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ON THE CAUCHY PROBLEM FOR KOWALEWSKI SYSTEMS

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The present paper is concerned with the global Cauchy problem for a Kowalewski system of partial differential equations of the form

$$\frac{\partial u_\mu}{\partial t} = \sum_{\nu=1}^{k} \left\{ \sum_{j=1}^{m} A_{\mu\nu j}(t,\, x) \frac{\partial u_\nu}{\partial x_j} + B_{\mu\nu}(t,\, x) u_\mu \right\} + f_\mu(t,\, x) \tag{0.1}$$

with the initial conditions

$$u_\mu(0,\, x) = \varphi_\mu(x)\,, \qquad \mu = 1, 2, \cdots, k \tag{0.2}$$

where $x=(x_1, \cdots, x_m)$ is the generic point in the m-dimensional Euclidean space R^m.

In this paper we shall not impose any condition on the characteristics of this system.

In [2] S. Mizohata studied the global uniqueness of solutions of the Cauchy problem within the class of tempered distributions under the conditions that the coefficients $A_{\mu\nu j}(t, x)$ and $B_{\mu\nu}(t, x)$ of (0.1) are bounded continuous functions in (t, x) whose Fourier transforms with respect to x are measures with compact supports. In [1] A.G. Kostjutschenko and G.E. Shilov considered the uniqueness of solutions of the Cauchy problem for the system of type (0.1)-(0.2) within a class of functions which satisfy the inequality $|u(x)| \leqq Me^{|x|^p}$ for some constants M and p, under the conditions that the coefficients $A_{\mu\nu j}(t, x)$ and $B_{\mu\nu}(t, x)$ of (0.1) are independent of t and are bounded continuous functions of x whose Fourier transforms are exponentially decreasing measures. On the other hand in [5] T. Yamanaka investigated the uniqueness of solutions of the Cauchy problem for the system (0.1) within a class of distributions with a finite growth order under the condition that the coefficients $B_{\mu\nu}(t, x)$ are of the form $B_{\mu\nu}(t, x) = P_{\mu\nu}(x)\, B'_{\mu\nu}(t, x)$ where $P_{\mu\nu}(x)$ are any polynomials in x and $B'_{\mu\nu}(t, x)$ are bounded continuous functions of (t, x) whose Fourier transforms with respect to x are exponentially decreasing measures, and the coefficients $A_{\mu\nu j}(t, x)$ are the same type of functions as $B'_{\mu\nu}(t, x)$.

In [4] the author studied the existence and the uniqueness of global solutions of the Cauchy problem for the system (0.1). The uniqueness of solutions was proved within a class of functions which satisfy the inequality

$$|u(t, x)| \leqq M \exp (ae^{b|x|}) \text{ in } [0, T] \times R^m$$

for some constants a, b and M, under some conditions on the coefficients of (0,1) specified in the main theorems.

The purpose of this paper is to give a revised and complete proof of the results obtained in [4]. The method of the existence proof used here is essentially based on that of M. Nagumo [2].

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1. Assumptions and Notations

We denote by R^m and C^m the m-dimensional real and complex Euclidean space respectively, and denote by $x=(x_1, \cdots, x_m)$ and $z=x+\sqrt{-1}y$ $=(x_1+\sqrt{-1}y_1, \cdots, x_m+\sqrt{-1}y_m)$ $(x, y\in R^m)$ their generic point respectively. For positive numbers T and γ, $D(T)$ and $\mathfrak{D}_\gamma(T)$ are defined as follows:

$$\begin{aligned} D(T) &= \{(t, x);\ 0\leqq t\leqq T,\ x\in R^m\} \\ \mathfrak{D}_\gamma(T) &= \{(t, x);\ 0\leqq t\leqq T,\ z\in C^m,\ |y_j|<\gamma,\ j=1, \cdots, m\}. \end{aligned}$$

For any non-negative integer h we denote by $C_{(t,z)}^h[\mathfrak{D}_\gamma(T)]$ the class of all complex valued functions whose derivatives of order up to h are continuous in $\mathfrak{D}_\gamma(T)$. By $A_{(z)}[\mathfrak{D}_\gamma(T)]$, we denote the class of all complex valued functions defined in $\mathfrak{D}_\gamma(T)$ and holomorphic with respect to z when t is fixed in $[0, T]$. For any positive constants a and b, the class of all continuously differentiable functions which satisfy the inequality $|f(t, x)| \leqq M \exp (ae^{b|x|})$ in $D(T)$ for some positive constant M is denoted by $\mathfrak{F}(a, b)$, M being allowed to be dependent on the individual f.

