

Title	Correction to : "On J_R -homomorphisms"
Author(s)	Minami, Haruo
Citation	Osaka Journal of Mathematics. 20(4) P.939-P.941
Issue Date	1983
Text Version	publisher
URL	https://doi.org/10.18910/5796
DOI	10.18910/5796
rights	
Note	

Osaka University Knowledge Archive : OUKA

<https://ir.library.osaka-u.ac.jp/>

Osaka University

CORRECTION TO
“ON J_R -HOMOMORPHISMS”

HARUO MINAMI

(Received December 2, 1982)

The proof of Theorem 4.1 in [3] is incorrect, for the relation $(ig) \circ \gamma \simeq_\tau \gamma \circ (if)$ made on p. 587 does not hold. The mistake was pointed out also by Professor Victor Snaith in Math. Rev. (1981). The following theorem for $G = Z/2$ proves immediately a modified form of Theorem 4.1 (Corollary) via the canonical transformation σ from $KR(-)$ to $KO_G(-)$. Because σ commutes with the p -th Adams operations for p odd (This is proved by a similar discussion as the usual realification homomorphism). This theorem seems to follow from [2], Theorem 11.4.1 and the result on p. 295. But the latter is not necessary for a proof of our case. So we give here a direct simple proof of it by using only Proposition 11.3.7 of [2].

We use the notations of [3] and [1]. Let X be a $\Sigma^{1,1}$ -suspension of a finite pointed G -complex, and let p be an odd prime. By ψ^p we denote the p -th Adams operation in KO_G -, KR - or KO -theory.

Theorem. *Let J_G denote the equivariant stable real J -homomorphism defined by the usual construction. Then, for any $x \in \widetilde{KO}_G^{-1}(X)$*

$$J_G(1 - \psi^p)(x) = 0$$

as an element of $\pi_s^{0,0}(X)_{(2)}$. ($L_{(2)}$ is the localization of a group L at the prime 2.)

Corollary. *For any $x \in \widetilde{KR}^{-1}(X)$*

$$J_R(1 - \psi^p)(x) = 0$$

as an element of $\pi_s^{0,0}(X)_{(2)}$.

1. Observe the composites

$$\widetilde{KO}_G^{-1}(X) \xrightarrow{\psi} \widetilde{KO}^{-1}(X) \xrightarrow{\delta} \widetilde{KO}_G^{-1}(X) \quad \text{and} \quad \pi_s^{0,0}(X) \xrightarrow{\psi} \pi_s^0(X) \xrightarrow{\delta} \pi_s^{0,0}(X)$$

then by the definitions of ψ and δ we have

$$(1) \quad \delta\psi = 1 - \rho \text{ times} \quad (\text{cf. [1], Lemma 12.6}).$$

Let $f: X \rightarrow SO(2n)$ be a pointed G -map. Here $SO(2n)$ denotes the rotation group of degree $2n$ with the involution $A \mapsto \tau A \tau$ where τ is the involution of $R^{n,n}$, and the unit matrix as basepoint. Define a G -map $\hat{f}: X \times R^{n,n} \rightarrow X \times R^{n,n}$ by $\hat{f}(x, u) = (x, f(x)u)$ for $x \in X, u \in R^{n,n}$. We denote by E_f a real G -vector bundle over $\Sigma^{0,1}X$ obtained from the clutching of trivial bundles $E_1 = C^+X \times R^{n,n}$ and $E_2 = C^-X \times R^{n,n}$ by $\hat{f}: E_1|_X \rightarrow E_2|_X$, where $C^\pm X = \{t \wedge x \in \Sigma^{0,1}X \mid \pm t \geq 0\}$ respectively. As is well known any element of $\widetilde{KO}_G(\Sigma^{0,1}X)$ is the stable equivalence class $\{E_f\}$ of such a G -vector bundle E_f .

Since p is odd we can set

$$\psi^p(\{E_f\}) = \{E_g\}$$

where g is a pointed G -map from X to $SO(2n)$.

By \tilde{F} we denote the fibrewise one-point compactification of a real G -vector bundle F . According to [2], Proposition 11.3.7 we may have

(2) There exists a fibrewise G -map $\tilde{\gamma}: \tilde{E}_f \rightarrow \tilde{E}_g$ with fibre-degree p^i for some $i \geq 0$, which preserves the basepoints of fibres.

2. Let γ be the restriction of $\tilde{\gamma}$ to the fibre on the basepoint of $\Sigma^{0,1}X$, which is a pointed G -map of $\Sigma^{n,n}$ into itself. Write $s + t\rho \in A(G) = Z[\rho]/(1 - \rho^2)$ for the element determined by γ (see [1], §§1 and 2). Then

$$s - t = p^i.$$

Moreover let $\tilde{\gamma}_k$ be the restrictions of $\tilde{\gamma}$ to \tilde{E}_k for $k=1, 2$. Then, as a fibrewise G -map

$$\tilde{\gamma}_k \simeq_G 1 \times \gamma$$

for $k=1, 2$ evidently. Considering γ to be the constant G -map from X to $\Omega^{n,n}\Sigma^{n,n}$ given by $X \mapsto \{\gamma\}$, and also f and g to be the G -maps from X to $\Omega^{n,n}\Sigma^{n,n}$ respectively through the natural inclusion $SO(2n) \subset \Omega^{n,n}\Sigma^{n,n}$, we therefore have

$$\gamma \circ f \simeq_G g \circ \gamma: X \rightarrow \Omega^{n,n}\Sigma^{n,n}$$

and hence

$$f \circ \gamma \simeq_G g \circ \gamma$$

as a G -map from X to $\Omega^{2n,2n}\Sigma^{2n,2n}$. Add $-\gamma$ to the both sides of this relation, then it follows from the definition of J_G that $(s + t\rho)J_G(1 - \psi^p)(\{E_f\}) = 0$. That is, we obtain

$$(3) \quad ((2t + p^i) + t(\rho - 1))J_G(1 - \psi^p)(x) = 0 \quad \text{for any } x \in \widetilde{KO}_G^{-1}(X).$$

Applying ψ to this equality we have

$$(4) \quad J_G(1 - \psi^p)(\psi(x))_{(2)} = 0$$

where J_o is the stable real J -homomorphism. (Here we write $z_{(2)}$ for $z \in L$ viewed as an element of $L_{(2)}$.)

By (3) and (1)

$$\begin{aligned} (2t+p^i)J_G(1-\psi^p)(x) &= t\delta\psi J_G(1-\psi^p)(x) \\ &= t\delta J_o(1-\psi^p)(\psi(x)). \end{aligned}$$

Thus, by (4)

$$J_G(1-\psi^p)(x)_{(2)} = 0$$

since $2t+p^i$ is odd. This completes the proof of Theorem.

References

- [1] S. Araki and M. Murayama: τ -Cohomology theories, Japan J. Math. **4** (1978), 363–416.
- [2] T. tom Dieck: Transformation groups and representation theory, Lecture Notes in Math. **766**, Springer-Verlag, 1979.
- [3] H. Minami: On J_R -homomorphisms, Osaka J. Math. **16** (1979), 581–588.

Department of Mathematics
Osaka City University
Sugimoto, Sumiyoshi-ku
Osaka 558, Japan

