Title	Mandatory Social Security Regime, Consumption and Retirement Behavior of Quasi-Hyperbolic Discounters
Author(s)	Zhang, Lin
Citation	大阪大学経済学. 2013, 63(2), p. 119-131
Version Type	VoR
URL	https://doi.org/10.18910/57127
rights	
Note	

Mandatory Social Security Regime, Consumption and Retirement Behavior of Quasi-Hyperbolic Discounters *

Lin Zhang †

Abstract

This paper proposes a mandatory social security contribution regime in order to adjust the consumption and retirement age of quasi-hyperbolic discounters to the optimal level in the long-run perspective, which is under exponential discounting. Within this mandatory pension contribution regime, this paper ascertains that the behavior of some generations can be adjusted to the optimal level while other generations cannot have both consumption and retirement age adjusted to the optimal level. With behavior adjusted to the optimal level, the adjusted generations' welfare is improved. However, the un-adjusted generations' welfare is deteriorated.

JEL Classification Numbers: E21, H55, D91

Keywords: Social security system, Consumption, Retirement, Quasi-hyperbolic discounting

1. Introduction

Quasi-hyperbolic discounters, with strong present bias, tend to consume at a higher level than the exponential discounters do, which is demonstrated by theoretical studies. And in order to support themselves after they stop working, quasi-hyperbolic discounters have to work for a longer time than exponential discounters. Since exponential discounters' preference is time-consistent, their decisions of consumption level and retirement age are regarded as the optimal outcome in the long-run perspective. Therefore, compared with exponential discounters, the present bias causes deviations of consumption level and retirement age of quasi-hyperbolic discounters from exponential discounters.

On the other hand, social security system, particularly pension system, reallocates consumption level across a consumer's life and is regarded as one of commitment devices to help individuals whose time preference is inconsistent commit their future behavior. It is of great interest to ask how a mandatory social security regime adjusts quasi-hyperbolic discounters' behavior which is considered

* I am very grateful to Shinsuke Ikeda for his invaluable guidance through my research. This research is supported by Grant-in-Aid for JSPS Fellows. All errors remain mine.

† Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan
Email address: kge017zl@mail2.econ.osaka-u.ac.jp

to be irrational in the long-run perspective. This paper investigates that whether social security system adjusts consumption level or retirement age of quasi-hyperbolic discounters, and to what extent it can adjust them.

Blake (2004) shows that pension promotes greater saving and encourages earlier retirement. Cremer et al. (2009) study the design of nonlinear social security schemes when individuals differ in productivity and in their degree of myopia and suggest that as the proportion of myopic individuals increases, there is less redistribution and more forced saving by numerical examples. Cremer and Pestieau (2011) explore how the presence of more or less myopic individuals affects pension design when individuals differ also in productivity.

The quasi-hyperbolic discounting model is widely applied to study people's behavior of saving for retirement, for its good approximation to hyperbolic discounting's accurate description of time-inconsistent impatience – discounting the near future much more heavily than the distant future for the same length of time period, which is inspired by experimental research and common intuitions.[1] The behavior in pension system under hyperbolic discounting (or quasi-hyperbolic discounting) has been included in theoretical studies. Schwarz and Sheshinski (2007) examine the effects of hyperbolic discounting on the comparison of alternative social security systems, and find that intergenerational transfers within a pay-as-you-go economy are usually secured by the social security system and independent of longevity, whereas this is not the case for the funded economy.

Previous studies of pension systems focus on the effects of pension system on the wealth accumulation and consumption after retirement, not including policy target. In these previous studies, social security system did not act initiatively. However, the pension system in this paper aims to modify the behavior of hyperbolic discounters' which is considered as irrational in the long run. This paper is intended to incorporate social security system as a policy instrument to adjust the consumption level and retirement age of hyperbolic discounters. The novelty of this paper includes: i) investigating the effects of mandatory pension system on the adjustment of consumption level and retirement age of quasi-hyperbolic discounters; ii) ascertain not only the welfare effects but also the behavior effects of pension adjustment.

An overlapping-generation model of quasi-hyperbolic discounters with endogenous labor supply is employed. A pension system is incorporated to adjust the over-consumption and late-retirement of quasi-hyperbolic discounters to the level that is obtained under exponential discounting and regarded as optimal in the long-run perspective.

By applying mandatory pension system to the adjustment of consumption level and retirement age of quasi-hyperbolic discounters in a small open economy, it is ascertained that as two policy target the consumption level and retirement age cannot be adjusted to the optimal level at the same time. For a second-best choice, only several generations' behavior can be adjusted, while other generation's welfare will be hurt. However, in order to lead to a Pareto Improvement there is path of contribution rates along which the welfare of each generation could be increased.

