Investigation on Moving Line Heat Source (Report I)†
—Numerical Calculation of Temperature Distribution—

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Abstract

Calculation based on heat conduction theory was made for moving line heat source of non-uniform input energy distribution and temperature distribution in the vicinity of the heat source was obtained.

These results are expressed in isothermal contour line maps.

These can be compared with the actual welding results such as arc welding, plasma welding, electron beam welding and so on.

1. Introduction

The heat input source during actual welding is neither a point, nor a line along which input energy is uniformly distributed, though these two cases have been used on theoretical treatment for the heat conduction pŀoblem of welding. The actual heat input source seems to lie midway between both cases.

When we measure the temperature at a position considerably distant from the heat source, it has enough accuracy for the purpose to use a point heat source approximation or a "so called" line heat source approximation.

When we discuss, however, the temperature surrounging the heat source, we frequently get unsatisfactory results from these extreme approximations. In this report, a medium case between a point heat source approximation and a line heat source approximation is formularized by introducing a dimensionless quantity α, and the temperature distribution is calculated numerically.

2. Equation for Temperature Distribution

2.1 Notation

We take the position of the heat source of the plate surface as the origin, O, the moving direction of the heat source as the X co-ordinate axis, the perpendicular directions to the X co-ordinate in the horizontal and the vertical plane as the Y and the Z co-ordinate respectively and assume the co-ordinate system which moves with the line heat source as shown in **Fig. 1.**

The other notation is as follows,

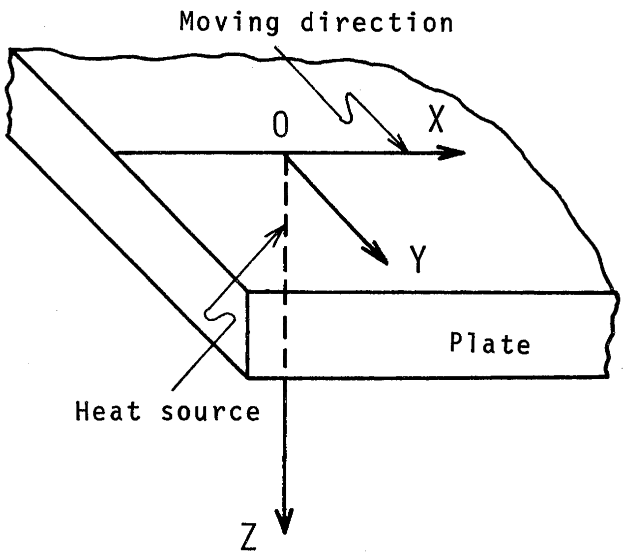

Fig. 1. The co-ordinate system.

Q : input energy per unit length along Z axis.
k : heat conductivity.
k_D: thermal diffusivity (or temperature conductivity) $=k/c\cdot\rho$
c : specific heat
ρ : density
v : moving speed of heat source
D: plate thickness

and the asterisk symbol $*$ expresses a dimensionless quantity in all cases.

2.2 Method of Calculation

The temperature T_p at the point (X, Y, Z) in the infinite medium due to the moving point heat source $Q(Z_l)dz_l$ can be calculated with eq. 1,

$$T_p = \frac{Q\,dZ_l}{4\pi k_D c \rho R}\, e^{-\frac{v}{2k_D}(X+R)} \quad \text{(1)}$$

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where Z_l is the position of the point heat source on the Z co-ordinate (only in this case, we can take the origin at the arbitrary position on the Z axis)

$$R^2 = X^2+Y^2+(Z-Z_l)^2$$
$$= R_l^2+(Z-Z_l)^2$$
$$R_l^2 = X^2+Y^2$$

We introduce the following dimensionless quantities,

$$X^*=\frac{X}{D},\quad Y^*=\frac{Y}{D},\quad Z^*=\frac{Z}{D}$$
$$Z_l^*=\frac{Z_l}{D},\quad R^*=\frac{R}{D},\quad R_l^*=\frac{R_l}{D} \quad \text{----- (2)}$$
$$T_\odot=\frac{Q(0)}{4\pi k}=\frac{Q(0)}{4\pi k_D c\rho},\quad Q^*(Z_l^*)=\frac{Q(Z_l^*)}{Q(0)}$$
$$v^*=\frac{v\cdot D}{2k_D}$$

