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Secure Implementation in Discrete and Excludable Public Good Economies*

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Abstract

This paper studies secure implementability [Saijo, T., T. Sjöström, and T.Yamato (2007), "Secure Implementation," *Theoretical Economics* 2, pp.203-229] on the provision of one discrete and excludable public good with cost shares. Our main result shows that only constant social choice functions are securely implementable in standard quasi-linear environments.

Key words: Secure implementation, Dominant strategy implementation, Nash implementation, Discrete public good, Strategy-proofness.
JEL classification: C72, D61, D71, H41

1 Introduction

Secure implementability, introduced by Saijo, Sjöström, and Yamato (2007), is a solution concept in implementation theory.[1] This requires that there exists a mechanism where (i) each dominant strategy equilibrium induces socially optimal outcome and (ii) each Nash equilibrium also induces socially optimal outcome, that is, double implementability in dominant strategy equilibria and Nash equilibria.[2] This concept is considered to be a benchmark of constructing mechanisms working well in laboratory experiments.[3]

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1 See Jackson (2001) and Maskin and Sjöström (2002) for implementation theory.

2 See Saijo, Sjöström, and Yamato (2007) for a formal definition of secure implementability and the motivation for secure implementation. See also Mizukami and Wakayama (2007) and Saijo, Sjöström, and Yamato (2007) for a characterization of dominant strategy implementable social choice functions and Maskin (1977) for a characterization of Nash implementable social choice functions.

3 See Cason, Saijo, Sjöström, and Yamato (2006) for an experimental result on secure implementation.

Saijo, Sjöström, and Yamato (2007) characterize securely implementable social choice functions by strategy-proofness, which is a necessary condition for dominant strategy implementation, and the rectangular property (Saijo, Sjöström, and Yamato, 2007), which is a stronger condition than non-bossiness (Satterthwaite and Sonnenschein, 1981).[4] Strategy-proofness requires that the truthful revelation is a weakly dominant strategy for each agent in the direct revelation mechanism associated with the social choice function. This concept is a standard incentive property in social choice theory.[5] The rectangular-property requires that if each agent cannot change his utility by his revelation, then the outcome cannot change by the all agents' revelation in the direct revelation mechanism associated with the social choice function. In direct revelation mechanism associated with a social choice function satisfying strategy-proofness, the rectangular property requires that if each agent has a best response which is different from truthful revelation, then the outcome induced by Nash equilibrium associated with such best responses coincides with the one induced by truthful revelations.

In the previous literature, secure implementability is considered in some environments: single-peaked voting environments (Berga and Moreno, 2009; Saijo, Sjöström, and Yamato, 2007), public good economies (Saijo, Sjöström, and Yamato, 2007), production economies (Kumar, 2009), allotment economies (Bochet and Sakai, 2010), Shapley-Scarf housing markets (Fujinaka and Wakayama, 2010), and the assignment of indivisible and private goods with monetary transfers (Fujinaka and Wakayama, 2008).[6] Unfortunately, almost all of these studies show negative results, that is, there is rarely non-trivial securely implementable social choice function. On receiving these results, it is interesting to investigate which environments have non-trivial securely implementable social choice functions. This paper studies such a problem.

This paper is closely related to two papers written by Saijo, Sjöström, and Yamato (2007) and Fujinaka and Wakayama (2008). Saijo, Sjöström, and Yamato (2007) consider the provision of one discrete and non-excludable public good with cost shares. They show that when each agent has a quasi-linear utility function with a concave valuation function, there is no securely implementable and surplus-maximizing social choice function.[7] In our model, we consider the provision of one discrete and excludable public good with cost shares. Examples of such provisions include public facilities (e.g. highways and museums) and public services (e.g. train, bus, and plane services per hour) as long as they are not congested. Information goods (e.g. software and audio-visual contents) and intellectual properties (e.g. patented technologies and copyrighted pieces) are also included in such examples. Theoretically, we have more strategy-proof social choice functions in excludable public good economies than non-excludable public good economies.[8] Moreover, we require each agent's

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4 See Mizukami and Wakayama (2008) for an alternative characterization of securely implementable social choice functions in terms of a version of monotonicity (Maskin, 1977).

