OUKA
The University of Osaka
Institutional Knowledge Archive

Title	Correction to : "Characteristic classes with values in complex cobordism''
Author(s)	Nakaoka, Minoru
Citation	Osaka Journal of Mathematics. 1975, 12(1), p. 215-216
Version Type	VoR
URL	https://doi.org/10.18910/6531
rights	
Note	

The University of Osaka Institutional Knowledge Archive : OUKA

https://ir.library.osaka-u.ac.jp/

The University of Osaka

Nakaoka, M.
Osaka J. Math.
12 (1975), 215-216

CORRECTION TO CHARACTERISTIC CLASSES WITH VALUES IN COMPLEX COBORDISM

MINORU NAKAOKA

This Journal, vol. 10 (1973), 521–543

(Received May 12, 1974)

In the proof of Theorem 7 it is asserted that there exists $\alpha \in U^*(B_G \times M)$ such that

$$\alpha q(\tau(M)) = w^{m'}, \quad r_0^*(\alpha) = 0 .$$

The proof of this assertion is not correct. Under a further assumption that $U_*(M)$ *is projective over* $U_*(pt)$, this is proved correctly as follows.

It follows from Lemma 2 and (7.1) that $i^*(id \underset{G}{\times} f^k)^* \Delta' = 0$ for $i^*: U^*(E_G \underset{G}{\times} M^k) \to U^*(E_G \underset{G}{\times} (M^k - M))$ induced by the inclusion. Therefore there exists $\alpha \in U^{i-2m(k-1)}(B_G \times M)$ such that

$$(id \underset{G}{\times} f^k)^* \Delta' = j^* \phi_{\nu_1}(\alpha) ,$$

where $\phi_{\nu_1}: U^*(B_G \times M) \cong U^*(E_G \underset{G}{\times} (M^k, M^k - M))$ is the Thom isomorphism, and $j^*: U^*(E_G \underset{G}{\times} (M^k, M^k - M)) \to U^*(E_G \underset{G}{\times} M^k)$ is induced by the inclusion. It is easily seen that the diagram

$$\begin{array}{ccccc}
U^{i-2m(k-1)}(M) & \xleftarrow{r_0^*} & U^{i-2m(k-1)}(B_G \times M) & \xrightarrow{\cdot e(\nu_1)} & U^i(B_G \times M) \\
\downarrow{\scriptstyle d_!} & & \downarrow{\scriptstyle j^* \circ \phi_{\nu_1}} & \nearrow{\scriptstyle (id \underset{G}{\times} d)^*} & \\
U^i(M^k) & \xleftarrow{r^*} & U^i(E_G \underset{G}{\times} M^k) & &
\end{array}$$

is commutative, where r and r_0 are the inclusions, and $d_!$ is the Gysin homomorphism induced by the diagonal map $d: M \to M^k$. Consequently we have

$$(id \underset{G}{\times} d)^*(id \underset{G}{\times} f^k)^* \Delta' = \alpha \cdot e(\nu_1) , \tag{8.1}$$

$$r^*(id \underset{G}{\times} f^k)^* \Delta' = d_! r_0^*(\alpha) . \tag{8.2}$$

It follows from (6.1), (6.3) and (8.1) that

$$\alpha q(\tau(M)) = (id \times f)^* q(\tau(M'))\,,$$

and from (8.2) that

$$d_! r_0^*(\alpha) = (f^k)^* r'^*(\Delta')\,,$$

where r': $M'^k \to E_G \underset{G}{\times} M'^k$ is the inclusion. Since f is null-homotopic, these imply that

$$\alpha q(\tau(M)) = w^{m'}\,, \quad d_! r_0^*(\alpha) = 0\,.$$

Thus it suffices to prove that $d_!$ is injective. Since $U_*(M)$ is projective over $U_*(pt)$, it holds that

$$U^*(M) \cong \mathrm{Hom}_{U_*(pt)}(U_*(M),\ U_*(pt))$$

(see [2]). Therefore, if $b \in U_*(M)$ is not zero then there exists $\beta \in U^*(M)$ such that $\langle \beta, b \rangle \neq 0$. Since $\langle \beta \times 1 \times \cdots \times 1, d_* b \rangle = \langle \beta, b \rangle$, it follows that d_* and hence $d_!$ is injective.