Collective States in Heavy Nuclei

Akitsu Ikeda

Department of Physics, Osaka University,

Toyonaka, Osaka

Table of Contents

Abstracts:

The generator coordinate method is formulated so as to be suitable for a description of the transition region nuclei. To take into account the pairing correlation the BCS wave function constructed with various single particle states in the deformed potential well is used as an intrinsic wave function, from which eigenstates of angular momentum are projected out. Using the quadrupole deformation parameters β, γ and the gap parameter Δ as generator coordinates, the trial wave function is constructed by superposing the projected wave functions with various values of β, γ and Δ. Such a choice of the generator coordinates enables us to treat the rotation, the surface-vibrations and the pairing vibration in a unified way. The generator function, which serves as a weight function, is obtained by solving an integral equation derived through the variational principle. It is shown that our projected wave funcitons have the same symmetry properties as those of wave functions given by the Bohr model. This method is first applied to a schematic pairing vibration problem. It seems that this method is powerful enough in the description of real nuclei. Second, the energy surface of Cd^{116} is calculated as a function of the deformation parameters β, γ.

§1. Introduction

It is well known that in the nuclei far from the double closed shell the law-lying excited states have collective nature. We restrict ourselves to the doubly even nuclei in this paper.

In the region of the spherical nuclei, the energy spectra and the electromagnetic properties of the low-lying excited states are at least qualitatively well described as the vibration in the frame work of the Bohr model with the harmonic potential of deformation[1, 2, 3, 4]. The microscopic treatment based on linearization approximation of the equation of motion[5,6] is equally successful in the description. There are observed, however, many deviations from the predicted harmonic vibration such as violation of the systematics of excitation energy, splitting of the two-phonon triplet states, missing of one or two members of the triplet and so on. Furthermore unexpectedly large quadrupole moments of the first 2^+ states have been observed in the typical vibrational nuclei[7]. It was shown by Tamura and Udagawa[8] that existing models, which start from harmonic vibrations around a spherical shape cannot explain such large quadrupole moments. Therefore, anharmonic effects must be taken into account more rigorously in a theoretical treatment[9].

The low-lying excited states of the deformed nuclei constitute several rotational bands whose energy spectra seem to obey the $I(I + 1)$ rule[1,2,4]. These spectra are understood as the rotation of a deformed nucleus as a whole, which is given as a limiting case in the Bohr model. A microscopic method to evaluate the moments of inertia is the cranking model[10]. In order to reproduce the order of magnitude of experimentally obtained moments of inertia it is essentially important[11] to use wave functions in which the pairing correlation is taken into account. Deviations of the energy spectra from the $I(I + 1)$ rule are always

observed in actual nuclei. Many works have been performed to explain the deviations from various points of view, which may be classified into three groups; (1) the rotation-vibration coupling including the centrifugal stretching[12, 13], (2) the Coriolis anti-pairing effect[12,14,15], (3) higher order effects of the cranking model[16]. However, if all of these effects are taken into account simultaneously[19], theoretical estimate of the deviation becomes too large. There are two defects in the usual treatments; (1) semi-classical nature of the cranking model and (2) the special assumption about the nuclear shape.

From the above-mentioned facts we can see that there are many difficulties in the usual treatments in which all nuclei are divided into two categories, i.e., spherical or deformed, and the approximation of harmonic vibration either around a spherical shape or a deformed shape.

Besides spherical and deformed nuclear regions of the periodic table there is a region of onset or disapperance of nuclear deformation, which is called the transition region. Properties of the low-lying excited states of nuclei in this region are very much different from the predictions of either the phonon model or the rotational model. The experimental evidences suggest a strong vibration-rotation coupling. It is expected that a more rigorous method to treat the low-lying excited states of these heavy nuclei should be developed.

There are experimental evidences that nuclear shapes change with not only mass number but also excitation energy. For example, at higher excitation energy rotational spectra[18] were observed in the double closed shell nuclei O^{16}. This fact shows that even the double closed shell nucleus can be deformed. The possibility of variation of the nuclear shape with excitation energy may be common in all nuclei. So there arises an interesting problem to investigate how the nuclear shape varies with increasing excitation energy.

Baranger and Kumar[19,20,21] made a lot of calculations to get nuclear energy

surfaces and investigated collective motions in the transition region nuclei with the aid of the Bohr model. The "pairing-plus-quadrupole model"[22] was employed. It is shown that dependence of the potential energy of deformation on non-axiality is very important to explain various properties of low-lying excited states of the transition region nuclei. Rather good agreements with experimental results were obtained. However, the method used has some defects: (1) The pairing-plus-quadrupole model is too simple to be a realistic description of nuclei because the other multipole components of the nuclear force than monopole and quadrupole ones are neglected. (2) Exchange characters of the nuclear force are not considered at all. (3) As they pointed out in their series (V), the fluctuations in the energy gap Δ are not considered.

In studying the properties of the low-lying excited states of heavy nuclei far from the double closed shell, the generator coordinate method (GCM)[23 - 28] seems most appropriate. This method is based on the variational method and is superior to the macroscopic treatment of the collective motion (Bohr model) because redundant variables inherent to the Bohr model are integrated out and interactions between nucleons are considered rigorously. In the GCM trial wave functions are eigenstates of angular momentum in contrast to the usual self-consistent field methods[29]. Equilibrium deformations and fluctuations around an equilibrium deformation in excited states can be described naturally. This method has been already applied to the transition region nuclei by Onishi and Yoshida[27] with restriction to axially symmetric deformation and to light nuclei by Une[28]. A purpose of the present paper is to generalize their formulation to axially non-symmetric deformation.

Since the generator coordinate method is based on the variational principle, it is very much important to choose a trial wave function which simulates the actual wave function as far as possible. In choice of generator coordinates we

should make full use of successful results obtained by many authors in the course of various attempts.

It has been established that the low-lying excited states of the heavy nuclei far from the double closed shell are closely connected with quadrupole deformation of the nuclear density as can be seen from the structure of the Bohr model[1,2]. Since the nuclear force is of short-range nature, we can reasonably expect that a nucleon feels a field of the same shape as the nuclear density. For the sake of simplicity, we employ the anisotropic harmonic oscillator potential well with various quadrupole deformations[30] without restriction to axial-symmetry as the generating field. The parameters β and γ which characterize deformations are used as generator coordiantes.

After Bohr, Mottelson and Pines[31] pointed out the existence of an energy gap in the intrinsic excitation spectrum of nuclei, it was proposed by Beliaev[32] to apply the BCS wave function for a description of nuclei. This wave function has attained many successes particularly in calculations of various matrix elements of transition[4,33] and moments of inertia. Though the BCS wave function, which is specified by a value of the energy gap, can explain many properties of nuclei, there remain residual interactions which are not taken into account in it. A more favourable wave function can be obtained by superposing BCS wave function with various energy gaps. In this way fluctuations around an equilibrium energy gap can be considered[34,35,36]. So we employ the energy gap Δ as another generator coordinate.

Intrinsic wave functions prepared in the above-mentioned way are not eigenstates of angular momentum. Using the projection operator introduced by Peierls and Yoccoz[24], eigenstates of angular momentum are projected out from the intrinsic wave functions. This kind of angular momentum projection is just a quantum mechanical treatment of nuclear rotations[37].

Fluctuations around an equilibrium deformation may be interpreted as the so-called surface vibrations including β-and γ-vibrations and fluctuations around an equilibrium energy gap is the pairing vibration. Our method enables us to treat the nuclear rotation, the surface vibrations and the pairing vibration in a unified way, and mutual couplings are taken account of automatically.

The Hamiltonian used consists of the single particle energy and the interaction between nucleons with all the exchange characters. The interaction can be of any non-singular radial dependence.

In Sec. II our formalism is developed. The present method is applied to a schematic problem of the pairing vibration in Sec III. In the subsequent sections an energy surface of Cd^{116} is calculated, and some discussions are given.

§2. Description of the Formalism

2-1. Single-Particle States in Quadrupole Field

As mentioned in Sec. 1, we use the following generating field to obtain single particle wave functions,

$$H_g = -\frac{\hbar^2}{2M}\nabla'^2 + \frac{M}{2}(\omega_x^2 x'^2 + \omega_y^2 y'^2 + \omega_z^2 z'^2) + C\bar{l}\cdot\bar{s} + D\bar{l}^2 \qquad (2\text{-}1)$$

$$\equiv H_{sph} + H_{def} ,$$

where M is the nucleon mass and H_{sph} stands for a spherical field and H_{def} for a deformed field. After a transformation of the variables

$$x = \sqrt{\frac{M}{\hbar}\omega_0(\beta,\gamma)}\; x' , \quad \cdots \qquad (2\text{-}2)$$

was performed, these are given as

$$H_{sph} = \frac{\hbar\omega_0(\beta,\gamma)}{2}[-\nabla^2 + r^2] + C\bar{l}\cdot\bar{s} + D\bar{l}^2 , \qquad (2\text{-}3)$$

$$H_{def} = \frac{\hbar\omega_0(\beta,\gamma)}{2}\left[\frac{\omega_x^2-\omega_0^2(\beta,\gamma)}{\omega_0^2(\beta,\gamma)}x^2 + \frac{\omega_y^2-\omega_0^2(\beta,\gamma)}{\omega_0^2(\beta,\gamma)}y^2 + \frac{\omega_z^2-\omega_0^2(\beta,\gamma)}{\omega_0^2(\beta,\gamma)}z^2\right] . \qquad (2\text{-}4)$$

Introducing parameters α's, H_{def} can be written as

$$H_{def} = -\hbar\omega_0(\beta,\gamma)r^2\sum_m \alpha_m Y_{2m}(\vartheta\varphi) . \qquad (2\text{-}5)$$

In the coordinate system whose axes coincide with the principal axes of the ellipsoid (2-1), α's can be chosen as

$$\alpha_2=\alpha_{-2}=\frac{\hat{\beta}}{\sqrt{2}}\sin\gamma,\quad \alpha_1=\alpha_{-1}=0,\quad \alpha_0=\beta\cos\gamma. \tag{2-6}$$

Then, in the β, γ-representation,

$$H_{def}=-\hbar\omega_0(\beta,\gamma)\sqrt{\frac{5}{4\pi}}\beta\left[\cos(\gamma-\tfrac{2\pi}{3})x^2+\cos(\gamma+\tfrac{2\pi}{3})y^2+\cos\gamma\cdot z^2\right], \tag{2-7}$$

and

$$\begin{aligned}\omega_x^2&=\omega_0^2(\beta,\gamma)\left[1-2\sqrt{\tfrac{5}{4\pi}}\beta\cos(\gamma-\tfrac{2\pi}{3})\right],\\ \omega_y^2&=\omega_0^2(\beta,\gamma)\left[1-2\sqrt{\tfrac{5}{4\pi}}\beta\cos(\gamma+\tfrac{2\pi}{3})\right],\\ \omega_z^2&=\omega_0^2(\beta,\gamma)\left[1-2\sqrt{\tfrac{5}{4\pi}}\beta\cos\gamma\right].\end{aligned} \tag{2-8}$$

From the volume conservation, $\omega_0(\beta,\gamma)$ is determined by

$$\omega_0(\beta,\gamma)=\omega_0(\beta=0)\left\{\left[1-2\sqrt{\tfrac{5}{4\pi}}\beta\cos(\gamma-\tfrac{2\pi}{3})\right]\cdot\left[1-2\sqrt{\tfrac{5}{4\pi}}\beta\cos(\gamma+\tfrac{2\pi}{3})\right]\left[1-2\sqrt{\tfrac{5}{4\pi}}\beta\cos\gamma\right]\right\}^{-1/6}, \tag{2-9}$$

where

$$\hbar\omega_0(\beta=0)=41\cdot A^{-1/3}\ \mathrm{MeV},$$

and

A = mass number .

We expand the eigenfunctions of H_g in terms of the ones of H_{sph} which are denoted as ϕ_α, α being an abbreviation of a set of quantum numbers $(n_\alpha l_\alpha j_\alpha m_\alpha)$. Since H_{def} causes $\Delta m = 2$ mixing, the energy matrix is divided into two parts, one of which belongs to $m = j, j-2, \cdots, -j+1$, and the other to $m = j-1, j-3, \cdots, -j$. Introducing a quantum number q, we assign $q = 1/2$ to the wave functions belonging to the former group and $q = -1/2$ to the remainings. Denoting the expansion coefficients by $a_{\sigma q,\alpha}(\beta,\gamma)$, the eigenfunctions $\psi_{\sigma q}(\beta,\gamma)$ of H_g are

given as

$$\psi_{\sigma g}(\beta,\gamma)=\sum_{\alpha} a_{\sigma g,\alpha}(\beta,\gamma)\phi_{\alpha} \quad , \tag{2-10}$$

where σ represents a set of other quantum numbers than g.

We write down the secular equations obeyed by a's,

$$\sum_{a,m_a>0} a_{\sigma 1/2,am_a}\langle a m_a|H_g|b m_\beta\rangle = E_{\sigma 1/2}\, a_{\sigma 1/2,bm_\beta} \quad , \tag{2-11a}$$

$$\sum_{a,m_a>0} a_{\sigma -1/2,a-m_a}\langle a-m_a|H_g|b-m_\beta\rangle = E_{\sigma -1/2}\, a_{\sigma -1/2,b-m_\beta} \quad . \tag{2-11b}$$

Here $m_\alpha > 0$ implies that the summation is made over $m_\alpha = \cdots, -3/2, 1/2, 5/2, \cdots$, and $\underline{a}$ stands for $(n_a l_a j_a)$. Noticing that $\langle \alpha|H_g|\beta\rangle$ is real and $\langle a-m_\alpha|H_g|b-m_\beta\rangle = (-)^{j_a-m_\alpha-(j_b-m_\beta)}\langle am_\alpha|H_g|bm_\beta\rangle$, we can see that $a_{\sigma 1/2,am_\alpha}$ and $(-)^{j_a-m_\alpha}\cdot a_{\sigma -1/2,a-m_\alpha}$ obey the same secluar equation and a's are real quantities. Then we get

$$E_{\sigma 1/2} = E_{\sigma -1/2} \equiv E_{\sigma} \quad , \tag{2-12}$$

and the following phase convention is used,

$$a_{\sigma 1/2,am_a} = (-1)^{j_a-m_a}\, a_{\sigma -1/2,a-m_a} \quad . \tag{2-13}$$

Under the above phase convention the time reversed state $\psi_{\sigma\tilde{g}}$ of $\psi_{\sigma g}$ is connected to $\psi_{\sigma -g}$ as follows,

$$\psi_{\sigma\tilde{g}} = (-)^{1/2-g}\,\psi_{\sigma -g} \quad . \tag{2-14}$$

2-2. Trial Wave Functions and Symmetry Properties

Since the pairing correlation plays an essentially important role in low-lying excited states of heavy nuclei as mentioned in Sec. 1, we employ wave functions taken account of the pairing correlation for intrinsic ones. These can be obtained by applying the BCS procedure to the following generating Hamiltonian,

$$H_p=\sum_{\sigma,q}(E_{\sigma q}-\lambda)C^{+}_{\sigma q}(\beta,\gamma)\,C_{\sigma q}(\beta,\gamma)-\frac{g}{4}\sum_{\sigma q\sigma' q'}C^{+}_{\sigma q}(\beta,\gamma)C^{+}_{\sigma\tilde{q}}(\beta,\gamma)C_{\sigma'\tilde{q}'}(\beta,\gamma)\,C_{\sigma' q'}(\beta,\gamma), \quad (2.15)$$

where $C^{+}_{\sigma q}(\beta,\gamma)$ is the creation operator of a nucleon in a state labelled by quantum number σ, q which assign an orbit in the deformed field specified by β and γ, and λ is the chemical potential which ensures the expectation value of the nucleon number operator to be the neutron or proton number. Here it should be noted that the Hamiltonian (2.15) is only a generating one, and g is the strength of the pairing interaction used to get wave functions taken account of the pairing correlation. Therefore g is a variable to generate wave functions with various energy gaps.

