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A NOTE ON AXIOMATIC DIRICHLET PROBLEM

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1. Introduction

Axiomatic Dirichlet problem was first discussed by M. Brelot in connection with a metrizable compactification of Green space Ω and a positive harmonic function h in Ω. In his paper [1] the theory was developed under the assumption $\mathfrak{A}_h$, that is, *all bounded continuous functions on the boundary are h-resolutive.* In our present paper we call a compactification with this property *h-resolutive*.

This axiomatic treatment of Dirichlet problem yields some complicated situations. For instance, Brelot gave many definitions for the regularity of boundary points, such as strongly h-regular, h-regular, weakly h-regular. A strongly h-regular boundary point is h-regular and weakly h-regular, but an h-regular boundary point is not weakly h-regular in general. It has been asked by M. Brelot [1] and L. Naïm [4] whether the complementary set of all h-regular boundary points is of h-harmonic measure zero (*h-négligeable*) or not. We can not yet give an answer to this question. However we can prove the following theorem:

Theorem. *Let $\hat{\Omega}$ be an arbitrary metrizable h-resolutive compactification of Green space Ω. Then there exists a metrizable h-resolutive compactification having $\hat{\Omega}$ as a quotient space and in which the complementary set of all h-regular and weakly h-regular boundary points is of h-harmonic measure zero.*

As a corollary of this theorem we can construct a family of filters $\{\mathcal{F}_x\}$ converging in $\hat{\Omega}$ and satisfying axioms

$\boldsymbol{A}_h$) *If s is subharmonic in Ω, s/h is bounded from above and* $\limsup_{\mathcal{F}} s/h \leq 0$ *for every $\mathcal{F}$ in $\{\mathcal{F}_x\}$, then $s \leq 0$.*

$\boldsymbol{B}_h'$) *Every filter in $\{\mathcal{F}_x\}$ is h-regular and weakly h-regular,*
where the latter is weaker than that of Brelot-Choquet [2].

2. Preliminaries

Let Ω be a Green space in the sense of Brelot-Choquet [2]. For a real valued function f defined in Ω we shall define a family $\bar{W}_f(\underline{W}_f)$ of superharmonic (subharmonic) functions s such that $s \geq f$ $(s \leq f)$ on $\Omega - K$, where K is a compact set depending on s in general. If $\overline{W}_f$ ($\underline{W}_f$) is not empty its lower (upper) envelope will be denoted by $\bar{d}_f(\underline{d}_f)$. $\bar{d}_f$ and $\underline{d}_f$ are harmonic and $\underline{d}_f \leq \bar{d}_f$. When $\underline{d}_f = \bar{d}_f$ they are denoted by d_f simply.

Throughout this paper we shall take a positive harmonic function h in Ω and fix it.

DEFINITION 1. A function f defined in Ω is *h-harmonizable* if the following conditions are satisfied:

1) there exists a superharmonic function s such that $|fh| \leq s$,
2) $\underline{d}_{fh} = \bar{d}_{fh}$

If f is h-harmonizable and $d_{fh}=0$ then f is termed an *h-Wiener potential*, and the class of all h-Wiener potentials is denoted by $W_{0,h}$[1].

Proposition 2.1. *Every $f \in W_{0,h}$ has a potential p such that $|fh| \leq p$.*

Let $\hat{\Omega}$ be a compactification of Ω, that is $\hat{\Omega}$ is compact and contains Ω as an everywhere dense subspace. Set $\Delta = \hat{\Omega} - \Omega$. In this paper it is always assumed that $\hat{\Omega}$ is *metrizable*.

For an arbitrary real valued function φ on Δ, which is permitted to take the values $\pm\infty$, $\overline{\mathcal{S}}_{\varphi,h}$ denotes the class of all superharmonic functions s such that

a) s/h is bounded from below,
b) $\varliminf_{a \to x} s(a)/h(a) \geq \varphi(x)$ for every $x \in \Delta$.

