. However, the phenomenon becomes quite different when the surface energy is less than 10 kN/m. Figure 6 is also normalized in the same manner as in the case of Fig. 3. The critical displacements are normalized with respect to the bonding strength and the interface energy in **Figs. 7** and **8**, respectively. It is seen from both figures that the

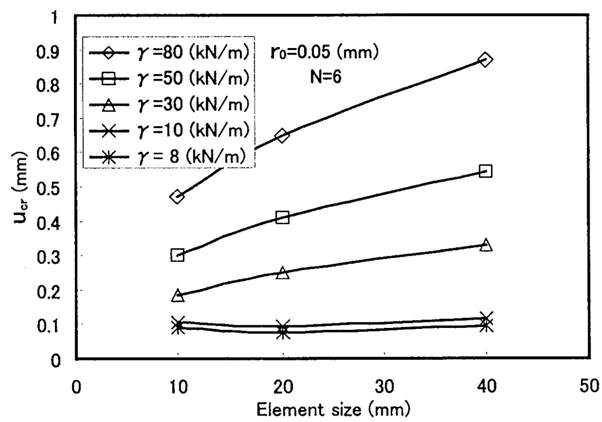

Fig.6 Influence of mesh size on critical displacement under static load.

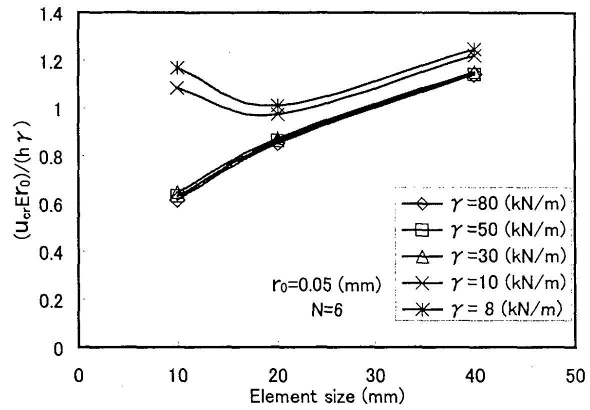

Fig.7 Influence of mesh division on normalized critical displacement under static load based on stress criterion.

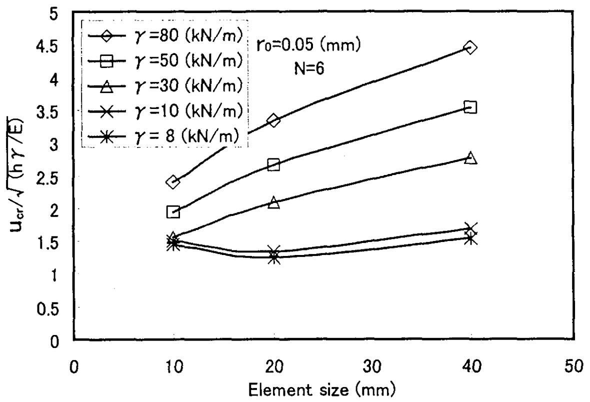

Fig.8 Influence of mesh division on normalized critical displacement under static load based on energy criterion.

curves are clearly divided into two groups depending on whether the interface energy is less than 10 kN/m or not. To clarify the reason for this, the deformations just before the loss of the static stability are closely examined. The deformations for the cases with γ=80 kN/m and 8 kN/m in which the size of the element is 10 mm are shown in **Fig. 9**. When the interface energy is 80 kN/m, no crack extension is observed. While the crack extends one element when the interface energy is 8 kN/m. This implies that when the size of the element is relatively small compared to the interface energy, the crack extents in a stable manner due to the stress concentration at the crack tip and the crack becomes unstable without the stable crack growth when the interface energy is relatively large.

In case of the real situation, it is more realistic to think that the crack extends till the critical length due to the strong stress concentration and the unstable crack growth occurs after the stable crack growth. Thus, the computed results with small interface energy and large