---

[1] Hereafter, the expressions "quasi-hyperbolic discounting" and "hyperbolic discounting" will be referred interchangeably.

The remainder of this paper is organized as follows. Section 2 introduces the theoretical model. Section 3 is devoted to the solutions. Section 4 discusses the adjustment effects of mandatory social security system. Section 5 discusses the results got from the foregoing sections. Section 6 makes the concluding remarks.

2. The Model

Consider a small open economy populated by overlapping generations of quasi-hyperbolic discounters who live for three ages with a population growth rate n. In this economy, the three-period model with endogenous retirement decision is employed. Besides, a social security system is incorporated into the model economy, in which the generation of individuals who are old enough, that is to say at age 3 of the whole life, receive a lump-sum pension benefit b. The time horizon is infinite.

Let's consider a generation who is born in period t-1 (hereafter generation t-1), and assume the population of this generation is unit. Therefore, for the generation who are born at period t-1, at age 1 they work and contribute to the social security system at some contribution rate k_{t-1}; and at age 2, they can choose when to retire with $0 \le l_{t-1} \le 1$ and contribute to social security system at a certain rate k_t; and at age 3, they retire and receive a lump-sum pension benefit b_{t-1}. The contribution rates of social security system vary over each period, but stay the same in one period. In each time period, the pension benefits of a generation of individuals are collected from the contributions of generation of individuals who are at age 1 and working at age 2. Hence for the generation t-1, they receive a lump-sum pension benefit at period t+1,

$$b_{t-1} = k_{t+1} w_1 (1+n)^2 + k_{t+1} w_2 (1+n) l \,. \tag{1}$$

Quasi-hyperbolic discounting (Laibson, 1997) forms the discounted utility function into

$$U_{t,j} = u(c_{t,j}) + \beta \sum_{\tau=1}^{3-j} [\delta^{\tau} u(c_{t,j+\tau})] - (\beta\delta)^{2-j} e(l_{t,2}), \qquad j = 1,2 \,, \tag{2}$$

where $0 < \beta < 1$ denotes the present bias parameter and $0 < \delta < 1$. $e(l)$ denotes the cost of working at age 2, including endogenous labor supply (Frogneux, 2009). When present bias βis less than 1, the marginal rates of substitution between c_2 and c_3 in period 1 differs from that in period 2. It causes time-inconsistency if this problem is solved forwardly without incorporating the future shifting of the inter-temporal marginal rate of substitution.

Assume the consumers to be constant relative risk aversion with the inverse of the elasticity of substitution equal to 1, which simplifies the utility function to be in the form of natural logarithm,

$$u(c_{t,j}) = \ln(c_{t,j}) \,. \tag{3}$$

Similarly, the cost function of working at age 2 is assumed to be

$$e(l_t) = \ln(l_t) \,. \tag{4}$$

The budget constraint faced by the consumers is

$$c_{t,3} = R^2 (w_1 - c_{t,1} - k_t w_1) + R w_2 (1 - k_{t+1}) l_t - R c_{t,2} + b_t, \tag{5}$$

where w_1 and w_2 are the wage rates of working at each age, and R>1 is the gross interest rate.

Characterized by their strong impatience for future utility, quasi-hyperbolic discounters are verified to consumer more than exponential discounters do, and retire later to support themselves after they stop working. It is the present bias that causes the deviation of consumption level and retirement age. Social security system is known for its reallocation effect on individual's consumption and it is of great interest to investigate whether the consumption level and retirement age can be adjusted by it.

3. The solutions

Following O'donoghue & Rabin (1999), the sophisticates and the naïfs are considered. The former are capable of realizing their self-control problem and incorporating it into the future plan while the later are unaware of their present-bias and behave time-inconsistently. However, the set-up of model in the foregoing section makes the consumption and labor-supply of both types of individuals coincide, that is to say that the naïfs just happen to correctly predict their consumption level and retirement age at age 1. Therefore, it is only necessary to consider the sophisticates (or the naïfs).

For the sophisticates, they fully predict the dynamic inconsistency of their own and solve the inter-temporal utility maximizing problem backwardly.