5 See Barberà (2010) for social choice theory related to strategy-proofness.

6 See also Saijo, Sjöström, and Yamato (2003) for examples of non-secure direct revelation mechanisms.

7 If the social choice function is surplus-maximizing, then it maximizes the sum of each agents' valuations of the public good.

8 See Deb and Razzolini (1999), Moulin (1994), and Ohseto (2000) for examples of strategy-proof social choice functions in excludable public good economies.

valuation functions of strict concavity and strict increasingness.[9] In such a model, we characterize securely implementable social choice functions. Fujinaka and Wakayama (2008) consider the assignment of indivisible and private goods with monetary transfers. They show that when each agent has a quasi-linear utility function and the set of profiles of valuation functions of private goods satisfies minimal richness (Fujinaka and Wakayama, 2008), only constant social choice functions are securely implementable. Theoretically, our model is different from theirs, so our result is not obvious. However, our approach is similar to theirs.

In this paper, we characterize the class of securely implementable social choice functions in our model: when each agent has a quasi-linear utility function with a strictly concave and strictly increasing valuation function, the social choice function is securely implementable if and only if it is constant. This result is stronger than the result of Saijo, Sjöström, and Yamato (2007) since each agent's utility functions are more restrictive in our model than theirs and our result is the characterization of securely implementable social choice functions in discrete and excludable public good economies.

This paper is organized according to the following sections. In Section 2, our model is introduced. We define properties of social choice functions related to secure implementability in Section 3. Some preliminary results on these properties are shown in Section 4. In Section 5, we show our main result. Conclusion is in Section 6.

2 Model

Let $I \equiv \{1, \ldots, n\}$ $(n \geq 2)$ be a set of **agents**. Let $Y \subseteq \mathbb{Z}_+$ be a set of **production levels of the public good** and $c\colon Y \to \mathbb{R}_+$ be a **cost function**. For each $i \in I$, let $y_i \in Y$ be **consumption of the public good for agent i**. By excludability of the public good, we allow for $y_i \neq y_j$ for some $i, j \in I$ with $i \neq j$. For each $i \in I$, let $x_i \in \mathbb{R}_+$ be a **cost share of the public good for agent i**. For each $i \in I$, let $(y_i, x_i) \in Y \times \mathbb{R}_+$ be a **consumption bundle for agent i**. Let

$$Z \equiv \left\{ ((y_1, x_1), \ldots, (y_n, x_n)) \;\middle|\; (y_i, x_i) \in Y \times \mathbb{R}_+ \text{ for each } i \in I \text{ and } c\left(\max\{y_i\}_{i \in I}\right) \leq \sum_{i \in I} x_i \right\}$$

be the set of **feasible allocations**.

For each $i \in I$, let $u_i\colon Y \times \mathbb{R}_+ \to \mathbb{R}$ be a **utility function for agent i**. We assume that for each $i \in I$, there exists $v_i\colon Y \to \mathbb{R}_+$, called a **valuation function of the public good for agent i**, such that for each $(y_i, x_i) \in Y \times \mathbb{R}_+$,

$$u_i(y_i, x_i) = v_i(y_i) - x_i.$$

We also assume that v_i is strictly concave and strictly increasing for each $i \in I$. For each $i \in I$, let V_i be the set of all valuation functions of the public good for agent i. Let $V \equiv \prod_{i \in I} V_i$ be the **domain** and $v \equiv (v_1, \ldots, v_n) \in V$ be a profile of valuation functions of the public good. For each $i \in I$, let $v_{-i} \equiv (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n) \in V_{-i} \equiv \prod_{j \neq i} V_j$ be a profile of valuation functions of the public

[9] Such requirements are standard in excludable public good economies except for quasi-linearity.

good other than agent i.

Let $f\colon V \to Z$ be a **social choice function**. For each $v \in V$ and each $i \in I$, let $(y_i(v), x_i(v)) \in Y \times \mathbb{R}_+$ be the consumption bundle for agent i associated with a social choice function f at v.