We now state the assumptions on the coefficients of (0,1) here.

Assumptions

(I) *$A_{\mu\nu j}(t, z)$, $B_{\mu\nu}(t, z)$ and $f_\mu(t, z)$ $(\mu, \nu=1, \cdots, k;\ j=1, \cdots, m)$ are continuous functions in $\mathfrak{D}_\gamma(T)$.*

(II) *$A_{\mu\nu j}(t, z)$ and $B_{\mu\nu}(t, z)$ $(\mu, \nu=1, \cdots, k;\ j=1, \cdots, m)$ are holomorphic functions with respect to z in the domain: $\{z\in C^m;\ -\infty<x_j<+\infty,\ |y_j|<\gamma,$ $j=1, 2, \cdots, m\}$ for each fixed t in $[0, T]$, and there exist positive constants A and*

B such that $|A_{\mu\nu j}(t, z)| \leqq A$, $|B_{\mu\nu}(t, z)| \leqq B$ *in* $\mathfrak{D}_\gamma(T)$.

(III) $f_\mu(t, z)$ $(\mu=1, 2, \cdots, k)$ *are holomorphic functions with respect to* z *in* $\{z\in C^m;\ -\infty<x_j<+\infty,\ |y_j|<\gamma, j=1,\cdots, m\}$ *for each* t *in* $[0, T]$, *and* $\varphi_\mu(z)$ $(\mu=1,\cdots, m)$ *are holomorphic functions in* $\{z\in C^m;\ -\infty<x_j<+\infty,\ |y_j|<\gamma, j=1,\cdots, m\}$.

2. Main Theorems

In this section we shall state an existence theorem and a uniqueness theorem for the system (0,1) with the initial conditions (0,2). Proofs of these theorems will be given in section 4.

Theorem 1. (existence of solutions)

Under the assumptions (I), (II) *and* (III) *on the coefficients of* (0,1) *and on the initial conditions* (0,2), *there exist positive numbers* T_1 *and* γ_1 $(0<T_1\leqq T,\ 0<\gamma_1<\gamma)$ *and a system of solutions* $u_\mu(t, z)$ $(\mu=1, \cdots, k)$ of (0,1) *satisfying the initail conditions* (0,2), *and belonging to* $C^1_{(t,z)}[\mathfrak{D}_{\gamma_1}(T_1)] \cap A_{(z)}[\mathfrak{D}_{\gamma_1}(T_1)]$.

Theorem 2. (uniqueness of solutions)

Suppose that the assumptions (I) *and* (II) *are satisfied. If* $u_\mu(t, x)$ *and* $v_\mu(t, x)$ $(\mu = 1, 2,\cdots, k)$ *are two continuously differentiable solutions of* (0,1) *in* $D(T)$ *satisfying the same initial conditions* (0,2) *and belonging to* $\mathfrak{F}(a, b)$ *for some constants* a *and* b, *then* $u_\mu(t, x)\equiv v_\mu(t, x)$ $(\mu=1, 2,\cdots, k)$ *in* $D(T)$.

3. Preliminary Lemmas

We begin this section with the following basic lemma.

