Measuring Theory of Three-dimensional Residual Stresses Using a Thinly Sliced Plate Perpendicular to Welded Line†

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Abstract

The authors developed the measuring theory of three-dimensional residual stresses, in which inherent strains, the source of residual stresses, are used as parameters, and demonstrated L_z method and nL_y method. In these methods, inherent strains are estimated using relaxed strains measured at many points. Necessarily, cutting is required many times. In the cutting process, additional inherent strains may be produced by the effect of cutting heat. Therefore, it is desirable to decrease the number of cutting as much as possible, which is also very important to improve the accuracy of measurement and to shorten the time required for experiments.

In this paper, in order to accurately estimate three-dimensional residual stresses produced in a region where residual stresses are uniform along the welded line, a measuring theory and its method using a thin plate cut out perpendicular to the welded line are presented.

The main results are as follows:

(1) *Using relaxed strains measured before and after the cutting of Specimen T, effective inherent strain, the source of residual stresses, can be estimated being divided into cross-sectional inherent strain components, $\{\epsilon_C^*\} = \{\epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}$, and one along the welded line, $\{\epsilon_x^*\}$.*

(2) *Giving the estimated effective inherent strains to the original three-dimensional object, residual stresses are calculated. By this method, three-dimensional welding residual stress at an arbitrary position including the inside of the object can be estimated.*

(3) *The reliability of this method was confirmed by a numerical experiment.*

(4) *By this method, the number of cutting has greatly been decreased. Accordingly, the time required for experiments can be greatly shortened and experimental expenses cut down.*

(5) *This method is applicable to measurement of three-dimensional residual stresses produced by any causes as well as welding, if the residual stresses are considered to be uniform in one direction.*

KEY WORDS: (Measuring Theory of Residual Stress) (Measurement of Residual Stress) (Welding Residual Stress) (Three-dimensional Residual Stress) (Inherent Strain) (Relaxation Method)

1. Introduction

In the tendency of constructing large welded structures, it has become more important to know three-dimensional welding residual stresses of welded joints in order to discuss strength and safety of welded structures.

On this kind of problem, a series of researches was carried out by the authors in order to accurately measure three-dimensional residual stresses. In this process, the authors showed[1)] the measuring principle of three-dimensional residual stresses in which inherent strains, the source of residual stresses, are dealt as parameters, and formulated a general measuring theory based on the finite element method[2)]. In this consequence, it became possible to measure three-dimensional residual stresses produced in structural elements by welding, heat treatment and the like. They also showed two measuring method, L_z method[3)] and nL_y method[4)], for three-dimensional welding residual stresses distributed uniformly in one direction. These methods were actually applied to measurement of three-dimensional residual stresses[3,5,6)].

Either L_z method or nL_y method uses a thin plate perpendicular to the weld line (Specimen T) and many thin plates parallel to the weld line (Specimens L_z or n

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pieces of Specimen L_y). The cutting of these specimens, to which careful attention must be paid lest plastic deformation should occur due to cutting heat, etc., requires a great amount of time and expenses for experiments. In order to make these measuring methods practical, cutting work should be greatly reduced.

In this paper, to meet the above mentioned demand, a theory to accurately measure three-dimensional welding residual stresses distributed uniformly along the weld line using only a thinly sliced plate perpendicular to the weld line (hereinafter called Specimen T) is developed. Application and measuring procedure based on this new method are shown. Validity of the new method is confirmed by numerical experiments. In this research, every analysis is performed by the finite element method.

2. Fundamental Theory of Measuring Method of Three-dimensional Residual Stresses in Which Inherent Strains are Dealt as Parameters

Simply stated below is the fundamental theory[2)] of measuring method of residual stresses by the finite element method, in which inherent strains are dealt as parameters.