According to the usual BCS theory, we perform the Bogoliubov-Valatin transformation,

$$\alpha^{+}_{\sigma q}=u_{\sigma q}C^{+}_{\sigma q}-v_{\sigma q}C_{\sigma\tilde{q}}\ , \quad (2.16)$$

$$C^{+}_{\sigma q}=u_{\sigma q}\alpha^{+}_{\sigma q}+v_{\sigma q}\alpha_{\sigma\tilde{q}}\ , \quad (2.17)$$

where $\alpha^{+}_{\sigma q}$ is the creation operator of a quasi-particle. The coefficients of the transformation, $u_{\sigma q}$ and $v_{\sigma q}$, are given as

$$|u_{\sigma\eta}|^2 = \frac{1}{2}\left\{1 + \frac{E_{\sigma\eta} - \lambda}{\sqrt{(E_{\sigma\eta}-\lambda)^2 + \Delta^2}}\right\},$$

$$|v_{\sigma\eta}|^2 = \frac{1}{2}\left\{1 - \frac{E_{\sigma\eta} - \lambda}{\sqrt{(E_{\sigma\eta}-\lambda)^2 + \Delta^2}}\right\}, \tag{2.18}$$

where Δ is called the energy gap. The quantities Δ and λ are determined by solving the BCS equations:

$$\frac{4}{g} = \sum_{\sigma\eta} \frac{1}{\sqrt{(E_{\sigma\eta}-\lambda)^2 + \Delta^2}}, \tag{2.19}$$

$$2N = \sum_{\sigma\eta}\left[1 - \frac{E_{\sigma\eta} - \lambda}{\sqrt{(E_{\sigma\eta}-\lambda)^2 + \Delta^2}}\right], \tag{2.20}$$

where N is the neutron or proton number. By operating the time reversal operation on eq. (2.16), we get

$$\alpha^{+}_{\sigma\tilde{\eta}} = u^{*}_{\sigma\eta} C^{+}_{\sigma\tilde{\eta}} + v^{*}_{\sigma\eta} C_{\sigma\eta}. \tag{2.21}$$

On the other hand, from the definition we have

$$\alpha^{+}_{\sigma\tilde{\eta}} = u_{\sigma\tilde{\eta}} C^{+}_{\sigma\tilde{\eta}} + v_{\sigma\tilde{\eta}} C_{\sigma\eta}. \tag{2.22}$$

From eqs. (2.21), (2.22), (2.18) and (2.12) it is clear that we can use the following phase convention

$$u_{\sigma\eta} = u_{\sigma\tilde{\eta}} \equiv u_{\sigma} = +\sqrt{\frac{1}{2}\left[1 + \frac{E_{\sigma} - \lambda}{\sqrt{(E_{\sigma}-\lambda)^2 + \Delta^2}}\right]},$$

$$v_{\sigma\eta} = v_{\sigma\tilde{\eta}} \equiv v_{\sigma} = +\sqrt{\frac{1}{2}\left[1 - \frac{E_{\sigma} - \lambda}{\sqrt{(E_{\sigma}-\lambda)^2 + \Delta^2}}\right]}. \tag{2.23}$$

The BCS wave function is given as

$$\phi_0(\beta\gamma\Delta) = \prod_\sigma \left(u_\sigma(\beta\gamma\Delta) + v_\sigma(\beta\gamma\Delta) C^+_{\sigma\frac{1}{2}}(\beta\gamma\Delta)\, C^+_{\sigma\tilde{\frac{1}{2}}}(\beta\gamma\Delta)\right)|0\rangle , \tag{2.24}$$

where $|0\rangle$ stands for the nucleon vacuum. This wave function is the quasi-particle vacuum; $\alpha\phi_0(\beta\gamma\Delta) = 0$. Thus the intrinsic wave functions are obtained.

The BCS wave function (2.24) can be written as follows without any approximation:

$$\phi_0(\beta\gamma\Delta) = \left[\prod_\sigma u_\sigma(\beta\gamma\Delta)\right] \exp\left[\sum_\sigma \frac{v_\sigma(\beta\gamma\Delta)}{u_\sigma(\beta\gamma\Delta)} C^+_{\sigma\frac{1}{2}}(\beta\gamma\Delta)\, C^+_{\sigma\tilde{\frac{1}{2}}}(\beta\gamma\Delta)\right]|0\rangle , \tag{2.25}$$

because extra terms appearing when the right-hand side is expanded in a power series vanish identically. (All u_σ are assumed not equal to zero; if some of them are zero, eq. (2.25) should be modified.) The expression (2.25) is of a form convenient to the later development. The nucleon operators in the exponent expressed on the deformed well basis are now transformed into the spherical basis. Any spherical basis may be used, but it is convenient to choose so that the single particle states with various values of deformation parameters may be expressed as simply as possible. We make an approximation to neglect the dependence of $\hbar\omega_0(\beta\ \gamma)$ on deformation parameters (eqs. (2.9) and (2.10)). In this case, as a spherical basis we can use the single particle states of H_{sph} in eq. (2.3) with replacement of $\omega_0(\beta\ \gamma)$ by $\omega_0(\beta = 0)$. The intrinsic wave function (2.25) may be rewritten in terms of the nucleon operator defined in the spherical potential well and the results are given by

$$\phi_0(\beta\gamma\Delta) = \left[\prod_\sigma u_\sigma(\beta\gamma\Delta)\right] \exp\left[\frac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta}(\beta\gamma\Delta) C^+_\alpha C^+_\beta\right]|0\rangle , \tag{2.26}$$

where C^+_α is the creation operator of a nucleon in an eigenstate of H_{sph} specified by quantum number $\alpha = (n_a l_a j_a m_\alpha)$. The relation C^+_α and $C^+_{\sigma q}(\beta\ \gamma)$ is just the same

as eq. (2.10), that is,

$$C^{+}_{\sigma q}(\beta,\gamma) = \sum_{\alpha} a_{\sigma q,\alpha}(\beta,\gamma)\, C^{+}_{\alpha}. \tag{2.27}$$

Here, $f_{\alpha\beta}(\beta\ \gamma\ \Delta)$ are given as

$$f_{\alpha\beta}(\beta\gamma\Delta) = \sum_{\sigma q} (-)^{\frac{1}{2}-q} a_{\sigma q,\alpha}(\beta\gamma)\, a_{\sigma -q,\beta}(\beta\gamma)\, \frac{v_{\sigma}(\beta\gamma\Delta)}{u_{\sigma}(\beta\gamma\Delta)}, \tag{2.28}$$

and have the following properties

$$f_{\alpha\beta}(\beta\gamma\Delta) = -f_{\beta\alpha}(\beta\gamma\Delta), \tag{2.29}$$

$$f_{a-m_a,b-m_\beta}(\beta\gamma\Delta) = (-1)^{j_a-m_\alpha+j_b-m_\beta} f_{\alpha\beta}(\beta\gamma\Delta), \tag{2.30}$$

where subscript $\underline{a}$ stands for $(n_a l_a j_a)$.

The intrinsic wave functions given by eq. (2.26) is not an eigenfunction of angular momentum. Therefore it is necessary to project out the wave function with definite angular momentum I and its z-component M. This projection can be performed by using the projection operator

$$P_{IMK} = \frac{2I+1}{8\pi^2}\int_0^{2\pi} d\varphi \int_0^{2\pi} d\chi \int_0^{\pi} \sin\theta\, d\theta\, \mathcal{D}^{I^{+}}_{KM}(\varphi\vartheta\chi)\, R_{\phi\theta\chi} \tag{2.31}$$

In eq. (2.31) the rotation operator $R_{\phi\theta\chi}$ is defined by

$$R_{\varphi\vartheta\chi} = e^{-i\varphi J_z}\, e^{-i\vartheta J_y}\, e^{-i\chi J_z}, \tag{2.32}$$

where J_z, J_y are z-, y-components of angular momentum operator and $D^{I^{+}}_{MK}(\phi\ \theta\ \chi)$ is

the hermitian conjugate of the ordinary representation of rotational group. The projected wave function is given by

$$\Phi_{IMK}(\beta\gamma\Delta) = P_{IMK}\phi_0(\beta\gamma\Delta)$$

$$= \int_0^{2\pi} d\varphi \int_0^{2\pi} d\chi \int_0^{\pi} \sin\theta d\theta \, \mathcal{D}^{I\dagger}_{KM}(\varphi\theta\chi)\, \phi_0(\beta\gamma\Delta;\Omega), \tag{2.33}$$

where

$$\phi_0(\beta\gamma\Delta;\Omega) = \left[\prod_\sigma U_\sigma(\beta\gamma\Delta)\right] \exp\left[\frac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta}(\beta\gamma\Delta;\Omega) C^+_\alpha C^+_\beta\right]|0\rangle, \tag{2.34}$$

and Ω stands for Eulerian angles (ϕ, θ, χ). The qunatities $f_{\alpha\beta}(\beta\ \gamma\ \Delta;\ \Omega)$ are written as

$$f_{\alpha\beta}(\beta\gamma\Delta;\Omega) = e^{i(m_\alpha+m_\beta)\varphi} \sum_{J, m'_\alpha, m'_\beta} e^{-i(m'_\alpha+m'_\beta)\chi} \langle j_a m_\alpha j_b m_\beta | J m_\alpha + m_\beta\rangle$$

$$\cdot \langle j_a m'_\alpha j_b m'_\beta | J m'_\alpha + m'_\beta\rangle\, d^J_{m_\alpha+m_\beta,\, m'_\alpha+m'_\beta}(\vartheta) f_{a m'_\alpha b m'_\beta}(\beta\gamma\Delta). \tag{2.35}$$

In eq. (2.35) $d^J_{MM'}(\theta)$ is defined by removing the ϕ and χ dependent factors from $D^J_{MM'}(\phi\ \theta\ \chi)$. The projected wave function (2.33) constitutes a trial wave function for variational calculation.

At this stage, it may be useful to investigate the symmetry properties of the projected wave functions (2.33).

(1) The intrinsic wave function (2.26) has the following property

$$\phi_0(\beta, -\gamma, \Delta) = R_{00\frac{\pi}{2}}\phi_0(\beta, \gamma, \Delta). \tag{2.36}$$

Proof: The explicit expressions of both sides of eq. (2.36) are given by

$$\phi_0(\beta,-\gamma,\Delta)=\left[\prod_\sigma u_\sigma(\beta,-\gamma,\Delta)\right]\exp\left[\tfrac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta}(\beta,-\gamma,\Delta)\,c_\alpha^+ c_\beta^+\right]|0\rangle, \tag{2.37a}$$

where

$$f_{\alpha\beta}(\beta,-\gamma,\Delta)=\sum_{\sigma q}(-1)^{\frac{1}{2}-q} a_{\sigma q,\alpha}(\beta,-\gamma)\,a_{\sigma,-q,\beta}(\beta,-\gamma)\,\frac{v_\sigma(\beta,-\gamma,\Delta)}{u_\sigma(\beta,-\gamma,\Delta)}, \tag{2.37b}$$

and

$$R_{00\frac{\pi}{2}}\phi_0(\beta,\gamma,\Delta)=\left[\prod_\sigma u_\sigma(\beta,\gamma,\Delta)\right]\exp\left[\tfrac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta}(\beta,\gamma,\Delta;00\tfrac{\pi}{2})\,c_\alpha^+ c_\beta^+\right]|0\rangle, \tag{2.38a}$$

where

$$f_{\alpha\beta}(\beta\gamma\Delta;00\tfrac{\pi}{2})=\sum_{\sigma q}(-1)^{\frac{1}{2}-q} a_{\sigma q,\alpha}(\beta,\gamma)\,a_{\sigma,-q,\beta}(\beta,\gamma)\,\frac{v_\sigma(\beta,\gamma,\Delta)}{u_\sigma(\beta,\gamma,\Delta)}\,e^{-i\frac{\pi}{2}(m_\alpha+m_\beta)} \tag{2.38b}$$

In order to prove eq. (2.36), we must know the relation between $a_{\sigma q,\alpha}(\beta,-\gamma)$ and $a_{\sigma q,\alpha}(\beta,\gamma)$. The secular equations for these quantities are

$$\sum_\alpha a_{\sigma q,\alpha}(\beta,\gamma)\langle\alpha|H_q(\beta,\gamma)|\beta\rangle=E_{\sigma q}(\beta,\gamma)\,a_{\sigma q,\beta}(\beta,\gamma), \tag{2.39a}$$

$$\sum_\alpha a_{\sigma q,\alpha}(\beta,-\gamma)\langle\alpha|H_q(\beta,-\gamma)|\beta\rangle=E_{\sigma q}(\beta,-\gamma)\,a_{\sigma q,\beta}(\beta,-\gamma), \tag{2.39b}$$

where the explicit form of $H_q(\beta,\gamma)$ is given in eq. (2.1). If we use the relation

$$H_q(\beta,\gamma)=R_{00\frac{\pi}{2}}\,H_q(\beta,-\gamma)\,R_{00\frac{\pi}{2}}^{-1}, \tag{2.40}$$

where R^{-1} is the inverse of R, the matrix element appearing in the left-hand side of eq. (2.39) becomes

$$\langle\alpha|\, H_g(\beta,-\gamma)|\beta\rangle = e^{-i\frac{\pi}{2}(m_\alpha - m_\beta)} \langle\alpha|\, H_g(\beta\gamma)|\beta\rangle . \tag{2.41}$$

If we insert eq. (2.41) into eq. (2.39b), it is seen that the two secular equations are identical. The quantities $a_{\sigma q,\alpha}(\beta, \gamma)$ and $a_{\sigma q,\alpha}(\beta, -\gamma)e^{-i\frac{\pi}{2}m_\alpha}$ are equal to each other except an arbitrary common phase. Since it is convenient to make both of them real, we use the following phase convention

$$a_{\sigma g,\alpha}(\beta,-\gamma)\, e^{-i\frac{\pi}{2}(m_\alpha - g)} = a_{\sigma g,\alpha}(\beta,\gamma) . \tag{2.42}$$

It is also clear that the energy eigenvalues of the secular equations (2.39a) and (2.39b) are equal to each other,

$$E_{\sigma g}(\beta,\gamma) = E_{\sigma g}(\beta,-\gamma) . \tag{2.43}$$

From eqs. (2.19), (2.20), (2.23) and (2.43), we get

$$\begin{bmatrix} V_\sigma(\beta,-\gamma,\Delta) \\ U_\sigma(\beta,-\gamma,\Delta) \end{bmatrix} = \begin{bmatrix} V_\sigma(\beta,\gamma,\Delta) \\ U_\sigma(\beta\gamma\Delta) \end{bmatrix} , \tag{2.44}$$

which is a short form of two equations. By making use of eqs. (2.42) and (2.44) it can be easily proved that

$$\prod_\sigma U_\sigma(\beta,\gamma,\Delta) = \prod_\sigma U_\sigma(\beta,-\gamma,\Delta) \tag{2.45}$$

and

$$f_{\alpha\beta}(\beta,-\gamma,\Delta) = f_{\alpha\beta}(\beta,\gamma,\Delta;00\tfrac{\pi}{2}) . \tag{2.46}$$

Thus eq.(2.36) is proved.

Operating the projection operator on the both sides of eq.(2.36), we get

$$P_{IMK}\Phi_0(\beta,-\gamma,\Delta) = (-i)^K P_{IMK}\Phi_0(\beta,\gamma,\Delta) , \tag{2.47}$$

where the expression of the right-hand side is obtained as follows:

$$\begin{aligned} P_{IMK} R_{00\frac{\pi}{2}} \Phi_0(\beta\gamma\Delta) &= \int d\Omega\, \mathcal{D}^{I\dagger}_{KM}(\Omega) R_\Omega R_{00\frac{\pi}{2}} \Phi_0(\beta\gamma\Delta) \\ &= \int d\Omega \sum_{K'} \mathcal{D}^{I\dagger}_{KK'}(\Omega) \mathcal{D}^{I\dagger}_{K'M}(00\tfrac{\pi}{2}) R_\Omega \Phi_0(\beta\gamma\Delta) \\ &= (-i)^K P_{IMK}\Phi_0(\beta\gamma\Delta) . \end{aligned} \tag{2.47a}$$

From eq. (2.47) it is said that there do not exist odd-K components in our projected wave functions. The symmetry property considered here corresponds to the R_2-symmetry of the Bohr model[1,2,21].