Lemma 1. *Let* $f(z_1, \cdots, z_m)$ *be a function which is holomorphic in the domain* $G(\delta)=\{z=x+\sqrt{-1}\,y\in C^m;\ |z_j|<\delta, j=1, 2, \cdots, m\}$ *and satisfies the inequality*

$$|f(z_1, \cdots, z_m)| \leqq M\rho^{-\alpha} \tag{3.1}$$

there for some positive constants M *and* α, *where* $\rho=\delta-\operatorname*{Max}_{1\leqq j\leqq m}|z_j|$. *Then the following inequality holds in* $G(\delta)$ *for each* $j=1, 2, \cdots, m$;

$$\left|\frac{\partial f}{\partial x_j}(x_1+\sqrt{-1}\,y_1, \cdots, x_m+\sqrt{-1}\,y_m)\right| \leqq \frac{(1+\alpha)^{1+\alpha}}{\alpha^\alpha} M\rho^{-\alpha-1} \tag{3.2}$$

Proof. For arbitrary $z^0=(z_1^0, \cdots, z_m^0)\in G(\delta)$ we take a circle C_j in the z_j-plane with center z_j^0 and raduis $\dfrac{\rho}{1+\alpha}$ where $\rho=\delta-\operatorname*{Max}_{1\leqq j\leqq m}|z_j|$. If $z_j\in C_j$ $(j=1, 2, \cdots, m)$, then $\delta-|z_j| \geqq \delta-|z_j^0|-|z_j-z_j^0| \geqq \dfrac{\alpha}{1+\alpha}\rho$ and hence we have the inequality

$$|f(z)| \leqq M\left(\rho-\frac{\alpha}{1+\alpha}\right)\rho^{-\alpha} = \frac{(1+\alpha)^{\alpha}}{\alpha^{\alpha}} M\rho^{-\alpha}.$$

Thus in view of Cauchy's integral formula we get the inequalities

$$\left|\frac{\partial f}{\partial x_j}(z^0)\right| \leqq \frac{(1+\alpha)^{1+\alpha}}{\alpha^{\alpha}} M\rho^{-\alpha-1}, \qquad j = 1, 2, \cdots, m$$

Q.E.D.

In the proof of Theorem 1 and Theorem 2 we may suppose without loss of generality that the initial values φ_μ all vanish. Then the system (0.1)–(0.2) is equivalent to the following system of integro-differential equations:

$$u_\mu(t, x) = \Phi_\mu[u(t, x)], \qquad \mu = 1, 2, \cdots, k \tag{3.3}$$

where for every μ

$$\Phi_\mu[u] = \sum_{\nu=1}^{k}\left\{\sum_{j=1}^{m}\int_0^t A_{\mu\nu j}(\tau, x)\frac{\partial u_\nu}{\partial x_j}(\tau, x)d\tau + \int_0^t B_{\mu\nu}(\tau, x)u_\nu(\tau, x)d\tau\right\} + \int_0^t f_\mu(\tau, x)d\tau.$$

Therefore in order to prove Theorem 1 and Theorem 2, it is sufficient to prove the existence and the uniqueness of solutions of (3.3) respectively.

First of all we shall prove the following local existence theorem.

Lemma 2. *Suppose that the assumptions* (**I**), (**II**) *and* (**III**) *are satisfied. Then for arbitrary $x^0 \in R^m$ there exists a solution $u(t, z)$ of* (3.3) *which is continuously differentiable in (t, z) and holomorphic with respect to z in*

$$\Delta(x^0) = \{(t, z);\ 0 \leqq t \leqq T_1,\ |z_j - x_j^0| < R_1 - L_1 t,\ j = 1, 2, \cdots, k\}$$

where $0 < R_1 < \mathrm{Min}\left\{1, \gamma, \left(\frac{1+\alpha}{\alpha}\right)^{1+\alpha}(1-\alpha)\frac{mA}{B}\right\}$

$$L_1 = \frac{mkA}{\kappa}\left(\frac{1+\alpha}{\alpha}\right)^{1+\alpha},$$

$$T_1 = \mathrm{Min}\{T, R_1/L_1\}$$

with any fixed constants α and κ satisfying $0 < \alpha < 1$ *and* $0 < \kappa < 1$.

