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ERRATUM TO THE ARTICLE "ZERO MEAN CURVATURE SURFACES IN LORENTZ–MINKOWKI 3-SPACE WHICH CHANGE TYPE ACROSS A LIGHT-LIKE LINE" OSAKA J. MATH. 52 (2015), 285–297

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In the paper [1] whose title is included in the above title, an error in one estimate was found, although the main results still remain valid. In fact, line 6 of p. 292 is incorrect, and the corrected line should read

$$= cM^{k-3}|y|^{k^*}\frac{432c^2}{M^4}\sum_{m=3}^{k-4}\sum_{n=3}^{k-m-1}\frac{k|3n-k+m-1|}{mn(m-1)(n-1)(k-m-n+1)^2}.$$

As a consequence, we have that

$$|kQ_k| \leq cM^{k-3}|y|^{k^*}\frac{432c^2}{M^4}\sum_{m=3}^{k-4}\sum_{n=3}^{k-m-1}\frac{k|3n-k+m-1|}{(m-1)^2(n-1)^2(k-m-n+1)^2}. \tag{1}$$

Theorem 1.1 and Corollary 1.2 of [1] remain true under this correction. To confirm this, it is sufficient to show the inequality at the bottom of [1, p. 292]:

$$|kQ_k| \leq \frac{c}{18\tau}M^{k-3}|y|^{k^*} \times 6\tau \leq \frac{c}{3}M^{k-3}|y|^{k^*}. \tag{2}$$

In fact, changing the original inequality in [1, line 6 of p. 292] to (1) affects only the proof of (2).

From here on out, we prove (2) assuming (1).

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Lemma 1. *For $k \geq 7$, the following inequality holds*:

$$\max_{\substack{3\leq m\leq k-4\\ 3\leq n\leq k-m-1}} (k|3n-k+m-1|) < 2(k-1)^2.$$

Proof. In fact,

$$\begin{aligned}\max_{\substack{3\leq m\leq k-4\\ 3\leq n\leq k-m-1}} |3n-k+m-1| &= \max_{(m,n)=(3,3),(3,k-4),(k-4,3)} |3n-k+m-1| \\ &= \max\{|-k+11|, 4, |2k-10|\} \leq 2(k-5).\end{aligned}$$

In particular, we have

$$\max_{\substack{3\leq m\leq k-4\\ 3\leq n\leq k-m-1}} (k|3n-k+m-1|) \leq 2k(k-5) < 2(k-1)^2,$$

proving the assertion. □

We set $p := m-1$, $q := n-1$ and $l := k-1$. Using (1), Lemma 1 and $432c^2/M^4 \leq 1/(36\tau)$ (cf. [1, (1.14)]), we have that

$$\begin{aligned}|kQ_k| &\leq \frac{c}{36\tau} M^{k-3}|y|^{k^*} \sum_{m=3}^{k-4}\sum_{n=3}^{k-m-1} \frac{2(k-1)^2}{(m-1)^2(n-1)^2(k-m-n+1)^2} \\ &= \frac{c}{18\tau} M^{k-3}|y|^{k^*} \sum_{p=2}^{l-4}\sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l-p-q)^2} \\ &\leq \frac{c}{18\tau} M^{k-3}|y|^{k^*} \sum_{p=2}^{l-2}\sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l-p-q)^2}.\end{aligned}$$

Thus, for $k \geq 7$, it holds that

$$|kQ_k| \leq \frac{c}{18\tau} M^{k-3}|y|^{k^*} \sum_{p=2}^{l-2}\sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l-p-q)^2}. \tag{3}$$

To get (2), we need the following assertion, which is a refinement of [1, Lemma A.2]:

Lemma 2. *For any integer $k \geq 4$, the following inequalities hold*:

$$\sum_{p=2}^{k-2}\sum_{q=2}^{k-p-2} \frac{k^2}{p^2q^2(k-p-q)^2} \leq \frac{6}{k}\int_{1/k}^{1-1/k} \frac{du}{u^2(1-u)^2} \leq 6\tau, \tag{4}$$

where τ is a positive constant satisfying [1, (A.3)].

Proof. The proof of [1, Lemma A.2] becomes a proof of the inequality (4) simply by replacing the upper limit "$k-5$" of the sum with "$k-2$". □

By (3) and (4), we have the desired inequality (2).

Finally we note the following typographical errors:

- In line 6 of p. 293, "$=$" should be replaced by "$\leq$".
- In the third line from the bottom of p. 294,

$$\int_{1/k}^{a-1/k} \frac{du}{u^2(a-u)^2}$$

should be

$$\int_{1/k}^{a-1/k} \frac{a^3\,du}{u^2(a-u)^2}.$$

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References

[1] S. Fujimori, Y.W. Kim, S.-E. Koh, W. Rossman, H. Shin, M. Umehara, K. Yamada and S.-D. Yang: *Zero mean curvature surfaces in Lorentz-Minkowski* 3-*space which change type across a light-like line*, Osaka J. Math. **52** (2015), 285–297.

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