(2) Then we can prove the following relation:

$$\Phi_0(\beta,\gamma,\Delta) = R_{0\pi\pi}\Phi_0(\beta,\gamma,\Delta) \tag{2.48}$$

Proof: We write down the right-hand side of eq. (2.48),

$$R_{0\pi\pi}\Phi(\beta,\gamma,\Delta) = \Big[\prod_\alpha u_\alpha(\beta\gamma\Delta)\Big] \exp\Big[\tfrac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta}(\beta\gamma\Delta;0\pi\pi) C^+_\alpha C^+_\beta\Big]|0\rangle .$$

Then, we proceed as follows;

$$f_{\omega\rho}(\beta\gamma\Delta;0\pi\pi)=\sum_{m'_\alpha m'_\beta} f_{am'_\alpha bm'_\beta}(\beta\gamma\Delta)\mathcal{D}^{j_a}_{m_\alpha m'_\alpha}(0\pi\pi)\mathcal{D}^{j_b}_{m_\beta m'_\beta}(0\pi\pi)$$

$$=\sum_{\sigma q}(-)^{\frac{1}{2}-q}e^{i\pi(m_\alpha+m_\beta)}(-1)^{j_a-m_\alpha}(-1)^{j_b-m_\beta}a_{\sigma q,a-m_\alpha}(\beta\gamma)\,a_{\sigma-q,b-m_\beta}(\beta\gamma)\frac{v_\sigma(\beta\gamma\Delta)}{u_\sigma(\beta\gamma\Delta)}$$

$$=f_{\omega\rho}(\beta\gamma\Delta), \tag{2.49}$$

where eq. (2.13) and the relation, $m_\alpha + m_\beta$ = even, for non-vanishing $f_{\alpha\beta}(\beta,\gamma,\Delta)$ were used. Thus eq. (2.48) is proved directly.

This equation presents another useful relation:

$$P_{IMK}\phi_0(\beta\gamma\Delta)=(-1)^I P_{IM-K}\phi_0(\beta\gamma\Delta). \tag{2.50}$$

In deriving the above equation, the same technique as used in eq. (2.47a) was used. It follows that the projected wave function with odd spin has not K = 0 component. There does not exist a state with I = 1 among the projected wave functions. The symmetry property (2.50) corresponds to R_1-symmetry of the Bohr model[1,2,21].

(3) Finally the symmetry property, which corresponds to the R_3-symmetry of the Bohr model[1,2,21], is proved. We start by proving the following relation:

$$\phi_0(\beta,\gamma+\tfrac{2}{3}\pi,\Delta)=R_{0\frac{\pi}{2}\frac{\pi}{2}}\phi_0(\beta,\gamma,\Delta). \tag{2.51}$$

Proof: We give here the expression of the right-hand side of eq. (2.51),

$$R_{0\frac{\pi}{2}\frac{\pi}{2}}\phi_0(\beta,\gamma,\Delta)=\Big[\prod_\sigma u_\sigma(\beta\gamma\Delta)\Big]\exp\Big[\tfrac{1}{2}\sum_{\alpha\beta}f_{\omega\rho}(\beta\gamma\Delta;0\tfrac{\pi}{2}\tfrac{\pi}{2})C^+_\alpha C^+_\beta\Big]|0\rangle, \tag{2.52a}$$

where

$$f_{\alpha\beta}(\beta\gamma\Delta;0\tfrac{\pi}{2}\tfrac{\pi}{2})=\sum_{m_\alpha' m_\beta'} f_{a m_\alpha' b m_\beta'}(\beta\gamma\Delta)\mathcal{D}^{j_a}_{m_\alpha m_\alpha'}(0\tfrac{\pi}{2}\tfrac{\pi}{2})\mathcal{D}^{j_b}_{m_\beta m_\beta'}(0\tfrac{\pi}{2}\tfrac{\pi}{2}). \tag{2.52b}$$

Since the deformed potential well $H_g(\beta, \gamma + \frac{2}{3}\pi)$ is connected with $H_g(\beta, \gamma)$ by

$$H_g(\beta,\gamma) = R_{0\frac{\pi}{2}\frac{\pi}{2}}\, H_g(\beta,\gamma+\tfrac{2}{3}\pi) R^{-1}_{0\frac{\pi}{2}\frac{\pi}{2}} \tag{2.53}$$

the secular equation for $a_{\sigma q,\alpha}(\beta, \gamma + \frac{2}{3}\pi)$ can be written as

$$\sum_\alpha \Big[\sum_{m_\alpha'} a_{\sigma q, a m_\alpha'}(\beta,\gamma+\tfrac{2}{3}\pi)\mathcal{D}^{j_a}_{m_\alpha m_\alpha'}(0\tfrac{\pi}{2}\tfrac{\pi}{2})\Big]\langle a m_\alpha' | H_g(\beta,\gamma)| b m_\beta\rangle$$

$$= E_{\sigma q}(\beta,\gamma+\tfrac{2}{3}\pi)\Big[\sum_{m_\beta'} a_{\sigma q, b m_\beta'}(\beta,\gamma+\tfrac{2}{3}\pi)\mathcal{D}^{j_b}_{m_\beta m_\beta'}(0\tfrac{\pi}{2}\tfrac{\pi}{2})\Big]. \tag{2.54}$$

This is identical with eq. (2.39a). Therefore the energy eigenvalues of eq. (2.54) are equal to those of eq. (2.39a),

$$E_{\sigma q}(\beta,\gamma+\tfrac{2}{3}\pi) = E_{\sigma q}(\beta,\gamma), \tag{2.55}$$

and, moreover, we get

$$a_{\sigma q,\alpha}(\beta,\gamma+\tfrac{2}{3}\pi)=\sum_{m_\alpha'} a_{\sigma q, a m_\alpha'}(\beta,\gamma)\mathcal{D}^{j_a}_{m_\alpha m_\alpha'}(0\tfrac{\pi}{2}\tfrac{\pi}{2})\mathcal{D}^{j_b}_{m_\beta m_\beta'}(0\tfrac{\pi}{2}\tfrac{\pi}{2}) e^{i\frac{\pi}{2}q}, \tag{2.56}$$

where the phase factor, $e^{i\frac{\pi}{2}q}$, was inserted to guarantee that both of $a_{\sigma q,\alpha}(\beta, \gamma)$ and $a_{\sigma q,\alpha}(\beta, \gamma + \frac{2}{3}\pi)$ be real. From eqs. (2.19), (2.20), (2.23) and (2.55), we get

$$\begin{bmatrix} v_\sigma(\beta,\gamma+\frac{2}{3}\pi) \\ u_\sigma(\beta,\gamma+\frac{2}{3}\pi) \end{bmatrix} = \begin{bmatrix} v_\sigma(\beta,\gamma) \\ u_\sigma(\beta,\gamma) \end{bmatrix}. \tag{2.57}$$

If we substitute eqs. (2.56) and (2.57) into eqs. (2.52a) and (2.52b), the validity of the (2.51) will be seen.

Using eq. (2.51), an important property of the projected wave function can be obtained,

$$P_{IMK}\phi_0(\beta,\gamma+\tfrac{2}{3}\pi,\Delta)=\sum_{K'}\mathcal{D}^{I}_{KK'}(0\tfrac{\pi}{2}\tfrac{\pi}{2})P_{IMK}\phi_0(\beta,\gamma,\Delta). \tag{2.58}$$

In the above consideration concerning the symmetry properties, only the quadrupole nature of the deformation was used. Therefore the results obtained are valid for any other single particle field with quadrupole deformation than the anisotropic harmonic potential well.

According to the method of the generator coordinate, the wave function which describes the collective motion is expressed as superposition of the projected wave functions $\Phi_{IMK}(\beta,\gamma,\Delta)$ with various values of the deformation parameters and the energy gap parameter. If the generator function is denoted by $f_K^{I,n}(\beta,\gamma,\Delta)$, then the wave function is given by

$$\Psi^{n}_{IM}=\sum_K\int_0^{\beta_m}d\beta\int_0^{\Delta_m}d\Delta\int_0^{\frac{1}{3}\pi}d\gamma\;\Phi_{IMK}(\beta,\gamma,\Delta)f^{I,n}_K(\beta\gamma\Delta), \tag{2.59}$$

where Δ_m and β_m are taken to be sufficiently large. The integral domain of the variable γ is reduced from $[0, 2\pi]$ to $[0, \frac{1}{3}\pi]$ by using eqs. (2.47) and (2.58)[21]. The quantum number n specifies the mode of the collective motion. The generator function $f_K^{I,n}(\beta,\gamma,\Delta)$ is determined by making the following expectation value of the energy stationary with respect to any variation of the generator function:

$$E^{I}_{n}=\frac{\langle\Psi^{n}_{IM}|H|\Psi^{n}_{IM}\rangle}{\langle\Psi^{n}_{IM}|\Psi^{n}_{IM}\rangle}\;, \tag{2.60}$$

where H represents the Hamiltonian of the system of nucleons under consideration. The condition making the above expectation value of the energy stationary leads to the integral equation

$$\sum_{K'} \int d\beta' d\gamma' d\Delta' \left[H^{I}_{KK'}(\beta\gamma\Delta, \beta'\gamma'\Delta') - E^{I}_{n} N^{I}_{KK'}(\beta\gamma\Delta, \beta'\gamma'\Delta') \right] f^{I,n}_{K'}(\beta'\gamma'\Delta') = 0 . \tag{2.61}$$

The projected energy kernel $H^{I}_{KK'}(\beta\gamma\Delta, \beta'\gamma'\Delta')$ and overlap kernel $N^{I}_{KK'}(\beta\gamma\Delta, \beta'\gamma'\Delta')$ are defined by

$$\begin{bmatrix} H^{I}_{KK'}(\beta\gamma\Delta, \beta'\gamma'\Delta') \\ N^{I}_{KK'}(\beta\gamma\Delta, \beta'\gamma'\Delta') \end{bmatrix} = \langle \Phi_{IMK}(\beta\gamma\Delta) | \begin{bmatrix} H \\ 1 \end{bmatrix} | \Phi_{IMK'}(\beta'\gamma'\Delta') \rangle \tag{2.62}$$

When we solve the integral equation (2.61), the symmetry properties (2.47), (2.50) and (2.58) provide sufficient boundary conditions.

2-3. Calculation of Kernels and Matrix Elements

This subsection is devoted mainly to the evaluation of the two kernels defined in (2.62). To avoid complexity, we use the abbreviated notation in the following discussions:

$$\delta \equiv (\beta\gamma\Delta) , \quad \int d\beta \int d\gamma \int d\Delta \equiv \int d\delta , \quad \Omega \equiv (\varphi, \vartheta, \chi) .$$

If eqs. (2.33) and (2.34) into eq. (2.62), then the following expression is obtained by using the invariance of the Hamiltonian under rotation and the closure property of the D-function.[29]:

$$\begin{bmatrix} N^{I}_{KK'}(\delta,\delta') \\ H^{I}_{KK'}(\delta,\delta') \end{bmatrix} = \int d\Omega \, \mathcal{D}^{I*}_{KK'}(\Omega) \begin{bmatrix} \eta(\delta,\delta';\Omega) \\ \mathcal{H}(\delta,\delta';\Omega) \end{bmatrix}, \tag{2.63}$$

where

$$\begin{bmatrix} \eta(\delta,\delta';\Omega) \\ \mathcal{H}(\delta,\delta';\Omega) \end{bmatrix} = N(\delta) N(\delta') \langle 0 | \exp\left[\tfrac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta}(\delta) c_\beta c_\alpha\right] \begin{bmatrix} 1 \\ H \end{bmatrix} \exp\left[\tfrac{1}{2}\sum_{\alpha'\beta'} f_{\alpha'\beta'}(\delta';\Omega) C^+_{\alpha'} C^+_{\beta'}\right] | 0 \rangle \tag{2.64}$$

In eq. (2.64) $N(\delta)$ is the normalization constant given by

$$N(\delta) = \prod_{\sigma} U_\sigma(\delta).$$

It is now necessary to give the explicit form of the Hamiltonian. Assuming that the effective interaction is a two-body interaction which has no hard core, it is represented as

$$H = \sum_{\alpha,\beta} \epsilon_{\alpha\beta} C^+_\alpha C_\beta + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} C^+_\alpha C^+_\beta C_\delta C_\gamma. \tag{2.65}$$

The single particle energy is expressed by $\epsilon_{\alpha\beta}$, which may contain the kinetic energy and other single particle potential such as the spin-orbit force if necessary. The matrix element of the effective interaction is given by $V_{\alpha\beta\delta\gamma}$, which satisfies the following antisymmetric relation:

$$V_{\alpha\beta\delta\gamma} = -V_{\beta\alpha\delta\gamma} = -V_{\alpha\beta\gamma\delta} = V_{\beta\alpha\gamma\delta} \tag{2.66}$$

The matrix element, $n(\beta\gamma\Delta, \beta'\gamma'\Delta'; \phi\theta\chi)$ is evaluated as follows[39]:

$$n(\delta,\delta';\Omega) = N(\delta)N(\delta')\exp\left[\tfrac{1}{2}\mathrm{Tr}\{\ln(1+M(\delta,\delta';\Omega))\}\right], \tag{2.67}$$

where

$$M_{\alpha\beta}(\delta,\delta';\Omega) = -\sum_{\xi} f_{\alpha\xi}(\delta';\Omega) f_{\xi\beta}(\delta). \tag{2.68}$$

According to the method given in ref. 39), the other matrix element $h(\delta, \delta', \Omega)$ is given by

$$h(\delta,\delta';\Omega)/n(\delta,\delta';\Omega) = \sum \varepsilon_{\alpha\beta}\rho_{\alpha\beta}(\delta,\delta';\Omega)$$

$$+\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\left[\frac{1}{2}\rho_{\alpha\gamma}(\delta,\delta';\Omega)\rho_{\beta\delta}(\delta,\delta';\Omega) + \bar{\kappa}_{\alpha\beta}(\delta,\delta';\Omega)\kappa_{\gamma\delta}(\delta,\delta';\Omega)\right], \tag{2.69}$$

where the generalized density and pair metrices are defined by

$$\rho_{\alpha\beta}(\delta,\delta';\Omega) = \left[\frac{M(\delta,\delta';\Omega)}{1+M(\delta,\delta';\Omega)}\right]_{\alpha\beta}, \tag{2.70a}$$

$$\bar{\kappa}_{\alpha\beta}(\delta,\delta';\Omega) = \sum_{\xi} f_{\alpha\xi}(\delta)\left[\frac{1}{1+M(\delta,\delta';\Omega)}\right]_{\xi\beta}, \tag{2.70b}$$

$$\kappa_{\alpha\beta}(\delta,\delta';\Omega) = \sum_{\xi}\left[\frac{1}{1+M(\delta,\delta';\Omega)}\right]_{\alpha\xi} f_{\xi\beta}(\delta';\Omega). \tag{2.70c}$$

The energy kernel, $h(\delta, \delta'; \Omega)$, can be brought into a form convenient to the numerical computations using the technique developed by Baranger[5]. The matrix element of the effective interaction is written in two ways; the one is convenient to treat the pairing type interaction

$$V_{\alpha\beta\gamma\delta} = -\frac{1}{2}\sum_{JM} G(abcd;J)\langle j_a m_\alpha j_b m_\beta|JM\rangle\langle j_c m_\gamma j_d m_\delta|JM\rangle, \tag{2.71a}$$

and the other is useful for describing the particle-hole interaction

$$V_{\alpha\beta\gamma\delta} = -\frac{1}{2}\sum_{JM} F(acdb;J) S_\gamma \langle j_a m_\alpha j_c -m_\gamma|JM\rangle S_\beta \langle j_d m_\delta j_b -m_\beta|JM\rangle, \tag{2.71b}$$

where $S_\gamma = (-1)^{j_c - m_\gamma}$. Next the density and the pair matrices are expanded as follows:

$$\rho_{ab;JM}(\delta,\delta';\Omega) = \sum_{m_\alpha m_\beta} (-1)^{j_b - m_\beta}\langle j_a m_\alpha j_b -m_\beta|JM\rangle \rho_{\alpha\beta}(\delta,\delta';\Omega) \tag{2.72}$$

$$\kappa_{ab;JM}(\delta,\delta';\Omega) = \sum_{m_\alpha m_\beta} \langle j_a m_\alpha j_b m_\beta|JM\rangle \kappa_{\alpha\beta}(\delta,\delta';\Omega). \tag{2.73}$$

Then, the energy kernel is expressed as

$$h(\delta,\delta';\Omega) = \sum_{\alpha\beta} \varepsilon_{\alpha\beta}\rho_{ab;00}$$

$$+\sum_{JM}\left[-\frac{1}{4}F(abdc;J)\cdot(-1)^{j_c - j_d + M}\rho_{ab;JM}\rho_{cd;J-M} + \frac{1}{8}G(abcd;J)\bar{\kappa}_{ab;JM}\kappa_{cd;JM}\right]. \tag{2.74}$$

The process of the numerical calculation is as follows: After the single particle states in the deformed field (2.1) are obtained, the BCS equations are solved. Then, the intrinsic wave functions are constructed. The energy kernel and the overlap kernel are calculated as a function of β, γ, Δ and the Eulerian angles. From these kernels the projected overlap and energy kernels are calculated by eq. (2.63). When the integral equation (2.61) is solved, the eigenfunction and the corresponding energy eigenvalue E_n^I are obtained.

