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CORRECTION TO  
**“ON  $J_R$ -HOMOMORPHISMS”**

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The proof of Theorem 4.1 in [3] is incorrect, for the relation  $(ig) \circ \gamma \simeq_\tau \gamma \circ (if)$  made on p. 587 does not hold. The mistake was pointed out also by Professor Victor Snaith in Math. Rev. (1981). The following theorem for  $G = Z/2$  proves immediately a modified form of Theorem 4.1 (Corollary) via the canonical transformation  $\sigma$  from  $KR(-)$  to  $KO_G(-)$ . Because  $\sigma$  commutes with the  $p$ -th Adams operations for  $p$  odd (This is proved by a similar discussion as the usual realification homomorphism). This theorem seems to follow from [2], Theorem 11.4.1 and the result on p. 295. But the latter is not necessary for a proof of our case. So we give here a direct simple proof of it by using only Proposition 11.3.7 of [2].

We use the notations of [3] and [1]. Let  $X$  be a  $\Sigma^{1,1}$ -suspension of a finite pointed  $G$ -complex, and let  $p$  be an odd prime. By  $\psi^p$  we denote the  $p$ -th Adams operation in  $KO_G$ -,  $KR$ - or  $KO$ -theory.

**Theorem.** *Let  $J_G$  denote the equivariant stable real  $J$ -homomorphism defined by the usual construction. Then, for any  $x \in \widetilde{KO}_G^{-1}(X)$*

$$J_G(1 - \psi^p)(x) = 0$$

as an element of  $\pi_s^{0,0}(X)_{(2)}$ . ( $L_{(2)}$  is the localization of a group  $L$  at the prime 2.)

**Corollary.** *For any  $x \in \widetilde{KR}^{-1}(X)$*

$$J_R(1 - \psi^p)(x) = 0$$

as an element of  $\pi_s^{0,0}(X)_{(2)}$ .

1. Observe the composites

$$\widetilde{KO}_G^{-1}(X) \xrightarrow{\psi} \widetilde{KO}^{-1}(X) \xrightarrow{\delta} \widetilde{KO}_G^{-1}(X) \quad \text{and} \quad \pi_s^{0,0}(X) \xrightarrow{\psi} \pi_s^0(X) \xrightarrow{\delta} \pi_s^{0,0}(X)$$

then by the definitions of  $\psi$  and  $\delta$  we have

$$(1) \quad \delta\psi = 1 - \rho \text{ times} \quad (\text{cf. [1], Lemma 12.6}).$$

Let  $f: X \rightarrow SO(2n)$  be a pointed  $G$ -map. Here  $SO(2n)$  denotes the rotation group of degree  $2n$  with the involution  $A \mapsto \tau A \tau$  where  $\tau$  is the involution of  $R^{n,n}$ , and the unit matrix as basepoint. Define a  $G$ -map  $\hat{f}: X \times R^{n,n} \rightarrow X \times R^{n,n}$  by  $\hat{f}(x, u) = (x, f(x)u)$  for  $x \in X, u \in R^{n,n}$ . We denote by  $E_f$  a real  $G$ -vector bundle over  $\Sigma^{0,1}X$  obtained from the clutching of trivial bundles  $E_1 = C^+X \times R^{n,n}$  and  $E_2 = C^-X \times R^{n,n}$  by  $\hat{f}: E_1|_X \rightarrow E_2|_X$ , where  $C^\pm X = \{t \wedge x \in \Sigma^{0,1}X \mid \pm t \geq 0\}$  respectively. As is well known any element of  $\widetilde{KO}_G(\Sigma^{0,1}X)$  is the stable equivalence class  $\{E_f\}$  of such a  $G$ -vector bundle  $E_f$ .

Since  $p$  is odd we can set

$$\psi^p(\{E_f\}) = \{E_g\}$$

where  $g$  is a pointed  $G$ -map from  $X$  to  $SO(2n)$ .

By  $\tilde{F}$  we denote the fibrewise one-point compactification of a real  $G$ -vector bundle  $F$ . According to [2], Proposition 11.3.7 we may have

(2) There exists a fibrewise  $G$ -map  $\tilde{\gamma}: \tilde{E}_f \rightarrow \tilde{E}_g$  with fibre-degree  $p^i$  for some  $i \geq 0$ , which preserves the basepoints of fibres.

2. Let  $\gamma$  be the restriction of  $\tilde{\gamma}$  to the fibre on the basepoint of  $\Sigma^{0,1}X$ , which is a pointed  $G$ -map of  $\Sigma^{n,n}$  into itself. Write  $s + t\rho \in A(G) = Z[\rho]/(1 - \rho^2)$  for the element determined by  $\gamma$  (see [1], §§1 and 2). Then

$$s - t = p^i.$$

Moreover let  $\tilde{\gamma}_k$  be the restrictions of  $\tilde{\gamma}$  to  $\tilde{E}_k$  for  $k=1, 2$ . Then, as a fibrewise  $G$ -map

$$\tilde{\gamma}_k \simeq_G 1 \times \gamma$$

for  $k=1, 2$  evidently. Considering  $\gamma$  to be the constant  $G$ -map from  $X$  to  $\Omega^{n,n}\Sigma^{n,n}$  given by  $X \mapsto \{\gamma\}$ , and also  $f$  and  $g$  to be the  $G$ -maps from  $X$  to  $\Omega^{n,n}\Sigma^{n,n}$  respectively through the natural inclusion  $SO(2n) \subset \Omega^{n,n}\Sigma^{n,n}$ , we therefore have

$$\gamma \circ f \simeq_G g \circ \gamma: X \rightarrow \Omega^{n,n}\Sigma^{n,n}$$

and hence

$$f \circ \gamma \simeq_G g \circ \gamma$$

as a  $G$ -map from  $X$  to  $\Omega^{2n,2n}\Sigma^{2n,2n}$ . Add  $-\gamma$  to the both sides of this relation, then it follows from the definition of  $J_G$  that  $(s + t\rho)J_G(1 - \psi^p)(\{E_f\}) = 0$ . That is, we obtain

$$(3) \quad ((2t + p^i) + t(\rho - 1))J_G(1 - \psi^p)(x) = 0 \quad \text{for any } x \in \widetilde{KO}_G^{-1}(X).$$

Applying  $\psi$  to this equality we have

$$(4) \quad J_G(1 - \psi^p)(\psi(x))_{(2)} = 0$$

where  $J_o$  is the stable real  $J$ -homomorphism. (Here we write  $z_{(2)}$  for  $z \in L$  viewed as an element of  $L_{(2)}$ .)

By (3) and (1)

$$\begin{aligned} (2t+p^i)J_G(1-\psi^p)(x) &= t\delta\psi J_G(1-\psi^p)(x) \\ &= t\delta J_o(1-\psi^p)(\psi(x)). \end{aligned}$$

Thus, by (4)

$$J_G(1-\psi^p)(x)_{(2)} = 0$$

since  $2t+p^i$  is odd. This completes the proof of Theorem.

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#### References

- [1] S. Araki and M. Murayama:  $\tau$ -Cohomology theories, Japan J. Math. **4** (1978), 363–416.
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