Similarly we define the class of subharmonic functions $\underline{\mathcal{S}}_{\varphi,h}$. When $\overline{\mathcal{S}}_{\varphi,h}$, $\underline{\mathcal{S}}_{\varphi,h}$ are not empty, we set

$$\overline{\mathcal{D}}_{\varphi,h} = \inf \{s;\, s \in \overline{\mathcal{S}}_{\varphi,h}\} ,$$
$$\underline{\mathcal{D}}_{\varphi,h} = \sup \{s;\, s \in \underline{\mathcal{S}}_{\varphi,h}\} .$$

$\underline{\mathcal{D}}_{\varphi,h}$ and $\overline{\mathcal{D}}_{\varphi,h}$ are both harmonic and $\underline{\mathcal{D}}_{\varphi,h} \leq \overline{\mathcal{D}}_{\varphi,h}$. When $\underline{\mathcal{D}}_{\varphi,h} = \overline{\mathcal{D}}_{\varphi,h}$, φ is called *h-resolutive* and the envelopes are denoted by $\mathcal{D}_{\varphi,h}$ simply.

DEFINITION 2. If all bounded continuous functions on Δ are h-resolutive, $\hat{\Omega}$ is called an *h-resolutive compactification* of Ω.

In the sequel, $\hat{\Omega}$ always denotes a metrizable h-resolutive compactification of Ω. Then, for $a \in \Omega$ there exists a Radon measure ω_h^a on Δ such that

$$\mathcal{D}_{\varphi,h} = \int \varphi \, d\omega_h^a \qquad \text{for every} \quad \varphi \in C(\Delta)^{2)} .$$

ω_h^a is called an *h-harmonic measure* (with respect to a).

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1) In the case that $h=1$ and Ω is a hyperbolic Riemann surface, this definition is slightly different from [3].

2) $C(\Delta)$ denotes the family of all bounded continuous functions on Δ.

Proposition 2.2. *Let F be bounded and continuous on $\hat{\Omega}$ and φ, f be its restrictions on Δ and on Ω respectively, then f is h-harmonizable and $d_{fh}=\mathfrak{D}_{\varphi,h}$.*

Proposition 2.3. *In order that an arbitrary compactification $\bar{\Omega}$ of Ω be h-resolutive, it is necessary and sufficient that for every bounded continuous function F on $\bar{\Omega}$, its restriction on Ω is h-harmonizable.*

DEFINITION 3. For potential p we set

$$\Gamma_{p,h}=\{x\in\Delta;\ \varliminf_{a\to x} p(a)/h(a)=0\},$$
$$\Gamma_h=\bigcap_p \Gamma_{p,h}.$$

Γ_h is called an *h-harmonic boundary*.
Γ_h is non-empty and compact.

Proposition 2.4. *If s is subharmonic in Ω such that s/h is bounded from above and $\varlimsup_{a\to x} s(a)/h(a)\leq 0$ for all $x\in\Gamma_h$ then $s\leq 0$.*

Proposition 2.5. *Let F be a bounded continuous function on $\hat{\Omega}$. The restriction of F on Ω is an h-Wiener potential if and only if F vanishes on Γ_h.*

Proposition 2.6. *Γ_h is the carrier of h-harmonic measure ω_h.*

In the case that $h=1$ and Ω is a hyperbolic Riemann surface, Constantinescu-Cornea [3] have given these propositions. Proofs of our propositions will be obtained from them with slight modifications.

3. *Q*-compactification of Green space

1. Let h be a positive harmonic function on Green space Ω and $\hat{\Omega}$ be an arbitrary *metrizable*, *h-resolutive* compactification of Ω. Set $\Delta=\hat{\Omega}-\Omega$.

For $F\in C(\hat{\Omega})$, its restrictions on Ω and on Δ are denoted by $F|_\Omega$ and $F|_\Delta$ respectively.

We set $Q_0'=\{F|_\Omega;\ F\in C(\hat{\Omega})\}$, $Q_0''=\{d_{fh}/h;\ f\in Q_0'\}$ and

$$Q_0=Q_0'\cup Q_0''\cup\left\{A\frac{\min(G_{a_0},h)}{h}+B\right\},$$

where G_{a_0} is a Green function of Ω with pole at a_0 and A, B are constants. The compactification Ω^{Q_0} of Ω is the one on which all functions of Q_0 are extended continuously and the boundary $\Delta^{Q_0}=\Omega^{Q_0}-\Omega$ is separated by functions in Q_0[3]. We have

Proposition 3.1. *Ω^{Q_0} is a metrizable h-resolutive compactification of Ω.*

3) We say functions in Q_0 separate points of Δ^{Q_0} if for every pair of distinct points x, y of Δ^{Q_0} there exists a function F in Q_0 such that $F(x)\neq F(y)$.