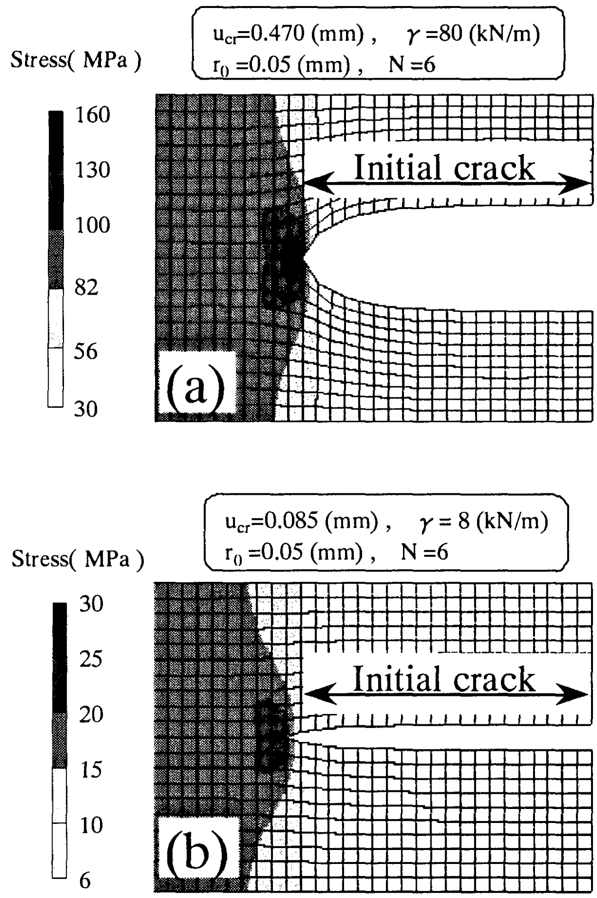

Fig.9 Deformation of the crack tip and stress distribution.

scale parameter r_0 are more reasonable between the two groups. Selecting the cases which satisfies this condition in Figs. 6, 7 and 8, the influence of the mesh size on the critical displacement u_{cr} is relatively small.

3.3 Effect of Pre-loading on crack propagation speed

Figure 10 shows the effect of pre-loading on the time histories of crack propagation speed when the element size is 10 mm. With an increase of pre-loading, the crack speed is accelerated quickly and reaches the steady state in a short time. On the contrary, it takes long time to reach the steady state when the pre-loading is small. The average values of the crack propagation speed between 350 μs and 450 μs are plotted in **Fig. 11.** When the pre-loading is small, such as the case with u_0=0.25 mm, the crack does not propagate. The crack speed in the steady state increases rapidly with preloading when u_0 is greater than 0.3 mm. As it is theoretically pre-dicted, It tends to converge to the Rayleigh wave speed[3), 4)]. In case of the plane strain problem Rayleigh wave speed is given by the following equation.

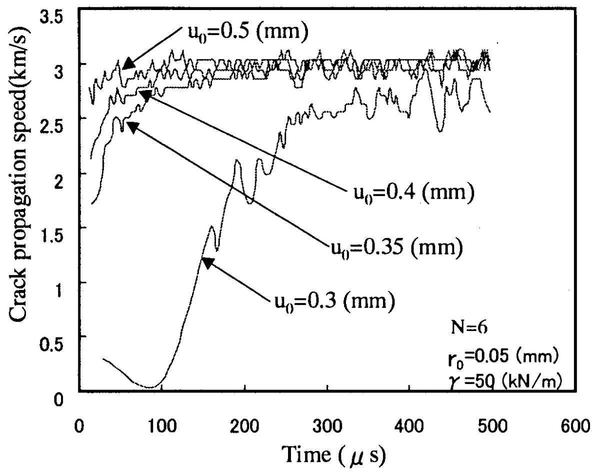

Fig.10 Influence of pre-stress on transient crack propagation speed.

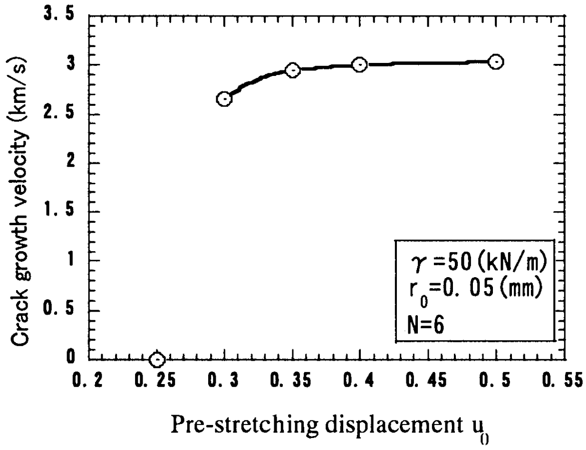

Fig.11 Relation between pre-stress and crack propagation speed.

$$V_R = \frac{(0.862 + 1.14\nu)}{(1+\nu)} \left\{ \frac{E}{2(1+\nu)\rho} \right\}^{\frac{1}{2}} \qquad (3)$$
$$= 2.97\text{km/s}$$

Though it does not directly correspond to the present three-dimensional model, it gives the information as a reference.

3.4 Influence of bonding strength on crack propagation

According to Eq.2, the bonding strength is inversely proportional to the scale parameter r_0. Thus, the influence of the bonding strength on the crack propagation is examined by changing r_0. The mesh size is 10 mm and the serial computations are done for cases

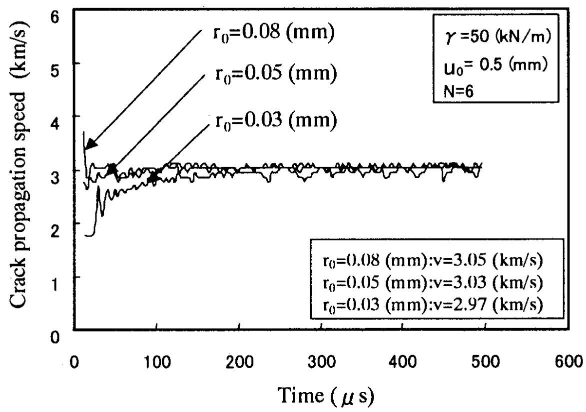

Fig.12 Influence of r_0 on transient crack propagation speed (γ=50 kN/m, u_0=0.5 mm).

