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Efficiency of skill training to acquire sector specific skills with search frictions*

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Abstract

This paper develops a simple search model in which sector-specific trainings are endogenously determined with or without a negotiation between a worker and an employer and characterizes the allocation of two types of training. If a worker and an employee can negotiate over the amount of skill training, the training hours to acquire skill of this employer's sector is longer in the decentralized allocation than in the social efficient allocation. Meanwhile, if they cannot negotiate, the training hours is shorter in the decentralized allocation than in the social efficient allocation.

JEL classification: J24; J64

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1. Introduction

The efficiency property of human capital investment has provoked a great deal of controversy. In search theory, it is well known that a worker's training effort to acquire a single-dimensional skill (general skill) is lower than what it would be in the socially efficient level[1]. To the best of our knowledge, the efficiency property of the investment to acquire multi-dimensional skills[2] has not been adequately discussed so far. Thus, in order to fill this gap, this paper constructs a search model with multi-dimensional skills and focuses on the efficiency of the allocation of (exogenous) training hours to acquire each skill.

In this model, there are two sectors, and each job in a sector requires a sector-specific skill for production. A newborn worker is initially assigned to a sector (which is called an initial sector)

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1 See Acemoglu (1997), Acemoglu and Shimer (1999), and Masters (forthcoming).

2 In recent years, Wasmer (2008) and Mukoyama and Sahin (2009) examined the skill investment problem with multi-dimensional skills with search friction. However, they did not analyze the efficiency property.

and conducts skill training to acquire necessary skills by spending exogenous training hours. More precisely, workers decide to allocate exogenous training hours in order to acquire these skills at birth. In the market equilibrium, workers acquire the skills of not only the initial sector but also another one, because if a worker allocates too many hours to acquire a skill specific to the initial sector, she/he produces only a small amount of output in her/his job when switching to another sector.

Previous papers have discussed the importance of considering the efficiency of skill investment in an environment where a worker and her/his employer can negotiate over the investment to acquire skills[3]. In response to these discussions, I analyze the following two cases: The first case is that a worker and her/his employer can negotiate the allocation of training hours, and the second case is that they cannot negotiate.

There are potentially two sources of inefficiency, and these sources have opposing effects on the allocation of training hours. The first source is the *hold-up problem*, which is due to a lack of complete contingent contracts of wages, which leads to over investment in training to acquire a skill of an initial sector. The second source is the *outside option* effect, by which workers determine the allocation to improve their outside option in wage bargaining, which leads to under investment in training to this skill. If an employed worker and her/his employer can negotiate the allocation of training hours, both sources arise, and the worker may allocate less hours to acquire the skill of an initial sector than what would be socially efficient. Meanwhile, if they can negotiate, only the hold-up problem arises, and then, the allocation of training hours for this skill must be more than what would be socially efficient.

The rest of this article is organized as follows. Section 2 presents the basic framework. Section 3 characterizes the socially efficient allocation. Section 4 defines the allocation characterized in market equilibrium, and section 5 concludes.

2. Environment

I consider a partial equilibrium and discrete-time model, and the measure of each period is one. Workers exit from the labor market with probability δ (death shock), and the same number of newborn workers enter the labor market in each period. The number of workers in the steady state is then a constant and normalized unity. There are two sectors, A and B, and two types of sector-specific skills, h_A and h_B, where $h_{j\in\{A,B\}}$ indicates the amount of a sector j specific skill. To simplify, an employed worker exogenously moves to unemployment with probability s (job destruction shock), and an unemployed worker meets a vacant job in sector j with exogenous probability $p_j \in [0,1]$ (and $p_A + p_B \leq 1$). Note that I assume that workers and firms do not discount the future utility and profits due to the death rate.

An employed worker with a skill vector $h = [h_A, h_B]$ produces $y_j(h) = y\left(h_j\right)$ in a sector $j \in \{A,B\}$, where $y' > 0$ and $y(0) = 0$. To guarantee an inner solution, y_j satisfies the following properties; $y'' < 0$, $y'(0) = \infty$, $y'(1) = 0$, and y'' is small enough. Moreover, I consider the equilibrium in which all workers acquire both h_A and h_B and then accept a job in both sectors.

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[3] See, for example, the survey by Acemoglu and Pischke (1999).

