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<th>F-FINITENESS OF HOMOMORPHISMS AND ITS DESCENT</th>
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<td>Author(s)</td>
<td>Hashimoto, Mitsuyasu</td>
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Abstract

Let $p$ be a prime number. We define the notion of $F$-finiteness of homomorphisms of $\mathbb{F}_p$-algebras, and discuss some basic properties. In particular, we prove a sort of descent theorem on $F$-finiteness of homomorphisms of $\mathbb{F}_p$-algebras. As a corollary, we prove the following. Let $g: B \to C$ be a homomorphism of Noetherian $\mathbb{F}_p$-algebras. If $g$ is faithfully flat reduced and $C$ is $F$-finite, then $B$ is $F$-finite. This is a generalization of Seydi’s result on excellent local rings of characteristic $p$.

1. Introduction

Throughout this paper, $p$ denotes a prime number, and $\mathbb{F}_p$ denotes the finite field with $p$ elements.

The notions of Nagata (pseudo-geometric, universally Japanese) and (quasi-)excellent rings give good frameworks to avoid pathologies which appear in the theory of Noetherian rings, see [20], [10], and [18].

In commutative algebra of characteristic $p$, $F$-finiteness of rings is commonly used for a general assumption which guarantees the “tameness” of the theory, as well as Nagata and (quasi-)excellent properties. A commutative ring $R$ of characteristic $p$ is said to be $F$-finite if the Frobenius map $F_R: R \to R (F_R(r) = r^p)$ is finite (that is, $R$ as the target of $F_R$ is a finite module over $R$ as the source of $F_R$). As the definition suggests, $F$-finiteness is important in studying ring theoretic properties defined via Frobenius maps, such as strong $F$-regularity [14]. Although $F$-finiteness for a Noetherian $\mathbb{F}_p$-algebra is stronger than excellence [16], $F$-finiteness is not so restrictive for practical use. A perfect field is $F$-finite. An algebra essentially of finite type over an $F$-finite ring is $F$-finite. An ideal-adic completion of a Noetherian $F$-finite ring is again $F$-finite. See Examples 3 and 9. It is known that an $F$-finite Noetherian ring is a homomorphic image of an $F$-finite regular ring of finite Krull dimension, and hence it has a dualizing complex [9, Remark 13.6].

In this paper, replacing the absolute Frobenius map by the relative one, we define the $F$-finiteness of homomorphism between rings of characteristic $p$. We say that an $\mathbb{F}_p$-algebra map $A \to B$ is $F$-finite (or $B$ is $F$-finite over $A$) if the relative Frobenius map (Radu–André homomorphism) $\Phi_1(A, B): B^{(1)} \otimes_{A^{(1)}} A \to B$ is finite (Definition 1, see Section 2 for the notation). Thus a ring $B$ of characteristic $p$ is $F$-finite if and

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only if it is $F$-finite over $\mathbb{F}_p$. Replacing absolute Frobenius by relative Frobenius, we get definitions and results on homomorphisms instead of rings. This is a common idea in [23], [2], [3], [4], [5], [7], [12], [6], and [13].

In Section 2, we discuss basic properties of $F$-finiteness of homomorphisms and rings. Some of well-known properties of $F$-finiteness of rings are naturally generalized to those for $F$-finiteness of homomorphisms. $F$-finiteness of homomorphisms has connections with that for rings. For example, if $A \rightarrow B$ is $F$-finite and $A$ is $F$-finite, then $B$ is $F$-finite (Lemma 2).

In Section 3, we prove the main theorem (Theorem 21). This is a sort of descent of $F$-finiteness. As a corollary, we prove that for a faithfully flat reduced homomorphism of Noetherian rings $g$: $B \rightarrow C$, if $C$ is $F$-finite, then $B$ is $F$-finite. Considering the case that $f$ is a completion of a Noetherian local ring, we recover Seydi’s result on excellent local rings of characteristic $p$ [26].

