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ERRATUM TO THE ARTICLE
“ZERO MEAN CURVATURE SURFACES IN
LORENTZ–MINKOWKI 3-SPACE WHICH
CHANGE TYPE ACROSS A LIGHT-LIKE LINE”

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In the paper [1] whose title is included in the above title, an error in one estimate was found, although the main results still remain valid. In fact, line 6 of p. 292 is incorrect, and the corrected line should read

$\frac{c^2 M^{k-3}}{A_0^4} \sum_{m=3}^{k-4} \sum_{n=3}^{k-m-1} \frac{k|3n - k + m - 1|}{mn(m-1)(n-1)(k - m - n + 1)^2}.$

As a consequence, we have that

$$|k Q_k| \leq c M^{k-3} |y|^k \frac{432 c^2}{M^4} \sum_{m=3}^{k-4} \sum_{n=3}^{k-m-1} \frac{k|3n - k + m - 1|}{(m-1)^2(n-1)^2(k - m - n + 1)^2}.$$

Theorem 1.1 and Corollary 1.2 of [1] remain true under this correction. To confirm this, it is sufficient to show the inequality at the bottom of [1, p. 292]:

$$|k Q_k| \leq \frac{c}{18 \tau} M^{k-3} |y|^k \times 6 \tau \leq \frac{c}{3} M^{k-3} |y|^k.$$

In fact, changing the original inequality in [1, line 6 of p. 292] to (1) affects only the proof of (2).

From here on out, we prove (2) assuming (1).
Lemma 1. For \( k \geq 7 \), the following inequality holds:

\[
\max_{3 \leq m \leq k-4, \ 3 \leq n \leq k-m-1} \ (k|3n - k + m - 1|) < 2(k - 1)^2.
\]

Proof. In fact, we have

\[
\max_{3 \leq m \leq k-4, \ 3 \leq n \leq k-m-1} \ (k|3n - k + m - 1|) = \max_{(m,n)=(3,3),(3,k-4),(k-4,3)} |3n - k + m - 1|
\]

\[
= \max\{|-k + 11|, 4, |2k - 10|\} \leq 2(k - 5).
\]

In particular, we have

\[
\max_{3 \leq m \leq k-4, \ 3 \leq n \leq k-m-1} \ (k|3n - k + m - 1|) \leq 2k(k - 5) < 2(k - 1)^2,
\]

proving the assertion.

We set \( p := m - 1 \), \( q := n - 1 \) and \( l := k - 1 \). Using (1), Lemma 1 and \( 432c^2/M^4 \leq 1/(36\tau) \) (cf. [1, (1.14)]), we have that

\[
|kQ_k| \leq \frac{c}{36\tau} M^{k-3} |y|^k \sum_{m=3}^{k-4} \sum_{n=3}^{k-m-1} \frac{2(k - 1)^2}{(m - 1)^2(n - 1)^2(k - m - n + 1)^2}
\]

\[
= \frac{c}{18\tau} M^{k-3} |y|^k \sum_{p=2}^{l-4} \sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l - p - q)^2}
\]

\[
\leq \frac{c}{18\tau} M^{k-3} |y|^k \sum_{p=2}^{l-2} \sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l - p - q)^2}.
\]

Thus, for \( k \geq 7 \), it holds that

\[
|kQ_k| \leq \frac{c}{18\tau} M^{k-3} |y|^k \sum_{p=2}^{l-2} \sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l - p - q)^2}.
\]

To get (2), we need the following assertion, which is a refinement of [1, Lemma A.2]:

Lemma 2. For any integer \( k \geq 4 \), the following inequalities hold:

\[
\sum_{p=2}^{k-2} \sum_{q=2}^{k-p-2} \frac{k^2}{p^2q^2(k - p - q)^2} \leq 6 \int_{1/k}^{1-1/k} \frac{du}{u^2(1-u)^2} \leq 6\tau,
\]

where \( \tau \) is a positive constant satisfying [1, (A.3)].
Proof. The proof of [1, Lemma A.2] becomes a proof of the inequality (4) simply by replacing the upper limit “$k - 5$” of the sum with “$k - 2$”.

By (3) and (4), we have the desired inequality (2).

Finally we note the following typographical errors:

- In line 6 of p. 293, “=” should be replaced by “≤”.
- In the third line from the bottom of p. 294,

\[ \int_{1/k}^{a-1/k} \frac{du}{u^2(a - u)^2} \]

should be

\[ \int_{1/k}^{a-1/k} \frac{a^3 du}{u^2(a - u)^2}. \]

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References

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