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NOTE ON A p -BLOCK OF A FINITE GROUP WITH ABELIAN DEFECT GROUP

Dedicated to Professor Tosihiro Tsuzuku on his 60th birthday

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(Received January 28, 1989)

1. Introduction

Let G be a finite group and B be a p -block of G with an abelian defect group D and with a root b in $C_D(D)$. Let $D_1 = C_D(T(b))$ and $\tilde{B}_1 = b^{C_G(D_1)}$, where $T(b)$ is the inertial group of b in $N_G(D)$. In [9, Theorem 1] we showed that the number $k(B)$ (resp. $l(B)$) of ordinary (resp. modular) irreducible characters in B is equal to that of ordinary (resp. modular) irreducible characters in \tilde{B}_1 . In this paper we continue our study to show further properties on the p -block \tilde{B}_1 . Let K be the algebraic closure of the p -adic number field, \mathfrak{o} the ring of local integers in K and F be the residue class field of \mathfrak{o} . For a subgroup H of G , we denote by Tr_H^G the relative trace map from $(FG)^H$ to the center $Z(FG)$ of the group ring FG , where $(FG)^H = \{x \in FG \mid hx = xh \text{ for any } h \in H\}$. For a p -subgroup Q of G , we denote by s_Q the Brauer homomorphism from $(FG)^Q$ to $FC_G(Q)$. The following is the main result.

Theorem 2. *Let E be the block idempotent of FG corresponding to B and \tilde{e}_1 be the block idempotent of $FC_G(D_1)$ corresponding to \tilde{B}_1 . Then the following hold.*

- (i) *The F -linear map f from $Z(FC_G(D_1))\tilde{e}_1$ to $Z(FG)E$ defined by $f(y) = \text{Tr}_{C_G(D_1)}^G(y)E$ is an isomorphism.*
- (ii) *The algebra homomorphism g from $Z(FG)E$ to $Z(FC_G(D_1))\tilde{e}_1$ defined by $g(z) = s_{D_1}(z)\tilde{e}_1$ is an isomorphism.*

2. Lower defect groups

One of our results (Theorem 1) concerns the multiplicities of lower defect groups. So we list some facts on lower defect groups that will be used in this paper (see [1], [2], [3, chapter V], [5], [6] and [7]). Let $B_1, B_2, \dots, B_r, \dots$ be the p -blocks of G and E_τ be the block idempotent of FG corresponding to B_τ . For a conjugacy class C of G , we also write C to denote the class sum in FG .

With each B_τ it is possible to associate $k(B_\tau)$ conjugacy classes C_j^τ , $1 \leq j \leq$

$k(B_\tau)$ such that the following conditions are satisfied.

- (i) Every conjugacy class of G is associated with exactly one p -block.
- (ii) $\{C_j^\tau E_\tau\}$ is an F -basis of $Z(FG)E_\tau$.

Let B be an arbitrary p -block of G and let $B=B_\tau$. For a p -subgroup Q we set $M(B, Q)=\{C_j^\tau \mid C_j^\tau \text{ has } Q \text{ as a defect group, } 1 \leq j \leq k(B_\tau)\}$ and $M(B, Q)'=\{C_j^\tau \mid C_j^\tau \text{ is a } p\text{-regular class and } C_j^\tau \text{ has } Q \text{ as a defect group, } 1 \leq j \leq k(B_\tau)\}$, and we denote by $m(B, Q)$ and $m(B, Q)'$ the cardinal numbers of $M(B, Q)$ and $M(B, Q)'$, respectively. When $m(B, Q) \neq 0$, Q is called a lower defect group of B . Let $Z_Q(FG)$ be the F -subspace of $Z(FG)$ spanned by the class sums C such that Q is a defect group of C and let $Z_Q(FG)'$ be the F -subspace of $Z(FG)$ spanned by the class sums C such that C is a p -regular class and Q is a defect group of C .

- (1) $m(B, Q) = \dim Z_Q(FG)E$ and $m(B, Q)' = \dim Z_Q(FG)'E$ if Q is normal in G ,

where $E=E_\tau$. Furthermore, $\{s_Q(C_j^\tau E) \mid C_j^\tau \in M(B, Q)\}$ is an F -basis of $Z_Q(FN_G(Q))s_Q(E)$ and $\{s_Q(C_j^\tau E) \mid C_j^\tau \in M(B, Q)'\}$ is an F -basis of $Z_Q(FN_G(Q))'s_Q(E)$. From this we have

- (2) $m(B, Q) = \sum_{\tilde{B}} m(\tilde{B}, Q)$ and $m(B, Q)' = \sum_{\tilde{B}} m(\tilde{B}, Q)'$,

where \tilde{B} ranges over the set $Bl(N_G(Q), B)$ of p -blocks of $N_G(Q)$ associated with B . On the other hand we have the following, which will be used to prove Theorem 2.

