



Title	Initial Fatigue Crack Growth Behavior in a Notched Component (Report II) : Evaluation of Fatigue Crack in a Notch Field by Elasto-Plastic Fracture Mechanics(Mechanics, Strength & Structural Design)
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# Initial Fatigue Crack Growth Behavior in a Notched Component (Report II)<sup>†</sup>

## — Evaluation of Fatigue Crack in a Notch Field by Elasto-Plastic Fracture Mechanics —

Kohsuke HORIKAWA\*, Sang-moung CHO\*\*

### Abstract

*In order to characterize the propagation rates of initial cracks in elasto-plastic notch fields by fatigue loads, it is inevitable to apply elasto-plastic fracture mechanics. In the present report, the evaluating method of  $\Delta J$  was developed for initial fatigue cracks in elasto-plastic notch field. The interpolation function for calculation of J-integral by Willson was modified for application to notch field. In the estimation of J-integral, stress gradient was considered in the notch field when crack is not present.*

*Fully reversed fatigue tests were carried out on the notched strips of HT80 and A5083-0. The crack closure behavior in the notch field was observed. The approximating formulas were derived to predict crack opening ratio in the notch fields. The fatigue crack growth rates in the notch fields were correlated to  $\Delta J$ . As the results, it was cleared that the present  $\Delta J$  is applicable to characterize the fatigue crack growth rates in both the elastic and elasto-plastic notch fields.*

*In the end of the report, the physical meaning of  $\Delta J$  was discussed.*

**KEY WORDS:** (Initial Fatigue Crack Growth) (Elasto-Plastic Notch Field) ( $\Delta J$ ) (Crack Opening Ratio)

### 1. Introduction

In the vicinity of notches (structural discontinuities) in welded structures subjected to high fatigue loads, local plastic deformation is developed. To assess the allowable size of weld defects existing in such notch fields, it is needed to characterize crack propagation rates from the initial defects. Also in service, when fatigue cracks are detected in notch fields, the evaluation of crack growth rates is needed to predict residual life of the components. In such cases, Elasto-Plastic Fracture Mechanics (EPFM) has been applied, and the application of  $\Delta J$  as the parameter has been implemented because of the violation of Linear Elastic Fracture Mechanics (LEFM)<sup>1-3</sup>). However, the evaluating method of  $\Delta J$  which is applicable to engineering structures, has not been settled till now.

The object of the present report is an initial fatigue crack in a notch field. And the purpose of the present study is to establish the evaluating method of  $\Delta J$  in that case, and to propose the approaching procedure for application to engineering structures.

The interpolation function for estimation of J-integral which had been derived by Willson for an edge crack in a semi-infinite body<sup>4</sup>), was modified for application to a crack in elasto-plastic notch field. In the modification of the interpolation function, stress gradient and plastic constraint in a notch field was taken into account. Also, experimental results on crack closure in notch field were modeled and formulated approximately. It was imple-

mented that the experimental formular made it possible to obtain effective stress range in practical structures by analysis rather than experimental measuring.  $\Delta J$  in a notch field was estimated by the effective stress range and by the modified interpolation function. And the validity of  $\Delta J$  was investigated by characterizing the fatigue crack propagation rates in the notch fields of high tensile strength steel (HT80) and aluminum alloy (A5083-0).

In the end of the report, the physical meaning and the limitation of  $\Delta J$  was also discussed.

### 2. Specimens and Experimental Procedure

The center notched specimens used in experiments and in numerical analysis are shown in Fig. 1. As the previous report<sup>5</sup>), the notch root radius  $\rho$  of circular hole was 2.5 mm ( $K_t = 2.7$ ), elliptical hole was 0.25 mm ( $K_t = 6.0$ ).

The materials used were high tensile strength steel (HT80), which has cyclic softening property, and aluminum alloy (A5083-0) which has cyclic hardening property. The mechanical properties of materials are shown in Table 1. Where, cyclic yield strength  $\sigma_{YC}$  and cyclic hardening exponent  $n'$  were obtained by the Companion specimens method.

Fatigue crack propagation tests were carried out by fully reversed axial load control ( $R = -1$ ) of sine wave, and load speed was 0.5 – 10.0 Hz.

Center Cracked Tension (CCT) specimens (Breadth

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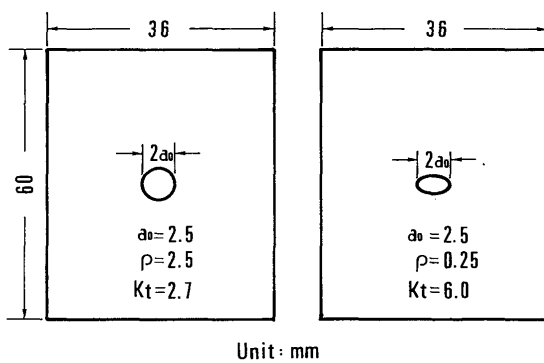


Fig. 1 Configuration of notched specimens

Table 1 Mechanical properties of HT80 and A5083-0

Material	Monotonic				Cyclic	
	E GPa	$\sigma_Y$ MPa	$\sigma_U$ MPa	n	$\sigma_{YC}$ MPa	n'
HT80	205.8	678.0	786.3	0.05	480.0	0.167
A5083-0	68.6	172.0	320.5	0.17	274.4	0.125

2B = 50.0 mm) of the two experimental materials were tested to obtain the long crack propagation properties used as the master curves. For the notched specimens, initial fatigue crack propagation tests were conducted.