and obtain the dimensionless temperature T_p^* $(=\frac{T_p}{T_\odot})$

$$T_p^*(v^*, X^*, R_l^*, Z^*-Z_l^*, Z_l^*)=\frac{Q^*(Z_l^*)dZ_l^*}{R^*}$$
$$\times e^{-v^*(X^*+\sqrt{R_l^{*2}+(Z^*-Z_l^*)^2})} \quad \text{---------------------- (3)}$$

In case the medium is of finite thickness, it is necessary to calculate the dimentionless temperature T_{pl}^* by the following equation.

$$T_{pl}^*(v^*, X^*, R_l^*, Z^*-Z_l^*, Z_l^*)$$
$$=T_p^*(v^*, X^*, R_l^*, Z^*-Z_l^*, Z_l^*)+T_p^*(v^*, X^*,$$
$$R_l^*, Z^*+Z_l^*, Z_l^*)+\sum_{n=1}^{\infty}\{T_p^*(v^*, X^*, R_l^*,$$
$$2n-z^*-Z_l^*, Z_l^*)+T_p^*(v^*, X^*, R_l^*, 2n+Z^*-$$
$$-Z_l^*, Z_l^*)+T_p^*(v^*, X^*, R_l^*, 2n-Z^*+Z_l^*, Z_l^*)$$
$$+T_p(v^*, X^*, R_l^*, 2n+Z^*+Z_l^*, Z_l^*)\}$$
$$=e^{-v^*\cdot X^*}\, T_{p2}(v^*, R_l^*, Z^*, Z_l^*)\,dZ_l^* \quad \text{---------- (4)}$$

When it is reguired to treat the line heat source whose input energy distribution function $Q^*(Z_l^*)$ changes along the Z co-ordinate axis, we need to sum up all the contribution of $Q^*(Z_l^*)dZ_l^*$, then, to obtain the integral as

$$T_{line}^*(\alpha, v^*, X^*, Y^*, Z^*)=e^{-v^*X^*}\int Q^*(Z_l^*)$$
$$\times T_{p2}^*(v^*, R_l^*, Z^*, Z_l^*)dZ_l^*=e^{-v^*X^*}$$
$$\times T^*(\alpha, v^*, R_l^*, Z^*) \quad \text{---------------------------- (5)}$$

where $Q^*(Z_l^*)$ may be chosen arbitrarily.

Considering the actual heat input situation in welding process, we put

$$Q^*(Z^*)=(1-Z^*)^\alpha \quad \text{-------------------------------- (6)}$$

where α is the index to the input energy distribution. Then, at $\alpha=0$,

$$Q^*(Z^*)=1 \quad \text{-- (7)}_1$$

and it corresponds to the line heat source of uniform input energy distribution, so called "line heat source" which has been employed so far.

at $\alpha\rightarrow\infty$

$$\left.\begin{aligned}Q^*(Z^*)&=1 \quad (Z^*=0)\\ &=0 \quad (Z^*\neq 0)\end{aligned}\right\} \quad \text{------------------------------ (7)}_2$$

and corresponds to the point heat source as is well known. Dependencv of $Q^*(Z^*)$ on Z^* is shown in **Fig. 2,** when α is in a certain value between both limits.

Substituting eq. (6) into eq. (5), $T^*(\alpha, v^*, R_l^*, Z^*)$ was calculated first, because on calculation of T_{line}, it is profitable to obtain T^*. Calculations were made on various combinations of α and v^* values. Calculations for integral in eq. (5) were performed numerically, using Simpson's formula.

Then, the temperature on the path of the heat source $T_x^*(\alpha, v^*, X^*, O, Z^*)$ was calculated for $X^*<0$.