Let's consider a generation who are born at period t-1(generation t-1). Since the sophisticates are considered, they can incorporate the dynamic inconsistency into plan, and behave according to it. To start with planning the optimal consumption level at each age and optimal retirement age, they begin with solve the utility maximizing problem at age 2. And assume that social security system covers every generation and every generation incorporates it the utility maximizing problem.

$$\text{Max}\, U_{t-1,2} = u_{t-1,2}(c_{t-1,2}) + \beta\delta u_{t-1,3}(c_{t-1,3}) - e(l_{t-1}) \tag{6}$$

$$\text{s.t.}\ \ c_{t-1,3} = R^2(w_1 - c_{t-1,1} - k_{t-1}w_1) + Rw_2(1-k_t)l_{t-1} - Rc_{t-1,2} + b_{t-1} \tag{7}$$

$$b_{t-1} = k_{t+1}w_1(1+n)^2 + k_{t+1}w_2(1+n)l_t \tag{8}$$

The first order condition at age 2 is

$$u_{t-1,2}' = \beta\delta R u_{t-1,3}', \tag{9}$$

$$e_{t-1}' = \beta\delta w_2[R(1-k_t) + k_{t+1}(1+n)]u_{t-1,3}'. \tag{10}$$

Similarly, the first order condition at age 1 is obtained by incorporating the first order condition at age 2,

$$u_{t-1,1}' + \beta\delta^2 u_{t-1,3}'\{\beta R\partial c_{t-1,2}/\partial c_{t-1,1} + \partial c_{t-1,3}/\partial c_{t-1,1} - \beta w_2[R(1-k_{t-1}) + k_t(1+n)]\partial l_{t-1}/\partial c_{t-1,1}\} = 0. \tag{11}$$

The first order conditions show that social security system does have effects on consumption through the endogenous labor supply.

And the optimal consumption levels and retirement age of generation t-1 are determined as

$$c_{t-1,1}{}^H = \frac{[R^2(1-k_{t-1}) + k_{t+1}(1+n)^2]w_1}{R^2(1+\beta\delta^2)}, \tag{12}$$

$$c_{t-1,2}{}^H = \frac{[R^2(1-k_{t-1}) + k_{t+1}(1+n)^2]\delta w_1}{R(1+\beta\delta^2)}, \tag{13}$$

$$c_{t-1,3}{}^H = \frac{[R^2(1-k_{t-1}) + k_{t+1}(1+n)^2]\beta\delta^2 w_1}{(1+\beta\delta^2)}, \tag{14}$$

$$l_{t-1}{}^H = \frac{[R^2(1-k_{t-1}) + k_{t+1}(1+n)^2]\delta w_1}{[R(1-k_t) + k_{t+1}(1+n)](1+\beta\delta^2)w_2}. \tag{15}$$

The generation t-1 chooses when to retire at age 2, which is period t. In the same period, they have to contribute to the social security system at rate of k_t. And k_t only has effects on the labor supply. The consumption level at each age is merely independent of the contribute rate of social security at age 1 and 3, those are period t-1 and period t+1. Consistent with intuition, consumption level decreases with the contribution rate which is in the period generation t-1 is at age 2, and increases with the contribution rate which is in the period generation t-1 is not working. It is noticeable that the consumption level at each age is independent of the contribution rate at age 2, even though consumers contribute to the social security system at age 2 when they are working.

4. A mandatory social security system

4.1 The adjusted generations

In the social security system, the consumption level and labor supply of generation t-1 are influenced by contributing to social security at rate k_{t-1} and k_t in each period and receive a lump-sum pension benefit whose amount is independent of the contribution rate in period t+1. And it is obtained that for generation t-1of quasi-hyperbolic discounters in the case without social security system which is denoted as "NH",

$$c_{t-1,1}{}^{HN} = w_1/(1+\beta\delta^2), \tag{16}$$

$$l_{t-1}{}^{HN} = R\delta w_1/(1+\beta\delta^2)w_2. \tag{17}$$

Following O'Donoghue & Rabin (1999), the exponential discounters' consumption level and retirement age are considered as the optimal outcome from the long-run perspective, for their preference is time consistent. And in this model, these are denoted as "E",

$$c_1{}^E = w_1/(1+\delta^2), \tag{18}$$

$$l^E = R\delta w_1/(1+\delta^2)w_2. \tag{19}$$

It is straightforward that the consumption level and retirement age of quasi-hyperbolic discounters without social security system are higher than those of exponential discounters, since $0<\beta<1$. These have been demonstrated as over-consumption and late-retirement of quasi-hyperbolic discounters.

In this paper, social security system is employed to modify the consumption level and labor supply of quasi-hyperbolic discounters, who are verified to over-consume or late-retire, to the level under exponential discounting without social security:

$$c_{t-1,1}{}^H = c_1{}^E \tag{20}$$

$$\text{and } l_{t-1}{}^H = l^E. \tag{21}$$

The adjustment effect of social security system for generation t-1 causes

$$k_t = k_{t-1}R/(1+n) - (1-\beta)\delta^2 R/(1+\delta^2)(1+n), \tag{22}$$

$$k_{t+1} = k_t R/(1+n). \tag{23}$$

Consider first-best optimal conditions that the consumption level and labor supply of every generation are adjusted. And similarly for generation t,

$$k_{t+1} = k_t R/(1+n) - (1-\beta)\delta^2 R/(1+\delta^2)(1+n), \tag{24}$$

$$k_{t+2} = k_{t+1}R/(1+n). \tag{25}$$

However, the first-best optimal conditions cannot be realized since they contradict to each other.