The crack closure behavior was measured over all crack propagation tests for CCT and notched specimens by strain gages and clip gages.

A servo-hydraulic closed-loop system was used throughout the tests. And all tests were performed at ambient room temperature. The others on testing procedure were performed in accordance with ASTM E647 (1983).

### 3. Estimation of $\Delta J$ for a Fatigue Crack in a Notch Field.

#### 3.1 J-integral for a crack in a notch field under static tensile loading

When a crack exists in a notch field, stress intensity factor  $K$  to Mode I can be calculated by nominal stress  $\sigma$  and total crack length ( $a_0 + a$ ) which is put together notch length  $a_0$  and crack length  $a$  from the notch tip<sup>6)</sup>. The  $K$  is given as follows,

$$K = \sigma \sqrt{\pi (a_0 + a)} \cdot F(a) \quad (1)$$

where,  $F(a)$  is the correction factor. Also in case of a crack in a notch field, it is said that, even though  $K$  is calculated by crack length  $a$  from a notch tip and local stress  $\sigma(a)$  as following Eq. (2), the accuracy of a certain extent may be guaranteed<sup>7)</sup>.

$$K = 1.1215 \sigma(a) \sqrt{\pi a} \quad (2)$$

where, 1.1215 is the correction factor for an edge crack in a semi-infinite body, and  $\sigma(a)$  is the stress at the corresponding point of crack tip when crack is not present. However, by using the formula such as Eq. (2) rather than

Eq. (1), the effect of local yielding due to stress concentration of notch on the crack extension force can be considered. Thus, the formula of Eq. (2) was applied to evaluate the crack extension force for a crack in a notch field. Moreover, the equivalent stress was used as local stress  $\sigma(a)$  in Eq. (2). Because, in this study the estimation of J-integral was tried for a crack in elasto-plastic notch field which had been cleared to be under multi-axial stress state<sup>5)</sup>. Namely, the formula of Eq. (2) was replaced with the following Eq. (3).

$$K = C_N \cdot \bar{\sigma}(a) \sqrt{\pi a} \quad (3)$$

where,  $C_N$  is the correction factor based on local equivalent stress  $\bar{\sigma}(a)$  in a notch field. In Fig. 2, dot-and-dashed line and broken line indicate the tendency on  $C_N$  in a elliptical notch field (major axis  $2a_0 = 5.0$  mm, root radius  $\rho = 0.156$  mm) and a circular notch field ( $\rho = 2.5$  mm) of an infinite plate, respectively. Now, the  $K$  was obtained by the results of Newman<sup>8)</sup>. And equivalent stress  $\bar{\sigma}(a)$  in the elliptical notch field was determined by the approximating formula<sup>9)</sup>, and in the circular notch field by analytic solution<sup>10)</sup>. The crack length  $a$  was normalized by  $\sqrt{a_0 \rho}$  because notch field had been defined as the range from notch tip to  $\sqrt{a_0 \rho}$  in the previous report.

Also, the tendency of  $C_N$  in the present finite plate was given as symbols in Fig. 2. The  $K$  in the present finite plate was calculated by elastic FEM, in which energy method and path integral (or J-integral) method were used together<sup>11)</sup>. And the equivalent stress  $\bar{\sigma}(a)$  was determined by the method of the previous report<sup>5)</sup>.

From the general tendency in Fig. 2, it was understood that the factor  $C_N$  is higher than 1.1215 in the finite plate as well as the infinite one, and varies with the shape of notch and specimen. If the maximum principal stress instead of the equivalent stress in Eq. (3) was used, the factor  $C_N$  would become somewhat low because of biaxial stress state (plane stress) in a notch field. However, it can be expected that the general tendency of  $C_N$  based on the maximum principal stress would be similar to that based on the equivalent stress. When the ratio of notch length  $2a_0$  to breadth  $2B$  of plate is small, even though the factor  $C_N$  for the infinite plate is used, considerably good results may be obtained.

The results of FEM on the factor  $C_N$  for the present finite plates were approximated to the solid lines in Fig. 2, which were used to calculate  $K$  hereinafter. By this procedure if stress intensity factor  $K$  is determined, then  $J_e$  which is J-integral in elastic condition, in case of plane stress, can be estimated from the relation of  $J_e = K^2/E$ .

Next, in elasto-plastic condition, J-integral was estimated by modifying the interpolation function by Willson<sup>4)</sup>, which had been derived for an edge crack in a semi-

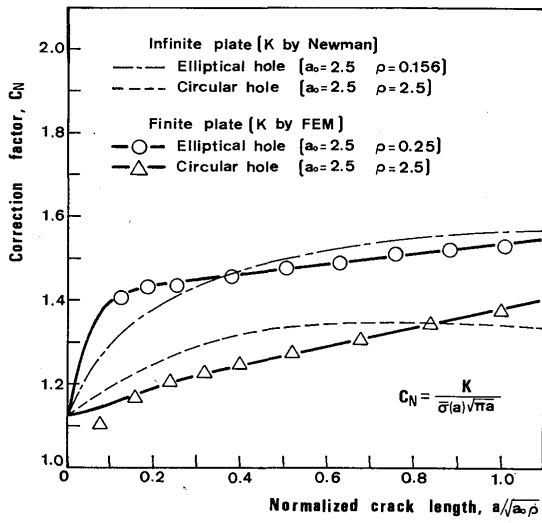


Fig. 2 Correction factor  $C_N$  of  $K$  as a function of crack length in notch fields.

infinite body.