From the obtained $T^*(\alpha, v^*, R_l^*, Z^*)$, $T_{line}^*(\alpha, v^*, X^*, Y^*, Z^*)$ was calculated and $T_m^*(\alpha, v^*, Y^*, Z^*)$ was obtained from eq. (8).

$$T_m^*(\alpha, v^*, Y^*, Z^*)=\text{Max.}\{T_{line}^*(\alpha, v^*, X^*, Y^*, Z^*)\}$$
$$=T_{line}^*(\alpha, v^*, X_m^*, Y^*, Z^*) \quad \text{-- (8)}$$

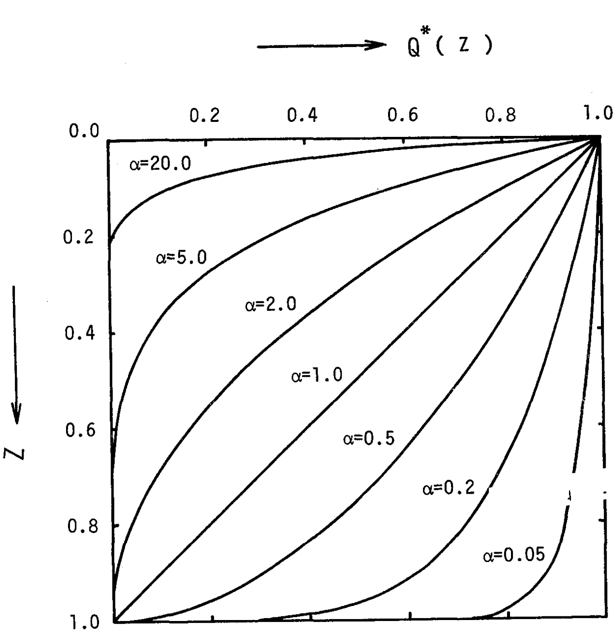
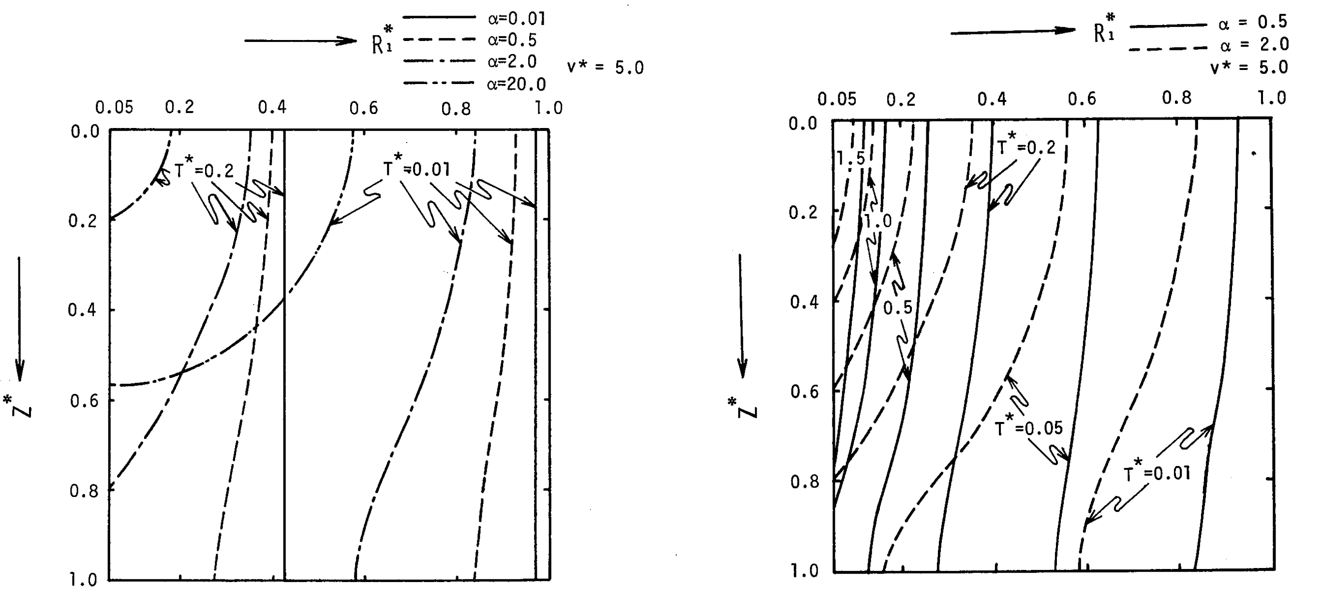


Fig. 2. The input energy distribution function $Q^*(Z)$.