2. $F$-finiteness of homomorphisms

Let $k$ be a perfect field of characteristic $p$, and $r \in \mathbb{Z}$. For a $k$-space $V$, the additive group $V$ with the new $k$-space structure $a \cdot v = a^{p^r}v$ is denoted by $V^{(r)}$. An element $v$ of $V$, viewed as an element of $V^{(r)}$ is (sometimes) denoted by $v^{(r)}$. If $A$ is a $k$-algebra, then $A^{(r)}$ is a $k$-algebra with the product $a^{(r)} \cdot b^{(r)} = (ab)^{(r)}$. We denote the Frobenius map $A \rightarrow A (a \mapsto a^p)$ by $F$ or $F_A$. Note that $F^e: A^{(r+e)} \rightarrow A^{(r)}$ is a $k$-algebra map. Throughout the article, we regard $A^{(r)}$ as an $A^{(r+e)}$-algebra through $F^e (A$ is viewed as $A^{(0)}$). For an $A$-module $M$, the action $a^{(r)} \cdot m^{(r)} = (am)^{(r)}$ makes $M^{(r)}$ an $A^{(r)}$-module. If $I$ is an ideal of $A$, then $I^{(r)}$ is an ideal of $A^{(r)}$. If $e \geq 0$, then $I^{(e)}A = I^{p^r}m$, where $I^{p^r}$ is the ideal of $A$ generated by $\{a^{p^e} | a \in I\}$. In commutative algebra, $A^{(r)}$ is also denoted by $eA$. We employ the notation more consistent with that in representation theory—the $e$-th Frobenius twist of $V$ is denoted by $V^{(e)}$, see [15]. We use this notation for $k = \mathbb{F}_p$.

Let $A \rightarrow B$ be an $\mathbb{F}_p$-algebra map, and $e \geq 0$. Then the relative Frobenius map (or Radu–André homomorphism) $\Phi_e(A, B)$: $B^{(e)} \otimes_{A^{(e)}} A \rightarrow B$ is defined by $\Phi_e(A, B)(b^{(e)} \otimes a) = b^{p^e}a$.

**Definition 1.** An $\mathbb{F}_p$-algebra map $A \rightarrow B$ is said to be $F$-finite if $\Phi_1(A, B): B^{(1)} \otimes_{A^{(1)}} A \rightarrow B$ is finite. That is, $B$ is a finitely generated $B^{(1)} \otimes_{A^{(1)}} A$-module through $\Phi_1(A, B)$. We also say that $B$ is $F$-finite over $A$.

**Lemma 2.** Let $f : A \rightarrow B$, $g : B \rightarrow C$, and $h : A \rightarrow \tilde{A}$ be $\mathbb{F}_p$-algebra maps, and $\tilde{B} := \tilde{A} \otimes_B A$.

(1) The following are equivalent.

(a) $f$ is $F$-finite. That is, $\Phi_1(A, B)$ is finite.

(b) For any $e > 0$, $\Phi_e(A, B)$ is finite.

(c) For some $e > 0$, $\Phi_e(A, B)$ is finite.
(2) If \( f \) and \( g \) are \( F \)-finite, then so is \( g f \).

(3) If \( g f \) is \( F \)-finite, then so is \( g \).

(4) The ring \( A \) is \( F \)-finite (that is, the Frobenius map \( F_A: A^{(1)} \to A \) is finite) if and only if the unique homomorphism \( \mathbb{F}_p \to A \) is \( F \)-finite.

(5) If \( f: A \to B \) is \( F \)-finite, then the base change \( \tilde{f}: \tilde{A} \to \tilde{B} \) is \( F \)-finite.

(6) If \( B \) is \( F \)-finite, then \( f \) is \( F \)-finite.

(7) If \( A \) and \( f \) are \( F \)-finite, then \( B \) is \( F \)-finite.

Proof. (1) This is immediate, using [12, Lemma 4.1, 2]. (2) and (3) follow from [12, Lemma 4.1, 1]. (4) follows from [12, Lemma 4.1, 5]. (5) follows from [12, Lemma 4.1, 4]. (6) follows from (3) and (4). (7) follows from (2) and (4).

Example 3. Let \( e \geq 1 \), and \( f: A \to B \) be an \( \mathbb{F}_p \)-algebra map.

(1) If \( B = A[x] \) is a polynomial ring, then it is \( F \)-finite over \( A \).

(2) If \( B = A_S \) is a localization of \( A \) by a multiplicatively closed subset \( S \) of \( A \), then \( \Phi_e(A, B) \) is an isomorphism. In particular, \( B \) is \( F \)-finite over \( A \).

