Survey of Methods of Analysis of the Strength of Stiffened and Unstiffened Plates Subjected to Combined In-Plane and Lateral Loads†

Yukio UEDA *, S.M.H. RASHED ** and W. SOSNOWSKI ***

Abstract

One of the main strength members of a ship structure is the bottom plates which are usually reinforced by stiffeners. These plates are subjected usually to lateral water pressure and in-plane loads due to bending of the structure. Therefore, it is very important to clarify the behavior and strength of stiffened and unstiffened plates under those loadings. In this paper, a survey was performed on methods of analysis of strength of stiffened and unstiffened plates subjected to combined in-plane and lateral loads.

KEY WORDS: (Stiffened Plates) (Combined Loads) (Ultimate Strength) (Buckling)

1. Introduction

The problem of the rectangular stiffened and unstiffened plates subjected to loads both normal to and in the plane of the plate is very important, particularly to ship designers, but only few papers mostly concerning limited cases of loading are known to the authors of this survey. Such situation is caused by the fact that in traditional design methods considering the elastic behavior of structures, the influence of lateral loading on buckling strength was usually neglected because of the assumption that a strengthening effect due to a bending configuration different from the buckling mode and a weakening effect due to lateral deflection cancel each other.

Although there exist comparatively many papers on the linear behavior of plates loaded in plane and laterally, large deflection analysis is usually limited to lateral load or in-plane shear and/or compression or to both in-plane uniaxial compression and lateral load (without taking shearing force into account).

However, out of plane bending, shear, tension and compression effects occur together in almost all practical ship structures and are not separable or additive.

The existing extensive literature on stiffened plates is directed mainly to plates loaded with compression, because of the very important buckling problem. A comprehensive survey of methodologies predominantly used for in-plane loaded plates was made by C.G. Soarez[2]. He described the basic principles and characteristics of the different analytical and numerical methods and made many comparisons among results by various authors.

In this survey, the attention is concentrated on the behavior of unstiffened and stiffened plates loaded with combined in-plane and lateral loads.

2. Unstiffened plates subjected to combined in-plane load and lateral pressure

In the simplest and oldest method which has been used for solving the equation for unstiffened plates under combined loading, the available elastic solutions for the plate under lateral pressure alone and the solutions for the plate under in-plane loading have been superimposed[3].

Such approach can be improved by the implementation of edge compression into the flexural equation for the lateral load so as to obtain the linearized solution in which the flexural stiffness is reduced by the effect of compressive in-plane load, the deflection and bending stresses are magnified. A solution obtained in this way[4]. still needs to be superimposed on the in-plane stresses estimated independently to give the final stress distribution.

The general theory of large deflection for the elastic and elastic-plastic behavior of rectangular plates which takes both membrane and bending actions of the plates

† Received on October 31, 1983
* Professor
** Senior Specialist, Century Research Center Corp., Osaka
*** Post-doctoral Researcher, Institute of Fundamental Technical Research, Polish Academy of Sciences, Poland

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into account as well as initial imperfections was developed in 5) and 6), but due to the complexity of the equations no explicit solutions were obtained.

In reference 7), work on the behavior of rectangular plates subjected to both lateral and in-plane loads is presented. Experimental work is described for two square and two rectangular plates and compared with the solutions given by Timoshenko[3], Bleich[4], Chang and Convey[8], limited to the small deflection theory. In reference 7), uniaxial and biaxial compressions are considered and the plate edges are fixed.

Large deflection analysis for an infinite plate strip subjected to lateral load and temperature variations across the width of the strip is presented in reference 9). Results are graphed for both clamped and simply supported edge conditions.

Simply supported rectangular plates subjected to the combined action of a uniformly distributed lateral load and compressive forces in the middle plane are analyzed by Burghgraff in 10). The analysis is based on a comparison of the results obtained from Marguerres[11] theory of large deflections and from Saint Venant's linear equation for small deflections of plates. A magnification factor is derived by means of a similar procedure as used by F. Bleich[4]. More accurate results are derived by taking a term neglected by Bleich into account.

An experimental and theoretical investigation of simply supported thin plates subjected to lateral load and uniaxial compression is described in 12). The Von Karmans' large deflection equations for thin plates in which the membrane loading terms satisfy the mid-plane edge conditions were solved by approximate Galerkin's integral technique. The results are valid in the pre- and post-buckling range.

Limited to the elastic range, Becker, et al.[39], conducted experimental studies described in 2). They observed that the lateral load increases the longitudinal compressive strength of rectangular plates. On the other hand it decreases its transverse compressive strength. These results from the difference of the deflection mode from the buckling mode in the longitudinal direction and the coincidence of the deflection mode with the buckling mode in the transverse direction of the plate.