Lemma 1. *Let $\{Q_1, Q_2, \dots, Q_r\}$ be a complete set of representatives for the G -conjugacy classes of lower defect groups of B . Then we have*

$$Z(FG)E = \sum_{i=1}^r \text{Tr}_{N_G(Q_i)}^G(Z_{Q_i}(FN_G(Q_i))s_{Q_i}(E))E.$$

Proof. Let $\{w_{ij} \mid j=1, 2, \dots, m(B, Q_i)\}$ be an F -basis of $Z_{Q_i}(FN_G(Q_i))s_{Q_i}(E)$. Since $\dim Z(FG)E = \sum_{i=1}^r m(B, Q_i)$, to prove the lemma it suffices to show that $\text{Tr}_{N_G(Q_i)}^G(w_{ij})E, j=1, 2, \dots, m(B, Q_i), i=1, 2, \dots, r$ are linearly independent. Suppose that

$$\sum_{i=1}^r \sum_{j=1}^{m_i} a_{ij} \text{Tr}_{N_G(Q_i)}^G(w_{ij})E = 0,$$

where $a_{ij} \in F$ and $m_i = m(B, Q_i)$. We assume that $|Q_1| \geq |Q_2| \geq \dots \geq |Q_r|$. Then we obtain by [1, (B3.1)]

$$s_{Q_1}(\sum_{i=1}^r \sum_{j=1}^{m_i} a_{ij} \text{Tr}_{N_G(Q_i)}^G(w_{ij})E) = s_{Q_1}(\sum_{j=1}^{m_1} a_{1j} \text{Tr}_{N_G(Q_1)}^G(w_{1j})E) = \sum_{j=1}^{m_1} a_{1j} w_{1j} = 0,$$

because $Z_{Q_1}(FN_G(Q_1)) = \text{Tr}_{Q_1^{G(Q_1)}}^{N_G(Q_1)}(FC_G(Q_1))$. Hence $a_{1j} = 0$ for any j . For $i(2 \leq$

$i \leq r$), let $a_{kj}=0$ for any $k \leq i-1$ and j . Then we have

$$s_{Q_i}(\sum_{j=1}^{m_i} a_{ij} \operatorname{Tr}_{N_G(Q_i)}^G(w_{ij}) E) = \sum_{j=1}^{m_i} a_{ij} w_{ij} = 0.$$

So we have $a_{ij}=0$ for any j . This completes the proof.

Let C_B be the Cartan matrix of B . We have

$$(3) \quad k(B) = \sum_Q m(B, Q), \quad l(B) = \sum_Q m(B, Q)' \quad \text{and} \quad |C_B| = \prod_Q |Q|^{m(B, Q)'},$$

where Q ranges over a complete set of representatives for the G -conjugacy classes of lower defect groups of B . In fact, the multiplicity of p^m as an elementary divisor of C_B is given by $\sum_Q m(B, Q)'$, where Q ranges over a complete set of representatives for the G -conjugacy classes of lower defect groups of B of order p^m .

We denote by $[Q, \tilde{B}]$ the pair of a p -subgroup Q and a p -block \tilde{B} of $N_G(Q)$ such that \tilde{B} belongs to $Bl(N_G(Q), B)$. Let \mathcal{S} be a complete set of representatives for the G -conjugacy classes of such pairs $[Q, \tilde{B}]$. From (2), (3) and Lemma 1, the following (4), (5) and (6) hold.

$$(4) \quad m(B, Q) = \sum_{[P, \mathfrak{b}]} m(\mathfrak{b}, P) \quad \text{and} \quad m(B, Q)' = \sum_{[P, \mathfrak{b}]} m(\mathfrak{b}, P)'$$

where $[P, \mathfrak{b}]$ ranges over all the elements of \mathcal{S} such that P is G -conjugate to Q .