By the way, as the relation of stress and total strain, piecewise power hardening rule was used.

$$\sigma = E\epsilon, \quad \sigma \leq \sigma_Y \quad (4-1)$$

$$(\sigma/\sigma_Y) = (\epsilon/\epsilon_Y)^n, \quad \sigma > \sigma_Y \quad (4-2)$$

Also, when nominal stress and its ratio to  $\sigma_Y$  are referred to as  $\sigma$  and  $\phi = \sigma/\sigma_Y$  respectively, the interpolation function by Willson can be written as follows<sup>4)</sup>.

$$J/J_e = 1 + f(r_p), \quad \phi \leq 1.0 \quad (5-1)$$

$$J/J_e = \left\{ 1 + f(r_p) \right\} \left( \frac{1}{\phi} \right)^2 + \frac{1}{1.25\pi} \left\{ \phi^{(1+n)/n} - 1 \right\} \left( \frac{1}{\phi} \right)^2 h(n), \quad \phi > 1.0 \quad (5-2)$$

where,  $f(r_p)$  is the term which depends upon plastic zone of crack, 1.25 is given from the square of 1.1215 which is the correction factor of an edge crack in a semi-infinite body,  $h(n)$  is the function of shape and material which is determined by the solution of He and Hutchinson<sup>12)</sup>.

In order to apply Eq. (5) to a crack in a notch field, local equivalent stress  $\bar{\sigma}(a)$  and the factor  $C_N^2$  were substituted for nominal stress  $\sigma$  and the factor 1.25, respectively. In the local plastic zone of a notch field, stress gradient exists and the rest is elastic region. Therefore, for a crack in a notch field under elasto-plastic condition it may be considered that the constraint for plastic deformation of crack tip is very severe. Accordingly, the term  $f(r_p)$  related to the plastic zone of crack was disregarded, and the function  $h(n)$  was multiplied by the plastic constraint factor  $c_p$  ( $c_p < 1.0$ ). The plastic constraint factor  $c_p$  was estimated by introduction of the coefficient  $(2.32n/\rho)$  which is related to plastic stress gradient in a notch field<sup>13)</sup>.

$$C_p = \tanh \left\{ \frac{1}{4} \sqrt{\frac{\rho}{2.32n}} \right\} \quad (6)$$

Consequently, in a notch field under elasto-plastic condition,  $J$ -integral was estimated by modifying as follows, when  $\phi_a = \bar{\sigma}(a)/\sigma_Y$  was defined.

$$J/J_e = 1.0, \quad \phi_a \leq 1.0 \quad (7-1)$$

$$J/J_e = 1.0 + \frac{1}{C_N^2 \pi} \left\{ \phi_a^{(1+n)/n} - 1 \right\} \frac{h(n)c_p}{\phi_a^2}, \quad \phi_a > 1.0 \quad (7-2)$$

where,  $J_e$  is determined from the relation to the  $K$  as mentioned above. Also, elasto-plastic equivalent stress  $\bar{\sigma}(a)$  can be calculated by the estimating method in the previous report using only elastic solution and material constants<sup>5)</sup>. Namely,  $J$ -integral in Eq. (7) would be estimated by only elastic analysis and material constants.

### 3.2 Estimation of $\Delta J$ to a fatigue crack in a notch field

When LEFM is applied to characterize fatigue crack propagation rates, the effective stress intensity factor range  $\Delta K_{\text{eff}}$  using effective stress range  $\Delta \sigma_{\text{eff}} = U \Delta \sigma$  ( $U$ : crack opening ratio) can be used<sup>14)</sup>. When  $\Delta K_{\text{eff}}$  is used as the crack extension force, it may be interpreted that only the stress range, while a crack is opened, contributes to the intensity of singular field in a crack tip. In this case, it may be regarded that the stress range, while a crack is closed, does not affect the singular field. According to this interpretation on the crack extension force, the effective stress range  $\Delta \sigma_{\text{eff}}$  was used for the estimation of  $\Delta J$ . That is to say, even though a cracked body subjected to cyclic load is under elasto-plastic condition, the crack closure behavior may occur. Also in this case, it can be interpreted that only the effective stress range contributes to the intensity of singular field in a crack tip. This consideration is also appeared in the study of Dowling who characterized fatigue crack propagation rates by  $\Delta J$ <sup>15)</sup>.

On the other hand, in order to evaluate  $\Delta J$  for a fatigue crack in notch field under elasto-plastic condition, crack opening ratio  $U$  has to be determined as the function of crack length.

Figure 3 (a) shows the disposition of strain gages, which were attached for measurement of crack opening ratio  $U$  in the notch field (Elliptical hole,  $\rho = 0.25$  of HT80). By the gage ① located above the periphery of notch was measured strain  $\epsilon_x$  perpendicular to load direction. And by the gage ② located near by crack was measured strain  $\epsilon_y$ . All strain gages had gage-length of 0.3 mm. Displacement  $\delta$  of load direction was measured by clip gage which was mounted to give gage-length of 50.0 mm.