In this context, the consumption level and labor supply of quasi-hyperbolic discounters cannot be adjusted to be the same level as exponential discounters for every generation.

Proposition 1 The consumption level and retirement age of *all* generations cannot be adjusted to the optimal levels at the same time.

Consider second-best optimal conditions that social security system adjusts not every generation's, but every other one generation's consumption level and labor supply, for generation t+1,

$$k_{t+2} = k_{t+1}R/(1+n) - (1-\beta)\delta^2 R/(1+\delta^2)(1+n)\,, \quad (26)$$

$$k_{t+3} = k_{t+2}R/((1+n)\,. \quad (27)$$

By considering the conditions for generation t-1 and t+1 simultaneously, a dynamic path of contribution rate of social security system can be achieved. In this context, the consumption level and retirement age of every other generation can be regulated.

Figure 1 to Figure 3 illustrate how contribute rate to social security in each period dynamically relates to each other's. Before moving to the analysis of figures, relative interest ratio $R/(1+n)$ is introduced, which is defined as the ratio of gross interest rate to the speed of population growth.

Definition 1 The ratio of gross interest rate to the speed of population growth $R/(1+n)$ is defined as *relative interest ratio*.

Based on the simplified assumptions that the population of generation t-1 is unit and the population growth rate is n, these figures illustrate the tendency of the path of contribution rate *qualitatively* rather than *quantitatively*.

When R>1+n, that is relative interest ratio is larger than 1, shown in Fig. 1, the contribute rate of social security grows on a path of divergence. R>1+n implies that the gross interest rate exceeds the speed of population growth. And in order to adjust every other generation's behavior, the contribution rate of social security has to grow on a path of divergence. However, there is a steady point on which every other generation's behavior can be adjusted to the optimal level, and when $R/(1+n) > [(1+\delta^2)/(1+\beta\delta^2)]^{\frac{1}{2}}$, the steady point of contribution rate is between 0 and 1. This is a steady state of contribution rate when the population grows sufficiently slow, which implies that in this steady state the social security system meets the pension benefit and adjust every other generation's consumption level and retirement age.

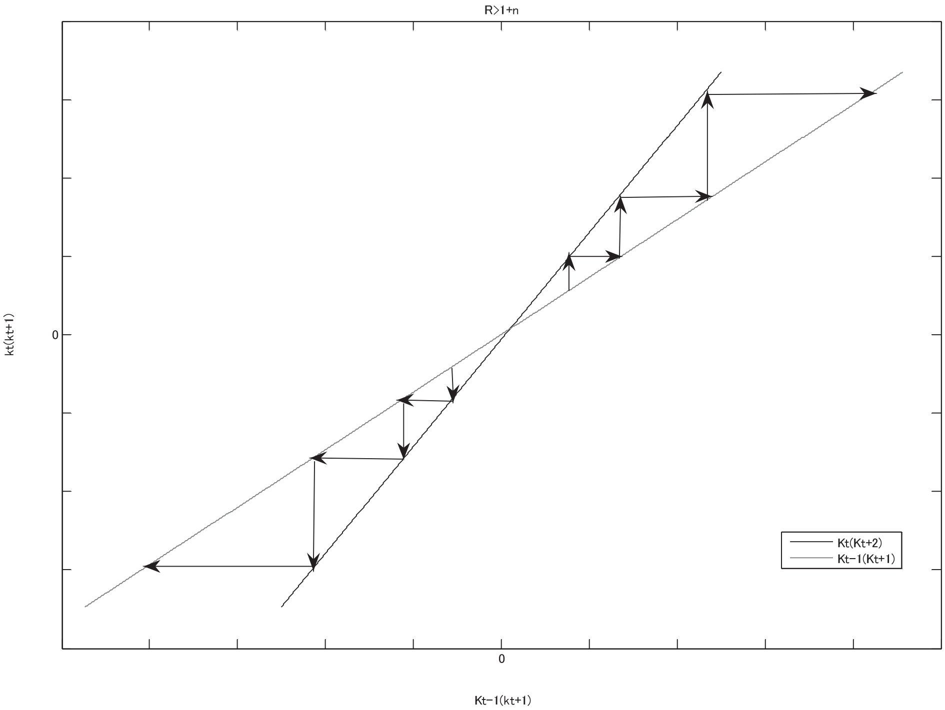

Fig. 1 The contribution rate of pension system: the case of R>1+n

Fig. 2 shows that when R<1+n, i.e. relative interest ratio is less than 1, the contribution rate of social security converges. R<1+n implies that the speed of population growth exceeds the gross interest rate. When population grows rapidly, benefitting from a large population in working age, the whole population doesn't have a growing burden of pension paid for the retired people. However, the steady point to which the contribution rate of social security converges is definitely negative, which implies people receive from rather than contribute to the social security system when they are working.