$\hat{\Omega}$ *is a quotient space of* Ω^{Q_0}.

To prove this proposition, we require some lemmas.

In $C(\hat{\Omega})$ we select a countable subfamily $\{F_k\}$ which is dense in the topology of uniform norm $(\|F\|=\sup_{a\in\hat{\Omega}}|F(a)|)$.

If we set $f_k=F_k|_\Omega$, f_k is h-harmonizable (Prop. 2.2). We form the family of a countable number of functions

$$Q=\{f_k\}\cup\{d_{f_k}/h\}\cup\left\{\frac{\min(G_{a_0},h)}{h}\right\},$$

which is a subfamily of Q_0.

The Q-compactification Ω^Q of Ω is compact and contains Ω as an everywhere dense subspace. Functions in Q are extended continuously on Ω^Q and separate two distinct points of $\Delta^Q=\Omega^Q-\Omega$.

Theory of general topology tells us Ω^Q is metrizable (for instance, N. Bourbaki: Topologie générale, Chap. IX, §2).

Lemma 3.1. *For every* $F\in C(\hat{\Omega})$, *if we set* $f=F|_\Omega$, *then* f *and* d_{fh}/h *are extended continuously on* Ω^Q.

Proof. (i) Case of f. It will be sufficient to show that for every $x\in\Delta^Q$ and for every sequence of points $\{a_n\}$ in Ω converging to x in the topology of Ω^Q $\{f(a_n)\}$ has the unique limit. If it were not, there should exist two sequences $\{a_n\}$, $\{b_n\}$ in Ω such that $a_n\to x$, $b_n\to x$ (in the topology of Ω^Q) and $\alpha=\lim_{n\to\infty}f(a_n)>\lim_{n\to\infty}f(b_n)=\beta$.

We take a positive number $\varepsilon=(\alpha-\beta)/4$. For this ε and $F\in C(\hat{\Omega})$ we can find F_k in our countable family such that

$$\sup_{\hat{\Omega}}|F_k-F|\leq\varepsilon\,.$$

Then we have

$$\alpha=\lim_{n\to\infty}f(a_n)\leq\overline{\lim_{n\to\infty}}f_k(a_n)+\varepsilon\,,$$

$$\underline{\lim_{n\to\infty}}f_k(b_n)-\varepsilon\leq\lim_{n\to\infty}f(b_n)=\beta\,.$$

where $f_k=F_k|_\Omega$. Since f_k is extended continuously on Ω^Q,

$$\alpha-\varepsilon\leq\lim_{n\to\infty}f_k(a_n)=\lim_{n\to\infty}f_k(b_n)\leq\beta+\varepsilon\,,$$

this leads to a contradiction $4\varepsilon=\alpha-\beta\leq2\varepsilon$.

(ii) Case of d_{fh}/h. We take f_k as above. Then we have

$$\frac{d_{f_kh}}{h}-\varepsilon\leq\frac{d_{fh}}{h}\leq\frac{d_{f_kh}}{h}+\varepsilon$$

and we can proceed quite in the same way as in (i).

Lemma 3.2. *Let $\mathcal{H}$ be a class of all functions F' each of which is bounded and continuous on Ω^{Q_0} and its restriction on Ω is h-harmonizable. Then $\mathcal{H}$ is dense in $C(\Omega^{Q_0})$ in the topology of uniform norm in Ω^{Q_0}.*

Proof. Clearly $\mathcal{H}$ contains all constant functions and $\mathcal{H}$ is a linear space. All functions in Q_0, are extended continuously on Ω^{Q_0} and these extended functions are contained in $\mathcal{H}$, therefore Ω^{Q_0} is separated by functions in $\mathcal{H}$. To see $\mathcal{H}$ is closed under the maximum and minimum operations, that is F_1', $F_2' \in \mathcal{H}$ implies $\max(F_1', F_2')$, $\min(F_1', F_2') \in \mathcal{H}$, let $F_1', F_2' \in \mathcal{H}$ and $f_i = F_i'|_\Omega$ $(i=1,2)$. $\min(F_1', F_2')|_\Omega = \min(f_1, f_2)$ and $d_{\min(f_1,f_2)h} = d_{\min(f_1h, f_2h)} = d_{f_1h} \wedge d_{f_2h}$, where $u \wedge v$ denotes the greatest harmonic function which is dominated by u and v. This means $\min(f_1, f_2)$ is h-harmonizable. By Stone's theorem[4] $\mathcal{H}$ is dense in $C(\Omega^{Q_0})$.