Since the intrinsic wave functions have various values of K, we are forced

to perform integration over all of the Eulerian angles. This procedure to evaluate the projected kernels requires a large amount of computation time. A method to reduce the computation time is developed by making use of some symmetry properties of the density and pair matrices, and explained in Appendix.

Once the wave function is obtained we can calculate the various nuclear quantities such as the magnetic moment, the quadrupole moment and the transition probability of electric and magnetic radiations. To calculate these quantities we need the matrix element of the general one-body tensor operator O^k_q, whose expression is given:

$$\langle \Psi^{n'}_{IM'} | O^k_q | \Psi^{n}_{IM} \rangle = \frac{8\pi^2}{2I'+1} \sum_{K_1,K_1'} \langle IM\, kq | I'M' \rangle \sum_{\mu,\nu} \langle I\mu\, k\nu | I'K_1' \rangle$$

$$\cdot \int \mathcal{D}^{I}_{\mu K_1}(\Omega)\, d\Omega \int f^{I,n'}_{K_1'}(\delta')\, f^{I,n}_{K_1}(\delta)\, d\delta\, d\delta' \langle a \| O^k \| b \rangle \rho_{ab;k\nu}(\delta',\delta;\Omega)\, \mathcal{N}(\delta',\delta;\Omega), \qquad (2.75)$$

where the reduced matrix element is given by

$$\langle a \| O^k \| b \rangle = \sqrt{2j_a+1} \sum_{m_\alpha q} \langle j_b\, m_\beta\, k\, q | j_a\, m_\alpha \rangle \langle j_a\, m_\alpha | O^k_q | j_b\, m_\beta \rangle \ . \qquad (2.76)$$

The method explained in Appendix to reduce the integral domain of the Eulerian angles can be used in the evaluation of the matrix element (2.75).

§3. Application of the GCM to a Schematic Pairing Vibration Problem

We apply the GCM to a system of n-particles moving in two non-degenerate j-shells and interacting via the pairing force. Being amenable to exact treatment, this two-level problem has been conveniently used to see how well various models work.

In this problem the Hamiltonian is given as

$$H=\sum_{i=1}^{2}\varepsilon_i c_i^{\dagger} c_i - \frac{G}{4}\sum_{i,j} c_i^{\dagger} c_{\bar{i}}^{\dagger} c_{\bar{j}} c_j \quad . \tag{3-1}$$

The energy of the system is obtained by solving the following integral equation,

$$\int [\langle \psi(\Delta)|H|\psi(\Delta')\rangle - E_n \langle \psi(\Delta)|\psi(\Delta')\rangle] f_n(\Delta')\, d\Delta' = 0, \tag{3-2}$$

where $f_n(\Delta)$ is the generator function corresponding to an energy eigenvalue E_n and $\psi(\Delta)$ is the BCS wave function of the generating Hamiltonian

$$H' = \sum_i (\varepsilon_i - \lambda) c_i^{\dagger} c_i - \frac{g}{4}\sum_{i,j} c_i^{\dagger} c_{\bar{i}}^{\dagger} c_{\bar{j}} c_j \, . \tag{3-3}$$

The corresponding wave function is given by

$$\Psi_n = \int \psi(\Delta) f_n(\Delta)\, d\Delta \, . \tag{3-4}$$

Expressions for the kernels are

$$\langle\psi(\Delta)|H|\psi(\Delta')\rangle = \langle\psi(\Delta)|\psi(\Delta')\rangle\Big[\sum_i \varepsilon_i \frac{v_i(\Delta)v_i(\Delta')}{u_i(\Delta)u_i(\Delta')+v_i(\Delta)v_i(\Delta')}$$

$$-\frac{G_1}{2}\sum_i\Big\{\frac{v_i(\Delta)v_i(\Delta')}{u_i(\Delta)u_i(\Delta')+v_i(\Delta)v_i(\Delta')}\Big\}^2-\frac{G_1}{4}\sum_{i\neq j}\frac{v_i(\Delta)u_i(\Delta')}{u_i(\Delta)u_i(\Delta')+v_i(\Delta)v_i(\Delta')}\cdot\frac{v_j(\Delta')u_j(\Delta)}{u_j(\Delta)u_j(\Delta')+v_j(\Delta)v_j(\Delta')}\Big], \tag{3-5}$$

$$\langle\psi(\Delta)|\psi(\Delta')\rangle = \prod_i\left(u_i(\Delta)u_i(\Delta')+v_i(\Delta)v_i(\Delta')\right). \tag{3-6}$$

Numerical calculations are made for the case of $j_1 = j_2 = 19/2$, $N = 20$ and $\varepsilon_2 - \varepsilon_1 = 0.5$ MeV. The excitation energy, $E_1 - E_0$, is plotted against G in Fig. 1 and the generator functions, $f_n(\Delta)$, are illustrated in Fig. 2. In Fig. 1 predictions of the linearization method are also given.

It can be seen that the GCM works well enough except the region where the energy gap corresponding to the Hamiltonian (3-1) takes very small values, while the linearization method breaks down in a physically interesting region where the energy gap assumes about 0.6 MeV. Since the values of energy gap lie $0.5 \sim 1.5$ MeV in nuclei far from the double closed shell, the GCM seems a very satisfying method to treat those nuclei.

§4. Energy Surface of Cd^{116}

In this section the method developed so far is applied to an actual nucleus. Because of the restricted memory capacity of the computers we can use, the full calculation is impossible at present. We compute the intrinsic energy

$$E(\beta,\gamma,\Delta)=\langle\phi_0(\beta,\gamma,\Delta)|H|\phi_0(\beta,\gamma,\Delta)\rangle\,, \qquad (4\text{-}1)$$

which has a simple form

$$=\sum_{\alpha\beta}\varepsilon_{\alpha\beta}\rho_{ab;00}+\sum_{JM}\left[-\frac{1}{4}F(abdc;J)(-1)^{j_c-j_d+M}\rho_{ab;JM}\rho_{cd;J-M}+\frac{1}{8}G(abcd;J)\kappa_{ab;JM}\kappa_{cd;JM}\right] \qquad (4\text{-}2)$$

The first term of eq.(4-2) is the single particle energy part. The second term is the contribution from the particle-hole interaction and the third term is that from the particle-particle interaction.

The values of the quadrupole deformation parameters, β and γ, and the energy gaps, Δ_p and Δ_n, at which the intrinsic energy is minimum can be interpreted to give the equilibrium deformation of the nucleus considered. The corresponding wave function is expected to be a pretty good approximation of that obtained with the Hartree-Bogoliubov calculation[40]. Moreover, the dependence of the energy surface, $E(\beta,\gamma,\Delta)$, on the parameters in the neighbourhood of the equilibrium gives us information concerning the nature of the vibration, because the energy surface is reasonably interpreted as the potential energy of the deformations.

The calculation is carried out for Cd^{116}. As mentioned in Sec.1, this nucleus has long been considered from the energy

spectrum as a typical spherical nucleus making harmonic vibration about spherical shape. However, the recently observed quadrupole moment of the 1st excited state is quite large, which reveals the existence of large anharmonic effects. Therefore it is important to investigate the nature of the existing anharmonicity by calculating the energy surface.

A critical point is the choice of the nucleon-nucleon interaction. Our formalism can be used for any nucleon-nucleon interaction which has no hard core. We choose the interaction used by Dietrich et al.[40] which is of Gaussian shape with reasonable range and of Rosenfeld exchange character. This interaction was used in the self-consistent calculation and succeeded in explaining the gradual appearance and disappearance of deformations as a function of particle number. We do not fix the strength of the interaction to the one used in ref.40), but vary it so that the energetically most favourable values of Δ_n and Δ_p may be close to experimental ones,

$$\Delta_n \sim 1.4 \text{ MeV}$$

$$\Delta_p \sim 1.6 \text{ MeV}.$$

Then the strength turns out to be two thirds of the one in ref.40). The parameters of the interaction are tabulated in Table I.

We include only the kinetic energy in the single particle energy matrix elements

$$\varepsilon_{\alpha\beta} = \langle \alpha | T | \beta \rangle$$

The phenomenological shifts of the single particle energy are

necessary in the calculation taking account of only one major shell[28], where the kinetic energy is independent of deformation. In the present investigation all single-particle states of principal quantum number $N=4,5$ for neutrons and $N=3,4$ for protons are included.

The condition of volume conservation (2-9) is not considered and the oscillator frequency is kept fixed for all values of deformation parameters. This approximation, whose main effect is some reduction of the kinetic energy at large deformation, is not at all serious in Cd^{116}, because the equilibrium deformation is spherical.

The intrinsic wave functions are constructed according to the procedure described in subsections (2-1) and (2-2) with the parameters written in Table II. The results of the calculation are given in Table III. They may be summarized as follows:

1) The kinetic energy part increses with increasing energy gaps as illustrated in Fig.3. This fact is easy to understand, because the diffuseness of the Fermi level means the increase in the number of particles in the upper shell.

2) The particle-particle interaction part decreases with increasing energy gaps as shown in Fig.4. This may be a reason why the so-called pairing force model works well.

3) The particle-hole interaction part decreases also with increasing energy gaps as shown in Fig.5, but the variation is small (~10%) compared with that of the particle-particle interaction.

Though the illustrations (Figs.3,4 and 5) are made at $\beta=0$ as examples, these characteristics are common to all deformation.

4) The most favourable values of the energy gap parameters are determined mainly by the kinetic energy and the particle-particle interaction parts. The total energy varies very slowly with energy gaps in the neighbourhood of a minimum (Fig.6). Therefore the energy surface as a function of quadrupole deformation parameters can be obtained with reasonable accuracy with the energy gap parameters determined through the eq.(2-15) in which the strengths of the pairing forces are fixed as $G_n=23.0/A$, $G_p=28.0/A$.

5) The kinetic energy part increases with increasing deformation parameter β (Fig.8).

6) The particle-particle and the particle-hole interaction parts decreases with increasing β, and the variations are comparable (Fig.9).

The conclusion of the present calculation is that the equilibrium shape of Cd^{116} is spherical and that the anharmonicity is γ-dependent(Figs.7a - 7d).

§5. Discussion

The calculation carried out in the preceding section surpasses the past calculations in that it uses the realistic nucleon-nucleon interaction with natural radial dependence and exchange character and at the same time allows for axially non-symmetric deformation.

The main results of the calculations made in Sects.3 and 4 may be summarized as follows:

1) The mode of pairing vibration can be treated successfully in the framework of the method developed in this paper in the case that the equilibrium values of the energy gaps are large. Otherwise, the quasi-particle excitations, which are not included in our BCS configurations, will be important.

2) The equilibrium values of the energy gaps are mainly determined as a result of the competition of the kinetic energy and the particle-particle interaction whereas the particle-hole interaction plays a negligible role. The dependence of the total energy on the gap parameters are very small.

3) It is not only the particle-hole- but particle-particle interaction that gaines energy by quadrupole deformation.

4) The equilibrium shape of Cd^{116} is spherical and the calculated energy surface is consistent with the energy spectrum. The deviations from the phonon spectrum may be attributed to the γ-dependent anharmonicity.

The core consisting of all nucleons in the lower orbits was assumed to be spherically symmetric in Sec.4. This is only an assumption. This assumption is of course not realistic and in the strongly deformed nucleus this leads to too small quadrupole

moments. The problem of how to treat the core more accurately is left for the future.

The preceding calculations were performed by making use of the computor HITAC 5020E at the University of Tokyo. The computing time required is about 5 minuites for each set of parameters.

Acknowledgements

The auther would like to express hearty thanks to Professor S. Yoshida for suggesting this problem and for valuable discussions and guidance. He is greatly indebted to Professor M. Muraoka for critical discussions and careful reading of the manuscript. He also expresses sincere thanks to Dr. N. Onishi for valuable discussions to and to Professor M. Kawai for reading the manuscript.

Appendix

The method developed to reduce the integral domain of the Eulerian angles is explained.

Operating a factor depending on ϕ among the three factors in $R_{\phi\theta\chi}$ onto the bra vector in eq. (2.32), the matrix M defined in (2.68) can be rewritten as

$$M_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)=-\sum_{\xi} F_{\alpha\xi}(\delta';\vartheta\chi)G_{\xi\beta}(\delta;\varphi), \tag{A.1}$$

where

$$F_{\alpha\xi}(\delta;\vartheta\chi)\equiv f_{\alpha\xi}(\delta;0,\vartheta,\chi), \tag{A.2}$$

$$G_{\xi\beta}(\delta;\varphi)\equiv f^{*}_{\xi\beta}(\delta;-\varphi,0,0). \tag{A.3}$$

Defining a matrix $M^{(1)}$ by

$$M^{(1)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)=-\sum_{\xi} F^{*}_{\alpha\xi}(\delta';\vartheta\chi)G_{\xi\beta}(\delta;\varphi), \tag{A.4}$$

another kind of the density and pair matrices connected with $M^{(1)}$ are introduced:

$$\rho^{(1)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi)=\left[\frac{M^{(1)}(\delta,\delta';\varphi\vartheta\chi)}{1+M^{(1)}(\delta,\delta';\varphi\vartheta\chi)}\right]_{\alpha\beta}, \tag{A.5}$$

$$\kappa^{(1)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)=\left[\frac{1}{1+M^{(1)}(\delta,\delta';\varphi\vartheta\chi)}F^{*}(\delta';\vartheta\chi)\right]_{\alpha\beta}, \tag{A.6}$$

$$\bar{\kappa}^{(1)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)=\left[G(\delta;\varphi)\frac{M^{(1)}(\delta,\delta';\varphi\vartheta\chi)}{1+M^{(1)}(\delta,\delta';\varphi\vartheta\chi)}\right]_{\alpha\beta}. \tag{A.7}$$

It can be easily seen from the definitions of $f_{\alpha\beta}$, (2.28), that there are relations

$$G_{1\alpha\beta}(\delta;2\pi-\varphi)=G_{1\alpha\beta}(\delta;\pi-\varphi)=G^{*}_{1\alpha\beta}(\delta;\varphi), \tag{A.8}$$

$$F_{\alpha\beta}(\delta;\vartheta,2\pi-\chi)=F_{\alpha\beta}(\delta;\vartheta,\pi-\chi)=F^{*}_{\alpha\beta}(\delta;\vartheta,\chi), \tag{A.9}$$

$$F_{\alpha\beta}(\delta;\pi-\vartheta,\chi)=(-1)^{m_\alpha+m_\beta}F^{*}_{\alpha\beta}(\delta;\vartheta,\chi). \tag{A.10}$$

Using eqs. (A.8) $\sim$ (A.10), the properties of M and $M^{(1)}$ are verified,

$$M_{\alpha\beta}(\delta,\delta';2\pi-\varphi,\vartheta,\chi)=M_{\alpha\beta}(\delta,\delta';\pi-\varphi,\vartheta,\chi)=M^{(1)*}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi), \tag{A.11}$$

$$M_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,2\pi-\chi)=M_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi)=M^{(1)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi). \tag{A.12}$$