Proof. In the first place we note that

$$g_\mu(t, z) \in C^1_{(t,z)}[\mathfrak{D}_\gamma(T)] \cap A_{(z)}[\mathfrak{D}_\gamma(T)]$$

implies $$\Phi_\mu[g(t, z)] \in C^1_{(t,z)}[\mathfrak{D}_\gamma(T)] \cap A_{(z)}[\mathfrak{D}_\gamma(T)].$$

Next consider the sequence of functions $u_\mu^{(n)}(t, z)$ defined inductively as follows:

$$u_\mu^{(0)}(t, z) \equiv 0$$
$$u_\mu^{(n+1)}(t, z) = \Phi_\mu[u^{(n)}(t, z)], \qquad n = 0, 1, \cdots \tag{3.4}$$

Then from $u_\mu^{(0)}(t, z) \in C^1_{(t,z)}[\mathfrak{D}_\gamma(T)] \cap A_{(z)}[\mathfrak{D}_\gamma(T)]$, it follows that $u_\mu^{(n+1)}(t, z) \in C^1_{(t,z)}[\mathfrak{D}_\gamma(T)] \cap A_{(z)}[\mathfrak{D}_\gamma(T)]$ for all positive integers n.

Let
$$\Psi_\mu[u] = \Phi_\mu[u] - \int_0^t f_\mu(\tau, z)d\tau .$$

Then $u_\mu^{(h+1)} - u_\mu^{(h)} = \Psi_\mu[u^{(h)} - u^{(h-1)}]$, $h = 1, 2, \cdots$. To demonstrate the convergence of the sequence $\{u_\mu^{(n)}(t, z)\}$ we consider the series:

$$\begin{aligned} u_\mu^{(n+1)}(t, z) &= \sum_{h=1}^{n} \{u_\mu^{(h+1)}(t, z) - u_\mu^{(h)}(t, z)\} + u_\mu^{(1)}(t, z) \\ &= \sum_{h=1}^{n} \Psi_\mu[u^{(h)} - u^{(h-1)}] + u_\mu^{(1)}(t, z) . \end{aligned}$$

It is obvious that for given α with $0 < \alpha < 1$, there exists a positive constant M such that

$$|u_\mu^{(1)} - u_\mu^{(0)}| \leqq \int_0^t |f_\mu(\tau, z)| d\tau \leqq M\rho^{-\alpha} \qquad \text{in } \Delta(x^0) \tag{3.5}$$

where $\rho = (R_1 - L_1 t - \operatorname*{Max}_{1 \leqq j \leqq m}\{|z_j - x_j^0|\})$, and hence we get

$$\int_0^t |u_\mu^{(1)} - u_\mu^{(0)}| d\tau \leqq \frac{M}{(1-\alpha)L_1} R_1^{1-\alpha} \qquad \text{in } \Delta(x^0) .$$

From Lemma 1 and (3.5) we obtain the inequality

$$\left| \int_0^t \frac{\partial(u_\nu^{(1)} - u_\nu^{(0)})}{\partial x_j} d\tau \right| \leqq \left(\frac{1+\alpha}{\alpha}\right)^{1+\alpha} \frac{M}{L_1}(\rho^{-\alpha} - R_1^{-\alpha}) .$$

Hence we easily get the following inequality:

$$|u_\mu^{(2)} - u_\mu^{(1)}| \leqq mkA\left(\frac{1+\alpha}{\alpha}\right)^{1+\alpha} \frac{M}{L_1}(\rho^{-\alpha} - R_1^{-\alpha}) + \frac{kBM}{(1-\alpha)L_1} R_1^{1-\alpha} .$$

The assumptions on the constants L_1 and R_1 lead to the estimates

$$|u_\mu^{(2)} - u_\mu^{(1)}| \leqq \kappa M\rho^{-\alpha} \qquad (\mu = 1, \cdots, k) \text{ in } \Delta(x^0) .$$

Therefore we obtain inductively for all positive integers n

$$|u_\mu^{(n+1)} - u_\mu^{(n)}| \leqq \kappa^n M\rho^{-\alpha} \qquad (\mu = 1, \cdots, k) \text{ in } \Delta(x^0) . \tag{3.6}$$

This proves that the sequence $\{u_\mu^{(n)}(t, z)\}$ converges uniformly to a function $u_\mu(t, z)$ on any closed subdomain of $\Delta(x^0)$, and therefore $u_\mu(t, z)$ belongs to $C^1_{(t,z)}[\Delta(x^0)] \cap A_{(z)}[\Delta(x^0)]$ and $u_\mu(t, z) = \Phi_\mu[u(t, z)]$ in $\Delta(x^0)$. Q.E.D.