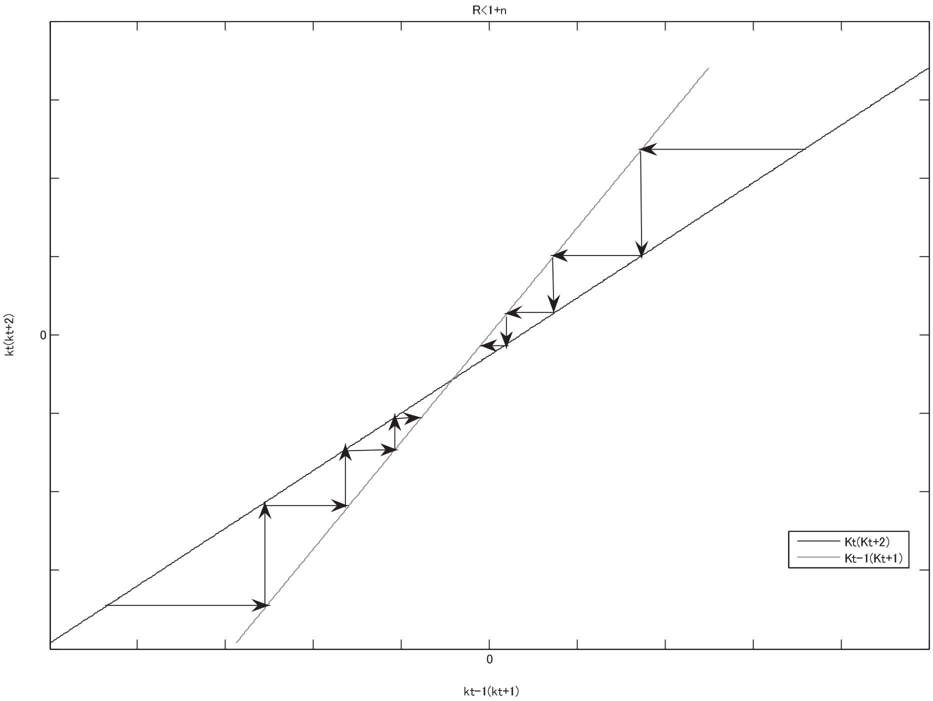

Fig. 2 The contribution rate of pension system: the case of R< 1 +n

Fig.3 shows that when R=1+n, i.e. relative interest ratio equals to 1, the contribution rate of social security system proportionally decreases and there is no steady point. R=1+n implies that the gross interest rate equals to the speed of population growth. In this context, people don't have a growing burden of contribution to social security system either.

Proposition 2 In pension system, in order to adjust the consumption and retirement behavior of some generations to the optimal level, the path of contribution rate to this system depends on the relative interest rate $R/(1+n)$.

Proposition 2 implies that the path of contribution rate to pension system depends on the relationship between the gross interest rate and the speed of population growth, i.e. the relative interest rate. Intuitively, in a pension system, the pension benefits of the retired generations are paid by the younger working generations. Therefore, it is strongly dependent on the population structure.

4.2 The un-adjusted generations

The behavior of the generations who have been adjusted by the social security system has been investigated; however, it is unknown what the behavior of un-adjusted generations is. The consumption level and retirement age of the un-adjusted generations are discussed in this sub-section.

Definition 2 The generations whose consumption level and retirement age are adjusted to the optimal levels by the mandatory pension system are referred as the *adjusted generations*; the other generations whose behavior is not adjusted are referred as the *un-adjusted generations*.