Figure 3 (b) indicates the hysteresis loops recorded

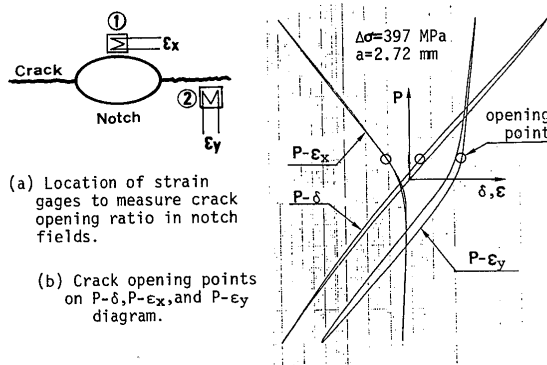


Fig. 3 Location of strain gages and hysteresis loops to measure crack opening ratio in notch fields of HT80 ( $\rho = 0.25$ ).

from the relation of  $P$  and  $\epsilon_x$ ,  $\epsilon_y$ ,  $\delta$ . And at that time, nominal stress range  $\Delta\sigma$  was 397 MPa, and crack length from the notch tip was 2.72 mm. In Fig. 3 (b), crack opening point  $P_{op}$  could be determined by  $P-\delta$  and  $P-\epsilon_y$  diagram respectively. Moreover,  $P_{op}$  was also able to be obtained clearly by  $P-\epsilon_x$  diagram. Particularly, when a crack was short, it was difficult to measure crack opening point  $P_{op}$  by  $P-\epsilon_y$  from gage ② or by  $P-\delta$ . However, by  $P-\epsilon_x$  from gage ① could be determined the  $P_{op}$ , because comparatively obvious folded point was appeared even when a crack was very short as 0.1 mm. The opening ratio  $U$  for initial fatigue crack measured by this procedure was plotted by symbols in Fig. 4. It is recognized that the opening ratio  $U$  of initial fatigue crack is affected considerably by stress level at the corresponding point of crack tip when crack is not present. Also, the similar trends to the present behavior would be seen in the results of experiments by Katoh et al.<sup>16)</sup> and Nishikawa et al.<sup>3)</sup>. Considering their experimental results, the approximating formula for estimation of crack opening ratio  $U$  was derived by using the present behavior as shown in Fig. 4. When the opening ratio for a long crack which LEFM can be applied is referred to as  $U_e$ , the opening ratio  $U$  for an initial crack in elasto-plastic notch field (stress ratio  $R = -1$ ) could be estimated as follows.

$$U = U_e, \quad (\epsilon_{com}/\epsilon_{YC}) \leq 0.4 \quad (8-1)$$

$$U = U_e + (1 - U_e) \tanh \left\{ 0.7 \cdot \log (\epsilon_{com}/\epsilon_{YC} + 0.6) \right\}, \quad (\epsilon_{com}/\epsilon_{YC}) > 0.4 \quad (8-2)$$

where,  $\epsilon_{YC} = \sigma_{YC}/E$ ,  $\epsilon_{com}$  is compressive strain at the corresponding point of crack tip when crack is not present. It has been known that the opening ratio  $U_e$  is affected by residual plastic deformation, oxide, roughness of fracture surfaces<sup>17)</sup>, and crack length<sup>18)</sup> etc. When the effect of environment can be disregarded,  $U_e$  would be estimated from the approximating formulas by Dugdale model<sup>18)</sup> or numerical simulation<sup>19)</sup>, and from empirical

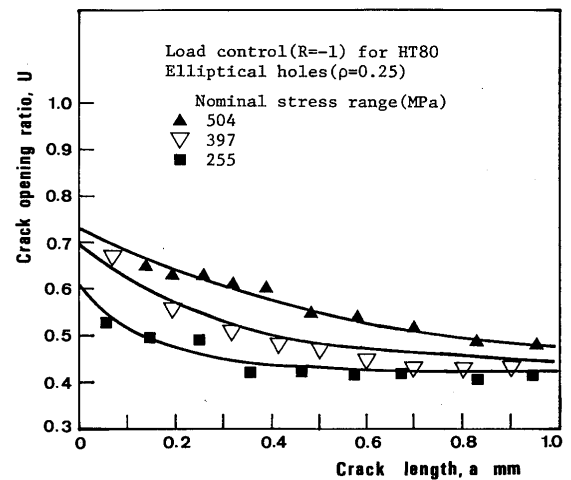


Fig. 4 Measured crack opening ratio in the notch fields of HT80.

formulars by experiments<sup>16)</sup>. And,  $U_e$  can be also determined by the experiment under the same condition with practical case. For instance, the empirical formula by Katoh et al. for long cracks (stress ratio  $R \leq 0.5$ ),  $U_e = (1.5-R)^{-1}$  was considered. From this empirical formula in the case of  $R = -1$ ,  $U_e$  is given as 0.4. But, from the present experiments,  $U_e = 0.39-0.44$  was obtained in both of HT80 and A5083-0. In practice to simplify the estimation, as the mean value of experimental results,  $U_e = 0.42$  was used in Eq. (8).

Using the previous estimating method<sup>5)</sup> by which elasto-plastic stress had been calculated in notch field from elastic solution and material constants, the opening ratio  $U$  estimated by Eq. (8) gives the solid lines in Fig. 4. Namely, the solid lines on  $U$  in Fig. 4 is the trends estimated by only elastic solution and material constants. If the opening ratio  $U$  for a crack in elasto-plastic notch field is obtained, then the effective stress range  $\Delta\bar{\sigma}_{eff}(a)$  is given.

For estimating  $\Delta J$  by application of Eq. (7) by which J-integral could be computed,  $\Delta\bar{\sigma}_{eff}(a)$  instead of  $\bar{\sigma}(a)$  was used in Eq. (7). Also, material constants that had been determined by cyclic loading tests as shown in Table 1, were used for estimating  $\Delta J$ . Besides the cyclic yield strength  $\sigma_{YC}$  had been obtained from the relation of cyclic stress and strain amplitude, but when the range of cyclic stress and strain is considered,  $\Delta\sigma_{YC}$  is given as  $2\sigma_{YC}$ . Accordingly,  $U \cdot \Delta\sigma_{YC} = U \cdot 2 \cdot \sigma_{YC}$  was used instead of  $\sigma_Y$  in Eq. (7).