(3) If \( B = A/I \) with \( I \) an ideal of \( A \), then

\[
B^{(e)} \otimes_{A^{(e)}} A \cong (A^{(e)}/I^{(e)}) \otimes_{A^{(e)}} A \cong A/I^{(e)} A = A/I^{[p^e]}.
\]

Under this identification, \( \Phi_e(A, B) \) is identified with the projection \( A/I^{[p^e]} \to A/I \). In particular, \( B \) is \( F \)-finite over \( A \).

(4) If \( B \) is essentially of finite type over \( A \), then \( B \) is \( F \)-finite over \( A \).

Proof. (1) The image of \( \Phi_1(A, B) \) is \( A[x^p] \), and hence \( B \) is generated by \( 1, x, \ldots, x^{p-1} \) over it. (2) Note that \( B^{(e)} \) is identified with \( (A^{(e)})_{S^{(e)}} \), where \( S^{(e)} = \{ s^{(e)} | s \in S \} \). So \( B^{(e)} \otimes_{A^{(e)}} A \) is identified with \( (A^{(e)})_{S^{(e)}} \otimes_{A^{(e)}} A \cong A_{S^{(e)}} \), and \( \Phi_e(A, B) \) is identified with the isomorphism \( A_{S^{(e)}} \cong A_S \). (3) is obvious. (4) This is a consequence of (1), (2), (3), and Lemma 2 (2).

Lemma 4. Let \( A \xrightarrow{f} B \xrightarrow{g} C \) be a sequence of \( \mathbb{F}_p \)-algebra maps. Then for \( e > 0 \), the diagram

\[
\begin{array}{ccc}
B^{(e)} \otimes_{A^{(e)}} A & \xrightarrow{\Phi_e(A, B)} & B \\
\downarrow \phi^{(e)} \otimes 1 & & \downarrow \downarrow g \\
C^{(e)} \otimes_{A^{(e)}} A & \xrightarrow{\Phi_e(A, C)} & C
\end{array}
\]

is commutative.

Proof. This is straightforward.
Lemma 5. Let $A \xrightarrow{f} B \xrightarrow{g} C$ be a sequence of $\mathbb{F}_p$-algebra maps, and assume that $C$ is $F$-finite over $A$. If $g$ is finite and injective, and $B^{(e)} \otimes_{A^{(e)}} A$ is Noetherian for some $e > 0$, then $B$ is $F$-finite over $A$.

Proof. By assumption, $C^{(e)} \otimes_{A^{(e)}} A$ is finite over $B^{(e)} \otimes_{A^{(e)}} A$, and $C$ is finite over $C^{(e)} \otimes_{A^{(e)}} A$. So $C$ is finite over $B^{(e)} \otimes_{A^{(e)}} A$. As $B$ is a $B^{(e)} \otimes_{A^{(e)}} A$-submodule of $C$ and $B^{(e)} \otimes_{A^{(e)}} A$ is Noetherian, $B$ is finite over $B^{(e)} \otimes_{A^{(e)}} A$. □

Lemma 6. Let $A \xrightarrow{f} B$ be a ring homomorphism, and $I$ a finitely generated nilpotent ideal of $B$. If $B/I$ is $A$-finite, then $B$ is $A$-finite.

Proof. As $I^i/I^{i+1}$ is $B/I$-finite for each $i$, it is also $A$-finite. So $B/I^r$ is $A$-finite for each $r$. Taking $r$ large, $B$ is $A$-finite. □

Lemma 7. Let $f: A \xrightarrow{} B$ be an $\mathbb{F}_p$-algebra map, and $I$ a finitely generated nilpotent ideal of $B$. If $B/I$ is $F$-finite over $A$, then $B$ is $F$-finite over $A$.

Proof. As $B/I$ is $F$-finite over $A$, $B/I$ is $(B^{(1)}/I^{(1)}) \otimes_{A^{(1)}} A$-finite. So $B/I$ is also $B^{(1)} \otimes_{A^{(1)}} A$-finite. By Lemma 6, $B$ is $B^{(1)} \otimes_{A^{(1)}} A$-finite. □

For the absolute $F$-finiteness, we have a better result.