Inherent strains are composed of effective and ineffective components to produce residual stresses (See 3.3). In this research, only the effective inherent strain component is taken into account and simply called inherent strain. The following elastic response relation equations are formulated among this inherent strain $\{\epsilon^*\}$ which is the source of residual stress, and elastic strain $\{\epsilon\}$ and residual stress $\{\sigma\}$ produced by $\{\epsilon^*\}$ at an arbitrary position of a three-dimensional object.

$$\begin{aligned}\{\epsilon\} &= [H^*]\{\epsilon^*\}\\ \{\sigma\} &= [D]\{\epsilon\} = [D]\,[H^*]\{\epsilon^*\}\end{aligned} \qquad (1)$$

where,

$[H^*]$: elastic response matrix

$[D]$: elasticity matrix

Using these elastic response equations (1), residual stress $\{\sigma\}$ can be accurately measured, if inherent strain $\{\epsilon^*\}$ is accurately estimated. Therefore, it is important for measurement of residual stresses by this measuring method to estimate inherent strain accurately. Invariability of inherent strain (the source of residual stress) is applied in this measuring method, i.e., when an object is cut, (1) residual stress varies in magnitude and distribution, but (2) inherent strain does not. Using these characteristics, inherent strains $\{\epsilon^*\}$ are estimated as follows:

First, elastic strains produced in a three-dimensional object are observed as many as possible by cutting the object. Thus observed strains are $\{{}_m\epsilon\}$ which may include various observation errors. Using Eq.(1), a measuring equation is derived as follows:

$$\{{}_m\epsilon\} - [H^*]\{\hat{\epsilon}^*\} = \{\nu\} \qquad (2)$$

where,

$\{\hat{\epsilon}^*\}$: the most probable value of inherent strain

$\{\nu\}$: residual

The most probable value of inherent strain $\{\hat{\epsilon}^*\}$ can be obtained under the condition that the sum of square of residual becomes the minimum, that is,

$$\{\hat{\epsilon}^*\} = ([H^*]^T[H^*])^{-1}[H^*]^T\{{}_m\epsilon\} \qquad (3)$$

Substituting this $\{\hat{\epsilon}^*\}$ for $\{\epsilon^*\}$ in Eq.(1), the most probable value of elastic strains $\{\hat{\epsilon}\}$ and residual stresses $\{\hat{\sigma}\}$ produced in an arbitrary position of a three-dimensional object can be calculated as follows:

$$\begin{aligned}\{\hat{\epsilon}\} &= [H^*]\{\hat{\epsilon}^*\}\\ \{\hat{\sigma}\} &= [D]\{\hat{\epsilon}\} = [D]\,[H^*]\{\hat{\epsilon}^*\}\end{aligned} \qquad (4)$$

3. Measuring Theory of Three-dimensional Residual Stresses Using Only Thinly Sliced Plate Perpendicular to Weld Line

It is considered that three-dimensional welding residual stresses produced in a joint by continuous welding with a long weld line under restraint uniform along the weld line distribute uniformly along the weld line except at the starting and finishing weld ends. In this case, inherent strains are also regarded uniform along the weld line. Taking account of this distribution characteristics of inherent strains, an accurate measuring method using a thinly sliced plate perpendicular to the weld line is developed for three-dimensional residual stresses produced in a portion where residual stresses are considered to distribute uniformly.

3.1 Object and assumptions used in the measuring theory

Measuring object R' ($L' \times B' \times h$) is shown in **Fig. 1.** When R' is too large or residual stresses produced in a welded joint of an actual large welded structure is to be observed, various difficulties may accompany the treatment such as to move or cut the object. In such a case, a new specimen R ($L \times B \times h; L$ may be set freely unless extremely short, but is desired to be longer than h) (Fig. 1 (b)) should be cut out from the portion where residual stresses are considered to distribute uniformly and inherent strains are produced (near the welded zone). Using this Specimen R, inherent strains can be estimated. This is because the distribution of inherent strains (invariants) in R is the same as that in R' unless new inherent strains are added, though stresses redistributed under the change of restraint by cutting are different from those in R'. Apply-