If we put eqs. (A.8) $\sim$ (A.12) into eqs. (2.70a) $\sim$ (2.70c) and (A.5) $\sim$ (A.7), then the following properties of the two kinds of density and pair matrices are verified;

$$\rho_{\alpha\beta}(\delta,\delta';2\pi-\varphi,\vartheta,\chi)=\rho_{\alpha\beta}(\delta,\delta';\pi-\varphi,\vartheta,\chi)=\rho^{(1)*}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi), \tag{A.13}$$

$$\rho_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,2\pi-\chi)=\rho_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\pi-\chi)=\rho^{(1)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi), \tag{A.14}$$

$$\kappa_{\alpha\beta}(\delta,\delta';2\pi-\varphi,\vartheta,\chi)=\kappa_{\alpha\beta}(\delta,\delta';\pi-\varphi,\vartheta,\chi)=\kappa^{(1)*}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi), \tag{A.15}$$

$$\kappa_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,2\pi-\chi)=\kappa_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\pi-\chi)=\kappa^{(1)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi) \tag{A.16}$$

$$\bar{\mathcal{N}}_{\alpha\beta}(\delta,\delta';2\pi-\varphi,\vartheta,\chi)=\bar{\mathcal{N}}_{\alpha\beta}(\delta,\delta';\pi-\varphi,\vartheta,\chi)=\bar{\mathcal{N}}^{(i)*}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi), \tag{A.17}$$

$$\bar{\mathcal{N}}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,2\pi-\chi)=\bar{\mathcal{N}}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\pi-\chi)=\bar{\mathcal{N}}^{(i)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi). \tag{A.18}$$

Eqs. (A.13) $\sim$ (A.18) are sufficient to reduce the integral domain of the variables ϕ and χ from $[0,2\pi]$ to $[0,\frac{\pi}{2}]$. Here we define the second kind of the overlap and energy kernels,

$$\mathcal{N}^{(i)}(\delta,\delta';\varphi\vartheta\chi)=N(\delta)N(\delta')\exp\left[\tfrac{1}{2}\mathrm{Tr}\left\{\ln(1+M^{(i)}(\delta,\delta';\varphi\vartheta\chi))\right\}\right], \tag{A.19}$$

$$\begin{aligned}\mathcal{K}^{(i)}(\delta,\delta';\varphi\vartheta\chi)&\equiv\mathcal{H}^{(i)}(\delta,\delta';\varphi\vartheta\chi)/\mathcal{N}^{(i)}(\delta,\delta';\varphi,\vartheta,\chi)\\&=\sum_{\alpha\beta}\varepsilon_{\alpha\beta}\rho^{(i)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)+\sum_{\alpha\beta\gamma\delta}V_{\alpha\beta\gamma\delta}\left[\tfrac{1}{2}\rho^{(i)}_{\alpha\gamma}(\delta,\delta';\varphi\vartheta\chi)\rho^{(i)}_{\beta\delta}(\delta,\delta';\varphi\vartheta\chi)+\tfrac{1}{4}\bar{\mathcal{N}}^{(i)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)\mathcal{N}^{(i)}_{\gamma\delta}(\delta,\delta';\varphi\vartheta\chi)\right]\end{aligned} \tag{A.20}$$

Then, the projected kernels of the integral eq. (2.61) is written as

$$\begin{aligned}&H^{I}_{KK'}(\delta,\delta')-E^{I}_{n}N^{I}_{KK'}(\delta,\delta')\\&=2\int_0^{\pi}\sin\vartheta\,d\vartheta\,d^{I}_{KK'}(\vartheta)\Big[\int_0^{\pi/2}d\varphi\,e^{iK\varphi}\int_0^{\pi/2}d\chi\Big\{(\mathcal{K}(\delta,\delta';\varphi\vartheta\chi)-E^{I}_{n})\mathcal{N}(\delta,\delta';\varphi\vartheta\chi)e^{iK'\chi}\\&+(\mathcal{K}^{(i)}(\delta,\delta';\varphi\vartheta\chi)-E^{I}_{n})\mathcal{N}^{(i)}(\delta,\delta';\varphi\vartheta\chi)e^{-iK'\chi}\Big\}+\mathrm{c.c.}\Big].\end{aligned} \tag{A.21}$$

Then we make the reduction concerning the variable θ. First, let us prove the following equation,

$$M_{\alpha\beta}(\delta,\delta';\varphi,\pi-\vartheta,\chi)=(-1)^{m_\alpha-m_\beta}M^{(i)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi). \tag{A.22}$$

Proof: From the definition the left-hand side of eq. (A.22) is written as

$$M_{\alpha\bar{\beta}}(\delta,\delta';\varphi,\pi-\vartheta,\chi)=-\sum_{\xi}F_{\alpha\xi}(\delta';\pi-\vartheta,\chi)G_{\xi\beta}(\delta;\varphi),$$

making use of eq. (A.10),

$$=-\sum_{\xi}(-1)^{m_\alpha+m_\xi}F^{*}_{\alpha\bar{\xi}}(\delta';\vartheta,\chi)G_{\xi\beta}(\delta;\varphi)$$

$$=(-1)^{m_\alpha-m_\beta}M^{(1)}_{\alpha\beta}(\delta,\delta';\varphi,\vartheta,\chi),$$

$\because\ m_\xi+m_\beta=$ even . for non-vanishing $G_{\xi\beta}$.

Thus eq. (A.22) is directly proved.

Then it is easy to prove the relation

$$\rho_{\alpha\beta}(\delta,\delta';\varphi,\pi-\vartheta,\chi)=(-1)^{m_\alpha-m_\beta}\rho^{(1)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi). \tag{A.23}$$

Proof: We expand the left-hand side of eq. (A.23) in a power series,

$$\rho_{\alpha\beta}(\delta,\delta';\varphi,\pi-\vartheta,\chi)=\sum_{n=1}^{\infty}(-1)^n\sum_{k_1,\cdots,k_{n-1}}M_{\alpha k_1}(\delta,\delta';\varphi,\pi-\vartheta,\chi)M_{k_1k_2}(\delta,\delta';\varphi,\pi-\vartheta,\chi)\cdots M_{k_{n-1}\beta}(\delta,\delta';\varphi,\pi-\vartheta,\chi)$$

then inserting the relation (A.22) into the right-hand side,

$$=\sum_{n=1}^{\infty}(-1)^n\sum_{k_1\cdots k_{n-1}}(-1)^{m_\alpha-m_{k_1}+m_{k_1}-m_{k_2}+\cdots-m_\beta}M^{(1)}_{\alpha k_1}(\delta,\delta';\varphi\vartheta\chi)M^{(1)}_{k_1k_2}(\delta,\delta';\varphi\vartheta\chi)\cdots M^{(1)}_{k_{n-1}\beta}(\delta,\delta';\varphi\vartheta\chi)$$

$$=(-1)^{m_\alpha-m_\beta}\rho^{(1)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi).$$

Thus the proof is completed. We can prove the similar relations of the pair matrices in just the same way,

$$\bar{\kappa}_{\alpha\beta}(\delta,\delta';\varphi,\pi-\vartheta,\chi)=(-1)^{m_\alpha+m_\beta}\bar{\kappa}^{(i)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi), \tag{A.24}$$

$$\kappa_{\alpha\beta}(\delta,\delta';\varphi,\pi-\vartheta,\chi)=(-1)^{m_\alpha+m_\beta}\kappa^{(i)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi). \tag{A.25}$$

The last step is to prove the relations

$$k(\delta,\delta';\varphi,\pi-\vartheta,\chi)=k^{(i)}(\delta,\delta';\varphi,\vartheta,\chi), \tag{A.26}$$

$$n(\delta,\delta';\varphi,\pi-\vartheta,\chi)=n^{(i)}(\delta,\delta';\varphi,\vartheta,\chi). \tag{A.27}$$

The proof of the second relation is easy. We prove here the first one.

Proof: Using eqs. (A.23) $\sim$ (A.25) the left-hand side of eq. (A.26) can be written as

$$k(\delta,\delta';\varphi,\pi-\vartheta,\chi)=\sum\varepsilon_{\alpha\beta}\rho^{(i)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)(-1)^{m_\alpha-m_\beta}$$

$$+\sum_{\alpha\beta\gamma\delta}V_{\alpha\beta\gamma\delta}\Big\{\frac{1}{2}(-1)^{m_\alpha-m_\delta}\rho^{(i)}_{\alpha\gamma}(\delta,\delta';\varphi\vartheta\chi)(-1)^{m_\beta-m_\delta}\rho^{(i)}_{\beta\delta}(\delta,\delta';\varphi\vartheta\chi)$$

$$+\frac{1}{4}(-1)^{m_\alpha+m_\beta}\bar{\kappa}^{(i)}_{\alpha\beta}(\delta,\delta';\varphi\vartheta\chi)(-1)^{m_\gamma+m_\delta}\kappa^{(i)}_{\delta\gamma}(\delta,\delta';\varphi,\vartheta,\chi).$$

Since the matrix elements $\varepsilon_{\alpha\beta}$ and $V_{\alpha\beta\gamma\delta}$ conserve the magnetic quantum number, all the phase factors appearing in the right-hand side disappear, and eq. (A.26) is shown to be valid.

All the preparations are completed. The kernel in the integral equation becomes

$$\left[H^{I}_{KK'}(\delta,\delta')-E^{I}_{n}N^{I}_{KK'}(\delta,\delta')\right]=2\int_0^{\pi/2}\sin\vartheta\,d\vartheta\left[d^{I}_{KK'}(\vartheta)A_{KK'}(\vartheta)+(-1)^{I}d^{I}_{K-K'}(\vartheta)A_{K-K'}(\vartheta)\right], \tag{A.28}$$

where

$$\mathcal{A}_{KK'}(\vartheta) \equiv \int_0^{\pi/2} d\varphi\, e^{iK\varphi} \int_0^{\pi/2} d\chi \Big[(k(\delta,\delta';\varphi\vartheta\chi) - E_n^I)\, n(\delta,\delta';\varphi\vartheta\chi) e^{iK'\chi}$$

$$+ (k^{(0)}(\delta,\delta';\varphi\vartheta\chi) - E_n^I)\, n^{(1)}(\delta,\delta';\varphi\vartheta\chi) e^{-iK'\chi} + \text{c.c.} \Big] . \qquad \text{(A.29)}$$

The computation time may be largely reduced by using this method.

References

1) A. Bohr, Mat. Fys. Medd. Dan. Vid. Selsk. 26, No. 14 (1952)

2) A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. 27, No. 16 (1953)

3) L. Wilets and M. Jean, Phys. Rev. 102 (1956) 788

4) O. Nathan and S. G. Nilsson, in Alpha-Beta and Gamma-Ray Spectroscopy, Vol. 1, edited by K. Siegbahn (North-Horlland Publ. Co., Amsterdam, 1965)

5) M. Baranger, Phys. Rev. 120 (1960) 957
S. Yoshida, Nuclear Physics 38 (1962) 380

6) A. M. Lane, Nuclear Theory, (Benjamin, New York, 1964), Chapter 8

7) D. H. Stelson, W. T. Milner, J. L. C. Ford, Jr., F. K. McGowan, and R. L. Robinson, Bull. Am. Phys. Soc. 10 (1965) 427
R. G. Stokstad and I. Hall, Bull. Am. Phys. Soc. 12 (1967) 35
J. J. Simpson, D. Eccleshall, M. J. L. Yates and N. J. Freeman, Nuclear Physics A94 (1967) 177
R. G. Stokstad, I. Hall, G. D. Symons and J. deBoer, Nuclear Physics A92 (1967) 319
A. Kleinfeld, R. Covello-Moro, J. deBoer and H. P. Lie, Bull. Am. Phys. Soc. 12 (1967) 564

8) T. Tamura and T. Udagawa, Phys. Rev. 150 (1966) 783

9) M. Muraoka, JAERI 1158 (1968) 71
T. Kishimoto and K. Ikeda, Progr. Theor. Phys. 40 (1968) 499
B. Sorensen, Nuclear Physics A97 (1967) 1

10) D. R. Inglis, Phys. Rev. 96 (1954) 1059

11) S. G. Nilsson and O. Prior, Mat. Fys. Medd. Dan. Vid. Selsk. 32 No. 16 (1960)

12) A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. 27 No. 16 (1953)
R. Sheline, Rev. Mod. Phys. 32 (1960) 1

T. Udagawa and R. Sheline, Phys. Rev. 147 (1966) 647

13) J. Krumlinde, Nuclear Physics A121 (1968) 306

14) B. R. Mottelson and J. G. Valatin, Phys. Rev. Letters 5 (1960) 511

15) M. Sano and M. Wakai, Nuclear Physics 67 (1965) 481; A90 (1967) 298

K. Y. Chan and J. G. Valatin, Nuclear Physics 82 (1966) 222

J. Krumlinde, Nuclear Physics A121 (1968) 306

16) S. M. Harris, Phys. Rev. Letters 13 (1964) 633

S. M. Harris, Phys. Rev. 138 (1965) B509

17) E. R. Marshalek, Phys. Rev. 139 (1965) B770

E. R. Marshalek and J. B. Milazzo, Phys. Rev. Letters 16 (1966) 190

18) H. Morinaga, Phys. Rev. 101 (1956) 254

E. B. Carter, G. E. Mitchell and R. H. Davis, Phys. Rev. 133 (1964) B1421

19) M. Baranger, in 1962 Cargese Lectures in Theoretical Physics, edited by M. Levy (Benjamin, New York, 1963)

20) M. Baranger and K. Kumar, Nuclear Physics 62 (1965) 113

M. Baranger and K. Kumar, Nuclear Physics A110 (1968) 490

K. Kumar and M. Baranger, Nuclear Physics A110 (1968) 529

M. Baranger and K. Kumar, preprint

K. Kumar and M. Baranger, preprint

21) K. Kumar and M. Baranger, Nuclear Physics A92 (1967) 608

K. Kumar, Nuclear Physics A92 (1967) 653

22) L. S. Kisslinger and R. A. Sorensen, Mat. Fys. Medd. Dan. Vid. Selsk. 32 No. 9 (1960)

23) J. J. Griffin and J. A. Wheeler, Phys. Rev. 108 (1957) 311

24) R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. (London) A70 (1957) 381

J. Yoccoz, Proc. Phys. Soc. (London) A70 (1957) 388

25) R. E. Peierls and D. J. Thouless, Nuclear Physics 38 (1962) 154

26) B. Jancovici and D. H. Schiff, Nuclear Physics 58 (1964) 678

27) N. Onishi and S. Yoshida, Nuclear Physics 80 (1966) 367

28) T. Une, Ph. D. Thesis, Osaka University, 1967, to be published

29) F. Villars, in Proceedings of the International School of Physics "Enrico Fermi", Course XXXVI, edited by C. Bloch (Academic Press, New York and London, 1966)

30) S. G. Nilsson, Dan. Mat. Fys. Medd. 29 No. 16 (1955)

31) A. Bohr, B. R. Mottelson and D. Pines, Phys. Rev. 110 (1958) 936

32) S. T. Belyaev, Mat. Fys. Medd. Dan. Vid. Selsk. 31 No. 11 (1959)

33) H. Ikegami and T. Udagawa, Phys. Rev. 124 (1961) 1518
S. Yoshida, Phys. Rev. 123 (1961) 2122

34) J. Högaasen-Feldman, Nuclear Physics 28 (1961) 258

35) D. R. Bes and R. A. Broglia, Nuclear Physics 80 (1966) 289

36) A. Bohr, Invited talk given at the International Symposium on Nuclear Structure, Dubna, July 1968.
O. Nathan, Invited talk given at the International Symposium on Nuclear Structure, Dubna, July 1968.

37) B. J. Verhaar, Nuclear Physics 41 (1963) 53; 45 (1963) 129; 54 (1964) 641

38) D. M. Brink and G. R. Satchler, Angular Momentum, second edition (Oxford University Press, 1968)

39) S. Yoshida, INS-PT-17 (1966)

40) K. Dietrich, H. J. Mang, and J. Pradal, Zeitschrift für Physik 190 (1966) 357

Figure Captions

Figure 1. Energy of the 1st excited state relative to the ground state is plotted against the pairing force strength G in eq.(3-1). The solid curve is the result of the GCM, the dashed curve is the one of the exact calculation and the dash-dot curve is the one of the linearization method.

Figures 2a-2g. The generating function, f(g), is plotted against g for the value of G written in the figures.

Figure 3. The kinetic energy part of eq.(4-2) in MeV versus the energy gaps at $\beta=0$.

Figure 4. The particle-particle interaction part of eq.(4-2) in MeV versus the energy gaps at $\beta=0$.

Figure 5. The particle-hole interaction part of eq.(4-2) in MeV versus the energy gaps at $\beta=0$.

Figure 6. The total energy of eq.(4-2) in MeV versus the energy gaps at $\beta=0$.

