

Title	On the imbedding problem of algebraic number fields
Author(s)	Nobusawa, Nobuo
Citation	Osaka Mathematical Journal. 1960, 12(2), p. 357-361
Version Type	VoR
URL	https://doi.org/10.18910/5753
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Osaka University

On the Imbedding Problem of Algebraic Number Fields

By Nobuo NOBUSAWA

The purpose of this note is to give another proof of Akagawa's theorem [1] on the imbedding problem of algebraic number fields: the imbedding problem for a Galois extension with a Galois group of order l^n (l is a prime) in case a relative Galois group is of order l can be solved if the corresponding local imbedding problem at each place of the ground field is solved; moreover we can find a solution such that its local completions coincide with the solutions given in advance of local problems at a finite number of places and that the mappings of local Galois groups to the global Galois group coincide with the canonical ones. The proof in this note, using Richter's "*Monodromiesatz*", seems to be a little simpler.

1. Local and global imbedding problems

Throughout this paper, let g be a Galois group of order l^{n-1} of a Galois extension K over a finite algebraic number field k , let \mathfrak{G} be a group of order l^n , and let \mathfrak{N} be a normal subgroup of \mathfrak{G} of order l such that \mathfrak{G} is a central extension of \mathfrak{N} by g :

$$(1) \quad \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} g.$$

The imbedding problem $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} g)$ is then to find a larger field L containing K such that L/k is a Galois extension with the Galois group \mathfrak{G} and that the homomorphism $\varphi^* = \varphi\lambda$ of \mathfrak{G} onto g (λ is the canonical homomorphism: $\mathfrak{G} \xrightarrow{\lambda} \mathfrak{G}/\mathfrak{N}$) coincides with the natural mapping of \mathfrak{G} onto g gained by the restriction of the Galois group \mathfrak{G} of L to the subfield K .

A local imbedding problem corresponding to $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} g)$ at a place (a prime divisor) \mathfrak{p} of k is defined as follows: Let \mathfrak{P} be a place of K over \mathfrak{p} , and let $\mathfrak{g}_{\mathfrak{P}}$ be the decomposition group at \mathfrak{P} . $\mathfrak{g}_{\mathfrak{P}}$ is considered to be the Galois group of $K_{\mathfrak{P}}/k_{\mathfrak{p}}$. Let $\mathfrak{Z}_{\mathfrak{P}}$ be a group and let $\nu_{\mathfrak{P}}$ be an isomorphism of $\mathfrak{Z}_{\mathfrak{P}}$ into \mathfrak{G} such that $\varphi^*\nu_{\mathfrak{P}}(\mathfrak{Z}_{\mathfrak{P}}) = \mathfrak{g}_{\mathfrak{P}}$. If the kernel of $\varphi^*\nu_{\mathfrak{P}}$ is $\mathfrak{N}_{\mathfrak{P}}$, we have

$$(2) \quad \mathfrak{B}_{\mathfrak{p}}/\mathfrak{N}_{\mathfrak{p}} \stackrel{\psi}{\cong} \mathfrak{z}_{\mathfrak{p}},$$

where ψ is gained naturally from $\varphi^* \nu_{\mathfrak{p}}$. An imbedding problem $P(K_{\mathfrak{p}}/k_{\mathfrak{p}}, \mathfrak{B}_{\mathfrak{p}}/\mathfrak{N}_{\mathfrak{p}} \stackrel{\psi}{\cong} \mathfrak{z}_{\mathfrak{p}})$ is called a local imbedding problem corresponding to $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ at a place \mathfrak{p} . This is of course not uniquely determined.