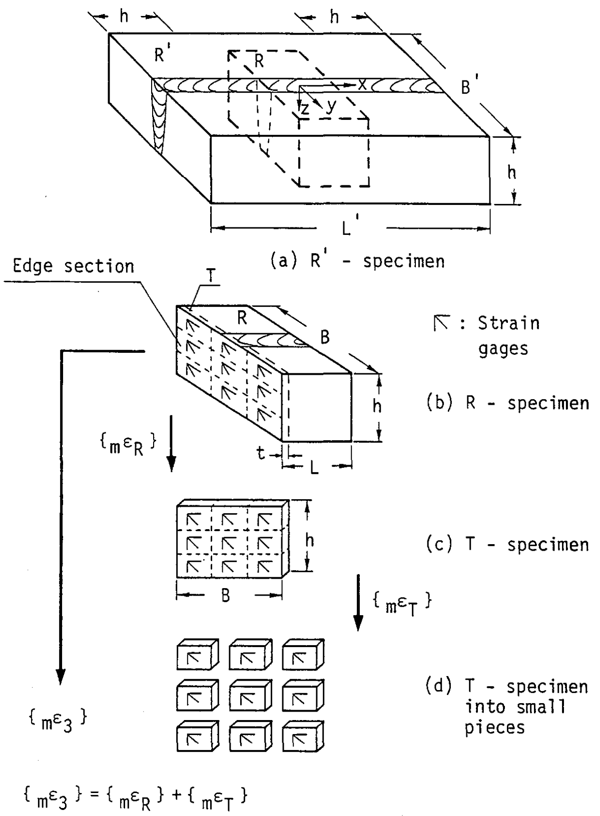


Fig. 1 Analysis model, location of strain gages and difinition of relaxed strains.

ing inherent strains estimated from this R either to R' or the welded joint of an actual welded structure, three-dimensional elastic analysis (this measuring method is based on the finite element method) is conducted and residual stresses are measured. This is one of the most characteristic merits of this measuring method of three-dimensional residual stresses, in which inherent strains are dealt as parameters.

The following assumptions are made for measurement of residual stresses.

(1) Cutting is accompanied by only elastic change of strains and does not produce any new inherent strains.

(2) Continuous welding is applied to R' at the middle of plate width ($y = 0$) along the x-axis. Among inherent strain components $\{\epsilon_x^*, \epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*, \gamma_{zx}^*, \gamma_{xy}^*\}$, the source of residual stresses, $\{\gamma_{zx}^*\}$ and $\{\gamma_{xy}^*\}$ which compose unsymmetric stresses may be neglected at the region inside from the starting and finishing weld ends by about the distance of plate thickness[3]. In this region, inherent strains are uniform along the weld line and their components are $\{\epsilon_x^*, \epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}$, the functions of only y and z.

(3) Inherent strain component perpendicular to the cross section of R', $\{\epsilon_x^*\}$, does not contribute to the stresses remain in thinly sliced Specimen T. Therefore, stresses remain in Specimen T are composed of only cross-sectional inherent strain components, $\{\epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}$, in the plane stress state.

Followings are the concrete method and procedure to estimate inherent strains $\{\epsilon^*\} = \{\epsilon_x^*, \epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}^T$, the source of residual stresses, by dividing into the cross-sectional inherent strain components $\{\epsilon_c^*\} = \{\epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}^T$ and the longitudinal one (in the direction of weld line) $\{\epsilon_x^*\}$.

3.2 Cutting sequence of Specimen R and strains to be observed

Prior to cutting Specimen T from Specimen R, strain gages are attached to the edge section (cross section) of R (Fig. 1 (b)). Specimen T is then sliced (Fig. 1 (c)). Relaxed strains by this cutting are observed. It is supposed that elastic strains with opposite sign to the observed strains existed before cutting. These elastic strains are denoted by $\{{}_m\epsilon_R\}$. Elastic strains $\{{}_m\epsilon_R\}$ are produced under the changes of restraint condition and boundary condition due to cutting Specimen T from R. Subdividing this Specimen T (Fig. 1 (d)), elastic strains $\{{}_m\epsilon_T\}$ and residual stresses $\{{}_m\sigma_T\}$ remained in Specimen T are observed. Consequently, elastic strains $\{{}_m\epsilon_3\}$ and residual stresses $\{{}_m\sigma_3\}$ existed on the edge section of R can be observed directly; that is,

$$\begin{aligned}\{{}_m\epsilon_3\} &= \{{}_m\epsilon_R\} + \{{}_m\epsilon_T\} \\ \{{}_m\sigma_3\} &= [D]\,\{{}_m\epsilon_3\}\end{aligned} \tag{5}$$

Stress $\{{}_m\sigma_3\}$ and strains $\{{}_m\epsilon_3\}$ observed on the edge section of R are produced by the inherent strain components $\{\epsilon_x^*\} = \{\epsilon_x^*, \epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}^T$ (Assumption (2)). On the other hand, stresses $\{{}_m\sigma_T\}$ and strains $\{{}_m\epsilon_T\}$ remaining in Specimen T are produced only by the cross-sectional inherent strain components $\{\epsilon_c^*\} = \{\epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}^T$, irrespective of the longitudinal inherent strain component $\{\epsilon_x^*\}$ (Assumption (3)).