As the results,  $\Delta J$  for a fatigue crack is elasto-plastic notch field could be estimated by Eq. (7) and Eq. (8). Moreover, if the distribution of elasto-plastic stress in a notch field when crack is not present could be estimated by elastic solution, as the previous report,  $\Delta J$  is able to be obtained by only elastic analysis.

## 4. Experimental Results and Discussion

### 4.1 Experimental results

In this section, initial fatigue crack propagation rates in the notch field of elliptical ( $\rho = 0.25$ ) and circular holes ( $\rho = 2.5$ ) were mentioned. The notch field had been defined as the range from notch tip to  $\sqrt{a_0 \rho}$ . Therefore, the crack propagation rates only up to  $a = 0.79$  mm in elliptical notches were characterized, and up to  $a = 2.5$  mm in circular notches.

Figure 5 indicates the long crack propagation trends examined for CCT specimens of HT80 and A5083-0. In that case,  $\Delta J$  was calculated from  $\Delta K_{\text{eff}}^2/E$ , which was determined by using effective stress range  $\Delta \sigma_{\text{eff}}$  obtained from  $U_e = 0.42$ , as stated above. It can be found out that the data on A5083-0 is more scattered than on HT80. The two solid lines in Fig. 5 were used as the master curves for two kinds of tested materials. The linear parts of the two master curves in Fig. 5 is similar to each other. But, in the region of low  $\Delta J$ , HT80, which has fatigue limit, and A5083-0, which has not fatigue limit, show very different behavior.

Figure 6 (a) (b) show initial fatigue crack propagation rates in notched specimens of HT80. These figures were plotted by  $\Delta K$  which was calculated from Eq. (1). Solid lines mean master curve for long crack of HT80.

Also, Fig. 7 (a) (b) indicate initial fatigue crack propagation rates on A5083-0.

In the Fig. 6 and Fig. 7, crack propagation rates were characterized by  $\Delta K$  based on LEFM in disregard of local

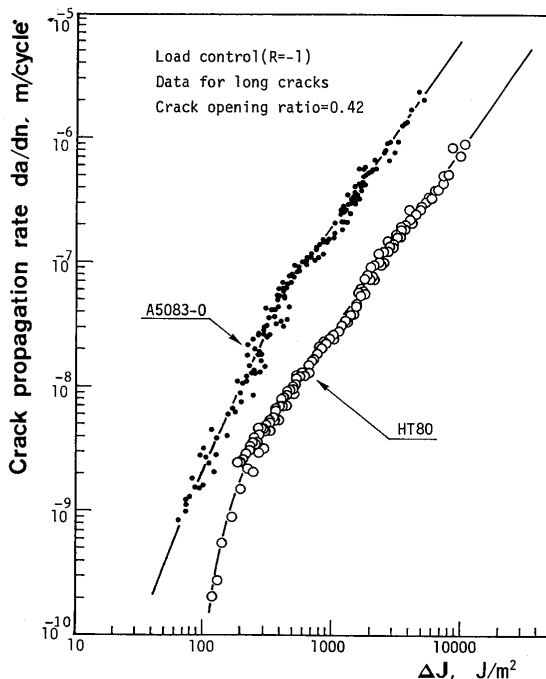
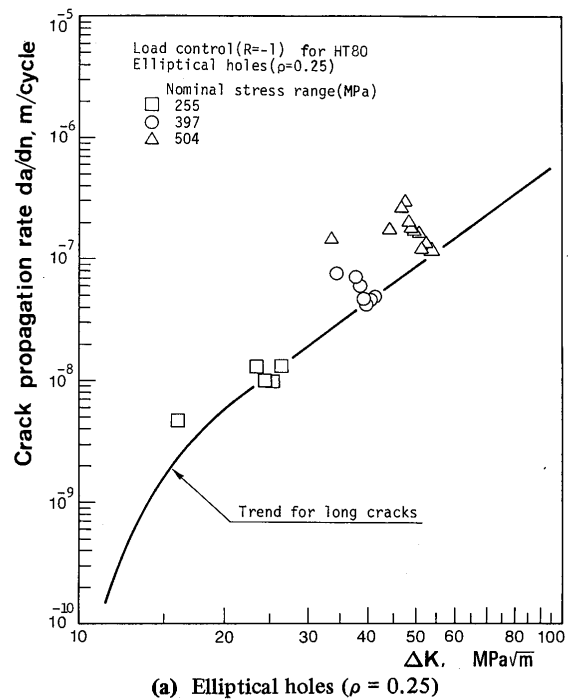
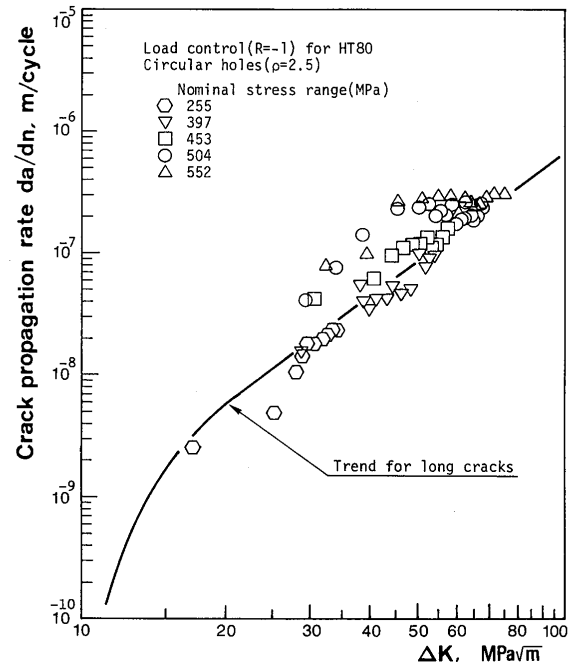


Fig. 5  $da/dn$  versus  $\Delta J$  evaluated by effective stress range for long cracks (HT80 and A5083-0).