(a) (b)

Fig. 3. The contour map of T* on the R_1^*—Z_l^* plane.

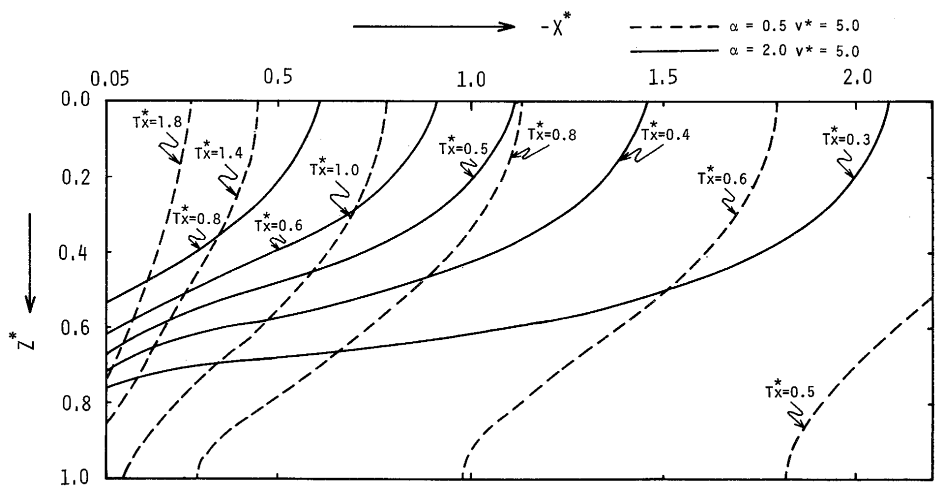


Fig. 4. The contour map of T_x^* on the X*—Z_l^* plane

where the value X* which makes T_{line}^* maximum is put to X_m^*.

These calculated results of T*, T_x^* and T_m^* were plotted on the R_l^*—Z*, the X*—Z* and the Y*—Z* plane respectively and the isothermal contour line maps were figured.

3. Results of Calculations

Figure 3 (a), (b) shows the contour line maps of T* for the constant value of v*(=5.0) and a few values of α.

It is seen from this figure that the pattern of the temperature distribution varies from that of a line heat source to that of a point heat source as α increases. An example of the T_x^* contour line map is shown in **Fig. 4** in which α and v* are the same values as in **Fig. 3 (b).**

The distribution of the maximum temperature T_m^* on the Y*—Z* plane and its dependency on α are shown in **Fig. 5 (a), (b), (c)** and **(d).** These are arranged in the same way as in **Fig. 3** for the constant value of v* and a few values of α. The dependency of the T* and the T_m^* distribution on the value of α are shown in **Fig. 6** and **Fig. 7.**

The values of T* are plotted on the R*—Z* plane for the constant value of α(=1.0) and a few values of v* in **Fig. 6 (a), (b)** and **(c).** The change of T*