The most probable value of cross-sectional inherent strain components, $\{\hat{\epsilon}_c^*\}$, is estimated using directly observed values of elastic strains remaining in Specimen T, $\{{}_m\epsilon_T\}$, and the most probable value of longitudinal inherent strain component, $\{\hat{\epsilon}_x^*\}$, is estimated using $\{\hat{\epsilon}_c^*\}$ and directly observed values of elastic strains produced on the edge section of R, $\{{}_m\epsilon_3\}$. Estimating method and procedure of $\{\epsilon_c^*\}$ and $\{\epsilon_x^*\}$ are mentioned in detail in 3.3 and 3.4 respectively. These procedures are shown by a flow chart in **Fig. 2**.

3.3 Estimating method and procedure of cross-sectional inherent strain components, $\{\epsilon_c^*\}$

It is possible by observing a cross-sectional macrostructure, etc., to estimate the region where cross-sectional inherent strains produced by welding exist, but it is hard to predict the region of inherent strains produced by other treatments such as plastic processing, quenching. For such general cases where the region of inherent strains

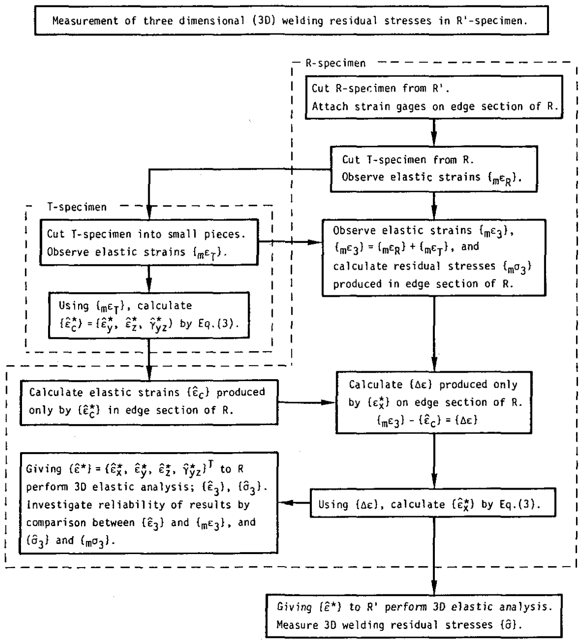


Fig. 2 Procedure for measurement of three-dimensional residual stresses.

is unknown (estimating procedure is the same even when the region is known), estimating method and procedure of cross-sectional inherent strain component $\{\epsilon_c^*\}$ are explained below.

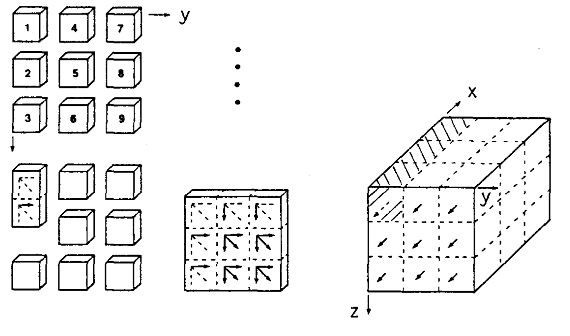
When the region of inherent strains is unknown, cross-sectional inherent strains are estimated considering that inherent strains are existing all over Specimen *T*. To begin with, strain gages are attached to Specimen *T* on the both sections at the same positions. In the next, Specimen *T* is divided into pieces including each gage on both sections in order to observe elastic strains remain in Specimen *T*. The average of the observed values on both sections at the same position is regarded as residual elastic strains $\{ _m\epsilon_T \}$. Substituting this directly observed value $\{ _m\epsilon_T \}$ for $\{ _m\epsilon \}$ in the right term of Eq.(3), the most probable value of cross-sectional inherent strain components, $\{ \hat{\epsilon}_c^* \} = \{ \hat{\epsilon}_y^*, \hat{\epsilon}_z^*, \hat{\gamma}_{yz}^* \}^T$, may be estimated. However, the following fact should be taken into consideration and consequently the estimating procedure be modified.