From the above proof of Lemma 2, we see obviously the following corollary.

Corollary 1. *Let the functions $u_\mu(t, z)$ $(\mu=1,\cdots,k)$ be a solution obtained in Lemma 2, then we have the following inequalities*:

$$|u_\mu(t, z)| \leqq \frac{M}{1-\kappa}\rho^{-\alpha} \qquad \textit{in } \Delta(x^0), \qquad \mu = 1,\cdots,k,$$

where $M = \sup_{\substack{(t,z)\in\Delta(x^0)\\ 1\leqq\mu\leqq k}} \{\rho^\alpha T_1 |f_\mu(t, z)|\}$.

From now on we shall denote by $u_\mu(t, z, x^0)$ the solution of (3.3) in $\Delta(x^0)$ constructed in the proof of Lemma 2.

4. Proof of Theorems

Proof of Theorem 1. We shall show that by the analytic continuation with respect to z of the local solutions whose existence was established in Lemma 2, we get a global solution of (3.3). For this purpose it suffices to prove that for arbitrary $z\in\Delta(x^0)\cap\Delta(x^1)(x^0, x^1\in R^m)$, $u_\mu(t, z, x^0)$ is equals to $u_\mu(t, z, x^1)$ $(\mu=1,\cdots,k)$.

Letting $\quad v_\mu(t, z) = u_\mu(t, z, x^0)-u_\mu(t, z, x^1)$, we have $v_\mu(0, z)\equiv 0$,

$$v_\mu(t, z) = \Psi_\mu[v(t, z)] \qquad \text{in } \Delta(x^0)\cap(x^1).$$

If R is a such positive number that

$$\Delta' = \left\{(t, z);\ 0\leqq t\leqq T_2, \left|z_j - \frac{x_j^0+x_j^1}{2}\right| < \tilde{R}-L_1 t, j = 1,\cdots,k\right\} \subset \Delta(x^0)\cap\Delta(x^1),$$

then we get the following inequalities as in the proof of Lemam 2:

$$|v_\mu(t, z)| = |\Psi_\mu[v(t, z)]| \leqq \kappa\tilde{M}\tilde{\rho}^{-\alpha} \qquad \text{in } \Delta',\ \mu = 1,\cdots,k$$

where $\tilde{\rho}=\left(\tilde{R}-L_1 t-\max_{1\leqq j\leqq m}\left\{\left|z_j-\frac{x_j^0+x_j^1}{2}\right|\right\}\right)$,

$$\tilde{M} = \sup_{\substack{(t,z)\in\Delta'\\ 1\leqq\mu\leqq k}} \{\tilde{\rho}^\alpha |v_\mu(t, z)|\}.$$

From these inequalities it follows that $\tilde{M}\tilde{\rho}^{-\alpha}\leqq\kappa\tilde{M}\tilde{\rho}^{-\alpha}$ for given κ such that $0<\kappa<1$. This shows that $\tilde{M}=0$ and hence $v_\mu(t, z)\equiv 0$ $(\mu=1,\cdots,k)$ in Δ'. Thus in view of the analyticity of v_μ with respect to z we get $v_\mu(t, z)\equiv 0$ in $\Delta(x^0)\cap\Delta(x^1)$ $(\mu=1,\cdots,k)$, obtaining a global solution $u_\mu(t, z)$ of (3.3) in $\mathfrak{D}_{\gamma_1}(T_1)$.