Without loss of generality, I assume that newborn workers do not have any skills and are exogenously allocated to a job in sector A (called an initial job). They conduct skill training to acquire skills before starting to produce in the initial job. Note that workers cannot conduct additional skill training after their training period, in order to focus on the allocation problem of training hours. The number of training hours to acquire h_j is αh_j, where α is the parameter indicating the effectiveness of skill training, and the training hours constraints that are then faced by newborn workers are $1 \geq \alpha \sum_j h_j$. Moreover, they must decide the allocation of training hours for h_A and h_B.

Timing in each period is as follows: (i) Newborn workers decide the allocation of training hours for h_A and h_B, (ii) employed workers produce outputs and conduct wage bargaining, (iii) the labor market is open, and (iv) job destruction and death shocks occur.

2.1. Flow conditions

Let $e_{j,t}$ and u_t denote the number of employed workers in sector j and the number of unemployed workers at period t respectively. Given p_A and p_B, $e_{A,t}$ has the following law of motion:

$$e_{A,t+1} = \delta + (1-\delta)\, p_A u_t + (1-\delta)(1-s)\, e_{A,t} \tag{1}$$

The first term of RHS represents the number of newborn workers, and the second term is the number of unemployed workers who meet a job in sector A, and the last term is the number of workers who do not lose their own jobs and die.

Similarly, the law of motion of $e_{B,t}$ is:

$$e_{B,t+1} = (1-\delta)\, p_B u_t + (1-\delta)(1-s)\, e_{B,t} \tag{2}$$

Finally, the law of motion of u_t is:

$$u_{t+1} = s(1-\delta)(1-u_t) + (1-\delta)(1-p_A-p_B)\, u_t \tag{3}$$

The first term of RHS is workers who are separated by job destruction shock and go into unemployment pool, and the second term is workers who cannot find a new employer.

From (3) and steady state condition $(u_{t+1} = u_t = u)$, the number of unemployment in steady state is:

$$u = \frac{s(1-\delta)}{1-(1-\delta)(1-p_A-p_B)+s(1-\delta)}, \tag{4}$$

from (1) and (4) the number of employed workers in sector A in steady state is:

$$e_A = \frac{(1-\delta)^2 s p_A + \delta\left[1-(1-\delta)(1-p_A-p_B)+s(1-\delta)\right]}{\left[1-(1-\delta)(1-s)\right]\left[1-(1-\delta)(1-p_A-p_B)+s(1-\delta)\right]}, \tag{5}$$

and from (2) and (4) the number of employed worker in sector B in steady state is:

$$e_B = \frac{(1-\delta)^2 sp_B}{[1-(1-\delta)(1-s)][1-(1-\delta)(1-p_A-p_B)+s(1-\delta)]}. \tag{6}$$

From (5) and (6), we obtain:

$$\frac{e_B}{e_A} = \frac{(1-\delta)^2 sp_B}{(1-\delta)^2 sp_A + \delta[1-(1-\delta)(1-s)+(1-\delta)(p_A+p_B)]}. \tag{7}$$

Equation (7) implies that $e_A > e_B$ if $p_A \geq p_B$, because newborn workers are initially allocated to jobs in sector A.

3. The social planner's problem

First, I characterize a problem faced by a social planner. The social planner determines the optimal allocation of training hours to maximize social surplus, subject to the training hours constraint. The social planner's problem is defined by:

$$\max_{h_A, h_A} \sum_{j\in\{A,B\}} y_j(h)e_j \text{ s.t.} 1 = \alpha \sum_{\mathrm{j}} \mathrm{h_j}.$$

The Lagrangian function associated with the above constrained optimization problem can be written as:

$$L = \sum_{j\in\{A,B\}} y_j(h)e_j + \lambda\left(1-\alpha\sum_j h_j\right),$$

where λ is the Lagrangian multiplier. The first order conditions are given by $y'\left(h_j\right)e_j = \lambda\alpha$. From (7), the first order condition can be rewritten as:

$$\frac{y'(h_A)}{y'(h_B)} = \frac{e_B}{e_A} = \frac{(1-\delta)^2 sp_B}{(1-\delta)^2 sp_A + \delta[1-(1-\delta)(1-s)+(1-\delta)(p_A+p_B)]}, \tag{8}$$

where $y'(h_j) = \partial y\left(h_j\right)/\partial h_j$. Given that $y_j'' < 0$, if $p_A = p_B$, then $h_A > h_B$, because in the steady state, workers are more likely to work in sector A than in sector B.