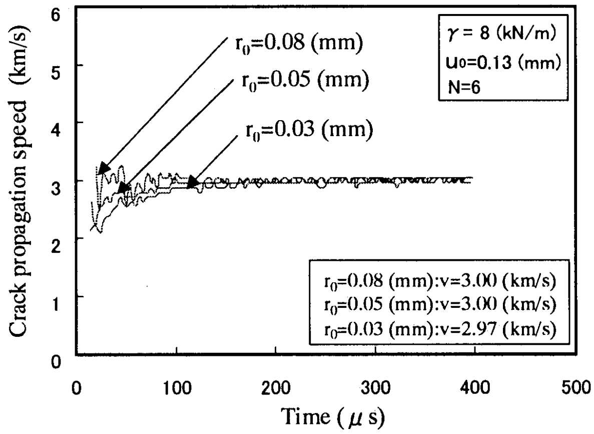

Fig.13 Influence of r_0 on transient crack propagation speed (γ=8 kN/m, u_0=0.13 mm).

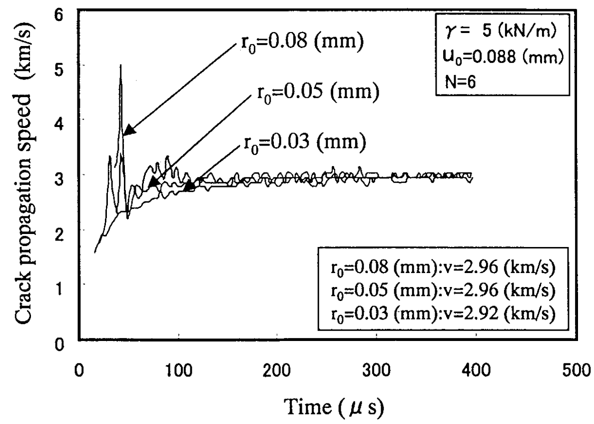

Fig.14 Influence of r_0 on transient crack propagation speed (γ=5 kN/m, u_0=0.088 mm).

with (γ=50 kN/m, u_0=0.5 mm), (γ=8 kN/m, u_0=0.13 mm) and (γ=5 kN/m, u_0=0.88 mm). **Figures 12, 13 and 14** show the time histories of crack speed for the three serial computations. It is generally observed that acceleration of the crack speed is slow when the scale parameter r_0 is small. The crack speeds shown in these figures are the average speed between 300 ms and 350 μs. When the scale parameters are large such as the case of r_0=0.08 mm and r_0=0.05 mm, the crack propagation speeds almost coincide each other, as shown in Figs. 12, 13 and 14. This implies that the influence of the scale parameter on the crack propagation is small when it is relatively large. Its influence becomes large when the scale parameter is less than 0.03 mm or when the interface energy is large such as in the case of γ=50 kN/m. This phenomenon can be related to those observed for the stability of the statically loaded crack. When the interface energy is small, the scale parameter is large and the element size is sufficiently small so that the stable crack extension occurs before the loss of stability. The dynamic crack propagation in the steady state is also governed by the interface energy as in the static peeling problem. In other words, the bonding strength σ_0 influences the crack propagation in the acceleration stage but its effect in the steady state is small.

4. Conclusions

The crack propagation in the center cracked elastic specimen is analyzed using the proposed interface element and the influences of various factors are closely examined. The following conclusions are drawn.

1. If the critical displacement is defined as the displacement at which the initial crack becomes unstable under the static loading, the types of critical state are divided into two groups. When the interface energy is large, the critical state is determined by the stress at the crack tip and it is proportional to the interface energy and inversely proportional to the scale parameter r_0. Conversely, when the interface energy is small, the critical state is determined by the energy release rate and the critical displacement is proportional to γ1/2.
2. The computed critical state can be divided into two types. In case of the first type, the crack becomes unstable without stable crack growth. In case of the second type, the crack becomes unstable after the stable crack growth. When the element size is small enough, the latter type appears. If the computed results are improved by reducing the size of the element, the critical state of the latter type can be expected in the fracture of real materials.
3. The speed of the crack under dynamic propagation accelerates quickly and reaches a steady state in a short time when the pre-loading is large. Conversely, it accelerates slowly when the pre-loading is small. Though the steady state of crack propagation may not be obtained when a model with finite width is used, the speed of crack propagation increases with pre-loading and tends to converge to Rayleigh wave speed.
4. When the interface energy is small, the scale parameter is large and the element size is sufficiently small so that the stable crack extension occurs before the loss of stability, the dynamic crack propagation is also governed by the interface energy, as in the peeling problem. The crack speed under steady propagation is almost the same regardless of the value of bonding strength when the interface energy is the same.

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