Figure 7a-7d. The energy surfaces are plotted as a function of β at each mesh of γ, 0°, 20°, 40° and 60°. The lowest one among the energies tabulated in Table III at each set of β and γ are adopted.

Figures 8-10. Plot of the kinetic energy part, the particle-particle interaction- and the particle-hole interaction part against β with $\gamma=0$, respectively.

Table Captions

Table I. Parameters of the interaction used in the calculation of Sec.4. The interaction is written as

$$V = (V_W + V_B P_\sigma - V_M P_\tau P_\sigma - V_H P_\tau) v(r),$$

$$v(r) = \exp[-r^2/r_0^2],$$

with obvious notations.

Table II. Parameters used in the construction of the intrinsic wave functions in Sec.4. κ and μ is of the same meaning as in ref.30).

Table III. Results of the calculation carried out in Sec.4. The columns β and γ give the values of the deformation parameters at which the computation is performed. The columns G'_n and G'_p give the pairing force parameters times mass number for neutrons and protons respectively, which are used in the construction of the intrinsic wave functions, and the columns Δ_n and Δ_p give the corresponding energy gaps. The columns E_k, E_{ph}, E_{pp} and E_t give the kinetic energy-, the particle-hole interaction-, the particle-particle interaction part and the total energy in MeV, respectively.

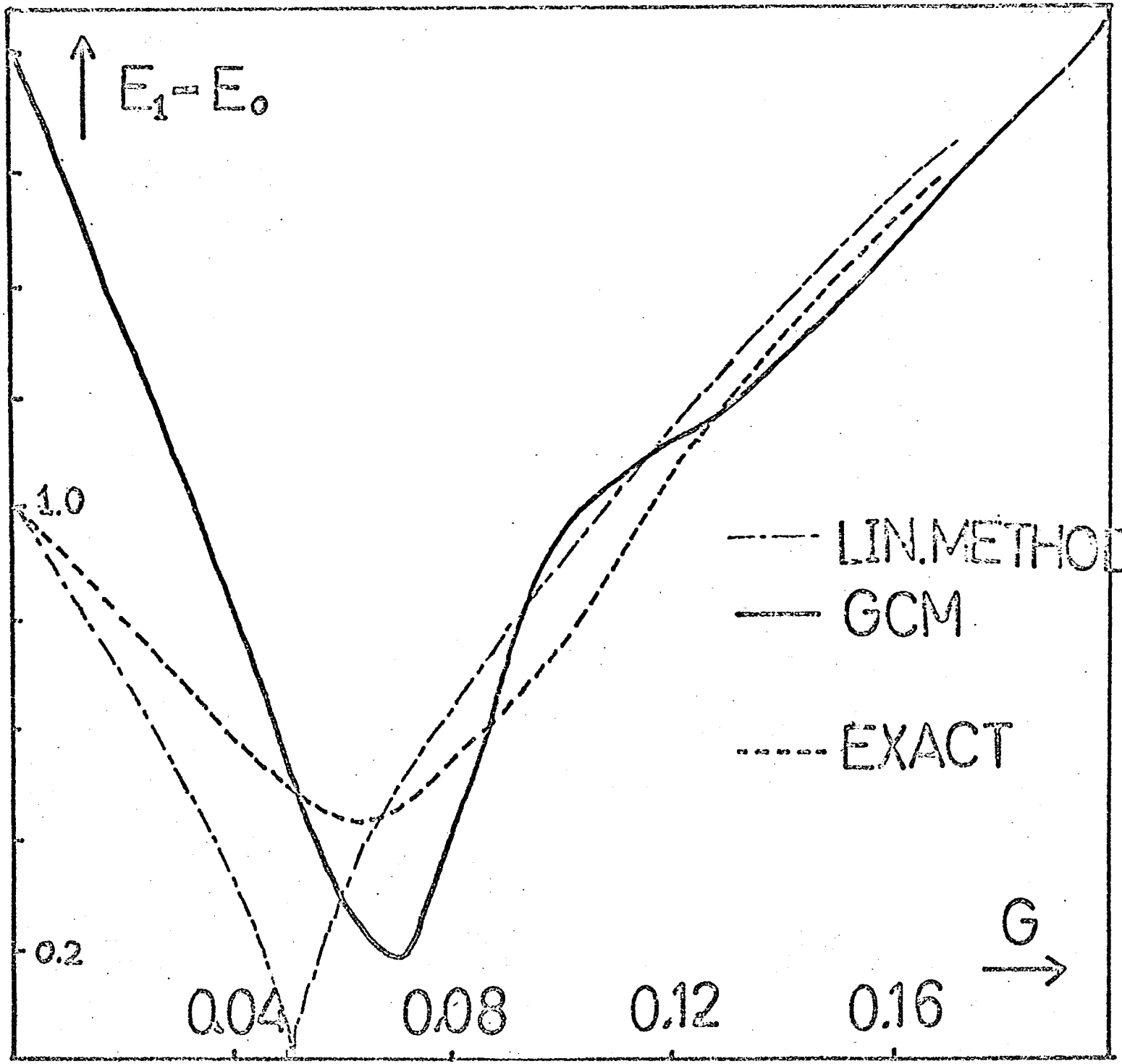


Fig.1

Fig. 2a.

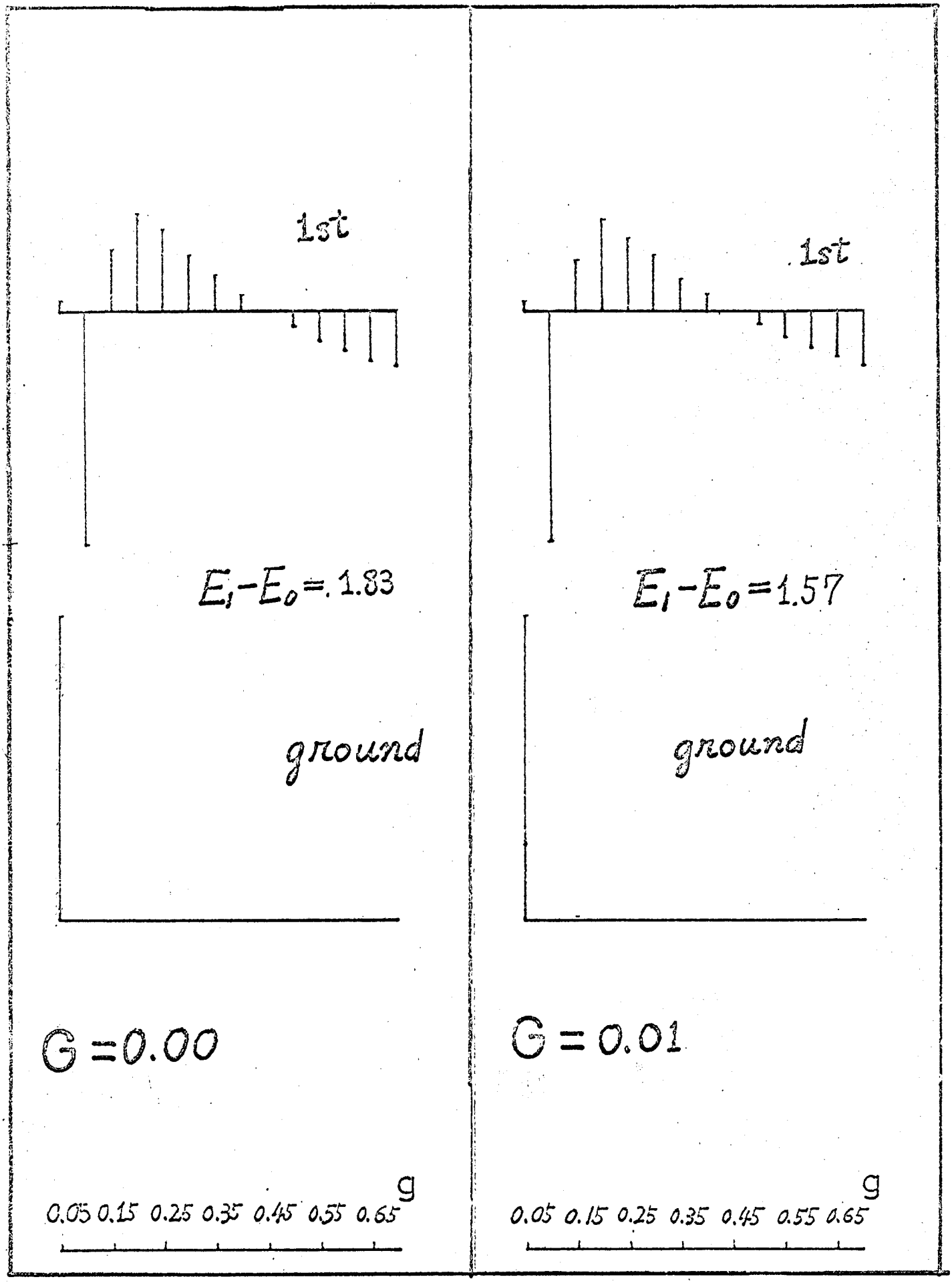
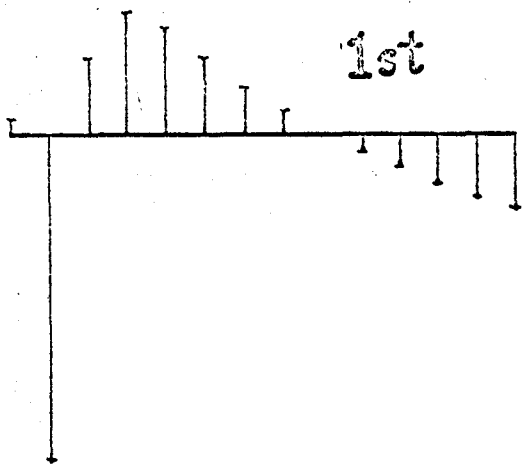
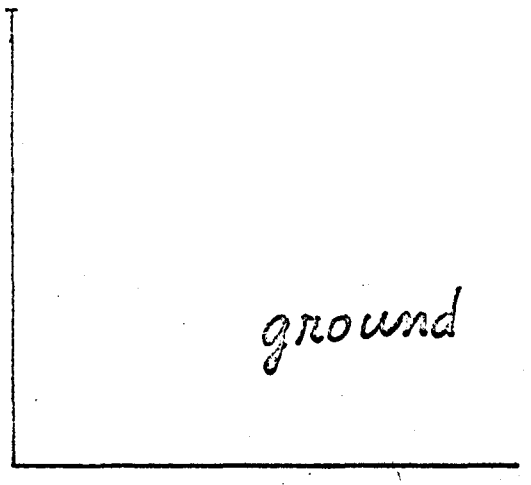

1st
$E_1 - E_0 = 1.83$
ground
G = 0.00
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65
1st
$E_1 - E_0 = 1.57$
ground
G = 0.01
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65

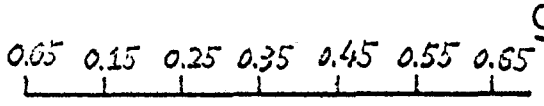
Fig. 2b.

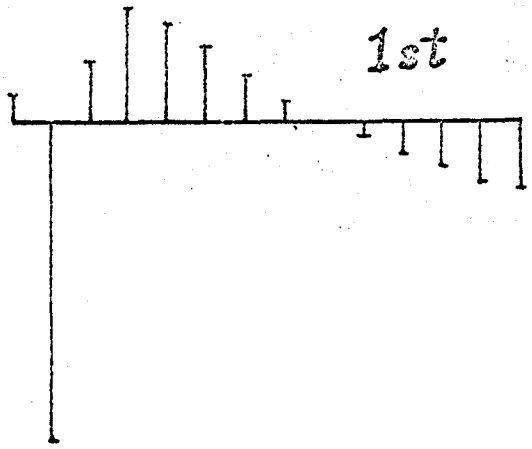


$E_1 - E_0 = 1.32$

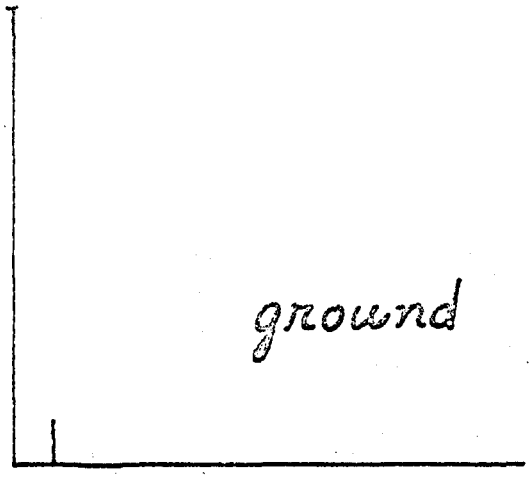


G = 0.02





$E_1 - E_0 = 1.06$



G = 0.03

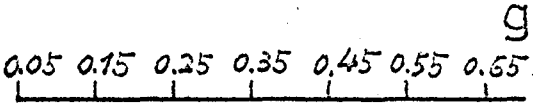


Fig. 2c.

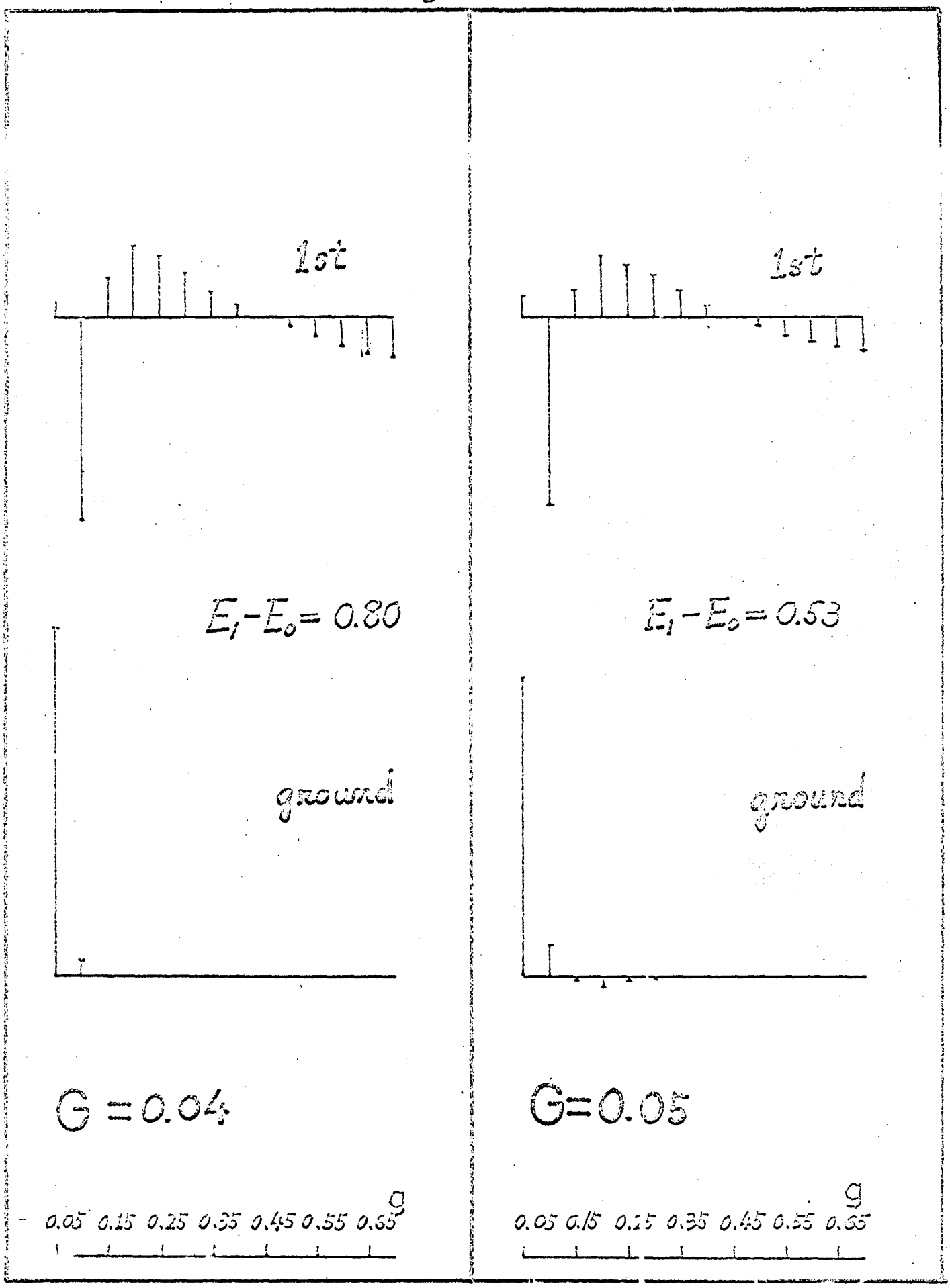

1st
$E_1 - E_0 = 0.80$
ground
G = 0.04
0.05 0.15 0.25 0.35 0.45 0.55 0.65 g
1st
$E_1 - E_0 = 0.53$
ground
G = 0.05
0.05 0.15 0.25 0.35 0.45 0.55 0.65 g

Fig. 2d.