4. Market equilibrium

This section solves the problem of market equilibrium. To do so, I first define the following value functions. According to Pissarides (2000), the value of unemployment is:

$$U(h) = (1-\delta)\left[\sum_j p_j W_j(h) + \left(1 - \sum_j p_j\right) U(h)\right], \tag{9}$$

The value of employment in sector j is:

$$W_j(h) = w_j(h) + (1-\delta)\left[sU(h) + (1-s)W_j(h)\right], \tag{10}$$

where $w_j(h)$ is wages, and the value of employer having a job filled is:

$$J_j(h) = y_j(h) - w_j(h) + (1-\delta)(1-s)J_j(h). \tag{11}$$

Note that I assume the value of employer after hitting job destruction shock is zero.

The wage is determined through the basic Nash bargaining. To simplify, assuming that the outside option of employer is zero, and then the first order condition of Nash bargaining yields:

$$W_j(h) - U(h) = b(W_j(h) + J_j(h) - U(h)), \tag{12}$$

where b is the parameter indicating the worker's bargaining power.

From (9) to (12), $U(h)$ can then be rewritten as:

$$\delta U(h) = \frac{(1-\delta)\, b \sum_j p_j y_j(h)}{1-(1-\delta)(1-s-b(p_A+p_B))}.$$

The marginal unemployed values of h_i are:

$$\frac{\partial U(h)}{\partial h_A} = \frac{1}{\delta}\frac{(1-\delta)\, b p_A}{1-(1-\delta)(1-s-b(p_A+p_B))} y'(h_A), \tag{13}$$

$$\frac{\partial U(h)}{\partial h_B} = \frac{1}{\delta}\frac{(1-\delta)\, b p_B}{1-(1-\delta)(1-s-b(p_A+p_B))} y'(h_B). \tag{14}$$

Next, I characterize the equilibrium allocation of the two types of training in the following two cases: (i) a non-negotiation case in which a worker decides the allocation of training hours to maximize her/his expected lifetime utility, and (ii) a negotiation case in which the Coase theorem holds for the allocation of training hours.

Note that the value of a vacancy does not affect market equilibrium in this model since job finding rate of unemployed worker and the outside option of employer is exogenous variable. Thus, I do not define the value function of a vacancy.

4.1 Skill training: The non-negotiation case

In this case, a newborn worker determines her/his allocation of training hours to maximize $W_A(h)$. Formally, the optimal problem can be written as:

$$\max_{h_A, h_B} W_A(h)\, s.t.\ \alpha \sum_{j \in \{A,B\}} h_j \leq 1.$$

Using the first order condition of Nash bargaining (12) , I rewrite the worker's problem as follows:

$$\max_{h_A, h_B} b\left(W_A(h) + J_A(h)\right) + (1-b)\, U(h) \;\; s.t.\ \alpha \sum_{j \in \{A,B\}} h_j \leq 1.$$

The above problem implies that a worker considers the effect on the outside option in Nash bargaining if $b < 1$, which is referred to as the **outside option effect**. Meanwhile, the social planner does not consider this effect because the level of the outside option only affects the share of output between a worker and an employer, but not the total output. Furthermore, when $p_A y'(h_A) < p_B y'(y_B)$ (this really happens in equilibrium), this effect leads to more training hours to acquire h_B than there would be in the socially efficient allocation.

Formally, from (10) and (11) the Lagrangian function associated with the above constrained worker's problem can be written as:

$$\begin{aligned} L &= \frac{b y_A(h) + (1-\delta)\, s b U(h) + [1-(1-\delta)(1-s)]\,(1-b)\, U(h)}{1-(1-\delta)(1-s)} + \lambda_1 \left(1 - \alpha \sum_j h_j\right) \\ &= \frac{b y_A(h) + [\delta(1-b) + (1-\delta)\, s]\, U(h)}{1-(1-\delta)(1-s)} + \lambda_1 \left(1 - \alpha \sum_j h_j\right), \end{aligned}$$

where λ_1 is the Lagrangian multiplier.