First, inherent strains are decomposed of effective components to produce stresses (incompatible inherent strains to produce elastic strains) and ineffective components (compatible inherent strains to produce only deformations such as uniform expansion-shrinkage or uniform shear). In order to estimate the most probable value of effective inherent strains, $\{ \hat{\epsilon}_c^* \}$, ineffective ones should be eliminated. To this end, a new method is developed, which is concretely shown below using Specimen *T*.

Specimen *T* is sliced and divided into elements on each of which elastic strains are observed. In the process of assembling these elements to the original Specimen *T* using the observed elastic strains, effective inherent strains can be estimated. The assembling sequence is according to the order of numbers put to the elements shown in **Fig. 3 (a)**; that is, from Element *1, 2, 3*, to *9*.

First, only Element *1* is considered. When cross-sectional inherent strain component $\{\epsilon_c^*\}$ is given to Element *1*, it only deforms by itself and does not produce stress; that is, $\{\epsilon_c^*\}$ in Element *1* is ineffective. Therefore, it can be assumed that inherent strain is zero in Element *1*. Secondly, Element *1* is assembled with Element *2*. If inherent strain $\{\epsilon_c^*\} = \{\epsilon_y^*, \epsilon_z^*, \gamma_{yz}^*\}^T$ is given to Element *2*, ϵ_z^* and γ_{yz}^* deform freely but ϵ_y^* does not owing to the restriction of Element *1*. That is to say, ϵ_y^* is effective, and ϵ_z^* and γ_{yz}^* are ineffective in Element *2*. In this way, as elements are assembled, inherent strains are classified into effective and ineffective components. These inherent strains are summed up and, using arrows, the direction of effective components is shown by solid lines and ineffective ones by broken lines in Fig. 3 (a). Slant arrows indicate shear components.

As a result, the total number of unknown effective inherent strains necessary to reproduce stresses remain in thinly sliced Specimen *T* is determined, and the most probable value of cross-sectional inherent strains, $\{ \hat{\epsilon}_c^* \}$, can be estimated by solving Eq.(3).



Number of total element = 9		
	Not effective	Effective
ϵ_y^*	3	6
ϵ_z^*	3	6
γ_{yz}^*	5	4

(a)

Number of total element = 27		
	Not effective	Effective
ϵ_x^*	3	24

(b)

Fig. 3 Estimating method of effective inherent strains.

3.4 Estimating method and procedure of longitudinal inherent strains, $\{ \epsilon_x^* \}$

The estimating method and procedure of $\{\epsilon_c^*\}$ have been shown in 3.3. On the assumption that $\{\epsilon_c^*\}$ is already estimated, longitudinal inherent strain component $\{\epsilon_x^*\}$ can be estimated in the following way.

The most probable value $\{\hat{\epsilon}_c^*\}$ of cross-sectional effective inherent strain components estimated from Specimen *T* is imposed in Specimen *R* to distribute uniformly along the weld line (*x*-axis) so as to conduct three-dimensional elastic analysis and calculate elastic strain produced only by $\{\hat{\epsilon}_c^*\}$ on the edge section of *R*. The difference between $\{\hat{\epsilon}_c\}$ and elastic strain $\{_m\epsilon_3\}$ produced on the edge section of R, formerly mentioned in 3.2, can be estimated as,

$$\{_m\epsilon_3\} - \{\hat{\epsilon}_c\} = \{\Delta\epsilon\} \quad (6)$$

This difference $\{\Delta\epsilon\}$ means the existence of longitudinal inherent strain component $\{\epsilon_x^*\}$; that is, the strain components $\{\Delta\epsilon_y, \Delta\epsilon_z, \Delta\gamma_{yz}\}^T$, produced only by $\{\epsilon_x^*\}$ on the edge section of R. Substituting $\{\Delta\epsilon\} = \{\Delta\epsilon_y, \Delta\epsilon_z, \Delta\gamma_{yz}\}^T$ as an observed value for $\{_m\epsilon\}$ in Eq.(3) and assuming $\{\hat{\epsilon}_c^*\}$ as zero, the most probable value of longitudinal effective inherent strains, $\{\hat{\epsilon}_x^*\}$, can be estimated.