3 Properties of Social Choice Functions

Saijo, Sjöström, and Yamato (2007) introduce secure implementation that is identical with double implementation in dominant strategy equilibria and Nash equilibria. They show that the social choice function is **securely implementable** if and only if it satisfies **strategy-proofness** and the **rectangular property** (Saijo, Sjöström, and Yamato, 2007). In this paper, we consider securely implementable social choice functions in our model.

Strategy-proofness requires that the truthful revelation is a weakly dominant strategy for each agent in the direct revelation mechanism associated with the social choice function.

Definition 1. The social choice function f satisfies **strategy-proofness** if for each $v, v' \in V$ and each $i \in I$,

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) \geq v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}).$$

The rectangular-property requires that if each agent cannot change his utility by his revelation, then the allocation cannot change by the all agents' revelation in the direct revelation mechanism associated with the social choice function.

Definition 2. The social choice function f satisfies the **rectangular property** if for each $v, v' \in V$,

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}) \text{ for each } i \in I \Rightarrow f(v) = f(v').$$

4 Preliminary Results

In what follows, we show some preliminary results on strategy-proofness and the rectangular property in our model.

Remark 1. We have four lemmas in this section. Lemmas 1, 2, and 4 does not depend on any properties of valuation functions. Lemma 3 only depends on strict increasingness of valuation functions.

4.1 Strategy-Proofness

Lemma 1 shows that each agent's cost shares of the public good depend on his consumption of the public good if the social choice function satisfies strategy-proofness.

Lemma 1. *If the social choice function f satisfies* ***strategy-proofness****, then for each $v, v' \in V$ and* *each* $i \in I$,

$$y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i}) \;\Rightarrow\; x_i(v_i, v'_{-i}) = x_i(v'_i, v'_{-i}).$$

Proof. Suppose, by contradiction, that there exist $v, v' \in V$ and $i \in I$ such that $y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i})$ and $x_i(v_i, v'_{-i}) \neq x_i(v'_i, v'_{-i})$. If $x_i(v_i, v'_{-i}) > x_i(v'_i, v'_{-i})$, then

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) < v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}),$$

which is a contradiction to **strategy-proofness**. If $x_i(v_i, v'_{-i}) < x_i(v'_i, v'_{-i})$, then

$$v'_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v'_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}),$$

which is a contradiction to **strategy-proofness**. □

By Lemma 1, we have the following lemma immediately.

Lemma 2. *If the social choice function* f *satisfies* ***strategy-proofness****, then for each* $v, v' \in V$ *and each* $i \in I$,

$$v_i(y(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v_i(y(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}) \;\Rightarrow\; y(v_i, v'_{-i}) \neq y(v'_i, v'_{-i}).$$

Proof. Suppose, by contradiction, that there exist $v, v' \in V$ and $i \in I$ such that $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$ and $y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i})$. By Lemma 1, we have $x_i(v_i, v'_{-i}) = x_i(v'_i, v'_{-i})$. This implies

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}),$$

which is a contradiction. □

Lemma 3 shows that the more each agent consumes the public good, the more he shares the cost of the public good if the social choice function satisfies strategy-proofness and his valuation functions of the public good are strictly increasing.

Lemma 3. *If the social choice function* f *satisfies* ***strategy-proofness****, then for each* $v, v' \in V$ *and each* $i \in I$,

$$y_i(v_i, v'_{-i}) < y_i(v'_i, v'_{-i}) \;\Rightarrow\; x_i(v_i, v'_{-i}) < x_i(v'_i, v'_{-i}).$$

Proof. Suppose, by contradiction, that there exist $v, v' \in V$ and $i \in I$ such that $y_i(v_i, v'_{-i}) < y_i(v'_i, v'_{-i})$ and $x_i(v_i, v'_{-i}) \geq x_i(v'_i, v'_{-i})$. Since $v_i(y_i(v_i, v'_{-i})) < v_i(y_i(v'_i, v'_{-i}))$ by strict increasingness of v_i, we have

$$v_i(y(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) < v_i(y(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}),$$

which is a contradiction to **strategy-proofness**. □

4.2 Rectangular Property

Lemma 4 shows that each agent's consumption of the public good depends on his utility if the social choice function satisfies the rectangular property.[10]