(a) Elliptical holes ( $\rho = 0.25$ )



(b) Circular holes ( $\rho = 2.5$ )

Fig. 6  $da/dn$  versus  $\Delta K$  evaluated by nominal stresses for initial fatigue cracks in notch fields (HT80).

stress states and crack opening ratio  $U$  in the notch fields. As the general tendency in these figures, the higher loading stress and the shorter crack length are, the more remarkable the deviation from the master curve is. That is to say, it can be noticed that the propagation rate of initial crack is higher than that of long crack even when the evaluated  $\Delta K$  is the same. Conversely, it can be mentioned that, to the same crack propagation rate,  $\Delta K$  of the initial fatigue crack is evaluated smaller than that of the long crack. However, as the initial crack become

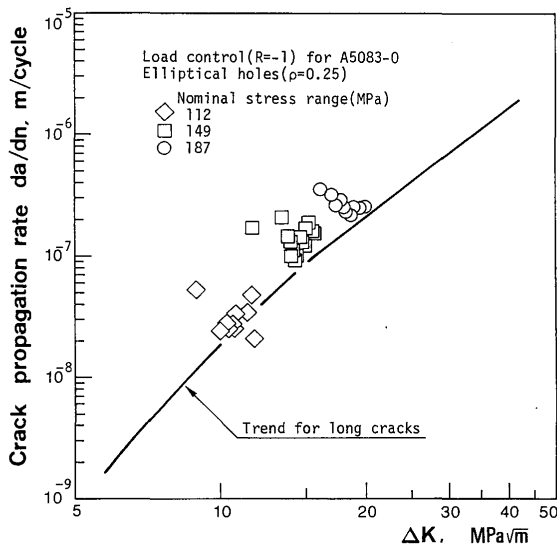
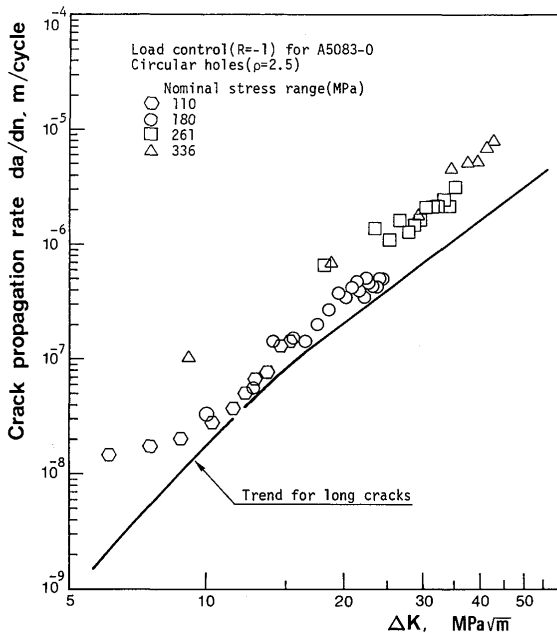
(a) Elliptical holes ( $\rho = 0.25$ )(b) Circular holes ( $\rho = 2.5$ )

Fig. 7  $da/dn$  versus  $\Delta K$  evaluated by nominal stress for initial fatigue cracks in notch fields (A5083-0).

longer, the propagation behavior is apt to approach the master curves. This is similar to the general tendency which appears in the case that initial fatigue cracks are characterized by  $\Delta K^{(3),20}$ .

Figure 8 (a) (b) are the results indicated by the relation of  $\Delta J$  and initial crack propagation rates for HT80.

In the same way, Fig. 9 (a) (b) are the results for A5083-0.

In Fig. 8 and Fig. 9,  $\Delta J$  was estimated by only using elastic solutions and cyclic material constants as mentioned above. Generally, the initial crack propagation rates in notch field show good coincidence with the trends for long cracks. Especially, it can be noticed that, regardless of stress level, from the case that notch tip is elastic to the case that it is plastic, one parameter of  $\Delta J$  may be applied