Lemma 8. Let $B$ be an $\mathbb{F}_p$-algebra, and $I$ a finitely generated ideal of $B$. If $B$ is $I$-adically complete and $B/I$ is $F$-finite, then $B$ is $F$-finite.

Proof. $B/I$ is $B^{(1)}/I^{(1)}$-finite. So $B/I^{(1)}B$ is $B^{(1)}$-finite by Lemma 6. As $\bigcap I^i = 0$, we have $\bigcap (I^{(1)})^j B = 0$. Moreover, $B^{(1)}$ is $I^{(1)}$-adically complete. Hence $B$ is $B^{(1)}$-finite by [19, Theorem 8.4]. □

Example 9. Let $A$ be an $\mathbb{F}_p$-algebra.

(1) If $A$ is $F$-finite, then the formal power series ring $A[[x]]$ is so.

(2) Let $J$ be an ideal of $A$. If $A$ is Noetherian and $A/J$ is $F$-finite, then the $J$-adic completion $A^e$ of $A$ is $F$-finite.

(3) If $(A, m)$ is complete local and $A/m$ is $F$-finite, then $A$ is $F$-finite.

Proof. For each of (1)–(3), we use Lemma 8. (1) Set $B = A[[x]]$ and $I = Bx$. Then $B/I \cong A$ is $F$-finite. (2) Set $B = A^e$ and $I = JB$. Then $B/I \cong A/J$ is $F$-finite. (3) is immediate. □

Remark 10. Let $A$ be a Noetherian ring and $I$ its ideal. If $A$ is $I$-adically complete and $A/I$ is Nagata, then $A$ is Nagata [17]. If $A$ is semi-local, $I$-adically complete, and $A/I$ is quasi-excellent, then $A$ is quasi-excellent [25]. See also [21].
Lemma 11. Let $A$ be an $\mathbb{F}_p$-algebra, and $B$ and $C$ be $A$-algebras. If $B$ and $C$ are $F$-finite over $A$, then

1. $B \otimes_A C$ is $F$-finite over $A$.
2. $B \times C$ is $F$-finite over $A$.

Proof. (1) $B$ is $F$-finite over $A$, and $B \otimes_A C$ is $F$-finite over $B$ by Lemma 2 (5). By Lemma 2 (2), $B \otimes_A C$ is $F$-finite over $A$. (2) Both $B$ and $C$ are finite over $(B \otimes C)^{(1)} \otimes_A A$, and so is $B \times C$.

Lemma 12. Let $A \to B$ be an $\mathbb{F}_p$-algebra map, and assume that $B$ and $B^{(e)} \otimes_{A^{(e)}} A$ are Noetherian for some $e > 0$. Then $B$ is $F$-finite over $A$ if and only if $B/P$ is $F$-finite over $A$ for every minimal prime $P$ of $B$.

Proof. The ‘only if’ part is obvious by Example 3 (3). We prove the converse. Let $\text{Min } B$ be the set of minimal primes of $B$. Then $\prod_{P \in \text{Min } B} B/P$ is $F$-finite over $A$ by Lemma 11. As $B_{\text{red}} \to \prod_{P \in \text{Min } B} B/P$ is finite injective, and $B^{(e)} \otimes_{A^{(e)}} A$ is Noetherian, $B_{\text{red}}$ is $F$-finite over $A$ by Lemma 5. As $B$ is Noetherian, $B$ is $F$-finite over $A$ by Lemma 7.

Remark 13. Fogarty asserted that an $\mathbb{F}_p$-algebra map $A \to B$ with $B$ Noetherian is $F$-finite if and only if the module of Kähler differentials $\Omega_{B/A}$ is a finite $B$-module [8, Proposition 1]. The ‘only if’ part is true and easy. The proof of ‘if’ part therein has a gap. Although $R_1$ in step (iii) is assumed to be Noetherian, it is not proved that $R'$ in step (iv) is Noetherian. The author does not know if this direction is true or not.

If, moreover, both $A$ and $B$ are Noetherian, the assertion is true. This is an immediate consequence of [1, Proposition 57].

3. Descent of $F$-finiteness

In this section, we prove a sort of descent theorem on $F$-finiteness of homomorphisms.