Usually, the region of cross-sectional inherent strains and that of longitudinal inherent strains may be the same. As for estimation of longitudinal inherent strains, their distributed region becomes known in most cases since the region of cross-sectional inherent strains is already known by the method shown in 3.3. In general, it is rare that inherent strains are produced all over the edge section of R. On estimation of $\{\epsilon_x^*\}$, such region as with no inherent strain can be counted at the early stage of calculation, so that the most probable value $\{\hat{\epsilon}_x^*\}$ of longitudinal effective inherent strains can be calculated by Eq.(3). If inherent strains are produced all over the edge section, it is necessary to eliminate ineffective inherent strain component in the same manner as in 3.3. In this case, like the example shown in **Fig. 3 (b)**, it may be assumed that longitudinal inherent strain does not exist in slanted elements.

3.5 Three-dimensional residual stresses and accuracy of measurement

Distributing the most probable value of inherent strains $\{\hat{\epsilon}^*\} = \{\hat{\epsilon}_x^*, \hat{\epsilon}_y^*, \hat{\epsilon}_z^*, \hat{\gamma}_{yz}^*\}^T$ estimated from Specimen R in the measuring object R', three-dimensional elastic analysis is conducted. Consequently, three-dimensional welding residual stresses $\{\hat{\sigma}\}$ produced in R' can be calculated by Eq.(4).

The accuracy of measured three-dimensional welding residual stresses can be evaluated as follows:

Using strain gages attached on the edge section of Specimen R, elastic strains $\{_m\epsilon_3\} = \{_m\epsilon_y, {_m\epsilon_z}, {_m\gamma_{yz}}\}^T$ and three-dimensional welding residual stresses $\{_m\sigma_3\}$, of which source is total inherent strain components, can be observed directly on the edge section of R. In the same manner, $\{_m\epsilon_x, {_m\epsilon_y}\}^T$ can also be observed directly on the top and bottom surfaces of R.

On the other hand, three-dimensional elastic analysis is conducted on Specimen R by imposing the most probable value $\{\hat{\epsilon}^*\}$ of inherent strain estimated by the method mentioned in 3.3 and 3.4. As a result, the most probable value of elastic strain $\{\hat{\epsilon}\} = \{\hat{\epsilon}_x, \hat{\epsilon}_y, \hat{\epsilon}_z, \hat{\gamma}_{yz}\}^T$, produced on the edge section and the top and bottom surfaces of R, and the most probable value of three-dimensional welding residual stresses $\{\hat{\sigma}\}$ are obtained. These most probable values $\{\hat{\epsilon}\}$ and $\{\hat{\sigma}\}$ are compared with directly observed values $\{_m\epsilon\}$ and $\{_m\sigma\}$. Consequently, the accuracy of the measured values can be evaluated. The accuracy of measurement can also be evaluated by using the unbiased estimate $\hat{s}$ of the standard deviation of measured residual stresses.

Thus the accuracy of measurement of three-dimensional welding residual stresses $\{\hat{\sigma}\}$ obtained from three-dimensional elastic analysis by distributing the most probable value of inherent strains $\{\hat{\epsilon}^*\}$ can be evaluated automatically.

4. Validity of the Method in Numerical Experiment

The validity of the afore-mentioned measuring theory of three-dimensional residual stresses, in which a thinly sliced plate perpendicular to the weld line is used, will be studied by a numerical experiment according to the above mentioned measuring procedure.

4.1 Model for numerical experiment

Measuring object R' which has length $L' = 600$ mm, width $B' = 250$ mm and plate thickness $h = 55$ mm (Fig. 1) is used in numerical experiments for measurement of three-dimensional welding residual stresses in the middle cross section. Specimen R, cut out from R' inside from the starting and finishing weld ends by more than a length of plate thickness, has the length $L = 180$ mm, the width $B = B' = 250$ mm and the plate thickness $h = 55$ mm. Hexahedral elements are used in the three-dimensional elastic analysis by the finite element method. Division of R into finite elements is shown in **Fig. 4**.