$$(5) \quad Z(FG)E = \sum_{[P, \mathfrak{b}]} \operatorname{Tr}_{N_G(P)}^G(Z_P(FN_G(P)) e_{\mathfrak{b}}) E,$$

$$(6) \quad k(B) = \sum_{[P, \mathfrak{b}]} m(\mathfrak{b}, P), \quad l(B) = \sum_{[P, \mathfrak{b}]} m(\mathfrak{b}, P)' \quad \text{and} \quad |C_B| = \sum_{[P, \mathfrak{b}]} |P|^{m(\mathfrak{b}, P)'},$$

where $e_{\mathfrak{b}}$ is the block idempotent of $FN_G(P)$ corresponding to \mathfrak{b} and $[P, \mathfrak{b}]$ ranges over \mathcal{S} . It is also clear that the multiplicity of p^m as an elementary divisor of C_B is given by $\sum_{[P, \mathfrak{b}]} m(\mathfrak{b}, P)'$, where $[P, \mathfrak{b}]$ ranges over the elements of \mathcal{S} such that $|P|=p^m$.

3. $Z(FG)E$ and $Z(FC_G(D_1))\tilde{e}_1$

In this section we prove our main result. First we prepare a lemma.

Lemma 2 (see [2, Proposition (1.5)]). *Let B be a p -block of G , \mathfrak{b} be a p -block of a normal subgroup N of G and T be a subgroup of G which contains the inertial group $T(\mathfrak{b})$. Then there exists a unique p -block \tilde{B} of T such that \tilde{B} covers \mathfrak{b} and \tilde{B} is associated with B . Let E be the block idempotent of FG corresponding to B and \tilde{E} be the block idempotent of FT corresponding to \tilde{B} .*

(i) *The F -linear map \tilde{f} from $Z(FT)\tilde{E}$ to $Z(FG)E$ defined by $\tilde{f}(y) = \operatorname{Tr}_T^G(y)$ is an isomorphism.*

(ii) The F -linear map \tilde{g} from $Z(FG)E$ to $Z(FT)\tilde{E}$ defined by $\tilde{g}(z)=z\tilde{E}$ is the inverse map of \tilde{f} .

Proof. The existence and the uniqueness of \tilde{B} follow from [3, chapter V, Theorem 2.5]. Furthermore $k(B)=k(\tilde{B})$, therefore $\dim Z(FG)E=\dim Z(FT)\tilde{E}$. Let $\{x_1, x_2, \dots, x_k\}$ be a set of representatives for the cosets of T in G . Since $T \supseteq T(b)$, $\tilde{E}^{x_i}\tilde{E}^{x_j}=0$ for any $i, j (i \neq j)$ and $E = \sum_{i=1}^k \tilde{E}^{x_i}$ (see [2, chapter V, §3]). Hence we have $\text{Tr}_T^G(y)E = (\sum_{i=1}^k y^{x_i} \tilde{E}^{x_i})E = \text{Tr}_T^G(y)$ and $\text{Tr}_T^G(y)\tilde{E} = (\sum_{i=1}^k y^{x_i} \tilde{E}^{x_i})\tilde{E} = y$ for any $y \in Z(FT)\tilde{E}$. This implies that \tilde{f} is well defined and \tilde{f} is an isomorphism, and \tilde{g} is well defined and \tilde{g} is the inverse map of \tilde{f} .

In the remainder of this paper we assume that B is a p -block of G with an abelian defect group D and b is a root of B in $C_G(D)$. If Q is a p -subgroup of G and \tilde{B} is a p -block of $N_G(Q)$ associated with B , then as is well known, a defect group of \tilde{B} is G -conjugate to D . Hence there is a subgroup P of D such that $[Q, \tilde{B}]$ and $[P, b^{N_G(P)}]$ are G -conjugate. Furthermore for subgroups P_1, P_2 of D , $[P_1, b^{N_G(P_1)}]$ and $[P_2, b^{N_G(P_2)}]$ are G -conjugate if and only if P_1 and P_2 are $T(b)$ -conjugate. Here we choose a complete set \mathfrak{P} of representatives for the $T(b)$ -conjugacy classes of subgroups of D and we fix it. Then $\{[P, b^{N_G(P)}] \mid P \in \mathfrak{P}\}$ is a complete set of representatives for the G -conjugacy classes of the pairs $[Q, \tilde{B}]$ such that \tilde{B} is a p -block of $N_G(Q)$ associated with B .