By Corollary 1 and Theorem 1 we can show without difficulty:

Corollary 2. *If* $|f_\mu(t, z)|\leqq M\exp(-ae^{b|x|})$ $(\mu=1,\cdots,k)$ *in* $\mathfrak{D}_\gamma(T)$ *for some positive constants a, b and M, then for any given positive number a' $(<a)$*

there exist positive numbers M' and T_1 such that the solution $u_\mu(t, x)$ $(\mu=1, \cdots, k)$ of (3.3) *satisfies the inequality*

$$|u_\mu(t, x)| \leqq M' \exp(-a' e^{b|x|}) \qquad \text{in} \quad D(T_1).$$

Lemma 3. *For arbitrarily given positive number ε and positive constants a and b, there exist positive numbers a', b' and γ such that the inequality*

$$\exp\{-(a+\varepsilon)e^{b|x|}\} \geqq \exp\{-a' \sum_{\nu=1}^{m} \cosh(b' z_\nu)\}$$

holds in $\mathfrak{D}_\gamma(T)$.

Proof.
$$|\exp\{-a' \cosh(b' z_\nu)\}| = \exp\{-a' \mathfrak{Re} \cosh(b' z_\mu)\}$$
$$\leqq \exp\left\{-\frac{a' \cos(b' y_\nu)}{2} e^{b'|x_\nu|}\right\}.$$

For fixed θ satisfying $0 < \theta < \dfrac{2}{\pi}$, $\cos(b' y_\nu) \geqq \cos\theta$ when $|y_\nu| \leqq \theta/b'$.

Thus
$$|\exp\{-a' \sum_{\nu=1}^{m} \cosh(b' z_\nu)\}| \leqq \exp\left\{-\frac{a' \cos\theta}{2} \sum_{\nu=1}^{m} e^{b'|x_\nu|}\right\}$$
$$\leqq \exp\left\{-\frac{a' \cos\theta}{2} e^{\frac{b'}{\sqrt{m}}|x|}\right\},$$

and setting $b' = \sqrt{m}\, b$ and $a' = \dfrac{2(a+\varepsilon)}{\cos\theta}$ we have

$$\exp\{-(a+\varepsilon)e^{b|x|}\} \geqq \exp\{-a' \sum_{\nu=1}^{m} \cosh(b' z_\nu)\} \quad \text{in} \quad \mathfrak{D}_\gamma(T)$$

when $|y_\nu| \leqq \gamma = \dfrac{\theta}{\sqrt{m}\, b}$, $\nu = 1, 2, \cdots, m$. Q.E.D.

Proof of Theorem 2. Set

$$L_\mu[u] = \frac{\partial u_\mu}{\partial t} - \sum_{\nu=1}^{k} \left\{\sum_{j=1}^{m} A_{\mu\nu j}(t, x) \frac{\partial u_\mu}{\partial x_j} + B_{\mu\nu}(t, x) u_\nu\right\}$$

and for every σ $(\sigma = 1, 2, \cdots, k)$

$$\tilde{L}_\mu^\sigma[u] = -\frac{\partial u_\mu}{\partial t} + \sum_{\nu=1}^{k} \left\{\sum_{j=1}^{m} \frac{\partial}{\partial x_j}[A_{\mu\nu j}(t, x_\nu) u_\mu] - B_{\mu\nu}(t, x) u_\nu\right\}$$
$$- e^{-\sqrt{-1} x \xi} \exp\{-a' \sum_{\nu=1}^{m} \cosh(b' x_\nu)\} \cdot \delta_{\sigma\mu}$$

where ξ is an arbitrary real vector and a', b', γ are positive constants such that the conclusion of Lemma 3 holds when $\varepsilon > 0$ was given in advance, and $\delta_{\mu\sigma}$ is the Kronecker's delta.

The system of equations $\tilde{L}_\mu^\sigma[u]=0$ $(\mu=1,\cdots,k)$ is of similar form to the system of equations considered in Theorem 1, and considering t in the negative direction in Theorem 1, we can conclude that there exist a positive number T_0 $(\leqq T_1)$ such that for any $T\in[0, T_0]$ there is a system of solutions $w_\mu(t, x)$ of $\tilde{L}_\mu^\sigma[u]=0$ $(\mu=1,\cdots,k)$ in $D(T)$ satisfying the initial condition $w_\mu(T, x)=0$. Moreover in view of the Corollary 2, we get the following inequalities:

$$|w_\mu(t, x)|\leqq M' \exp\left\{-\left(a+\frac{\varepsilon}{2}\right)e^{b|x|}\right\} \qquad \text{in} \quad D(T) \tag{4.1}$$

$(\mu=1,\cdots,k)$ for some positive constant M depending on ε, if we choose the constant a in that Corollary appropriately.