As an example of assumed distribution of inherent strains, the distribution of longitudinal inherent strain $\{\epsilon_x^*\}$ is shown in **Fig. 5**. This distribution was determined on the basis of the actual inherent strain distributions estimated by the measurement of three-dimensional welding residual stresses produced by electron beam welding using the presented measuring method. Details of the result of the measurement will be reported in the near future. Giving longitudinal inherent strain $\{\epsilon_x^*\}$ and cross-sectional inherent strain $\{\epsilon_c^*\}$ (linearly varied ones in each

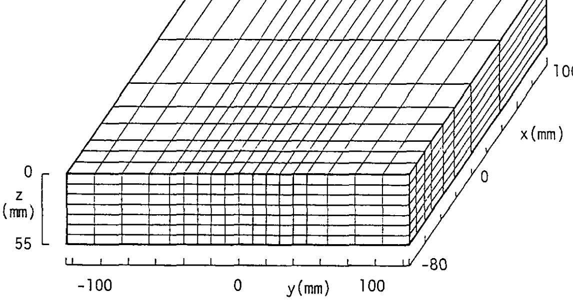


Fig. 4 Division of R-specimen into finite elements.

finite element) uniformly along the weld line (x-axis), three-dimensional elastic analysis is conducted. Obtained residual stress distributions are shown by solid lines in **Fig. 6**. Regarding these as the exact three-dimensional welding residual stresses $\{\sigma\}$, a numerical experiment is performed.

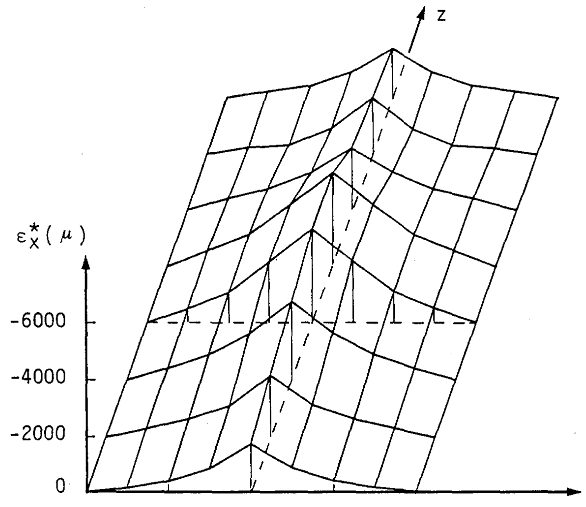


Fig. 5 Inherent strain, $\{\epsilon_x^*\}$, distribution assumed in numerical experiment.

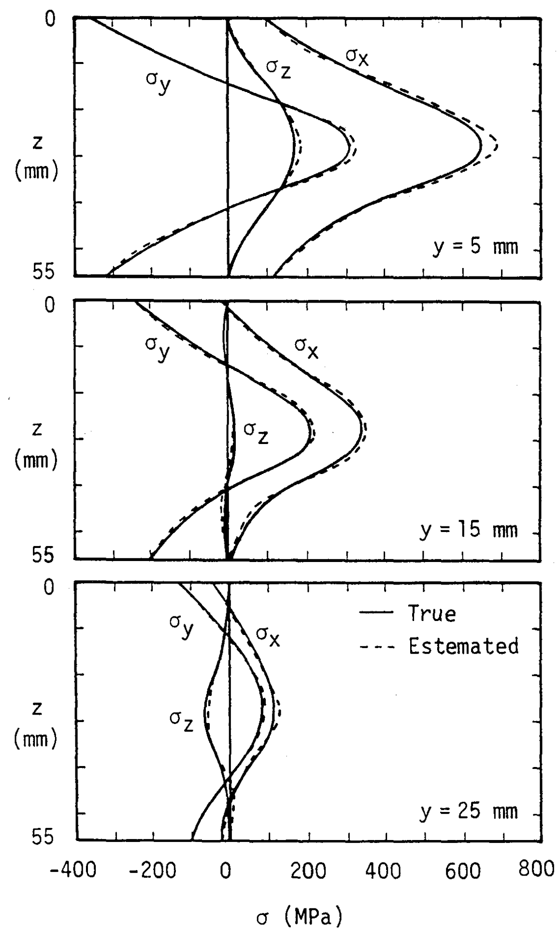


Fig. 6 True and estimated three-dimensional residual stresses.