An interesting remark may be added, there exist some results given in 2), showing similar effect in the presence of transverse compression on longitudinal compressive strength of plates.

Buckling strength of long rectangular plates under compression and hydrostatic pressure was studied by Okada et al.[13] by means of analytical methods. The effect of aspect ratio of plates on their buckling strength under such combined loads was also discussed. In reference 14), they studied the behavior of similarly loaded plates up to the ultimate strength by means of the following two methods:

a) assuming that a plate reaches its ultimate strength when the normal stress in the direction of compression at the edges of the plate becomes equal to the yield stress of the material and

b) assuming that a plate reaches its ultimate strength when the plate satisfies the yield criterion.

In later papers 15–20), different aspects of behaviors of similarly loaded plates were theoretically and experimentally studied.

From a practical point of view, a very useful work is presented by Aalami, et al.[21-23]. They described an experimental as well as theoretical ultimate strength analysis of isotropic plates under in-plane compression and lateral pressure.

In paper 22), *87* square plates with a breadth to thickness ratio of *50 : 1* subjected to varying ratios of lateral pressure to edge compression were tested. Interaction curves between the in-plane loading and the transverse pressure have been obtained (see **Fig. 1**) which show the ultimate strength of the panels. It is found, that under certain combinations of loading (*Q/N<25*), the plate

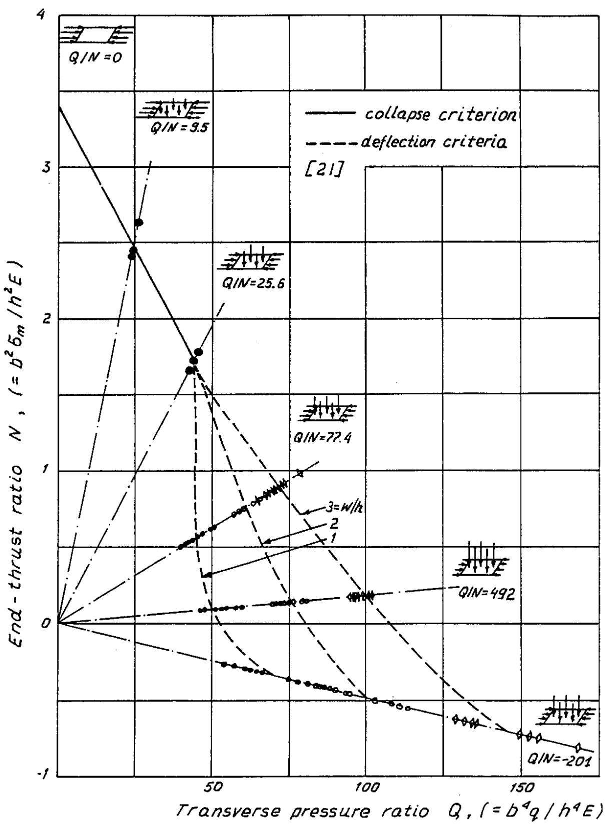

Fig. 1 Interaction curves between end-thrust and transverse pressure for panel failure

undergoes a sudden collapse due to plastic instability, while under other load combinations the plate exhibits increasing plastic deformation with load increments. The effects of residual stresses and initial deformations were investigated, too.

In refs. 21) and 23), large deflection behavior of rectangular orthotropic and isotropic plates subjected to lateral load and in-plane compression are investigated by the finite difference method. The stress distribution is shown to be greatly affected by the in-plane as well as the flexural boundary conditions. Several other important conclusions are presented especially concerning the influence of different boundary conditions and lateral load on deflection, restraints and stress distribution. It is found particularly, that

- in simply supported plates under edge compression and lateral loads of low values the central deflection and moments are very sensitive to the magnitude of the applied in-plane loading but under lateral loads of higher values this sensitivity diminishes
- the flexural behavior of simply supported square plates under uniform biaxial edge compression closely resembles that of a similar plate subjected to uniaxial edge compression.

A similar problem was analyzed later by Fujita, et al.[24] by means of analytical methods. The obtained results for plates with various aspect ratios ($\beta = 1, 2$ and 3) are in good agreement with experimental ones conducted by Yoshiki Yamamoto, et al.[25-26].

Plate panels of ship structures do not act in isolation. It is very important to properly choose suitable membranes and flexural boundary conditions. The former is in the plane of the plate perpendicular to the edge and/or in the plane of the plate parallel to the edge, and the latter is rotational and/or normal to the plane of the plate. Some finite element analysis solutions like those presented in 27) or 28) for instance, may be used directly for the analysis of plates under complex loading including lateral pressure. However, boundary conditions must be defined carefully in order to obtain realistic results.