Now we recall the Brauer-Richter-Reichardt's theory. Let N be a generator of \mathfrak{N} . The group extension (1) is uniquely determined by a factor set $\{N^{a_{u,v}}\}$ since it is a central extension, where u and $v \in \mathfrak{g}$ and $a_{u,v}$ are rational integers. On the other hand, let ζ be a primitive l -th root of the unity. Assume $K \ni \zeta$ (and hence $k \ni \zeta$), and $\{\zeta^{a_{u,v}}\}$ is a factor set of K/k . Then we can make a crossed product A of K/k by \mathfrak{g} with a factor set $\{\zeta^{a_{u,v}}\}$. The next theorem is a special case of Brauer's theorem [2].

Theorem 1. *Assume that the group extension (1) does not split. When $K \ni \zeta$, the imbedding problem $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ is solved if and only if the crossed product A splits.*

From this theorem, a special case of Richter's theorem [5] is gained :

Theorem 2. *Assume $K \ni \zeta$. $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ is solved if and only if at least a local imbedding problem $P(K_{\mathfrak{p}}/k_{\mathfrak{p}}, \mathfrak{B}_{\mathfrak{p}}/\mathfrak{N}_{\mathfrak{p}} \stackrel{\psi}{\cong} \mathfrak{z}_{\mathfrak{p}})$ corresponding to it at each place \mathfrak{p} of k is solved.*

When $K \not\ni \zeta$, we put $\bar{k} = k(\zeta)$ and $\bar{K} = K(\zeta)$. Then \bar{K}/\bar{k} is a Galois extension with the Galois group \mathfrak{g} . Denote by \bar{A} a crossed product of \bar{K}/\bar{k} by \mathfrak{g} with the factor set $\{\zeta^{a_{u,v}}\}$. The proof of the next theorem is gained by Reichardt [4].

Theorem 3. *Assume (1) does not split. If \bar{A} splits, then $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ is solved.*

The next theorem is a natural consequence of the above three theorems.

Theorem 4. *$P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ is solved if and only if at least a local problem corresponding to it is solved at each place of k , whether K contains ζ or not.*

Proof. The theorem is clear when $K \ni \zeta$. If (1) splits, $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ is always solved. Hence assume that $K \not\ni \zeta$ and that (1) does not split. If $P(K_{\mathfrak{p}}/k_{\mathfrak{p}}, \mathfrak{B}_{\mathfrak{p}}/\mathfrak{N}_{\mathfrak{p}} \stackrel{\psi}{\cong} \mathfrak{z}_{\mathfrak{p}})$ is solved, then $P(\bar{K}_{\mathfrak{p}}/\bar{k}_{\mathfrak{p}}, \mathfrak{B}_{\mathfrak{p}}/\mathfrak{N}_{\mathfrak{p}} \stackrel{\psi}{\cong} \mathfrak{z}_{\mathfrak{p}})$ is solved, as is easily seen. By Theorem 2, $P(\bar{K}/\bar{k}, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ is solved, and hence

\bar{A} splits by Theorem 1, that is, $P(K/k, \mathfrak{G}/\mathfrak{N} \cong \mathfrak{g})^{\mathcal{P}}$ is solved by Theorem 3.

2. The exact imbedding problem

According to Akagawa [1], the exact imbedding problem is defined as follows: Let $P(K/k, \mathfrak{G}/\mathfrak{N} \cong \mathfrak{g})^{\mathcal{P}}$ be a global imbedding problem, and let S be a set of a finite number of places \mathfrak{p} of k . We assume that a local imbedding problem $P(K_{\mathfrak{p}}/k_{\mathfrak{p}}, \mathfrak{Z}_{\mathfrak{p}}/\mathfrak{N}_{\mathfrak{p}} \cong \mathfrak{z}_{\mathfrak{p}})^{\psi}$ is given and has a solution $F_{\mathfrak{p}}$ at each place \mathfrak{p} of S . Then arises a problem to find a solution L of $P(K/k, \mathfrak{G}/\mathfrak{N} \cong \mathfrak{g})^{\mathcal{P}}$ such that its \mathfrak{p} -completion $L_{\mathfrak{p}}$ is isomorphic to $F_{\mathfrak{p}}$ and that $\nu_{\mathfrak{p}}$ coincides with the canonical mapping (the inclusion mapping as a decomposition group) of the Galois group $\mathfrak{Z}_{\mathfrak{p}}$ of $L_{\mathfrak{p}}/k_{\mathfrak{p}}$ (which we may identify with $F_{\mathfrak{p}}/k_{\mathfrak{p}}$) into the Galois group \mathfrak{G} of L/k . This problem is called *an exact imbedding problem* and will be denoted by $P(\mathfrak{G}/\mathfrak{N} \cong \mathfrak{g}; \mathfrak{Z}_{\mathfrak{p}}, \nu_{\mathfrak{p}}, F_{\mathfrak{p}})$. (We always fix K and a set S).