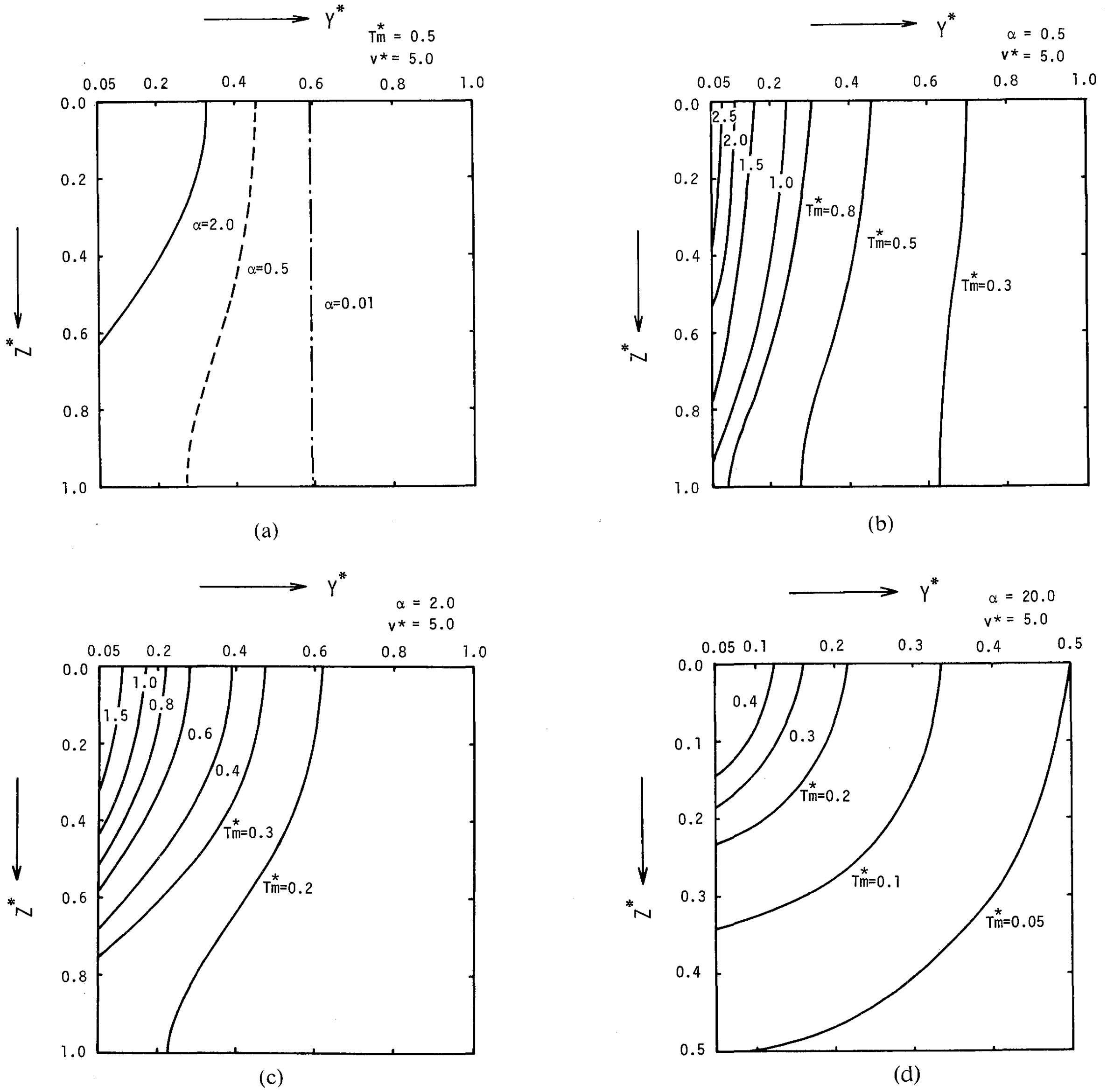


Fig. 5. The contour map of T_m^* on the Y^*—Z^* plane.