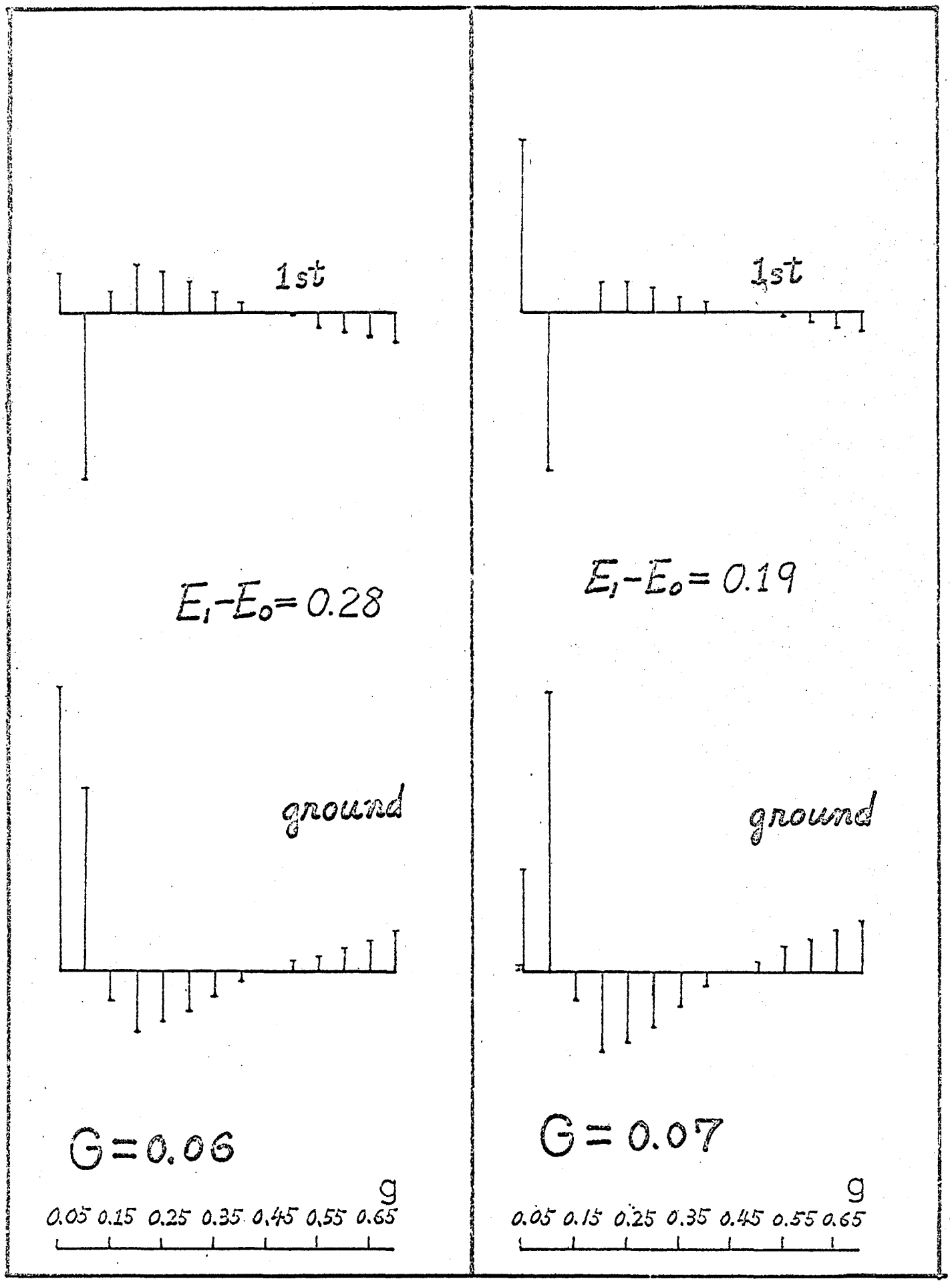

1st
$E_1 - E_0 = 0.28$
ground
G = 0.06
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65
1st
$E_1 - E_0 = 0.19$
ground
G = 0.07
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65

Fig.2e.

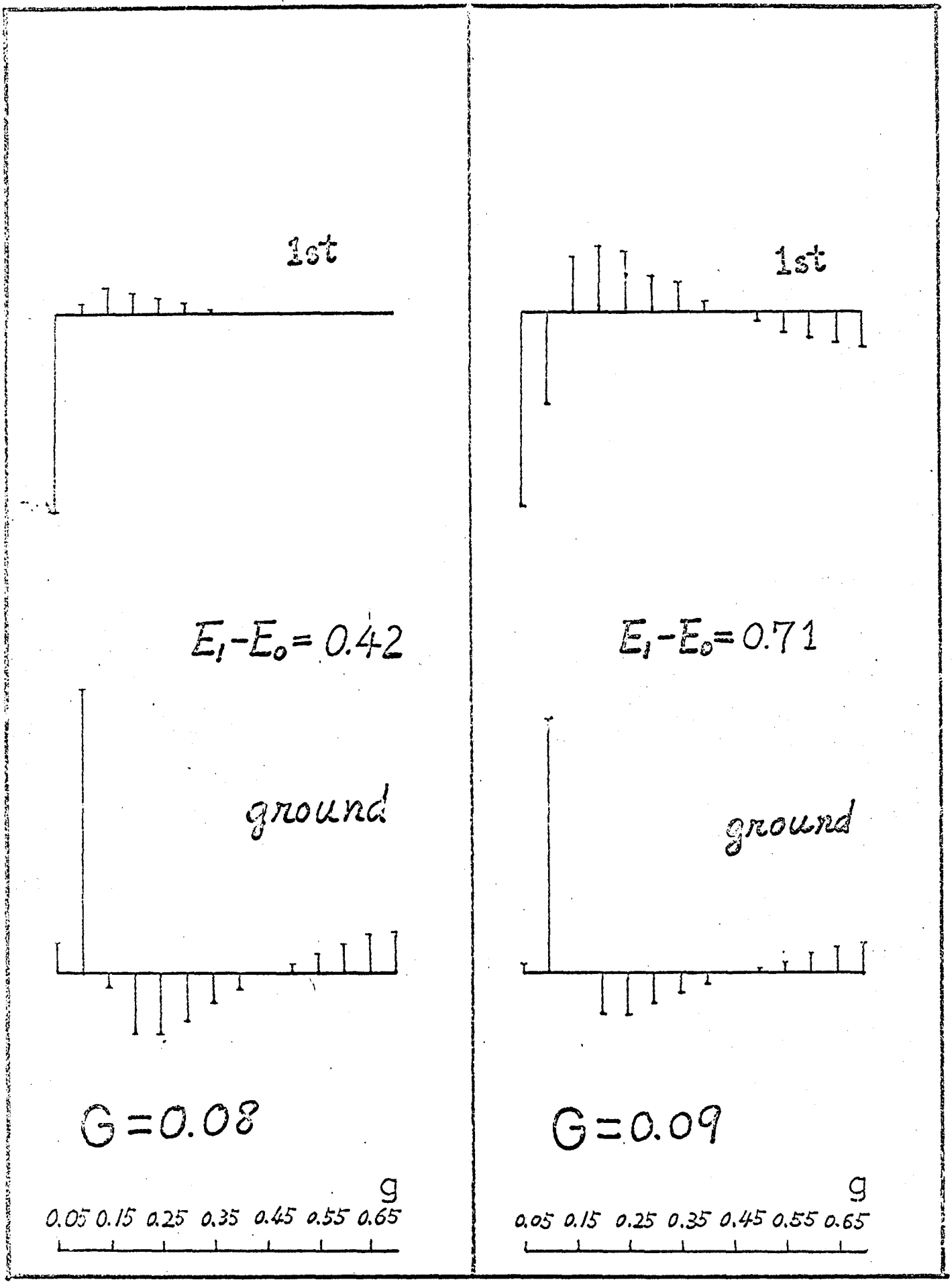

1st
$E_1 - E_0 = 0.42$
ground
G=0.08
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65
1st
$E_1 - E_0 = 0.71$
ground
G=0.09
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65

Fig. 2f.

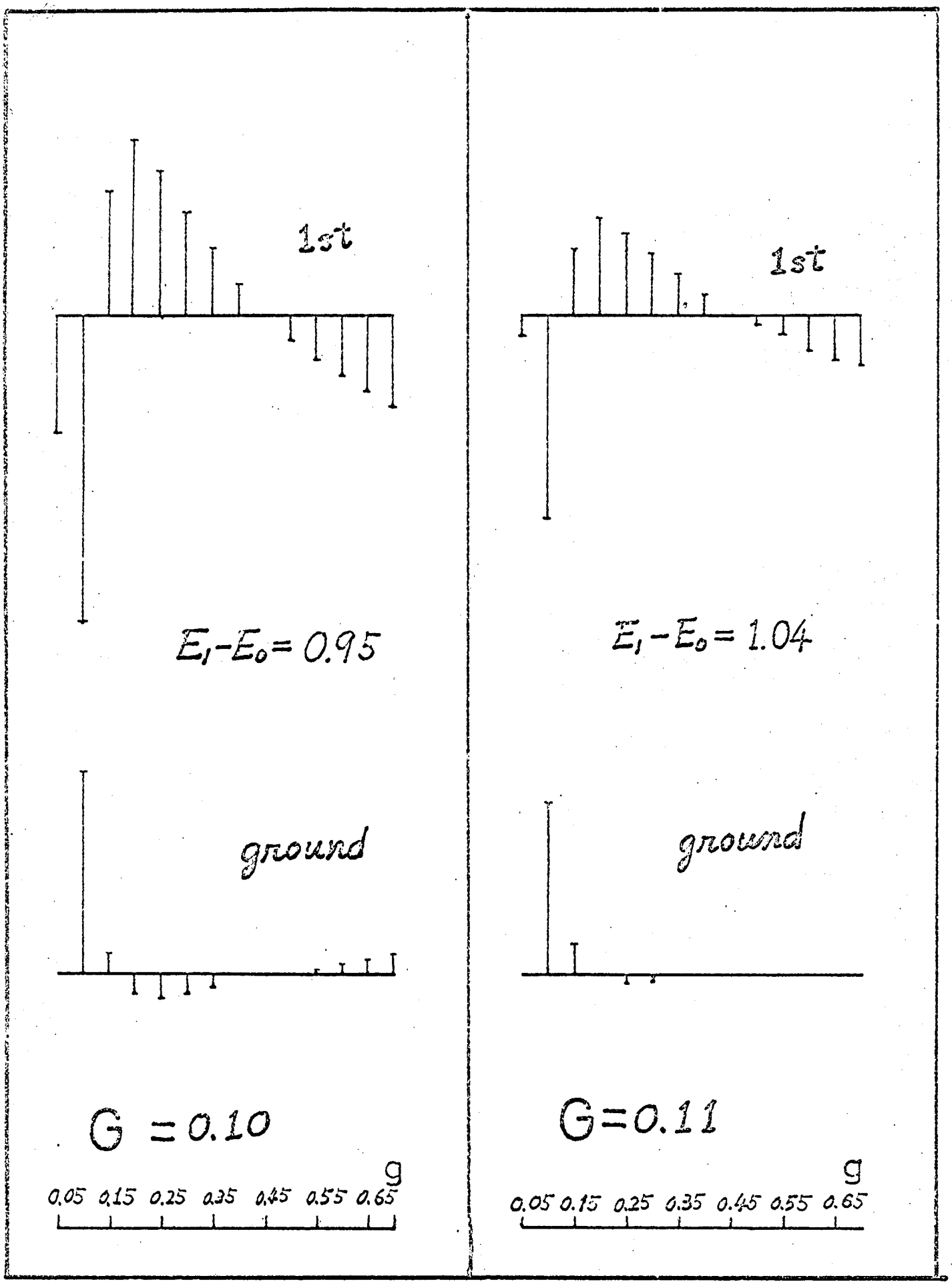
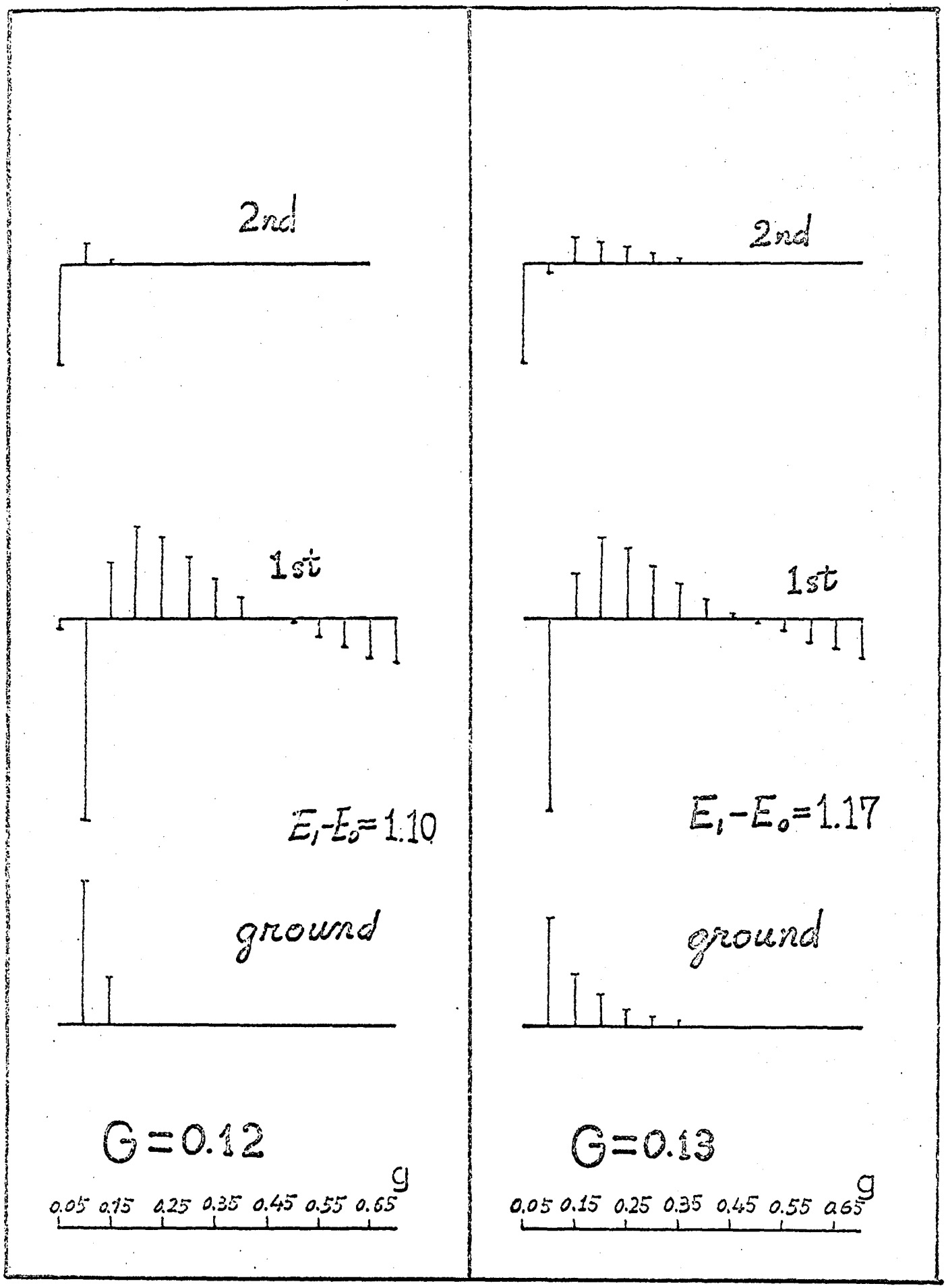