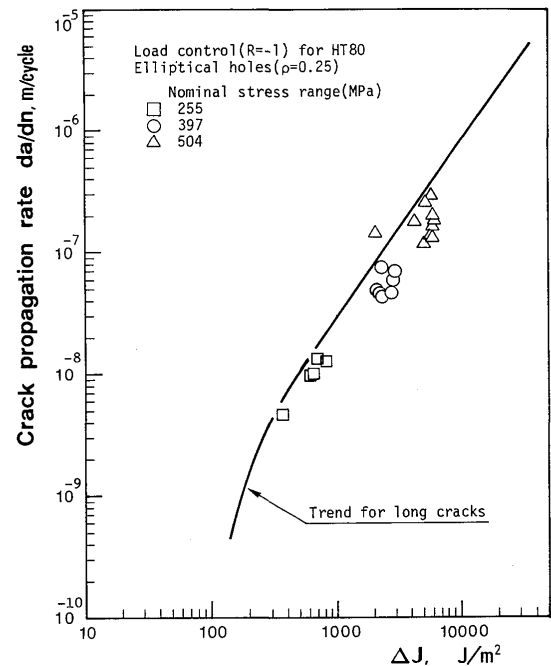
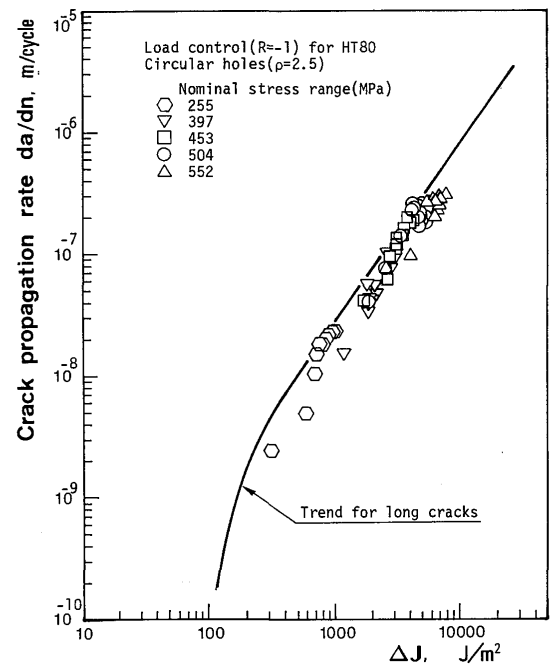
(a) Elliptical holes ( $\rho = 0.25$ )(b) Circular holes ( $\rho = 2.5$ )

Fig. 8  $da/dn$  versus  $\Delta J$  evaluated by local stresses for initial fatigue cracks in notch fields (HT80).

to characterize the initial crack propagation rates. Even though plastic zone of notch is not developed, as shown in Fig. 4 and Eq. (8), crack opening ratio  $U$  increases according as local stress increases. Therefore, in the case of low crack propagation rates, the experimental points in the left side of the master curve in Fig. 6 and Fig. 7 plotted by  $\Delta K$ , were shifted to the right side of the master curve in Fig. 8 and Fig. 9 plotted by  $\Delta J$ .

The material constants obtained by cyclic loading tests were used to estimate  $\Delta J$ . If the material constants by

static tests were used, as shown in Eq. (7),  $\Delta J$  would be calculated low in HT80 to have cyclic softening property, but high in A5083-0 to have cyclic hardening property. Namely, in order to estimate  $\Delta J$  for a crack subjected to high stress, cyclic material constants may have to be used. By this approach, cyclic hardening characteristics of materials would be considered in calculation of crack extension force such as  $\Delta J$ .

#### 4.2 Discussion on physical meaning of $\Delta J$

Under static tensile loading, stress intensity factor  $K$  and J-integral are calculated by supposing that a crack is opened with start of loading. From this consideration, when closure behavior in a fatigue crack appears,  $\Delta K_{\text{eff}}$  would have physical meaning that only stress range  $\Delta \sigma_{\text{eff}}$

while a crack is opened, contributes to the crack extension force.

As shown in Fig. 10 (a) it is assumed that the relation of effective stress range  $\Delta \sigma_{\text{eff}}$  and the strain range  $\Delta \epsilon_{\text{eff}}$  is not linear. Now, the constant amplitude load is considered, and the case that the relation between cyclic stress and strain of materials is stabilized, is presumed.

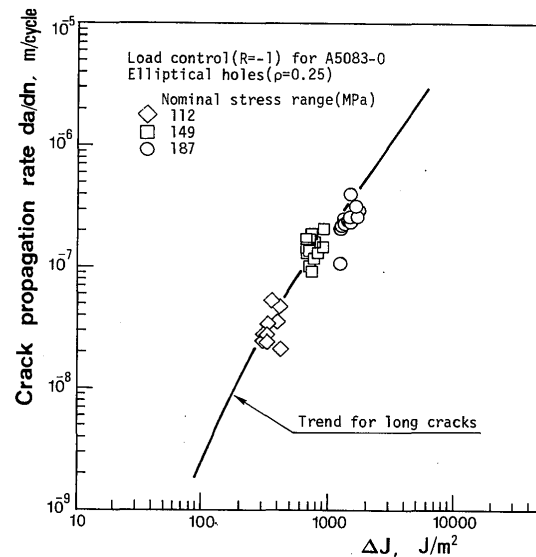
Because  $\Delta K_{\text{eff}}$  based on LEFM, in this case, can not be applied,  $\Delta J$  instead of it was used in this study. Thus, the physical meaning of  $\Delta J$  would be described below.

J-integral in static loading had been derived by considering only the loading path OAB in Fig. 10 (a) of nonlinear elastic materials. If this J-integral is applied to a cracked body subjected to cyclic load, the relation of stress and strain in unloading path would be BAO in the figure. However, unloading path of elasto-plastic material is given as BCO in Fig. 10(a). These unloading paths make the difference between nonlinear and elasto-plastic behavior of material.