Let $R$ be a commutative ring, and $f: M \to N$ an $R$-linear map between $R$-modules. We say that $f$ is pure, if $1_W \otimes f: W \otimes_R M \to W \otimes_R N$ is injective for any $R$-module $W$. When we need to clarify the base ring $R$, we also say that $f$ is $R$-pure. A homomorphism of rings $A \to B$ is said to be pure (without mentioning the base ring), if it is $A$-pure (i.e., pure as an $A$-linear map).

Lemma 14. Let $R$ be a commutative ring, $\varphi: M \to N$ and $h: F \to G$ be $R$-linear maps. If $\varphi$ is $R$-pure and $1_N \otimes h: N \otimes F \to N \otimes G$ is surjective, then $1_M \otimes h: M \otimes F \to M \otimes G$ is surjective.
Proof. Let \( C := \text{Coker} \, h \). Then by assumption, \( N \otimes C = 0 \). By the injectivity of \( \varphi \otimes 1_C : M \otimes C \to N \otimes C \), we have that \( M \otimes C = 0 \).

Corollary 15. Let \( A \to B \) be a pure ring homomorphism, and \( h : F \to G \) an \( A \)-linear map. If \( 1_B \otimes h : B \otimes_A F \to B \otimes_A G \) is surjective, then \( h \) is surjective.

Lemma 16. Let \( A \to B \) be a pure ring homomorphism, and \( G \) an \( A \)-module. If \( B \otimes_A G \) is a finitely generated \( B \)-module, then \( G \) is finitely generated as an \( A \)-module.

Proof. Let \( \theta_1, \ldots, \theta_r \) be generators of \( B \otimes_A G \). Then we can write \( \theta_j = \sum_{i=1}^s b_{ij} \otimes g_{ij} \) for some \( s > 0 \), \( b_{ij} \in B \), and \( g_{ij} \in G \). Let \( F \) be the \( A \)-free module with the basis \( \{ f_{ij} : 1 \leq i \leq s, 1 \leq j \leq r \} \), and \( h : F \to G \) be the \( A \)-linear map given by \( f_{ij} \mapsto g_{ij} \). Then by construction, \( 1_B \otimes h \) is surjective. By Corollary 15, \( h \) is surjective, and hence \( G \) is finitely generated.

Definition 17 (cf. [13, (2.7)]). Let \( e > 0 \) be an integer. An \( \mathbb{F}_p \)-algebra map \( A \to B \) is said to be \( e \)-Dumitrescu if \( \Phi_e(A, B) \) is \( A \)-pure.

Lemma 18. Let \( e, e' > 0 \). If \( A \to B \) is both \( e \)-Dumitrescu and \( e' \)-Dumitrescu, then it is \((e + e')\)-Dumitrescu. In particular, an \( e \)-Dumitrescu map is \( er \)-Dumitrescu map for \( r > 0 \).

Proof. This follows from [12, Lemma 4.1, 2].

So a \( 1 \)-Dumitrescu map is Dumitrescu (that is, \( e \)-Dumitrescu for all \( e > 0 \)), see [13, Lemma 2.9].

Lemma 19. Let \( e > 0 \).

(1) [13, Lemma 2.8],
(2) [13, Lemma 2.12], and
(3) [13, Corollary 2.13] hold true when we replace all the ‘Dumitrescu’ therein by ‘\( e \)-Dumitrescu’.

The proof is straightforward, and is left to the reader.

Remark 20. The precise statement of Lemma 19 for (2) is as follows.

Let \( f : A \to B \) be a ring homomorphism between rings of characteristic \( p \), and \( e > 0 \) an integer. Assume that \( A \) is Noetherian, and the image of the associated map \( ^a f : \text{Spec} \, B \to \text{Spec} \, A \) contains \( \text{Max}(A) \), the set of maximal ideals of \( A \). If \( f \) is \( e \)-Dumitrescu, then \( f \) is pure.
Theorem 21. Let \( f : A \to B \) and \( g : B \to C \) be \( \mathbb{F}_p \)-algebra maps, and \( e > 0 \). Assume that \( g \) is \( e \)-Dumitrescu, and the image of the associated map \( \Phi_g : \text{Spec} \ B \to \text{Spec} \ C \) contains the set of maximal ideals \( \text{Max} \ B \) of \( B \). If \( gf \) is \( F \)-finite, and \( B \) and \( C^{(e)} \otimes_{A^{(e)}} A \) are Noetherian, then \( f \) is \( F \)-finite.