The first order condition is:

$$\frac{1}{1-(1-\delta)(1-s)} \left[b \frac{\partial y_A(h)}{\partial h_j} + [\delta(1-b) + (1-\delta)\, s]\, \frac{\partial U(h)}{\partial h_j} \right] = \alpha \lambda_1. \tag{15}$$

Given that $y_A(h) = y(h_A)$, (15) implies that the benefit gained from an increase in h_B comes through the improvement of $U(h)$. From (14), an increase in b increases this benefit, because the worker's share of output determined by Nash bargaining increases. In other words, the **hold-up problem** arises, and a worker undervalues an increase in $U(h)$, thereby leading to underinvestment to acquire h_B when $p_A y'(h_A) < p_B y'(y_B)$.

From (15), (13), and (14), the equilibrium allocation in this case is:

$$\frac{y'(h_A)}{y'(h_B)} = \frac{(1-\delta)\left[\delta(1-b)+(1-\delta)s\right]p_B}{(1-\delta)\left[\delta(1-b)+(1-\delta)s\right]p_A+\delta\left[1-(1-\delta)(1-s-b(p_A+p_B))\right]}. \tag{16}$$

Comparing (16) with (8), the equilibrium allocation is the same as the socially efficient allocation if and only if $b=1$. Moreover, because the RHS of (16) is a decreasing function of b, I can state the following proposition.

Proposition 1 *If* $0 \leq b < 1$*, the market allocation of training hours is inefficient, and workers allocate excess training hours to acquire* h_B.

Intuitively, there are two sources of inefficiency in the allocation problem, the outside option effect and the hold-up problem. These two sources have opposing effects on the training allocation. If $b<1$, workers allocate excess training hours to acquire h_B in the market equilibrium because the outside option effect always dominates the hold-up problem.

4.2 Skill training: The negotiation case

Following the Coase theorem, the allocation of training hours is determined to maximize to the sum of $W_A(h)$ and $J_A(h)$. The allocation of training hours is then determined by the solution to the following problem:

$$\max_h \; W_A(h)+J_A(h), \;\text{s.t.}\alpha \sum_{j\in\{A,B\}} h_j \leq 1.$$

From (10), (11), the Lagrangian function associated with the above constrained optimal problem can be written as:

$$L = \frac{y_A(h)+(1-\delta)sU(h)}{1-(1-\delta)(1-s)} + \lambda_2\left(1-\alpha\sum_j h_j\right),$$

where λ_2 is the Lagrangian multiplier.

The first order condition is:

$$\frac{1}{1-(1-\delta)(1-s)}\left[\frac{\partial y_A(h)}{\partial h_j}+(1-\delta)s\frac{\partial U(h)}{\partial h_j}\right] = \alpha\lambda_2.$$

From (13) and (14), the optimal condition is:

$$\frac{y'(h_A)}{y'(h_B)} = \frac{b(1-\delta)^2 sp_B}{b(1-\delta)^2 sp_A+\delta\left[1-(1-\delta)(1-s-b(p_A+p_B))\right]}. \tag{17}$$

A comparison of (16) and (17) shows that a worker allocates more training hours to acquire h_A, because the outside option effect is eliminated. The RHS of (17) is an increase function of b, and similar to the allocation in the non-negotiation case, the equilibrium allocation is the same as the socially efficient allocation if $b = 1$. Thus, I can state the following proposition.

Proposition 2 *If $b < 1$, h_B is less in the equilibrium allocation than in the socially optimal allocation.*

Intuitively, while the outside option effect is eliminated because of the Coase theorem, the hold-up problem remains. Then, workers underestimate the benefit from improving outputs in sector B. Moreover, workers allocate all training hours to acquire h_A if $b = 0$, and the skill training is socially optimal if $b = 1$ since the hold-up problem is also eliminated.

5. Conclusion

This paper investigates the allocation problem of training hours. There are two sources of inefficiency in this allocation: the first is that workers consider an effect on the outside option in the wage bargaining, and the second is the hold-up problem. When newborn workers are initially assigned to sector A, the first source induces them to acquire the sector B specific skill, and the second source discourages them from acquiring this skill. If a newborn worker and her/his employer can negotiate, only the hold-up problem arises, and training hours to acquire the sector A specific skill are longer than the socially optimal hours. Meanwhile, if they cannot negotiate, the hold-up problem and the outside option effect occur, and the acquisition level of the sector A specific skill is lower in the decentralized solution than in the socially optimal level.

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