Corresponding to the definition of the product of group extensions (that is, the multiplication in 2-cohomology groups), we define a product of two exact imbedding problems $P(\mathfrak{G}^{(1)}/\mathfrak{N} \cong \mathfrak{g}; \mathfrak{Z}_{\mathfrak{p}}^{(1)}, \nu_{\mathfrak{p}}^{(1)}, F_{\mathfrak{p}}^{(1)})$ and $P(\mathfrak{G}^{(2)}/\mathfrak{N} \cong \mathfrak{g}; \mathfrak{Z}_{\mathfrak{p}}^{(2)}, \nu_{\mathfrak{p}}^{(2)}, F_{\mathfrak{p}}^{(2)})$ as follows: Put $(\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}} = \{(A^{(1)}, A^{(2)}) \text{ with } A^{(1)} \in \mathfrak{G}^{(1)} \text{ and } A^{(2)} \in \mathfrak{G}^{(2)} \text{ such that } \varphi_1^*(A^{(1)}) = \varphi_2^*(A^{(2)})\}$. Then $(\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}}$ contains a normal subgroup $\tilde{\mathfrak{N}}$ which is generated with (N, N) . Put $\mathfrak{G}^{(3)} = (\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}} / \tilde{\mathfrak{N}}$. $\mathfrak{G}^{(3)}$ contains $(\mathfrak{N} \times \mathfrak{N}) / \tilde{\mathfrak{N}}$, which is isomorphic with \mathfrak{N} by the natural way and we identify with \mathfrak{N} . Then we have

$$\mathfrak{G}^{(3)}/\mathfrak{N} \cong \mathfrak{g},$$

where φ_3 is defined such that $\varphi_3((A^{(1)}, A^{(2)}) \text{ mod } \mathfrak{N}) = \varphi_1^*(A^{(1)}) (= \varphi_2^*(A^{(2)}))$. We shall next determine $\mathfrak{Z}_{\mathfrak{p}}^{(3)}, \nu_{\mathfrak{p}}^{(3)}$ and $F_{\mathfrak{p}}^{(3)}$. Put $\bar{F}_{\mathfrak{p}} = F_{\mathfrak{p}}^{(1)} \cup F_{\mathfrak{p}}^{(2)}$ and $\bar{\mathfrak{Z}}_{\mathfrak{p}} = \mathfrak{G}(F_{\mathfrak{p}}^{(1)} \cup F_{\mathfrak{p}}^{(2)}/k_{\mathfrak{p}})$ (the Galois group of $F_{\mathfrak{p}}^{(1)} \cup F_{\mathfrak{p}}^{(2)}/k_{\mathfrak{p}}$). Let $\bar{\nu}_{\mathfrak{p}}$ be an isomorphism of $\bar{\mathfrak{Z}}_{\mathfrak{p}}$ into $(\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}}$ defined such that, for an element \bar{A} of $\bar{\mathfrak{Z}}_{\mathfrak{p}}$, $\bar{\nu}_{\mathfrak{p}}(\bar{A}) = (\nu_{\mathfrak{p}}^{(1)}(A^{(1)}), \nu_{\mathfrak{p}}^{(2)}(A^{(2)}))$ where $A^{(i)}$ are elements of $\mathfrak{Z}_{\mathfrak{p}}^{(i)}$ gained by the restriction of \bar{A} to $F_{\mathfrak{p}}^{(i)}$. Now let $F_{\mathfrak{p}}^{(3)}$ be the fixed subfield of $F_{\mathfrak{p}}^{(1)} \cup F_{\mathfrak{p}}^{(2)}$ of $\bar{\nu}_{\mathfrak{p}}^{-1}(\bar{\nu}_{\mathfrak{p}}(\bar{\mathfrak{Z}}_{\mathfrak{p}}) \cap \mathfrak{N})$, let $\mathfrak{Z}_{\mathfrak{p}}^{(3)}$ be the Galois group $\mathfrak{G}(F_{\mathfrak{p}}^{(3)}/k_{\mathfrak{p}})$, and let $\nu_{\mathfrak{p}}^{(3)}$ be the isomorphism of $\mathfrak{Z}_{\mathfrak{p}}^{(3)}$ into $\mathfrak{G}^{(3)}$ gained naturally from $\bar{\nu}_{\mathfrak{p}}$. In this case, we put

