Minami, H.
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CORRECTION TO

"ON J_R-HOMOMORPHISMS"

HARUO MINAMI

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The proof of Theorem 4.1 in [3] is incorrect, for the relation $(ig)\circ\gamma\simeq_{\tau}\gamma\circ(if)$ made on p. 587 does not hold. The mistake was pointed out also by Professor Victor Snaith in Math. Rev. (1981). The following theorem for $G=Z/2$ proves immediately a modified form of Theorem 4.1 (Corollary) via the canonical transformation σ from $KR(-)$ to $KO_G(-)$. Because σ commutes with the p-th Adams operations for p odd (This is proved by a similar discussion as the usual realification homomorphism). This theorem seems to follow from [2], Theorem 11.4.1 and the result on p. 295. But the latter is not necessary for a proof of our case. So we give here a direct simple proof of it by using only Proposition 11.3.7 of [2].

We use the notations of [3] and [1]. Let X be a $\Sigma^{1,1}$-*suspension* of a finite pointed G-complex, and let p be an *odd* prime. By ψ^p we denote the p-th Adams operation in KO_G-, KR- or KO-theory.

Theorem. *Let J_G denote the equivariant stable real J-homomorphism defined by the usual construction. Then, for any $x\in\widetilde{KO}_G^{-1}(X)$*

$$J_G(1-\psi^p)(x)=0$$

as an element of $\pi_S^{0,0}(X)_{(2)}$. ($L_{(2)}$ is the localization of a group L at the prime 2.)

Corollary. *For any $x\in\widetilde{KR}^{-1}(X)$*

$$J_R(1-\psi^p)(x)=0$$

as an element of $\pi_S^{0,0}(X)_{(2)}$.

1. Observe the composites

$$\widetilde{KO}_G^{-1}(X)\xrightarrow{\psi}\widetilde{KO}^{-1}(X)\xrightarrow{\delta}\widetilde{KO}_G^{-1}(X)\ \text{ and }\ \pi_S^{0,0}(X)\xrightarrow{\psi}\pi_S^{0}(X)\xrightarrow{\delta}\pi_S^{0,0}(X)$$

then by the definitions of ψ and δ we have

(1) $$\delta\psi=1-\rho \text{ times} \quad (\text{cf. [1], Lemma 12.6}).$$

Let f: $X \rightarrow SO(2n)$ be a pointed G-map. Here $SO(2n)$ denotes the rotation group of degree $2n$ with the involution $A \mapsto \tau A \tau$ where τ is the involution of $R^{n,n}$, and the unit matrix as basepoint. Define a G-map $\hat{f}$: $X \times R^{n,n} \rightarrow X \times R^{n,n}$ by $\hat{f}(x, u) = (x, f(x)u)$ for $x \in X$, $u \in R^{n,n}$. We denote by E_f a real G-vector bundle over $\Sigma^{0,1}X$ obtained from the clutching of trivial bundles $E_1 = C^+X \times R^{n,n}$ and $E_2 = C^-X \times R^{n,n}$ by $\hat{f}$: $E_1|_X \rightarrow E_2|_X$, where $C^{\pm}X = \{t \wedge x \in \Sigma^{0,1}X \mid \pm t \geq 0\}$ respectively. As is well known any element of $\widetilde{KO}_G(\Sigma^{0,1}X)$ is the stable equivalence class $\{E_f\}$ of such a G-vector bundle E_f.

Since p is odd we can set

$$\psi^p(\{E_f\}) = \{E_g\}$$

where g is a pointed G-map from X to $SO(2n)$.

By $\tilde{F}$ we denote the fibrewise one-point compactification of a real G-vector bundle F. According to [2], Proposition 11.3.7 we may have

(2) There exists a fibrewise G-map $\tilde{\gamma}$: $\tilde{E}_f \rightarrow \tilde{E}_g$ with fibre-degree p^i for some $i \geq 0$, which preserves the basepoints of fibres.

2. Let γ be the restriction of $\tilde{\gamma}$ to the fibre on the basepoint of $\Sigma^{0,1}X$, which is a pointed G-map of $\Sigma^{n,n}$ into itself. Write $s + t\rho \in A(G) = Z[\rho]/(1-\rho^2)$ for the element determined by γ (see [1], §§1 and 2). Then

$$s - t = p^i .$$

Moreover let $\tilde{\gamma}_k$ be the restrictions of $\tilde{\gamma}$ to $\tilde{E}_k$ for $k=1, 2$. Then, as a fibrewise G-map

$$\tilde{\gamma}_k \simeq_G 1 \times \gamma$$

for $k=1, 2$ evidently. Considering γ to be the constant G-map from X to $\Omega^{n,n}\Sigma^{n,n}$ given by $X \mapsto \{\gamma\}$, and also f and g to be the G-maps from X to $\Omega^{n,n}\Sigma^{n,n}$ respectively through the natural inclusion $SO(2n) \subset \Omega^{n,n}\Sigma^{n,n}$, we therefore have

$$\gamma \circ f \simeq_G g \circ \gamma : X \rightarrow \Omega^{n,n}\Sigma^{n,n}$$

and hence

$$f \circ \gamma \simeq_G g \circ \gamma$$

as a G-map from X to $\Omega^{2n,2n}\Sigma^{2n,2n}$. Add $-\gamma$ to the both sides of this relation, then it follows from the definition of J_G that $(s+t\rho)J_G(1-\psi^p)(\{E_f\})=0$ That is, we obtain

$$((2t+p^i)+t(\rho-1))J_G(1-\psi^p)(x) = 0 \quad \text{for any } x \in \widetilde{KO}_G^{-1}(X) . \tag{3}$$

Applying ψ to this equality we have

$$J_o(1-\psi^p)(\psi(x))_{(2)} = 0 \tag{4}$$

where J_o is the stable real J-homomorphism. (Here we wite $z_{(2)}$ for $z \in L$ viewed as an element of $L_{(2)}$.)

By (3) and (1)

$$(2t+p^i)J_G(1-\psi^p)(x) = t\delta\psi^r J_G(1-\psi^p)(x) = t\delta J_o(1-\psi^p)(\psi^r(x)).$$

Thus, by (4)

$$J_G(1-\psi^p)(x)_{(2)} = 0$$

since $2t+p^i$ is odd. This completes the proof of Theorem.

References

[1] S. Araki and M. Murayama: *τ-Cohomology theories,* Japan J. Math. **4** (1978), 363–416.
[2] T. tom Dieck: Transformation groups and representation theory, Lecture Notes in Math. **766**, Springer-Verlag, 1979.
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Department of Mathematics
Osaka City University
Sugimoto, Sumiyoshi-ku
Osaka 558, Japan