Proof. Note that \( \Phi_e(A, C) : C^{(e)} \otimes_{A^{(e)}} A \to C \) is a finite map. Note also that \( C^{(e)} \otimes_{B^{(e)}} B \) is a \( C^{(e)} \otimes_{A^{(e)}} A \)-submodule of \( C \) through \( \Phi_e(B, C) \), since \( \Phi_e(B, C) \) is \( B \)-pure and hence is injective. As \( C^{(e)} \otimes_{A^{(e)}} A \) is Noetherian, \( C^{(e)} \otimes_{B^{(e)}} B \), which is a submodule of the finite module \( C \), is a finite \( C^{(e)} \otimes_{A^{(e)}} A \)-module. Since \( g^{(e)} : B^{(e)} \to C^{(e)} \) is pure by Lemma 19 (2) (see Remark 20), \( B^{(e)} \otimes_{A^{(e)}} A \to C^{(e)} \otimes_{A^{(e)}} A \) is also pure. Since

\[
C^{(e)} \otimes_{B^{(e)}} B \cong (C^{(e)} \otimes_{A^{(e)}} A) \otimes_{B^{(e)} \otimes_{A^{(e)}} A} B
\]

is a finite \( C^{(e)} \otimes_{A^{(e)}} A \)-module, \( B \) is a finite \( B^{(e)} \otimes_{A^{(e)}} A \)-module by Lemma 16. \( \square \)

A homomorphism \( f : A \to B \) between Noetherian rings is said to be reduced if \( f \) is flat with geometrically reduced fibers.

Corollary 22. Let \( g : B \to C \) be a faithfully flat reduced homomorphism between Noetherian \( \mathbb{F}_p \)-algebras. If \( C \) is \( F \)-finite, then \( B \) is \( F \)-finite.

Proof. By [5, Theorem 3], \( g \) is Dumitrescu. As \( g \) is faithfully flat, \( ^{\text{a}}g : \text{Spec} \ C \to \text{Spec} \ B \) is surjective. Letting \( A = \mathbb{F}_p \) and \( f : A \to B \) be the unique map, the assumptions of Theorem 21 are satisfied, and hence \( f \) is \( F \)-finite. That is, \( B \) is \( F \)-finite. \( \square \)

Corollary 23 (Seydi [26]). Let \( (B, \mathfrak{m}) \) be a Nagata local ring with the \( F \)-finite residue field \( k = B/\mathfrak{m} \). Then \( B \) is \( F \)-finite. In particular, \( B \) is excellent, and is a homomorphic image of an \( F \)-finite regular local ring. So \( B \) has a dualizing complex.

Proof. Let \( g : B \to C = \hat{B} \) be the completion of \( B \). Then \( C \) is a complete local ring with the residue field \( k \). By Example 9 (3), \( C \) is \( F \)-finite. As \( g \) is reduced by [10, (7.6.4), (7.7.2)], \( B \) is \( F \)-finite by Corollary 22.

Now \( B \) is excellent by [16, Theorem 2.5] and is a homomorphic image of an \( F \)-finite regular local ring by [9, Remark 13.6]. The last assertion follows from the fact that a homomorphic image of a Gorenstein ring has a dualizing complex if it is of finite Krull dimension. For dualizing complexes, see [11]. \( \square \)

Even if \( A \to B \) is a faithfully flat reduced homomorphism and \( B \) is excellent, \( A \) need not be quasi-excellent. There is a Nagata local ring \( A \) which is not quasi-excellent [24], [22], and its completion \( A \to \hat{A} = B \) is an example. On the other hand, if \( A \to B \) is a faithfully flat regular homomorphism and \( B \) is quasi-excellent, then \( A \) is quasi-excellent [19, Theorem 32.2].
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References


Graduate School of Mathematics
Nagoya University
Chikusa-ku, Nagoya 464–8602
Japan

Current address:
Department of Mathematics
Faculty of Science
Okayama University
3-1-1 Tsushima-Naka, Kita-ku
Okayama 700–8530
Japan
e-mail: mh@okayama-u.ac.jp