Proof of Proposition 3.1. On account of Lemma 3.1 all functions of Q_0 are extended continuously on Ω^Q. Thus Ω^{Q_0} is homeomorphic to Ω^Q and therefore Ω^{Q_0} is metrizable. Since $\hat{\Omega}$ is homeomorphic to $\Omega^{Q_0'}$, $\hat{\Omega}$ is a quotient space of Ω^{Q_0}. For arbitrary $F' \in C(\Omega^{Q_0})$ and any positive number ε, by Lemma 3.2 we can find $F_0' \in \mathcal{H}$ such that

$$\sup_{\Omega^{Q_0}} |F' - F_0'| \leq \varepsilon .$$

Setting $f = F'|_\Omega$, $f_0 = F_0'|_\Omega$ we have

$$\underline{d}_{f_0h} - \varepsilon h \leq \underline{d}_{fh} \leq \bar{d}_{fh} \leq \bar{d}_{f_0h} + \varepsilon h .$$

Since f_0 is h-harmonizable we get $0 \leq \bar{d}_{fh} - \underline{d}_{fh} \leq 2\varepsilon h$. f is h-harmonizable, and by Proposition 2.3 Ω^{Q_0} is h-resolutive.

2. For an arbitrary metrizable h-resolutive compactification $\hat{\Omega}$ of Ω we have constructed Ω^{Q_0} of the same type which contains $\hat{\Omega}$ as a quotient space. If we start from Ω^{Q_0} it will be expected that we can arrive at a new larger compactification of the same type, but this is not so, that is

Proposition 3.2. *Let Ω^{Q_0} be the compactification of Ω constructed in the above paragraph. If we set $Q_1' = \{f = F|_\Omega;\ F \in C(\Omega^{Q_0})\}$, $Q_1'' = \left\{\frac{d_{fh}}{h};\ f \in Q_1'\right\}$ and $Q_1 = Q_1' \cup Q_1''$ the compactification Ω^{Q_1} is homeomorphic to Ω^{Q_0}.*

Before proving this propostition we remark the following:

Lemma 3.3. *For every $f \in Q_1'$, and for every positive number ε there exists $g \in Q_0'$ such that*

$$\sup_{\Omega} \left| \frac{d_{fh}}{h} - \frac{d_{gh}}{h} \right| \leq \varepsilon .$$

4) Cf. [3], p. 5.

Proof. For arbitrary distinct points x_1, x_2 in Ω^{Q_0} and for any numbers α_1, α_2 there exists a function $\lambda\in C(\Omega^{Q_0})$ which satisfies the following conditions:

1) $\lambda|_\Omega\in Q_0$.

2) $\lambda(x_i)=\alpha_i \quad (i=1,2)$.

Since continuous extensions of functions in Q_0 separate points of Ω^{Q_0} we can find $l\in C(\Omega^{Q_0})$ with $l(x_1)\neq l(x_2)$ among these extensions. Thus, either (i) $l|_\Omega=f\in Q_0'$ or (ii) $l|_\Omega=d_{fh}/h$ for some $f\in Q_0'$ or (iii) $l|_\Omega=A\dfrac{\min(G_{a_0},h)}{h}+B$. In cases (i) and (iii) we have

$$\lambda(x)=\frac{\alpha_1-\alpha_2}{l(x_1)-l(x_2)}l(x)-\frac{\alpha_1 l(x_2)-\alpha_2 l(x_1)}{l(x_1)-l(x_2)},$$

in the case (ii) we take, as λ, the continuous extension on Ω^{Q_0} of d_{f_0h}/h, where

$$f_0=\frac{\alpha_1-\alpha_2}{l(x_1)-l(x_2)}f-\frac{\alpha_1 l(x_2)-\alpha_2 l(x_1)}{l(x_1)-l(x_2)}\in Q_0'.$$

Let $F\in C(\Omega^{Q_0})$, $f=F|_\Omega$, $\varepsilon>0$. For arbitrary x, $y\in\Omega^{Q_0}$ we can take $\lambda_{xy}\in C(\Omega^{Q_0})$ satisfying the following:

1) $\lambda_{xy}|_\Omega\in Q_0$.

2) $\lambda_{xy}(x)=F(x)$, $\lambda_{xy}(y)=F(y)$.