Suppose that $u(t, x)$ and $v(t, x)$ are two solutions of the system (0.1) with the initial conditions (0.2). Suppose also that $u(t, x)$ and $v(t, x)$ belong to $\mathfrak{F}(a, b)$ for some positive numbers a and b. Then the function $u-v$ satisfies

$$L_\mu[u-v]=0\,,$$
$$[u_\mu-v_\mu](0, x)=0 \qquad (\mu=1,\cdots,k)$$

and the inequalities

$$|[u_\mu(t, x)-v_\mu(t, x)]|\leqq K\cdot\exp(ae^{b|x|}) \tag{4.2}$$

$(\mu=1,\cdots,k)$ in $D(T)$ for some positive constant K and for any $T\in[0, T_0]$.

Since

$$\sum_{\mu=1}^{k}\iint_{D(T)}\{w_\mu L_\mu[u-v]-(u_\mu-v_\mu)\tilde{L}_\mu^\sigma[w]\}\,dx\,dt=0$$

we have

$$\int_0^t dt\int_{R^m} e^{-\sqrt{-1}x\xi}[(u_\sigma-v_\sigma)\exp\{-a'\sum_{\nu=1}^{m}\cosh(b'|x_\nu|)\}]\,dx=0$$

for any $\xi\in R^m$ and any $t\in[0, T_0]$. Thus for any $\xi\in R^m$ and any $t\in[0, T_0]$, we obtain

$$\int_{R^m} e^{\sqrt{-1}x\xi}[(u_\sigma(t, x)-v_\sigma(t, x)\exp\{-a'\sum_{\nu=1}^{m}\cosh(b'|x_\nu|)\}]\,dx=0 \tag{4.3}$$

Since $\quad |(u_\sigma-v_\sigma)\exp\{-a'\sum_{\nu=1}^{m}\cosh(b'|x_\nu|)\}|\leqq\exp\{-\varepsilon e^{b|x|}\},$

(4.3) means that the Fourier Transform of the integrable continuous function $(u_\sigma-v_\sigma)\exp\{-a'\sum_{\nu=1}^{m}\cosh(b'|x_\nu|)\}$ vanishes identically in R^m for each $t\in[0, T_0]$. And since $\exp\{-a'\sum_{\nu=1}^{m}\cosh(b'|x_\nu|)\}\neq 0$ in R^m, we get

$$u_\sigma(t, x)-v_\sigma(t, x)\equiv 0 \quad \text{in} \quad D(T_0)\,.$$

As σ is arbitrary, $u_\sigma(t, x) \equiv v_\sigma(t, x)$ in $D(T_0)$ for every σ ($\sigma = 1, \cdots, k$).

Now suppose that there exists a $T' \in [0, T]$ such that $u_\mu(T', x) - v_\mu(T', x) \not\equiv 0$ in R^m for some μ, and let T_2 be the infimum of such T_1. Then $u_\mu(t, x) \equiv v_\mu(t, x)$ in $D(T_2)$. Taking T_2' and T_3 such that $T_3 - T_2 \leqq T_0$ and $T_2' < T_2 < T_3$, and repeating the above argument in the interval $[T_2', T_3]$, we get $u_\mu(t, x) = v_\mu(t, x)$ for $(t, x) \in \{D(T_3) - D(T_2')\} = \{(t, x); T_2' < t \leqq T_3, x \in R^m\}$. This contradicts the definition of T_2,and hence we get the conclusion

$$u_\mu(t, x) \equiv v_\mu(t, x) \quad \text{in} \quad D(T)$$

for every μ ($\mu = 1, \cdots, k$). Q.E.D.

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