Theorem 1. Under the above notations let $D_1=C_D(T(b))$. For any $P \in \mathfrak{P}$, we have

$$(7) \quad m(b^{N_G(P)}, P) = m(b^{C_G(D_1) \cap N_G(P)}, P), \\ m(b^{N_G(P)}, P)' = m(b^{C_G(D_1) \cap N_G(P)}, P)'.$$

In particular, if $P \not\supseteq D_1$ then $m(b^{N_G(P)}, P)=0$.

Proof. Let $P \in \mathfrak{P}$. First we assume that $P \supseteq D_1$. Then $b^{C_G(D_1) \cap N_G(P)}$ covers $b^{C_G(P)}$ and the inertial group $T(b^{C_G(P)})$ of $b^{C_G(P)}$ in $N_G(P)$ is contained in $(T(b) \cap N_G(P)) C_G(P)$, and hence $T(b^{C_G(P)}) \subseteq C_G(D_1) \cap N_G(P)$. Therefore if ε_P is the block idempotent of $FN_G(P)$ corresponding to $b^{N_G(P)}$ and δ_P is the block idempotent of $F(C_G(D_1) \cap N_G(P))$ corresponding to $b^{C_G(D_1) \cap N_G(P)}$, $\text{Tr}_{C_G(D_1) \cap N_G(P)}^{N_G(P)}$ induces an isomorphism from $Z(F(C_G(D_1) \cap N_G(P))) \delta_P$ to $Z(FN_G(P)) \varepsilon_P$ and $\text{Tr}_{C_G(D_1) \cap N_G(P)}^{N_G(P)}(w) \delta_P = w$ for $w \in Z(F(C_G(D_1) \cap N_G(P))) \delta_P$ by Lemma 2. Further we have

$$\text{Tr}_{C_G(D_1) \cap N_G(P)}^{N_G(P)}(Z_P(F(C_G(D_1) \cap N_G(P)) \delta_P) = Z_P(FN_G(P)) \varepsilon_P.$$

Hence from (1) we obtain (7) for P with $P \supseteq D_1$.

Since $C_G(D_1) \supseteq T(b)$, we have the following from (6).

$$k(b^{C_G(D_1)}) = \sum_{P \in \mathfrak{P}} m(b^{C_G(D_1) \cap N_G(P)}, P),$$

$$l(b^{C_G(D_1)}) = \sum_{P \in \mathfrak{P}} m(b^{C_G(D_1) \cap N_G(P)}, P)' .$$

Note that if $P \in \mathfrak{P}$ and $P \not\supseteq D_1$, then $m(b^{C_G(D_1) \cap N_G(P)}, P) = 0$ and $m(b^{C_G(D_1) \cap N_G(P)}, P)' = 0$. On the other hand, we have

$$k(B) = \sum_{P \in \mathfrak{P}} m(b^{N_G(P)}, P) \quad \text{and} \quad l(B) = \sum_{P \in \mathfrak{P}} m(b^{N_G(P)}, P)' .$$

But since $k(B) = k(b^{C_G(D_1)})$ and $l(B) = l(b^{C_G(D_1)})$ by [9, Theorem 1], we conclude from the aboves that if $P \not\supseteq D_1$, then $m(b^{N_G(P)}, P) = 0$ and $m(b^{N_G(P)}, P)' = 0$. This completes the proof of the theorem.

We set $\tilde{B}_1 = b^{C_G(D_1)}$. From Theorem 1 and (6), we obtain the following.

Corollary. *The elementary divisors of C_B are the same as those of $C_{\tilde{B}_1}$ (counting multiplicities). In particular $|C_B| = |C_{\tilde{B}_1}|$.*

Proof of Theorem 2. Let $\{P_1, P_2, \dots, P_s\}$ be the set of elements P of \mathfrak{P} such that $P \supseteq D_1$ and $m(b^{C_G(D_1) \cap N_G(P)}, P) \neq 0$ and we denote by δ_i, ε_i the block idempotents $\delta_{P_i}, \varepsilon_{P_i}$ in the proof of Theorem 1. Let $\{w_{i_1}, w_{i_2}, \dots, w_{i_{m_i}}\}$ be an F -basis of $Z_{P_i}(F(C_G(D_1) \cap N_G(P_i))) \delta_i$, where $m_i = m(b^{C_G(D_1) \cap N_G(P_i)}, P_i)$. From the proof of Lemma 1 and (5), $\text{Tr}_{C_G(D_1) \cap N_G(P_i)}^{C_G(D_1)}(w_{ij}) \tilde{e}_1, j=1, 2, \dots, m_i, i=1, 2, \dots, s$ form an F -basis of $Z(FC_G(D_1)) \tilde{e}_1$. We set $z_{ij} = f(\text{Tr}_{C_G(D_1) \cap N_G(P_i)}^{C_G(D_1)}(w_{ij}) \tilde{e}_1) (1 \leq j \leq m_i, 1 \leq i \leq s)$. Since $\dim Z(FG)E = \dim Z(FC_G(D_1)) \tilde{e}_1$, to prove (i) it suffices to show that $z_{ij}, j=1, 2, \dots, m_i, i=1, 2, \dots, s$ are linearly independent. We show this by the same way as in the proof of Lemma 1.