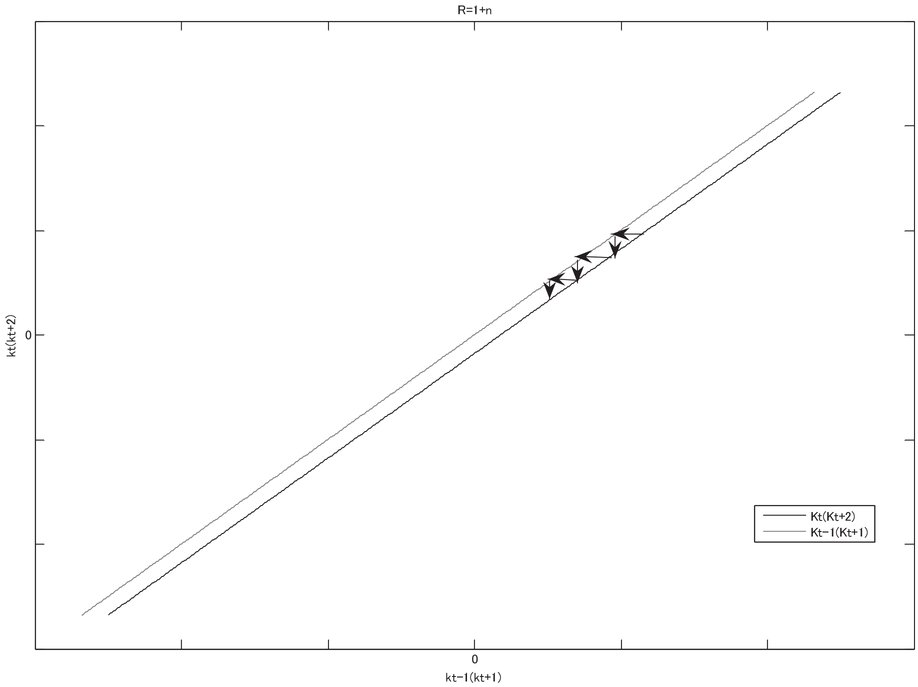

Fig. 3 The contribution rate of pension system: the case of R=1+n

For generation t, whose consumption level and labor supply aren't adjusted,

$$c_{t,1}{}^{H}=\frac{[R^2(1-k_t)+k_{t+2}(1+n)^2]w_1}{R^2(1+\beta\delta^2)}, \tag{28}$$

$$l_t{}^{H}=\frac{[R^2(1-k_t)+k_{t+2}(1+n)^2]\delta w_1}{[R(1-k_{t+1})+k_{t+2}(1+n)](1+\beta\delta^2)w_2}. \tag{29}$$

It is has been assumed that every generation incorporates the social security system into the life-time utility maximizing problem even though their behavior are not adjusted by the system. Therefore, the social security system still has indirect effects on the un-adjusted generations. And it is can be calculated that how the consumption level and retirement age are affected by the social security system by making subtractions.

For the consumption level,

$$c_{t,1}{}^{H}-c_{t,1}{}^{NH}=-\frac{w_1\delta^2(1-\beta)(1+n)}{R(1+\beta\delta^2)(1+\delta^2)}<0, \tag{30}$$

$$c_{t,1}{}^{H}-c_1{}^{E}=\frac{w_1\delta^2(1-\beta)(R-1-n)}{R(1+\delta^2)(1+\beta\delta^2)}. \tag{31}$$

As the result of subtraction shows, the consumption level of un-adjusted generation t is definitely lower than the case without social security system. In this context, the social security system does correct the over-consumption of quasi-hyperbolic discounters to some degree even if it does not adjust the consumption to the optimal level.

By comparing with the optimal consumption level, it can be found that whether the consumption level is corrected depends on the relationship between gross interest rate and the speed of population growth.

When R>1+n, $c_{t,1}^{H} > c_{1}^{E}$, the generation t still over-consume. However, they still consume less than the case without social security system. When R<1+n, $c_{t,1}^{H} < c_{1}^{E}$, the generation t under-consume. The social security system over-corrects the consumption level and results in that it is under the proper limit. When R=1+n, $c_{t,1}^{H} = c_{1}^{E}$, the generation t consume at an appropriate level. In this case, the consumption of un-adjusted generation is corrected to the optimal level even though the social security level does not intend to do so.

Proposition 3 For the un-adjusted generations, they always consume less than the case without social security system; and when relative interest ratio $R/(1+n)$ > (<) 1, they consume more (less) than the optimal level.

For the retirement age,

$$l_t^{H} - l_t^{NH} = \frac{\delta^3 w_1 (1-\beta)(R-1-n)]}{(1+\beta\delta^2)^2 w_2}, \tag{32}$$

$$l_t^{H} - l^{E} = \frac{\delta w_1 [R(1+\beta\delta^2) + (R-1-n)(1+\delta^2)]}{w_2 (1+\beta\delta^2)(1+\delta^2)}. \tag{33}$$

The results of subtractions show that the correction extent of un-adjusted generation's behavior depends on the relationship between gross interest rate and the speed of population growth, as well as the present bias parameter β and long-run discounter factor δ.

Compared with the case without social security system, when R=1+n it distinguishes between that generation t within social security system retire earlier or later than those without it. And R=1+n implies that social security system has no effects on the retirement age of quasi-hyperbolic discounters within it, since they work for the same time as those without the system.