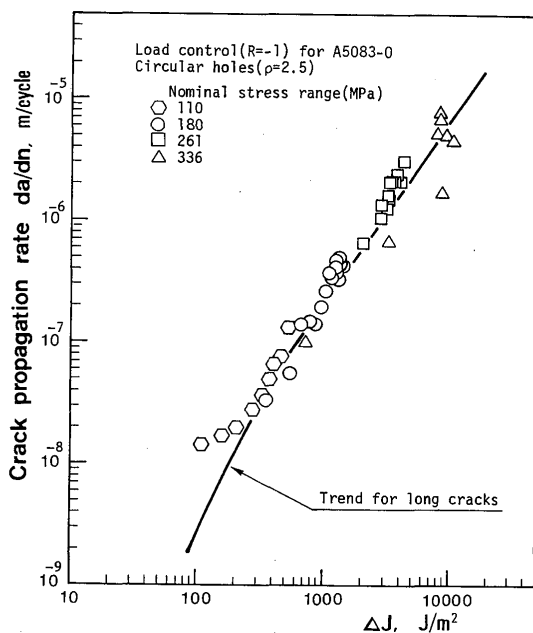
The typical  $\Delta J$  evaluated from nonlinear behaviour, as solid line shown in Fig. 10(b), varies with  $\Delta \sigma_{\text{eff}}^2$ . Namely,  $\Delta J$  undergoes the change as O ① ② ① O in Fig. 10 (b).

Besides, J-integral may be also interpreted as strain energy release rate for a crack in linear and nonlinear elastic body. From this interpretation on J-integral, in Fig. 10 (b), according as  $\Delta \sigma_{\text{eff}}^2$  increases and decreases, energy release rate increases as O ① ② and then decreases as ② ① O.

Next, in the elasto-plastic material, unloading path as BCO shown in Fig. 10(a) is considered. Even though energy release rate  $\Delta J$  in this case can not be given analytically, but would be indicated diagrammatically as path ② ③ ④ O in Fig. 10 (b). Now, the energy, which correspond to the inside area of the loop OABC in Fig. 10(a), is dispersed as heat energy due to plastic deformation. This dispersed energy can be interpreted to correspond with path ④ O in Fig. 10 (b). The difference of  $\Delta J$  in

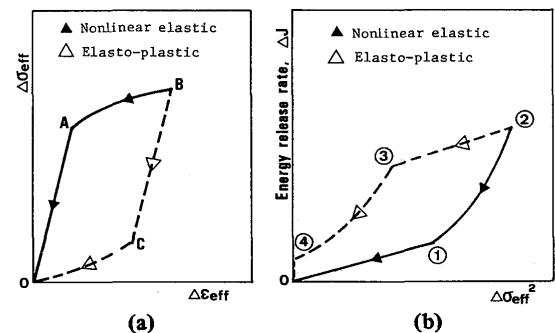


(a) Elliptical holes ( $\rho = 0.25$ )



(b) Circular holes ( $\rho = 2.5$ )

Fig. 9  $da/dn$  versus  $\Delta J$  evaluated by local stresses for initial fatigue cracks in notch fields (A5083-0).



(a)  $\Delta \sigma_{\text{eff}} - \Delta \epsilon_{\text{eff}}$  for nonlinear elastic and elasto-plastic materials.

(b)  $\Delta J - \Delta \sigma_{\text{eff}}^2$  for nonlinear elastic and elasto-plastic materials.

Fig. 10 Difference between nonlinear elastic and elasto-plastic behavior of materials.



unloading path is caused by the material behaviour. In practice, if material constants are given, and  $\Delta J$  is determined by loading path, then the variation of the  $\Delta J$  in unloading path would be nearly controlled by loading path.

It is well known that the strains near a crack can be correlated to J-integral<sup>21)</sup>. Also, there is an experimental report that fatigue crack propagation rate depends upon the strain range near a crack tip<sup>22)</sup>.

It may be stated that  $\Delta J$  controls the strain range near a crack tip. Accordingly,  $\Delta J$  can be recognized to be a parameter of fatigue crack extension force in elasto-plastic material.

In the case as that creep at high temperature and cyclic creep affect fatigue crack propagation, loading and unloading path of stress have the important effect upon the crack propagation rate. Under these conditions, fatigue crack propagation rates may not be characterized by application of  $\Delta J$ .

## 5. Conclusions

In order to evaluate the propagation rates of initial cracks or defects in elasto-plastic notch fields subjected to fatigue loads, the estimating method of  $\Delta J$  was proposed, and the validity of it was investigated by experiments. The initial fatigue crack propagation rates and crack closure behavior were examined for center notched specimens of HT80 and A5083-0.

The following conclusions could be drawn.

- (1) When stress intensity factor  $K$  for a crack in notch field was calculated as Eq. (3), the correction factor  $C_N$  was more than 1.1215 in the case of the notch with finite root radius. And it was confirmed that the factor  $C_N$  varied also with the configuration of specimen and notch. (Fig. 2)
- (2) Through the experiments, fatigue crack closure behavior in notch field was turned out to depend upon stress level (Fig. 4), and this tendency was formulated approximately as Eq. (8).
- (3) The  $\Delta J$  for fatigue crack in notch field was estimated by modifying the interpolation function by Willson, and by using the effective stress range. The relation between initial crack propagation rate and the present  $\Delta J$  in notch field agreed well with it of long crack. Namely, it would be stated that the present estimating method of  $\Delta J$  was reasonable. (Fig. 8, Fig. 9)
- (4) To evaluate  $\Delta J$  for both HT80 with cyclic softening property and A5083-0 with cyclic hardening property, the material constants obtained by cyclic loading tests were used. By this consideration, cyclic hardening characteristics of materials could be considered in calculation of crack extension force

such as  $\Delta J$ .

As described above, the present  $\Delta J$  was implemented to be calculated by only material constants and elastic analysis in notch field. Practically, for the present study, simple FORTRAN program of about 150 lines was used, and the application to engineering structures would be expected.

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