$$\begin{aligned} &P(\mathfrak{G}^{(1)}/\mathfrak{N} \cong \mathfrak{g}; \mathfrak{Z}_{\mathfrak{p}}^{(1)}, \nu_{\mathfrak{p}}^{(1)}, F_{\mathfrak{p}}^{(1)}) + P(\mathfrak{G}^{(2)}/\mathfrak{N} \cong \mathfrak{g}; \mathfrak{Z}_{\mathfrak{p}}^{(2)}, \nu_{\mathfrak{p}}^{(2)}, F_{\mathfrak{p}}^{(2)}) \\ &= P(\mathfrak{G}^{(3)}/\mathfrak{N} \cong \mathfrak{g}; \mathfrak{Z}_{\mathfrak{p}}^{(3)}, \nu_{\mathfrak{p}}^{(3)}, F_{\mathfrak{p}}^{(3)}). \end{aligned}$$

It will be verified that all exact imbedding problems (for fixed K and S) form an additive group by this product.

Theorem 5. *When the group extension (1) splits, every exact imbedding problem $P(\mathfrak{G}/\mathfrak{N} \cong_{\mathcal{P}} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}, \nu_{\mathfrak{S}}, F_{\mathfrak{S}})$ has infinitely many solutions for arbitrary $\mathfrak{Z}_{\mathfrak{S}}, \nu_{\mathfrak{S}}$, and $F_{\mathfrak{S}}$.*