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[10] This lemma holds even if the rectangular property is replaced by non-bossiness (Saijo, Sjöström, and Yamato, 2007), which is weaker than the rectangular property. The social choice function f satisfies **non-bossiness** if for each $v, v' \in V$ and each $i \in I$, $v_i(y_i(v_i, v_{-i})) - x_i(v_i, v_{-i}) = v_i(y_i(v'_i, v_{-i})) - x_i(v'_i, v_{-i}) \Rightarrow (y_i(v_i, v_{-i}), x_i(v_i, v_{-i})) =$

Lemma 4. *If the social choice function f satisfies the* **rectangular property***, then for each $v, v' \in V$ and each $i \in I$,*

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}) \;\Rightarrow\; y_i(v_i, v'_{-i}) = y_i(v'_i, v'_{-i}).$$

Proof. Suppose, by contradiction, that there exist $v, v' \in V$ and $i \in I$ such that

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}) \tag{1}$$

and $y_i(v_i, v'_{-i}) \neq y_i(v'_i, v'_{-i})$. Let $v'' \equiv (v''_i, v''_{-i})$ be such that $(v''_i, v''_{-i}) = (v_i, v'_{-i})$. For i, since (1) holds and $v''_i = v_i$, we have

$$v''_i(y_i(v''_i, v'_{-i})) - x_i(v''_i, v'_{-i}) = v''_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}). \tag{2}$$

For each $j \in I \setminus \{i\}$, since $v''_j = v'_j$, we have

$$v''_j(y_j(v''_j, v'_{-j})) - x_j(v''_j, v'_{-j}) = v''_j(y_j(v'_j, v'_{-j})) - x_j(v'_j, v'_{-j}). \tag{3}$$

By (2), (3), and the **rectangular property**, we have $f(v'') = f(v')$. This implies $y_i(v''_i, v''_{-i}) = y_i(v'_i, v'_{-i})$, which is a contradiction to $y_i(v_i, v'_{-i}) \neq y_i(v'_i, v'_{-i})$. □

5 Main Result

To show our main result, we introduce some definitions. For each $i \in I$ and each $v'_{-i} \in V_{-i}$, let

$$O_i(v'_{-i}) \equiv \{y_i \in Y \mid \text{there exists } v_i \in V_i \text{ such that } y_i(v_i, v'_{-i}) = y_i\}$$

be the **option set for agent i given v'_{-i}**, that is, the set of consumption of the public good that agent i can induce given v'_{-i}. For each $i \in I$, each $(y_i, x_i) \in Y \times \mathbb{R}_+$, and each $v_i \in V_i$, let

$$ID(y_i, x_i; v_i) \equiv \{(y'_i, x'_i) \in Y \times \mathbb{R}_+ \mid v_i(y_i) - x_i = v_i(y'_i) - x'_i\}$$

be the **indifferent set for agent i with v_i at (y_i, x_i)**. Given $i \in I$, $(y_i, x_i) \in Y \times \mathbb{R}_+$, and $v_i \in V_i$, we have $x'_i = v_i(y'_i) - v_i(y) + x_i$ for each $(y'_i, x'_i) \in ID(y_i, x_i; v_i)$. Since x'_i depends on y'_i given (y_i, x_i) and v_i, let

$$\boldsymbol{x}_i(y'_i; (y_i, x_i), v_i) \equiv v_i(y'_i) - v_i(y_i) + x_i.$$

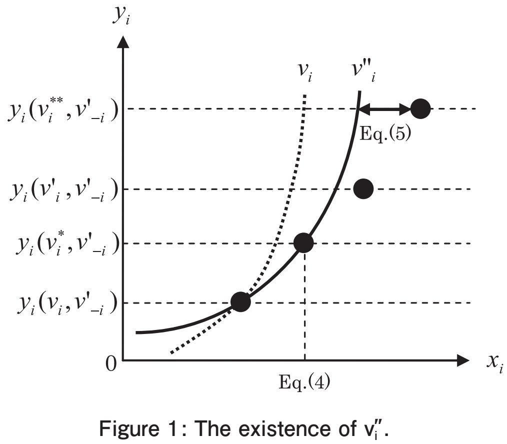

Figure 1: The existence of v''_i.