$U_{xy}=\{z\in\Omega^{Q_0};\ \lambda_{xy}(z)<F(z)+\varepsilon\}$ is open and contains x, y. From an open covering $\{U_{xy};\ y\in\Omega^{Q_0}\}$ of Ω^{Q_0} we select a finite subcovering $\{U_{xy_j};\ j=1,2,\cdots,n\}$. Set

$$u_x=\min_{1\leq j\leq n}\lambda_{xy_j},$$

where λ_{xy_j} is a function corresponding to U_{xy_j} $(j=1,2,\cdots,n)$. $u_x<F+\varepsilon$ on Ω^{Q_0} and $u_x(x)=F(x)$. Then, there exists a function g_0 of Q_0' such that $d_{u_xh}=d_{g_0h}$. In fact, let $\lambda_{xy_j}|_\Omega$ be $f_1,f_2,\cdots,f_k$; $\dfrac{d_{f_{k+1}h}}{h},\dfrac{d_{f_{k+2}h}}{h},\cdots,\dfrac{d_{f_{k+l}h}}{h}$; $A_{k+l+1}\dfrac{\min(G_{a_0},h)}{h}+B_{k+l+1},\cdots,A_n\dfrac{\min(G_{a_0},h)}{h}+B_n$, then

$$\begin{aligned}d_{u_xh}&=d(\min_{1\leq j\leq n}\lambda_{xy_j}h)=\bigwedge_{j=1}^{n}d_{\lambda_{xy_j}h}=(\bigwedge_{j=1}^{k}d_{f_jh})\wedge(\bigwedge_{j=k+1}^{k+l}d_{f_jh})\wedge(\min_{k+l+1\leq j\leq n}B_j)h\\&=(\bigwedge_{j=1}^{n+l}d_{f_jh})\wedge(\bigwedge_{j=k+l+1}^{n}d_{B_jh})=d_{g_0h},\end{aligned}$$

where $g_0=\min(\min_{1\leq j\leq k+l}f_j,\ \min_{k+l+1\leq j\leq n}B_j)\in Q_0'$. Since $U_x=\{z\in\Omega^{Q_0};\ u_x(z)>F(z)-\varepsilon\}$ is open and contains x, we can form a finite subcovering $\{U_{x_j};\ j=1,2,\cdots,l'\}$ of Ω^{Q_0}. Setting $v=\max_{1\leq j\leq l'}u_{x_j}$, where u_{x_j} is a function corresponding to U_{x_j} $(j=1,2,\cdots,l')$, we have $|u-F|<\varepsilon$ on Ω^{Q_0} and as above we can find $g\in Q_0'$ such that $d_{vh}=d_{gh}$.

$$d_{vh}-\varepsilon h\leq d_{fh}\leq d_{vh}+\varepsilon h$$

means $\left|\frac{d_{fh}-d_{gh}}{h}\right|=\left|\frac{d_{fh}-d_{vh}}{h}\right|\leq\varepsilon$, *q.e.d.*

Proof of Proposition 3.2. Since all functions of Q_0'' are extended continuously on Ω^{Q_0} we have $Q_0''\subset Q_1'$. The closure $\overline{Q_0''}$ of Q_0'' in the topology of uniform norm ($\|f\|=\sup_{\Omega}|f|$) is contained in Q_1'. On the other hand, above lemma tells us $Q_1''\subset\overline{Q_0''}$. We have thus $Q_1''=\overline{Q_0''}\subset Q_1'$ which implies $Q_1=Q_1'$ and the proposition follows.

4. Regularity of boundary points

Let $\hat{\Omega}$ be an arbitrary metrizable h-resolutive compactification of Ω, and $\Delta=\hat{\Omega}-\Omega$.

In this section we give a proof of theorem stated in the introduction. For definiteness we recall the definition of regularity of boundary points.

DEFINITION 4. A filter $\mathcal{F}$ on Ω converging to a boundary point x is called *strongly h-regular* if there exists an open neighbourhood δ of x and a positive superharmonic function s in $\delta\cap\Omega$ such that $s/h\underset{\mathcal{F}}{\rightarrow}0$ and the infimum of s/h outside of arbitrary open neighbourhood of x contained in δ is positive.