First we calculate $s_{P_i}(z_{ij})$. We can set

$$w_{ij} = \text{Tr}_{P_i^{C_G(D_1) \cap N_G(P_i)}}^{C_G(D_1)}(u_{ij}), \quad u_{ij} \in FC_G(P_i) .$$

By [1, (3B)], we have

$$\begin{aligned} s_{P_i}(z_{ij}) &= s_{P_i}(\text{Tr}_{C_G(D_1)}^C(\text{Tr}_{C_G(D_1) \cap N_G(P_i)}^{C_G(D_1)}(w_{ij}) \tilde{e}_1) E) \\ &= s_{P_i}(\text{Tr}_{P_i}^C(u_{ij} \tilde{e}_1)) s_{P_i}(E) = \text{Tr}_{P_i^{N_G(P_i)}}^{N_G(P_i)}(u_{ij} s_{P_i}(\tilde{e}_1)) s_{P_i}(E) \\ &= \text{Tr}_{C_G(D_1) \cap N_G(P_i)}^{N_G(P_i)}(\text{Tr}_{P_i^{C_G(D_1) \cap N_G(P_i)}}^{C_G(D_1)}(u_{ij} s_{P_i}(\tilde{e}_1))) s_{P_i}(E) \\ &= \text{Tr}_{C_G(D_1) \cap N_G(P_i)}^{N_G(P_i)}(w_{ij} s_{P_i}(\tilde{e}_1)) s_{P_i}(E) \\ &= \text{Tr}_{C_G(D_1) \cap N_G(P_i)}^{N_G(P_i)}(w_{ij}) . \end{aligned}$$

In particular $s_{P_i}(z_{ij}) \in Z(FN_G(P_i)) \varepsilon_i$ and $s_{P_i}(z_{ij}) \delta_i = w_{ij}$ by Lemma 2. Furthermore if P_i and P_k are G -conjugate and $k \neq i$, then $s_{P_i}(z_{kj}) \delta_i = 0$. In fact, let $P_i = P_k^x$ for some $x \in G$. Since we have

$$s_{P_i}(z_{kj}) = (s_{P_k}(z_{kj}))^x = (\text{Tr}_{C_G(D_1) \cap N_G(P_k)}^{N_G(P_k)}(w_{kj}))^x$$

and since $\varepsilon_i \neq \varepsilon_k^x, s_{P_i}(z_{kj}) \delta_i = s_{P_k}(z_{kj}) \varepsilon_k^x \delta_i = 0$.

Suppose that $|P_1| \geq |P_2| \geq \dots \geq |P_s|$. We assume

$$\sum_{i=1}^s \sum_{j=1}^{m_i} a_{ij} z_{ij} = 0, \quad a_{ij} \in F.$$

Then we have

$$s_{P_1} \left(\sum_{i=1}^s \sum_{j=1}^{m_i} a_{ij} z_{ij} \right) \delta_1 = \sum_{j=1}^{m_1} a_{1j} w_{1j} = 0.$$

So $a_{1j} = 0$ for any j . For $i (2 \leq i \leq s)$, let $a_{kj} = 0$ for any $k \leq i - 1$ and any j . Then we obtain

$$s_{P_i} \left(\sum_{j=1}^{m_i} a_{ij} z_{ij} \right) \delta_i = \sum_{j=1}^{m_i} a_{ij} w_{ij} = 0.$$

Therefore $a_{ij} = 0$.