$(y_i(v'_i, v_{-i}), x_i(v'_i, v_{-i}))$. See Saijo, Sjöström, and Yamato (2007) for an alternative characterization of securely implementable social choice functions in terms of non-bossiness.

The social choice function satisfies **constancy** if for each $v, v' \in V$, $f(v) = f(v')$.

Proposition 1. *For each $i \in I$ and each $v_i \in V_i$, suppose that v_i is strictly concave and strictly increasing. The social choice function f satisfies* ***strategy-proofness*** *and the* ***rectangular property*** *if and only if it satisfies* ***constancy****.*

Proof. Since it is obvious that f satisfies strategy-proofness and the rectangular property if f satisfies constancy, we show that f satisfies constancy if f satisfies strategy-proofness and the rectangular property.

Suppose, by contradiction, that there exist $v, v' \in V$ such that $f(v) \neq f(v')$. By the **rectangular property**, there exists $i \in I$ such that $v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) \neq v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i})$. By **strategy-proofness**, we have

$$v_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v_i(y_i(v'_i, v'_{-i})) - x_i(v'_i, v'_{-i}).$$

By Lemma 2, we have $y_i(v_i, v'_{-i}) \neq y_i(v'_i, v'_{-i})$. By Lemma 1, there exists $x_i(v_i, v'_{-i}) \in \mathbb{R}_+$ corresponding to $y_i(v_i, v'_{-i})$.

Suppose that $y_i(v_i, v'_{-i}) < y_i(v'_i, v'_{-i})$. This implies that $O_i(v'_{-i}) \setminus [0, y_i(v_i, v'_{-i})]$ is not empty since $y_i(v'_i, v'_{-i}) \in O_i(v'_{-i})$. Let

$$y_i(v^*_i, v'_{-i}) \equiv \min O_i(v'_{-i}) \setminus [0, y_i(v_i, v'_{-i})].$$

By Lemma 1, there exists $x_i(v^*_i, v'_{-i}) \in \mathbb{R}_+$ corresponding to $y_i(v^*_i, v'_{-i})$. Let $y_i(v^{**}_i, v'_{-i}) \in O_i(v'_{-i}) \setminus \{y_i(v_i, v'_{-i}), y_i(v^*_i, v'_{-i})\}$. By Lemma 1, there exists $x_i(v^{**}_i, v'_{-i}) \in \mathbb{R}_+$ corresponding to $y_i(v^{**}_i, v'_{-i})$. Since all valuation functions for agent i are strictly concave and strictly increasing, there exists $v''_i \in V_i$ such that

$$\boldsymbol{x}_i(y_i(v^*_i, v'_{-i}); (y_i(v_i, v'_{-i}), x_i(v_i, v'_{-i})), v''_i) = x_i(v^*_i, v'_{-i}), \tag{4}$$

$$\boldsymbol{x}_i(y_i(v^{**}_i, v'_{-i}); (y_i(v_i, v'_{-i}), x_i(v_i, v'_{-i})), v''_i) < x_i(v^{**}_i, v'_{-i}). \tag{5}$$

Notice that (5) is guaranteed by the definition of $y_i(v^*_i, v'_{-i})$ and Lemma 3 since there is no consumption of the public good for agent i between $y_i(v_i, v'_{-i})$ and $y_i(v^*_i, v'_{-i})$, which is induced by herself and $x_i(v^*_i, v'_{-i}) < x_i(v^{**}_i, v'_{-i})$ holds if $y_i(v^*_i, v'_{-i}) < y_i(v^{**}_i, v'_{-i})$ (See Figure 1). By (4) and the definition of $\boldsymbol{x}_i$, we have

$$v''_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) = v''_i(y_i(v^*_i, v'_{-i})) - x_i(v^*_i, v'_{-i}). \tag{6}$$