The comparison of retirement age between generation t within social security system and exponential discounters implies that whether $R/(1+n) > 1/[1+(1+\beta\delta^2)/(1+\delta^2)]$ distinguishes that whether generation t retire later than the exponential discounters do or not.

Proposition 4 For the un-adjusted generations, when $R/(1+n)$> (<) $1/[1+(1+\beta\delta^2)/(1+\delta^2)]$, they retire later (earlier) than the optimal level; when $R/(1+n)$> (<) 1, they retire later (earlier) than the case without social security system.

5. Discussions

5.1 The adjustment effects

Thus far the conditions distinguishing how social security system corrects quasi-hyperbolic discounters' consumption level and retirement age has been demonstrated separately. And a comprehensive analysis is made in this section.

As it has been shown, the behavior of generation t-i (i = ±(2k+1), k=0, 1, 2...) it adjusted to the optimal level by the social security system. However, the extent to which the behavior of generation t-i (i=±2k, k=0, 1, 2...) is corrected depends on several conditions. The conditions are divided into 3 regions by the relationship of the gross interest rate and the speed of population growth.

When R>1+n, the contribution rate of social security system grows on a divergence path. And in this context, the un-adjusted generations still consume more than the optimal level even though their consumption is corrected by the pension system to a lower level. Meanwhile, they have to retire later than both the optimal level and those without the pension system. In the region of $1/[1+(1+\beta\delta^2)/(1+\delta^2)] < R/(1+n) < 1$, where the contribution rate converges, the un-adjusted generation t-i (i=$\pm$2k, k=0, 1, 2...) consume even less than the optimal level. And at the same time, they retire later than the optimal level but earlier than those without pension system. When $1/[1+(1+\beta\delta^2)/(1+\delta^2)] > R/(1+n)$, the un-adjusted generation t-i (i=$\pm$2k, k=0, 1, 2...) still consumer at a low level, and in the mean time they retire earlier than both the optimal level and those without the pension system.

Let's notice 2 critical points where R=1+n and $1/[1+(1+\beta\delta^2)/(1+\delta^2)] = R/(1+n)$. When R=1+n, the gross interest rate coincides with the speed of population growth, and the un-adjusted generations consume at the optimal level. However, they retire later than the optimal level and the pension system has no effect on retirement age. When $1/[1+(1+\beta\delta^2)/(1+\delta^2)] = R/(1+n)$, the un-adjusted generations consume at a lower level than the optimal one, but retirement age is at the optimal level. And in this context, they retire earlier than those without social security system. To conclude, the consumption level and retirement age cannot be corrected to the optimal level at the same time. The policy makers have to confront with a tradeoff between the optimal consumption level and the optimal labor supply.

The critical value which distinguishes whether the un-adjusted generations retire at the optimal age deals with the time preference parameters: the long-run discount factor δ and the present bias parameter β. A smaller β which implies stronger present bias increases the region where the un-adjusted generations get retired earlier than the optimal level. In this context, the time preference parameters do not only have effects on determining the labor supply level but also on the degree of the un-adjusted generations' retirement age being corrected.

5.2 The welfare effects

This sub-section discusses welfare consequences of each generation after the mandatory contribution rate of social security system. From a long-run perspective, variations on life discounted utility after compulsory pension contribution rate regime are investigated. The behavior of exponential discounters is regarded as rational since their time-consistent preferences. And therefore compulsory pension contribution regime is designed in the foregoing section to adjust the consumption and retirement age to the optimal level which is under exponential discounting. It has been ascertained that for some generations consumption and retirement age can be adjusted to the optimal level while for other generations the optimal adjustment cannot be realized at the same time. And it is of interest to investigate what effects does the compulsory pension contribution regime have on each generation's welfare.

Following O'Donoghue & Rabin (2006), ax-ante welfare with exponential discounting is evaluated. Welfare is considered to be discounted utility from one's point of view that is at the beginning of each generation in a long-run perspective. For example, the welfare of generation t is defined as

$$V_t = u_1 + \delta(u_2 - e) + \delta^2 u_3 . \quad (34)$$

By making subtraction, let's compare welfare level between quasi-hyperbolic discounters within mandatory pension contribution regime and those without it.