(ii) By the above argument, $s_{P_i}(g(z_{ij})) \delta_i = s_{P_i}(s_{D_1}(z_{ij}) \tilde{e}_1) \delta_i = s_{P_i}(z_{ij}) \delta_i = w_{ij}$. Furthermore, if P_i and P_k are G -conjugate and $i \neq k$, then $s_{P_i}(g(z_{kj})) \delta_i = s_{P_i}(z_{kj}) \delta_i = 0$. From these facts, we can show $\text{Ker } g = \{0\}$ by the same way as in the proof of (i). This completes the proof of the theorem.

4. Indecomposable modules belonging to B and to \tilde{B}_1

Let R be \mathfrak{o} or F . In this section we show that there is a vertex-preserving bijection between the set of isomorphism classes of indecomposable $RC_G(D_1)$ -modules belonging to \tilde{B}_1 with vertex containing D_1 and a set of isomorphism classes of indecomposable RG -modules belonging to B . Let M be an indecomposable RG -module belonging to B with vertex Q . Since D is abelian, by [4, Theorem 2] and [8, Corollary 1 and Theorem 3] there exists a unique (up to $T(b)$ -conjugacy) subgroup P of D such that P is G -conjugate to Q and that the Green correspondent of M with respect to $(G, P, N_G(P))$ belongs to $b^{N_G(P)}$.

Proposition. *Let P be a subgroup of D , H be a subgroup of G containing $C_G(D_1)$ and $C_G(P)$ and let $\tilde{B}_2 = b^H$. Then the following hold.*

(i) *Let N be an indecomposable RH -module belonging to \tilde{B}_2 with vertex P such that the Green correspondent of N with respect to $(H, P, N_H(P))$ belongs to $b^{N_H(P)}$. Then there exists a unique (up to isomorphism) indecomposable component M of N^G such that M has P as a vertex. Further M belongs to B and the Green correspondent of M with respect to $(G, P, N_G(P))$ belongs to $b^{N_G(P)}$. Set $M = \psi(N)$.*

(ii) *ψ defines a one to one correspondence between the set of isomorphism classes of indecomposable RH -modules belonging to \tilde{B}_2 with vertex P whose Green correspondents with respect to $(H, P, N_H(P))$ belong to $b^{N_H(P)}$ and the set of isomorphism classes of indecomposable RG -modules belonging to B with vertex P whose Green correspondents with respect to $(G, P, N_G(P))$ belong to $b^{N_G(P)}$.*

Proof. By the assumption that $C_G(P) \subseteq H$ and $C_G(D_1) \subseteq H$, the inertial

group $T(b^{c_{\mathcal{G}(P)}})$ is contained in $N_H(P)$. Therefore “induction” gives a one to one correspondence between the set of isomorphism classes of indecomposable $RN_H(P)$ -modules belonging to $b^{N_H(P)}$ and the set of isomorphism classes of indecomposable $RN_G(P)$ -modules belonging to $b^{N_G(P)}$.

(i) Let N_1 be the Green correspondent of N with respect to $(H, P, N_H(P))$. Then $N_1^{N_G(P)}$ is an indecomposable $RN_G(P)$ -module belonging to $b^{N_G(P)}$ and P is a vertex of $N_1^{N_G(P)}$. By [4, Theorem 2], there exists a unique indecomposable component M of $N_1^{\mathcal{G}}$ such that M has P as a vertex. From [8, Corollary 1 and Theorem 3], M belongs to B and $N_1^{N_G(P)}$ is the Green correspondent of M . On the other hand, N is a component of N_1^H , and so N^G is a component of $N_1^{\mathcal{G}}$. Since P is a vertex of N , an indecomposable component of N^G has P as a vertex. Therefore M is a component of N^G . This establishes (i).

(ii) Let M be an indecomposable RG -module belonging to B with vertex P . We assume that the Green correspondent M_1 of M with respect to $(G, P, N_G(P))$ belongs to $b^{N_G(P)}$. Then there is an indecomposable $RN_H(P)$ -module N_1 belonging to $b^{N_H(P)}$ such that $M_1 = N_1^{N_G(P)}$. Let N be an indecomposable RH -module with vertex P and with the Green correspondent N_1 . Then N belongs to \tilde{B}_2 and M is a component of N^G from the argument of (i). Let N' be an indecomposable RH -module belonging to \tilde{B}_2 with vertex P and with the Green correspondent N'_1 belonging to $b^{N_H(P)}$. If M is a component of N'^G , then $N'_1^{N_G(P)}$ is the Green correspondent of M . Hence N'_1 and N_1 are isomorphic, and so N' and N are isomorphic. This completes the proof of the proposition.

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