By (5) and the definition of $\boldsymbol{x}_i$, we have

$$v''_i(y_i(v_i, v'_{-i})) - x_i(v_i, v'_{-i}) > v''_i(y_i(v^{**}_i, v'_{-i})) - x_i(v^{**}_i, v'_{-i}). \tag{7}$$

By (6), (7), and **strategy-proofness**, we have $y_i(v''_i, v'_{-i}) = y_i(v_i, v'_{-i})$ or $y_i(v''_i, v'_{-i}) = y_i(v^*_i, v'_{-i})$. If $y_i(v''_i, v'_{-i}) = y_i(v_i, v'_{-i})$, then, by (6), we have $v''_i(y_i(v''_i, v'_{-i})) - x_i(v''_i, v'_{-i}) = v''_i(y_i(v^*_i, v'_{-i})) - x_i(v^*_i, v'_{-i})$. This implies $y_i(v''_i, v'_{-i}) = y_i(v^*_i, v'_{-i})$ by Lemma 4, which is a contradiction since $y_i(v_i, v'_{-i}) \neq y_i(v^*_i, v'_{-i})$. By the same argument, we have a contradiction if $y_i(v''_i, v'_{-i}) = y_i(v^*_i, v'_{-i})$.

Suppose that $y_i(v_i, v'_{-i}) > y_i(v'_i, v'_{-i})$. In this case, we define $y_i(v^*_i, v'_{-i})$ as $\max O_i(v'_{-i}) \setminus [y_i(v_i, v'_{-i}), \infty)$ and have a contradiction by the same argument as the case of $y_i(v_i, v'_{-i}) < y_i(v'_i, v'_{-i})$. □

Proposition 1 is tight: Example 1 shows that strategy-proofness is necessary for Proposition 1 and Example 2 shows that the rectangular property is necessary for Proposition 1.

Example 1. Suppose that $Y = \mathbb{Z}_{++}$ and for each $i \in I$ and each $v_i \in V_i$, there exists $\theta_i \in \mathbb{Z}_{++}$ and $v_i(y_i) = \theta_i y_i^2$ for each $y_i \in Y$. Let f be such that for each $v \in V$, $f_1(v) = (\theta_1, 1/\theta_1)$ and for each $j \in I \setminus \{1\}$,

$$f_j(v) = \begin{cases} (0, \frac{c(\theta_1)-1/\theta_1}{n-1}) & \text{if } c(\theta_1) - 1/\theta_1 > 0, \\ (0,0) & \text{if } c(\theta_1) - 1/\theta_1 \leq 0. \end{cases}$$

We know that f satisfies the rectangular property but not strategy-proofness since the value of f is determined by agent 1's revelation alone and agent 1's utility is monotonically increasing in his consumption of the public good, which is assigned by f.

Example 2. Suppose the same environment as Example 1. Let f be such that for each $v \in V$, $f_1(v) = (\theta_n, c(\max\{\theta_i\}_{i \in I \setminus \{1\}}))$ and for each $j \in I \setminus \{1\}$, $f_j(v) = (\theta_{j-1}, c(\max\{\theta_i\}_{i \in I \setminus \{j\}}))$. We know that f satisfies strategy-proofness but not the rectangular property since each agent's consumption bundle assigned by f depends on other agents' revelation but does not change by her own revelation.

By Proposition 1 and a characterization of securely implementable social choice functions by Saijo, Sjöström, and Yamato (2007), we have the following constancy theorem of secure implementation.

Theorem 1. *For each $i \in I$ and each $v_i \in V_i$, suppose that v_i is strictly concave and strictly increasing. The social choice function f is **securely implementable** if and only if it satisfies **constancy**.*

6 Conclusion

In this paper, we consider secure implementability on the provision of one discrete and excludable public good with cost shares. Our main result shows that only constant social choice functions are securely implementable in standard quasi-linear environments. By applying the observations of Cason, Saijo, Sjöström, and Yamato (2006), our main result suggests that almost all of strategy-proof direct revelation mechanisms do not work well in discrete and excludable public good economies.

This paper does not sufficiently investigate domain-richness conditions related to secure implementability. It is open to shed light on the maximal domain on which securely implementable social choice functions are constant.

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