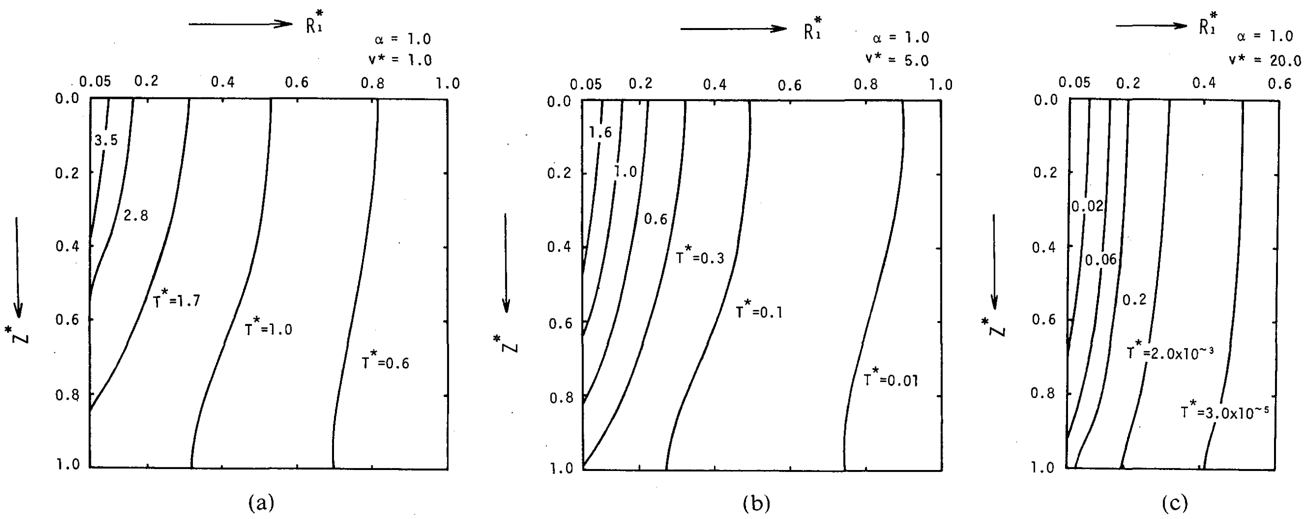
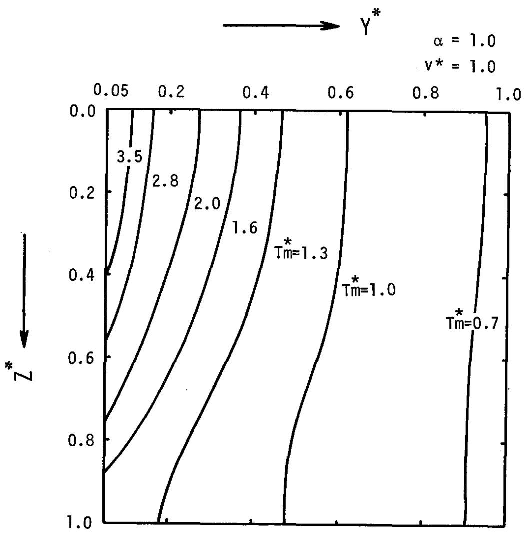
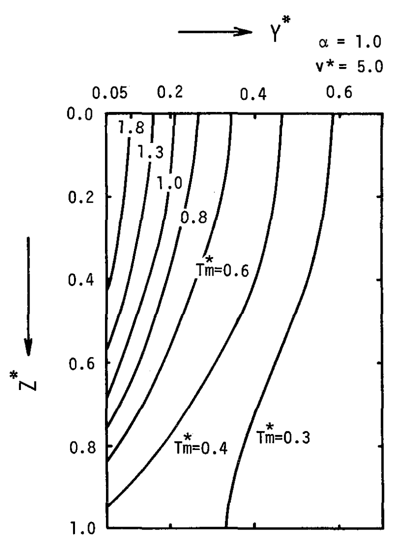


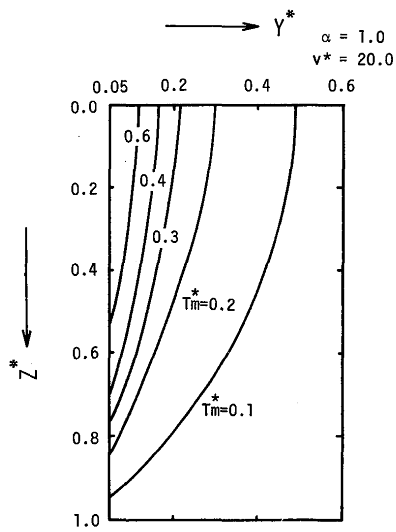
Fig. 6. The contour map of T^* on the R_1^*—Z^* plane.



(a)



(b)



(c)

Fig. 7. The contour map of T_m* on the Y*—Z* plane.

becomes remarkable as the value v* increases in this figure. The value of T_m* are plotted on the Y*—Z* plane in the same way as in **Fig. 6** as shown in **Fig. 7 (a), (b)** and **(c).**

In this figure, the remarkable tendency as is in **Fig. 6** is not seen.

4. Summary

Calculations, made in this report, will prove their worth when comparing with various welding results.

This is the problem in which α is to be determined under the given value of v* Some experimental approaches should be undertaken for this purpose.