4.2 Estimation of cross-sectional inherent strain component

In the experiment, it is necessary to observe elastic strains remaining in Specimen T by the method shown in 3.3. In this numerical experiment, however, giving cross-sectional inherent strain component $\{\epsilon_c^*\}$ to Specimen T (thickness $t = 10$ mm), two-dimensional elastic analysis is conducted and elastic strain $\{\epsilon_g\}$ at the center of gravity in each finite element is obtained. These elastic strains are assumed to be uniform in each element and used as directly observed values of residual elastic strains. Substituting this $\{\epsilon_g\}$ for $\{_m\epsilon\}$ in Eq.(3), the most probable value of cross-sectional inherent strain $\{\hat{\epsilon}_c^*\}$, being assumed to be uniform in each element, is estimated.

4.3 Estimation of longitudinal inherent strain component

In the first, inherent strain $\{\epsilon^*\}$ mentioned in 4.1 is given to Specimen R and three-dimensional elastic analysis is conducted. Elastic strains produced on the edge surface of R is obtained. These obtained elastic strains are regarded as $\{_m\epsilon_3\}$. Then the most probable value of cross-sectional effective inherent strains, $\{\hat{\epsilon}_c^*\}$, estimated in 4.2, being regarded as uniform in finite elements, is given to Specimen R along the weld line so as to conduct three-dimensional elastic analysis. Elastic strain $\{\hat{\epsilon}_c\}$ produced only by $\{\hat{\epsilon}_c^*\}$ on the edge surface of R is obtained. In the next, the difference $\{\Delta\epsilon\}$ between $\{_m\epsilon_3\}$ and $\{\hat{\epsilon}_c\}$ is calculated by Eq.(6) and substituted for $\{_m\epsilon\}$ in Eq.(3). As a result, on the assumption that inherent strains are uniform in elements, the most probable value of longitudinal effective inherent strain, $\{\hat{\epsilon}_x^*\}$, is estimated.

4.4 Reproduced three-dimensional residual stresses and validity of the method

According to the result of three-dimensional elastic analysis conducted by giving to Specimen R the most probable value of effective inherent strain, $\{\hat{\epsilon}^*\}$, estimated by the afore-mentioned method, the unbiased estimate $\hat{s}$ of elastic strains produced on the edge surface of R is 13μ. Residual stresses obtained by giving this $\{\hat{\epsilon}^*\}$ to the object R', that is, the most probable value of three-dimensional welding residual stresses, $\{\hat{\sigma}\}$, are shown by broken lines in Fig. 6.

As mentioned in 4.1, true three-dimensional welding residual stresses are calculated by linearly varying inherent strains in each element. The most probable values of observed elastic strains and inherent strains, though assumed to be uniform in elements, reproduce accurately three-dimensional welding residual stresses which are almost the same as those assumed in this paper.

5. Conclusion

In this paper, an accurate measuring theory of three-dimensional welding residual stresses produced uniformly along the weld line of an object and the concrete measuring procedure, in which only a thinly sliced plate perpendicular to the weld line is used, are presented.

The main results are as follows:

(1) The leading merit of the measuring theory of three-dimensional residual stresses, in which inherent strains are dealt as parameters, is availed of in this new measuring method. That is, an estimating method of effective inherent strains, the source of residual stresses, in which effective inherent strains are decomposed into cross-sectional component and longitudinal component, is demonstrated. Every inherent strain component is estimated and three-dimensional elastic analysis is conducted. As a result, it is possible to know three-dimensional welding residual stresses at an arbitrary position including.

(2) The validity of the presented method is confirmed by a numerical experiment.

(3) In this measuring method, different from the conventional methods (L_z method and nL_y method), it is necessary to calculate cross-sectional inherent strain component $\{\epsilon_c^*\}$ using observed elastic strains produced in Specimen T and to give $\{\epsilon_c^*\}$ to Specimen R for three-dimensional elastic analysis. Instead, cutting of many Specimens L and observation of longitudinal elastic strain $\{\epsilon_x\}$, required in the conventional method, are no more necessary. Consequently, experimental time and expenses are greatly saved.

(4) This method can be applied to measurement of any three-dimensional residual stresses produced by various treatments including welding, if three-dimensional residual stresses are considered to be uniform in one-direction.

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