Proof. Since $\mathfrak{G}/\mathfrak{N} \cong_{\mathcal{P}} \mathfrak{g}$ splits, $\mathfrak{G} = \mathfrak{A} \times \mathfrak{N}$ with $\mathfrak{A} \cong \mathfrak{g}$. It suffices to find a field k_1 such that K and k_1 are independent over k , that k_1/k is a Galois extension with the Galois group \mathfrak{N} and that Kk_1 suffices the condition of a solution. Then we may assume $\mathfrak{A} (\cong \mathfrak{g})$ is the Galois group of K/k . First, assume $\nu_{\mathfrak{S}}(\mathfrak{Z}_{\mathfrak{S}}) \supset \mathfrak{N}$. Then $\mathfrak{Z}_{\mathfrak{S}} = \mathfrak{Z}' \times \mathfrak{N}'$ with $\nu_{\mathfrak{S}}(\mathfrak{Z}') \subset \mathfrak{A}$ and $\nu_{\mathfrak{S}}(\mathfrak{N}') = \mathfrak{N}$, and $F_{\mathfrak{S}} = K_{\mathfrak{S}} \times k'_p$ where $K_{\mathfrak{S}}$ is the invariant field of \mathfrak{N}' and k'_p of \mathfrak{Z}' . From the assumption, $\nu_{\mathfrak{S}}$ is uniquely determined for each element of \mathfrak{Z}' (the canonical mapping). So the problem is reduced to find a field k_1 independent of K over k such that k_1/k is a Galois extension with the Galois group \mathfrak{N} , that its completion $k_{1,p} \cong k'_p$, and that the mapping $\nu_{\mathfrak{S}}$ of the Galois group \mathfrak{N}' of $k_{1,p}/k_p (=k'_p/k_p)$ onto \mathfrak{N} coincides with the canonical one. Next, assume that $\nu_{\mathfrak{S}}(\mathfrak{Z}_{\mathfrak{S}}) \not\supset \mathfrak{N}$. Then $\mathfrak{Z}_{\mathfrak{S}} \cong \mathfrak{z}_{\mathfrak{S}}$, and hence $F_{\mathfrak{S}} = K_{\mathfrak{S}}$. If we put $\nu_{\mathfrak{S}}(\mathfrak{Z}_{\mathfrak{S}}) = \mathfrak{A}'$, $\mathfrak{A}' = \mathfrak{z}_{\mathfrak{S}}$ or $\mathfrak{z}_{\mathfrak{S}} \cap \mathfrak{A}'$ has index l in $\mathfrak{z}_{\mathfrak{S}}$. Let k'_p be the invariant subfield of $\mathfrak{z}_{\mathfrak{S}} \cap \mathfrak{A}'$. We can define an isomorphism $\nu'_{\mathfrak{S}}$ of the Galois group $\mathfrak{G}(k'_p/k_p)$ into \mathfrak{N} such that, if A' is an element of $\mathfrak{G}(k'_p/k_p)$ gained by the restriction of $\bar{A} \in \mathfrak{G}(F_{\mathfrak{S}}/k_p)$, we put $\nu'_{\mathfrak{S}}(A') = N^a$ where $\nu_{\mathfrak{S}}(\bar{A}) = (A, N^a) \in \mathfrak{A} \times \mathfrak{N}$. $\nu'_{\mathfrak{S}}$ is then defined independent of the choice of \bar{A} as is seen from the definition of k'_p . (When $\mathfrak{z}_{\mathfrak{S}} = \mathfrak{A}'$, $\nu'_{\mathfrak{S}}$ is the trivial mapping of the unity.) In this case, the problem is reduced to find a field k_1 independent of K over k such that k_1/k is a Galois extension with the Galois group \mathfrak{N} , that its local completion $k_{1,p}$ is isomorphic to k'_p , and that the mapping $\nu'_{\mathfrak{S}}$ coincides with the canonical one. To find k_1 of these properties at a finite number of places is always possible by infinitely many ways by Hasse [2]. (Cf. also [1].)

Theorem 6. *Let*

$$\begin{aligned}
 &P(\mathfrak{G}^{(1)}/\mathfrak{N} \cong_{\mathcal{P}_1} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}^{(1)}, \nu_{\mathfrak{S}}^{(1)}, F_{\mathfrak{S}}^{(1)}) + P(\mathfrak{G}^{(2)}/\mathfrak{N} \cong_{\mathcal{P}_2} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}^{(2)}, \nu_{\mathfrak{S}}^{(2)}, F_{\mathfrak{S}}^{(2)}) \\
 &= P(\mathfrak{G}^{(3)}/\mathfrak{N} \cong_{\mathcal{P}_3} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}^{(3)}, \nu_{\mathfrak{S}}^{(3)}, F_{\mathfrak{S}}^{(3)}).
 \end{aligned}$$

If $P(\mathfrak{G}^{(1)}/\mathfrak{N} \cong_{\mathcal{P}_1} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}^{(1)}, \nu_{\mathfrak{S}}^{(1)}, F_{\mathfrak{S}}^{(1)})$ and $P(\mathfrak{G}^{(2)}/\mathfrak{N} \cong_{\mathcal{P}_2} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}^{(2)}, \nu_{\mathfrak{S}}^{(2)}, F_{\mathfrak{S}}^{(2)})$ have solutions independent of each other, then the third problem $P(\mathfrak{G}^{(3)}/\mathfrak{N} \cong_{\mathcal{P}_3} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{S}}^{(3)}, \nu_{\mathfrak{S}}^{(3)}, F_{\mathfrak{S}}^{(3)})$ has a solution.