For generations whose behavior is adjusted to the optimal level, say generation t-i (i = ±(2k+1), k=0, 1, 2...), the welfare difference is

$$V_{t-i}^{H} - V_{t-i}^{NH} = (1+\delta^2)\ln[(1+\beta\delta^2)/(1+\delta^2)] - \delta^2 \ln\beta \text{ (i = ±(2k+1), k=0, 1, 2...).} \quad (35)$$

And it is provable that $V_{t-i}^{H} - V_{t-i}^{NH} > 0$, $\forall \beta \in (0,1)$. This welfare improvement straightforwardly holds since the optimal consumption level and retirement age are considered to be those under exponential discounting which is represents the long-run perspective,

For generations whose behavior is not adjusted to the optimal level, i.e. generation t-i (i=±2k, k=0, 1, 2...), the welfare difference is

$$V_{t-i}^{H} - V_{t-i}^{NH} = (1+\delta^2)\ln[1 - \frac{\delta^2(1-\beta)(1+n)}{R(1+\delta^2)}] + \delta\ln[(1+\beta\delta^2)/(1+\delta^2)]$$

(i=±2k, k=0, 1, 2...),

(36)

which is definitely negative. This implies that the mandatory pension contribution regime cause welfare deterioration to the un-adjusted generations.

Proposition 5 After employing the mandatory pension system, the welfare of the adjusted generations is improved; however, the welfare of the un-adjusted generations is deteriorated.

So far it has been ascertained that under mandatory pension contribution regime proposed by this paper for some generations the consumption and retirement age can be adjusted to the optimal level while other generations have to suffer from welfare deterioration caused by this regime even though their behavior is adjusted to some extent. In the behavioral context, the policy maker has to confront with a tradeoff between the optimal consumption level and the optimal labor supply. And in the welfare context, the policy maker has to consider the fairness – which generations' welfare to be sacrificed.

However, it is still available to find out a mandatory pension regime to raise the welfare level of each generation. Consider a certain generation i of hyperbolic discounters, pension system causes Pareto improvement of welfare implies

$$U_i^{H} - U_i^{NH} \geq 0 . \quad (37)$$

A sufficient but not necessary condition of this improvement is

$$R^2(1-k_i) + k_{i+2}(1+n)^2 \geq R^2 \quad (38)$$

$$\text{and } R(1-k_{i+1}) + k_{i+2}(1+n) \geq R , \quad (39)$$

which imply that

$$k_{i+2} \geq [R/(1+n)]^2 k_i \quad (40)$$

$$\text{and } k_{i+2} \geq [R/(1+n)]k_{i+1} . \quad (41)$$

This sufficient but not necessary condition implies that in order to cause Pareto improvement the contribution rate of pension should at least keep proportionally to ratio of gross interest rate over population growth rate, which is referred as relative interest ratio.

When the relative interest ratio $R/(1+n)$ is larger than 1, which implies that the gross interest rate is higher than the population growth speed, the population grows at a relatively slow rate. In this context, the boundary condition of leading a Pareto improvement on contribution rate to pension system converging. While on the other hand, when the relative interest ratio is less than 1, the boundary condition is diverging. They both depend on the relative interest ratio.

6. Conclusions

This paper proposes a mandatory social security contribution regime in order to adjust the consumption and retirement age of quasi-hyperbolic discounters to the optimal level in the long-run perspective which is under exponential discounting. Within this mandatory pension contribution regime, this paper ascertains that behavior of all generations cannot be adjusted to the optimal levels at the same time. With behavior adjusted to the optimal level, the adjusted generations' welfare is improved. However, the un-adjusted generations' welfare is deteriorated.

There are some points of this paper that need to be further developed. This model is based on the specific set up of a small open economy in which the gross interest rate is exogenously given. What is more, by incorporating the mandatory pension system, the adjusted and un-adjusted generations are separated. With different welfare conditions, the fairness is out of the consideration of this paper.

References

Blake. D. (2004) "The impact of wealth on consumption and retirement behaviour in the UK", *Applied Financial Economics*, 14, pp. 555–576

Cremer, H., De Donder, P., Maldonado, D., Pestieau, P. (2009) "Forced Saving, redistribution, and nonlinear social security schemes", *Southern Economic Journal*, 76 (1), pp. 86-98

Cremer, H., Pestieau, P. (2011) "Myopia, redistribution and pensions", *European Economic Review*, 55 (2), pp. 165-175

Frogneux, V. (2009) "Retirement Decision and Quasi-hyperbolic Discounting", *Research report of FUNDP*.

Laibson, D. (1997) "Golden eggs and hyperbolic discounting", *Quarterly Journal of Economics* 112, pp. 443–477.

O'Donoghue, T and M. Rabin (1999) "Doing it now or later", *American Economic Review*, Vol. 89, No. 1 , pp. 103-124

—— and —— (2006) "Optimal Sin Tax", *Journal of Public Economics*, 90, pp. 1825–1849.

Schwarz. M. E and E. Sheshinski (2007) "Quasi-hyperbolic discounting and social security systems", *European Economic Review*, 51, pp. 1247–1262