Ship designers used to neglect lateral load in plate buckling analysis assuming that its different effects should cancel each other. However, application of such practice to the post-buckling behavior and ultimate strength cannot be approved.

3. Stiffened plates under combined in-plane and lateral load

The buckling and post-buckling behavior of a stiffened plate subjected to combined loads depends mainly on two

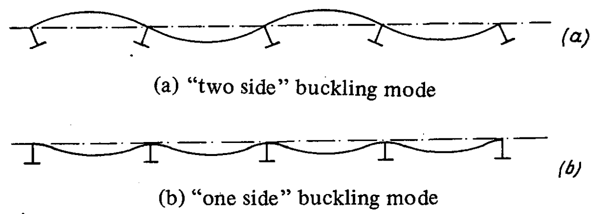

Fig. 2 Stiffened plate under combined load:

factors:

1. The proportions between in-plane and lateral load components
2. The stiffness ratio of the stiffeners to the plate.

As for the first, the characteristic ratio of in-plane force to lateral load may be defined, for which "two sides buckling mode" is replaced by "one side buckling mode" (**Fig. 2**)[38), 29)].

The second factor was fully analyzed for in-plane load by Ueda, et al., in 28). The authors defined two significant stiffness ratios of a stiffener to the plate:

γ^B_{min}, which defines a limiting value of the buckling strength and γ^B_{min}, at which the collapse mode changes from overall to local one.

Where γ characterizes the relation between the stiffness of the stiffener and the plate respectively.

The collapse mode of the stiffened plate is classified as follows:

MODE OO ($\gamma < \gamma^B_{min}$): overall collapse after overall buckling,

MODE LO ($\gamma^B_{min} < \gamma < \gamma^U_{min}$): overall collapse after local buckling,

MODE LL ($\gamma^U_{min} < \gamma$): local collapse after local buckling.

The effects of initial imperfections such as welding residual stresses and initial deflections on ultimate strength are also discussed.

Two commonly used approximate solutions of stiffened plating subjected to lateral load acting alone or combined with in-plane load are:

(a) by treating the structure as an equivalent *orthotropic plate*, whose flexural and torsional rigidities in the orthogonal directions represent the combined strength of stiffeners and plates.
(b) by a *discrete beam idealization* in which plating is represented by effective flanges acting with the stiffeners according to the simple beam theory.

3.1 "Orthotropic plate" model of analysis

An orthotropic plate model has been used by Mansour [30-32]. He examined the behavior of stiffened plates subjected to uniform lateral pressure and in-plane compression and shear, assuming that the stiffeners in either direction are equally stiff and spaced closely enough so that the plate elements may be considered fully effective. An equivalent homogenous orthotropic plate of constant thickness has been considered with appropriate rigidity factors in both directions. Mansour's approach is, however, limited to the elastic range.

This kind of approach was used by other authors to make comparisons with the discrete beam idealization [35),36)] or experimental results[45)]. Information about these works are given in chaps. 3.2 and 3.3.

3.2 Discrete beam idealization

Discrete beam idealization has an advantage to be applied to structures with irregular boundaries, hatch openings or such structures in which beams are non-uniform along their lengths.

Existing application of the thin plate theory to the analysis of stiffened plates has been confined mainly to flat panels with symmetrical stiffeners in which the stiffeners are assumed to behave according to the simple beam theory. Such technique has been employed by Timoshenko[3)] and Schade[1)] to investigate the effective breadth problem.

One of the first who developed the beam-colum method for approximate analyses of stiffened plates subjected to combined loads is Ostapenko[33)]. He considered the response of plates stiffened on one side in the pre-buckling, buckling, post-buckling and ultimate strength ranges under various combinations of geometric proportions and load intensities. The main assumption of his approach was that due to a large plate width: the interaction of the stiffeners can be neglected so that the behavior of one representative stiffener with a tributary plate width can represent the whole plate.

Smith[34)] described a generalized plate-beam analysis in which the stiffened plate is treated as a plane system of interconnected beams loaded laterally.

This method of analysis was extended later by the same author in references 35) and 36) to take the effects of combined lateral and axial loading into account. The discrete beam idealization used for analyses of ship bottom shell, side shell, bulkhead and deck stiffened structures was compared with the orthotropic plate solutions to suggest the use of these methods in design. A more approximate method of the analysis based on the orthotropic plate idealization has also been examined. This formulation is, however, restricted to the elastic behavior.