A filter $\mathcal{F}$ on Ω converging to a boundary point x is called *h-regular* if for every bounded continuous function φ on Δ we have $\frac{1}{h}\mathcal{D}_{\varphi,h}\underset{\mathcal{F}}{\rightarrow}\varphi(x)$.

A filter $\mathcal{F}$ on Ω converging to a boundary point x is called *weakly h-regular* if there exists a positive superharmonic function s such that $s/h\underset{\mathcal{F}}{\rightarrow}0$.

A boundary point x is called *strongly h-regular, h-regular* and *weakly h-regular* according as the filter formed by the trace on Ω of filter of neighbourhoods of x is strongly h-regular, h-regular and weakly h-regular respectively.

It is known that a strongly h-regular filter is h-regular and weakly h-regular. However an example of one-point compactification of Ω shows us that an h-regular filter is not necessarily weakly h-regular.

Since by Proposition 2.6 $\Delta^{Q_0}-\Gamma_h^{Q_0}$ is of h-harmonic measure zero, to prove our theorem it will be sufficient to show the following proposition:

Proposition 4.1. *Let Ω^{Q_0} be the compactification constructed in the preceding section and let $\Delta^{Q_0}=\Omega^{Q_0}-\Omega$. Every point of the h-harmonic boundary $\Gamma_h^{Q_0}$ of* Δ^{Q_0} is *h-regular and weakly h-regular.*

Proof. We use the same notations as in the preceding section. Let $x\in\Gamma_h^{Q_0}$ and $\varphi\in C(\Delta^{Q_0})$. Let F be a bounded continuous extension of φ on Ω^{Q_0} and set $f=F|_{\Omega}$.

Since $f \in Q_1'$, and $d_{fh}/h \in Q_1''$, f and d_{fh}/h can be extended continuously onto Ω^{Q_1}. By Proposition 3.2 Ω^{Q_1} is homeomorphic to Ω^{Q_0}, therefore f and d_{fh}/h are extended continuously onto Ω^{Q_0}. This is also ture for $g=f-d_{fh}/h$. Since $d_{gh}=0$, g is an h-Wiener potential and by Proposition 2.1 there exists potential p such that $|gh| \leq p$. For an arbitrary sequence of points $\{a_n\}$ in Ω converging to x we have

$$\lim_{n\to\infty} |g(a_n)| \leq \lim_{n\to\infty} \frac{p(a_n)}{h(a_n)} = 0 .$$

Hence

$$\lim_{n\to\infty} \left[f(a_n) - \frac{d_{fh}(a_n)}{h(a_n)} \right] = 0 ,$$

which means $\lim_{a\to x} \dfrac{\mathcal{D}_{\varphi,h}(a)}{h(a)} = \varphi(x)$. Thus, all points of $\Gamma_h^{Q_0}$ are h-regular.

Since $\min(G_{a_0}, h)/h$ is extended continuously on Ω^{Q_0}, this function assumes the value zero on $\Gamma_h^{Q_0}$, therefore all points of $\Gamma_h^{Q_0}$ are weakly h-regular, *q.e.d.*

If we take at every point $x \in \Gamma_h^{Q_0}$ the filter formed by the trace on Ω of neighbourhoods of x in Ω^{Q_0}, we obtain the family $\{\mathcal{F}_x\}$ of filters converging in $\hat{\Omega}$ and satisfying the following axioms:

$\boldsymbol{A_h}$) *If s is subharmonic in Ω, s/h is bounded from above and* $\limsup_{\mathcal{F}} s/h \leq 0$ *for every $\mathcal{F}$ in $\{\mathcal{F}_x\}$, then $s \leq 0$.*

$\boldsymbol{B_h'}$) *Every filters in $\{\mathcal{F}_x\}$ is h-regular and weakly h-regular.*

Indeed, A_h) follows from Proposition 2.4 and B_h') is a consequence of the above proposition.

The second axiom B_h') is weaker than the following axiom of Brelot-Choquet [2]:

$\boldsymbol{B_h}$) *Every filter in $\{\mathcal{F}_x\}$ is strongly h-regular.*

Thus, we have

Proposition 4.2. *Let $\hat{\Omega}$ be an arbitrary metrizable h-resolutive compactification of Ω. Then, there exists a family of filters in Ω converging in $\hat{\Omega}$ and satisfying the axiom A_h), B_h').*

L. Naïm gave a family of filters satisfying the axiom A_h), B_h) by using fine neighbourhoods on Martin space. Our filter is quite different from it.

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