Proof. If L_1 and L_2 are independent solutions of the first and second

problems, then $L_1 \cup L_2$ is a Galois extension with the Galois group $(\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}}$. If L_3 is the invariant subfield of $\mathfrak{N}(\langle (\mathfrak{G}^{(1)} \times \mathfrak{G}^{(2)})_{\mathfrak{g}} \rangle)$, then L_3 is a solution of the third problem.

Now we conclude this paper with Akagawa's theorem.

Theorem 7. *If S contains all ramified places at K/k , then every exact imbedding problem $P(\mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}, F_{\mathfrak{P}})$ has infinitely many solutions.*

Proof. The existence of $\mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}$ and $F_{\mathfrak{P}}$ at all ramified places implies the solvability of a local imbedding problem at each place of k , since the solvability at unramified places is clear. Then by Theorem 4 the global imbedding problem $P(K/k, \mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g})$ has a solution L . If we denote the decomposition group, the canonical mapping and the local completion of L by $\mathfrak{Z}_{\mathfrak{P}}(L), \nu_{\mathfrak{P}}$ and $L_{\mathfrak{P}}$, then $P(\mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}(L), \nu_{\mathfrak{P}}, L_{\mathfrak{P}})$ has a solution L . Put

$$\begin{aligned} P(\mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}(L), \nu_{\mathfrak{P}}, L_{\mathfrak{P}}) &= P(\mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}, F_{\mathfrak{P}}) \\ &= P(\mathfrak{G}^{(0)}/\mathfrak{N} \stackrel{\mathcal{P}_0}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}^{(0)}, \nu_{\mathfrak{P}}^{(0)}, F_{\mathfrak{P}}^{(0)}). \end{aligned}$$

Then $\mathfrak{G}^{(0)}/\mathfrak{N} \stackrel{\mathcal{P}_0}{\cong} \mathfrak{g}$ splits, and hence by Theorem 5 and 6 we can find infinitely many solutions of $P(\mathfrak{G}^{(0)}/\mathfrak{N} \stackrel{\mathcal{P}_0}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}^{(0)}, \nu_{\mathfrak{P}}^{(0)}, F_{\mathfrak{P}}^{(0)})$ and also infinitely many solutions of $P(\mathfrak{G}/\mathfrak{N} \stackrel{\mathcal{P}}{\cong} \mathfrak{g}; \mathfrak{Z}_{\mathfrak{P}}, \nu_{\mathfrak{P}}, F_{\mathfrak{P}})$.

(Received September 12, 1960)

References

- [1] Y. Akagawa: The extension of groups and the imbedding of fields, Osaka Math. J. **12** (1960), 195-215.
- [2] R. Brauer: Über die Konstruktion der Schiefkörper, die von endlichem Rang in Bezug auf ein gegebenes Zentrum sind, J. Reine Angew. Math. **168** (1932), 44-64.
- [3] H. Hasse: Zum Existenzsatz von Grunwald in der Klassenkörpertheorie, J. Reine Angew. Math. **188** (1950), 40-64.
- [4] H. Reichardt: Konstruktion von Zahlkörpern mit gegebener Galoisgruppe von Primzahlpotenzordnung, J. Reine Angew. Math. **177** (1937), 1-5.
- [5] H. Richter: Über die Lösbarkeit einiger nicht-Abelscher Einbettungsprobleme, Math. Ann. **112** (1936), 69-84.