An important contribution to the influence of lateral pressure on the behavior of compressed stiffened plates was made by Faulkner[37)], who reported that lateral pressure in a ship slightly raises the critical load though it will not prevent the usual buckled pattern, but can affect the effective width equations in the post-buckling range and considerably decrease the ultimate strength. He also reported an extensive review of effective plating. Much more information about the influence of lateral load may be found in ref. 38), by Faulkner, et al. They reported that the interaction relationship of "column" failure of stiffeners in a stiffened plate structure, assumed as ultimate strength, can be shown as in (**Fig. 3**). In the region where the collapse is predominantly due to lateral load, the ultimate in-plane copressive strength diminishes linearly as the lateral load increases.

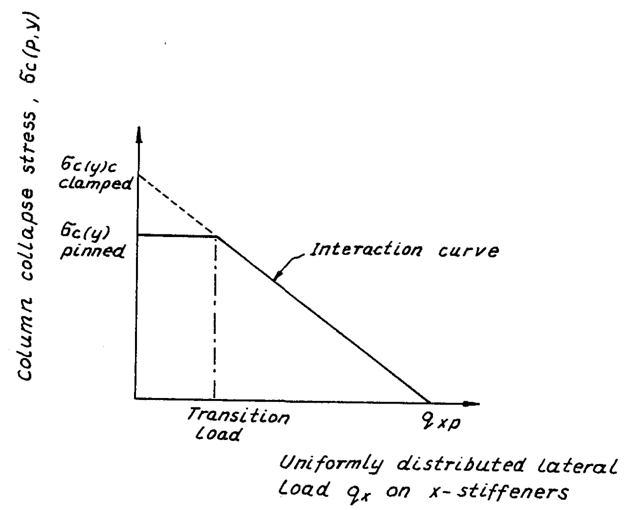

Fig. 3 Interaction curve for column collapse under combined lateral and in-plane loading

One of Faulkner's important conclusions supported by experimental data is that below some certain critical value of lateral pressure, the ultimate strength should be calculated assuming pinned edges of the plate, while above this value, clamped boundaries are more appropriate. This critical value is defined as $q_{cr} = \sigma_o^2/E\beta$ where $\beta = \frac{b}{t}\sqrt{\sigma_o/E}$ is the slenderness ratio, σ_o is the yield stress, b and t distance between stiffeners and thickness of the plate respectively.

The ultimate strength of a rectangular stiffened plate under any combination of compressive, in-plane bending, shearing and lateral pressure load are studies in recent years by S. Nishihara and T. Fukuoka[40)] and by S. Nishihara in reference 41–42). The applied approximate analytical method consists of two parts. The first is the elastic large deformation analysis which takes into account the plastification of the stiffener in a simplified way. The second is the plastic analysis in which a collapse mechanism is assumed. In reference 41), particular attention is given to the efficiency of plates stiffened on one side and subjected to lateral pressure combined with compression. Faulkner's effective width formula is used. In reference 42) in which the same problem is analyzed, initial deflections and residual stresses are taken into account. Approximate formulas developed for calculating the ultimate strength are in good agreement with past experimental results.

3.3 Other references related to this survey

The procedure of the finite element method was used for study on behavior of stiffened plates in the ultimate limit state by Soreide, et al., in 43). They used Von Karman's large-deflection strain displacement relations accounting for initial imperfections, plastic behavior for general loading and boundary conditions. Design criteria are discussed and optimum plate dimensions are recommended for simultanous local buckling of the plate and global buckling of the whole structure.

The post buckling behavior of panels for axial and bending load was studied by Murray in ref. 44) by developing comprehensive collapse mechanism for the plate and the stiffener. Comparison is made with tests on large stiffeners panels.

Experiments conducted on the one eighth scale model of the double-bottom structure by applying longitudinal compressive stress and distributed normal pressure is described in ref. 45). The comparisons with numerical calculations for the orthotropic plate model are made.

Formulas for buckling strength of stiffened plates subjected to lateral pressure and in-plane load may be found in 46).

4. Conclusions

The general conclusion is that the problem of plates and stiffened plates subjected to combined loads is still not exhausted. Commonly accepted procedure in ship design, which allows to neglect lateral load acting on plates subjected primarily to in-plane loads may be approved in buckling analysis, but cannot be justified in consideration of post-buckling behavior, ultimate strength and postultimate strength stiffness of the structure. The problem may be analyzed by means of *FEM* or *FSM*. However, these methods are very expensive. Analytical solutions are available for few cases. More research is necessary in order to develop more efficient methods of analysis and fully understand the effects of lateral load on the behavior of stiffened plates subjected to combined loads.

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