1st
$E_1 - E_0 = 0.95$
ground
G = 0.10
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65
1st
$E_1 - E_0 = 1.04$
ground
G = 0.11
g
0.05 0.15 0.25 0.35 0.45 0.55 0.65

Fig. 2g


2nd
1st
$E_1-E_0=1.10$
ground
G=0.12
0.05 0.15 0.25 0.35 0.45 0.55 0.65
g
2nd
1st
$E_1-E_0=1.17$
ground
G=0.13
0.05 0.15 0.25 0.35 0.45 0.55 0.65
g

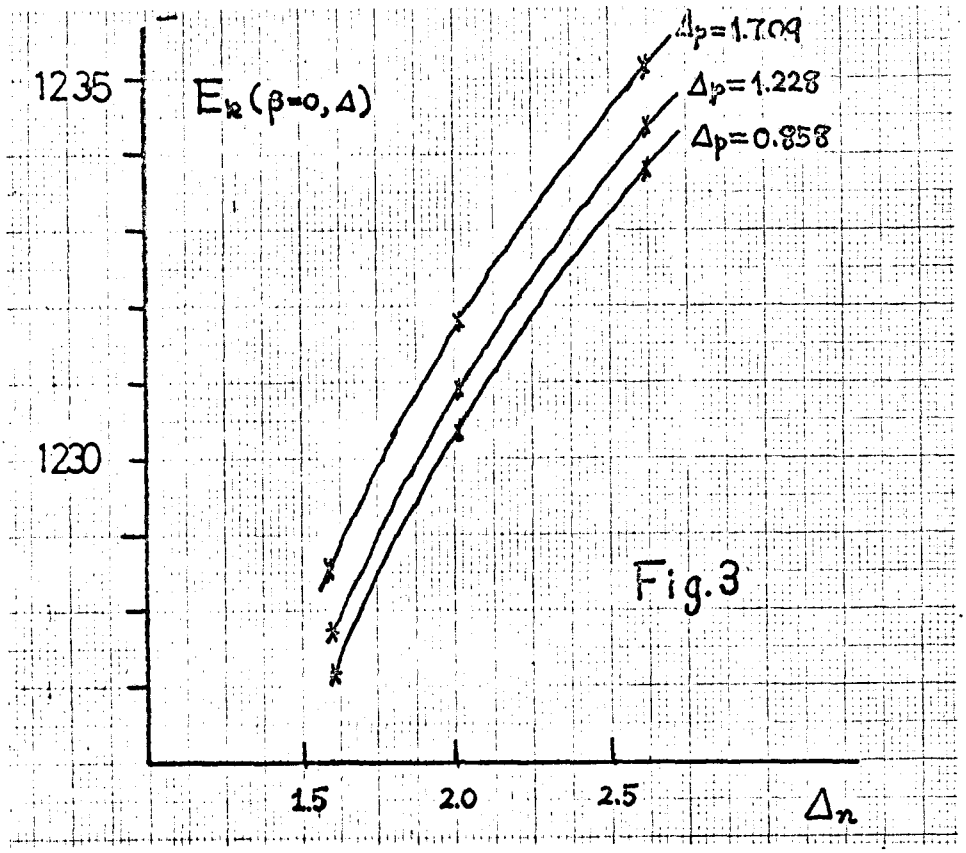

1235
1230
$E_R(\beta=0,\Delta)$
$\Delta_p=1.709$
$\Delta_p=1.228$
$\Delta_p=0.858$
1.5
2.0
2.5
Δ_n

Fig.3

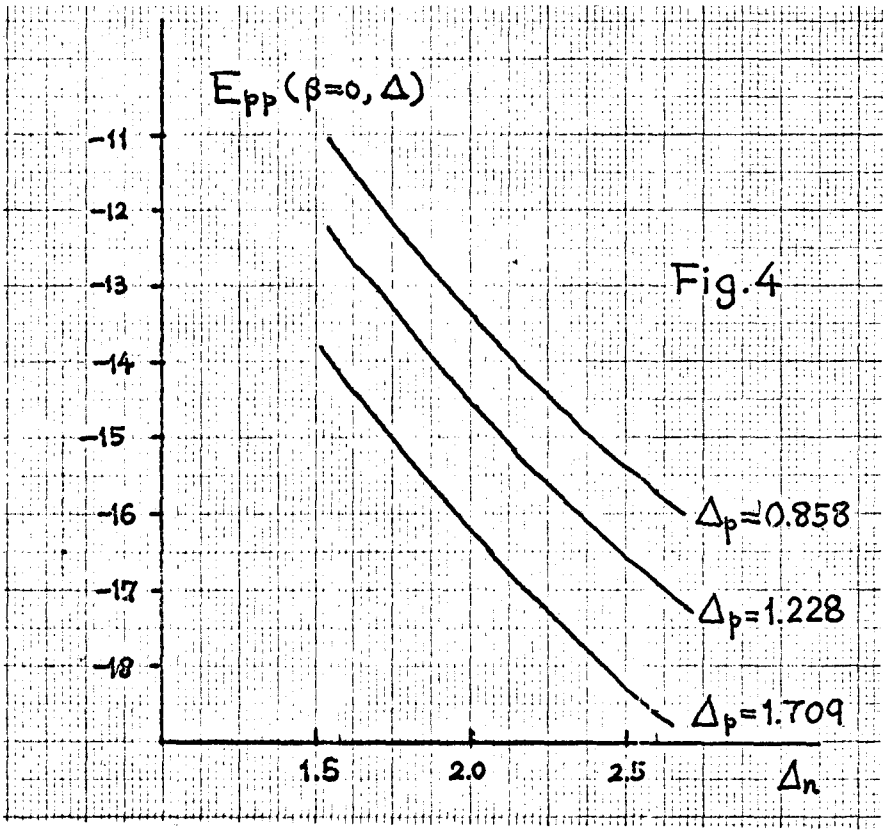

$E_{pp}(\beta=0,\Delta)$
-11
-12
-13
-14
-15
-16
-17
-18
$\Delta_p=0.858$
$\Delta_p=1.228$
$\Delta_p=1.709$
1.5
2.0
2.5
Δ_n

Fig.4

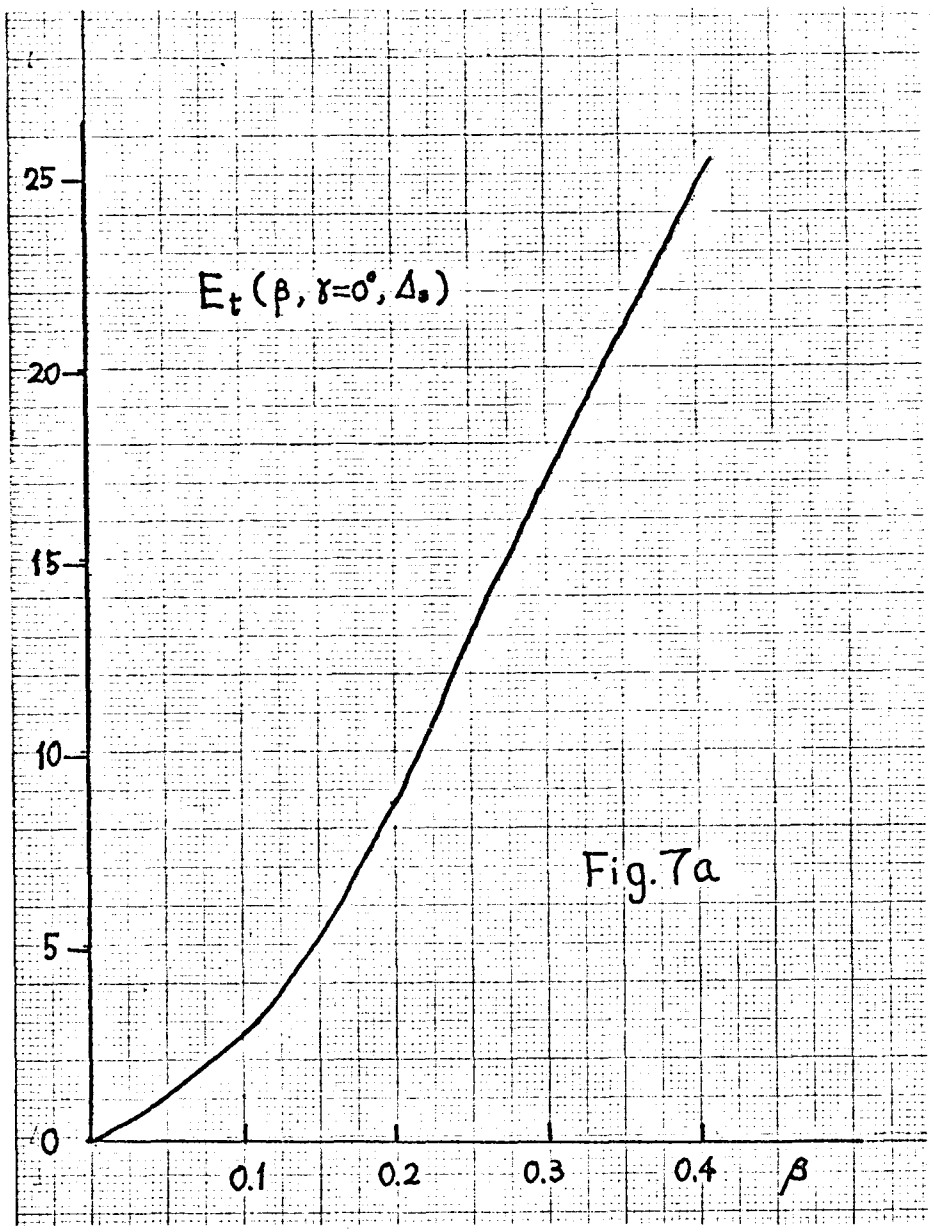


Fig. 7a

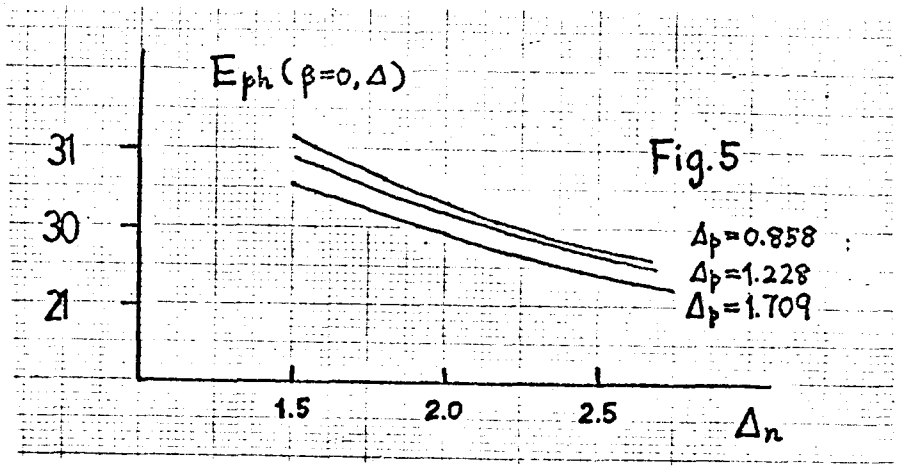


Fig. 5

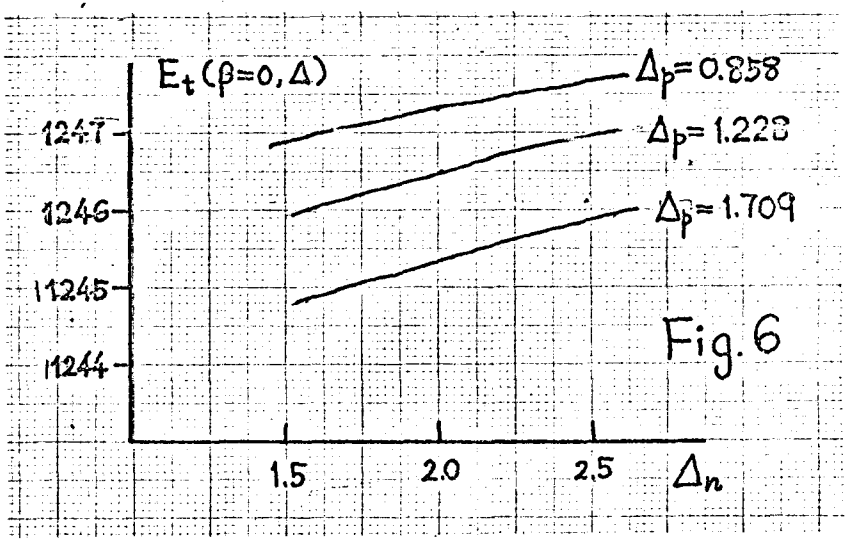


Fig. 6

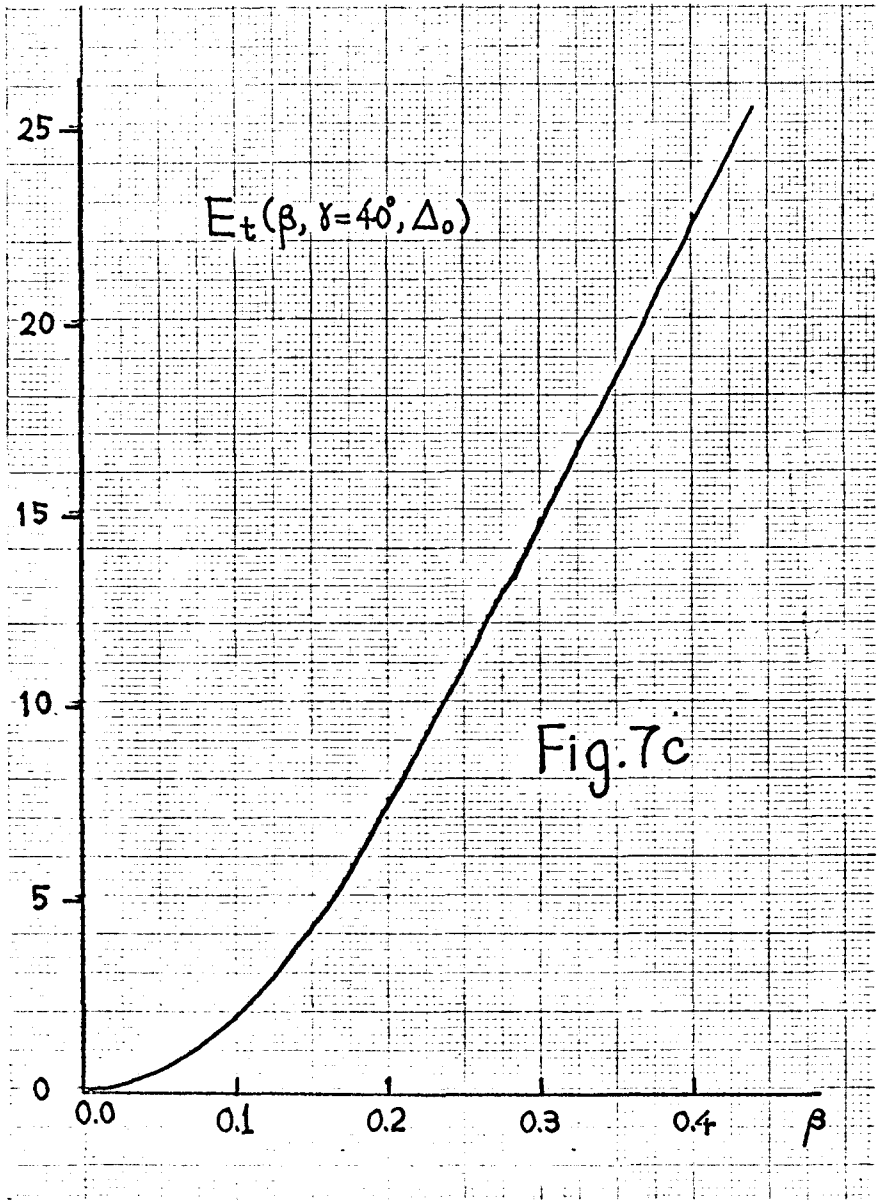

$E_t(\beta, \gamma=40^\circ, \Delta_o)$
25
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
β

Fig. 7c

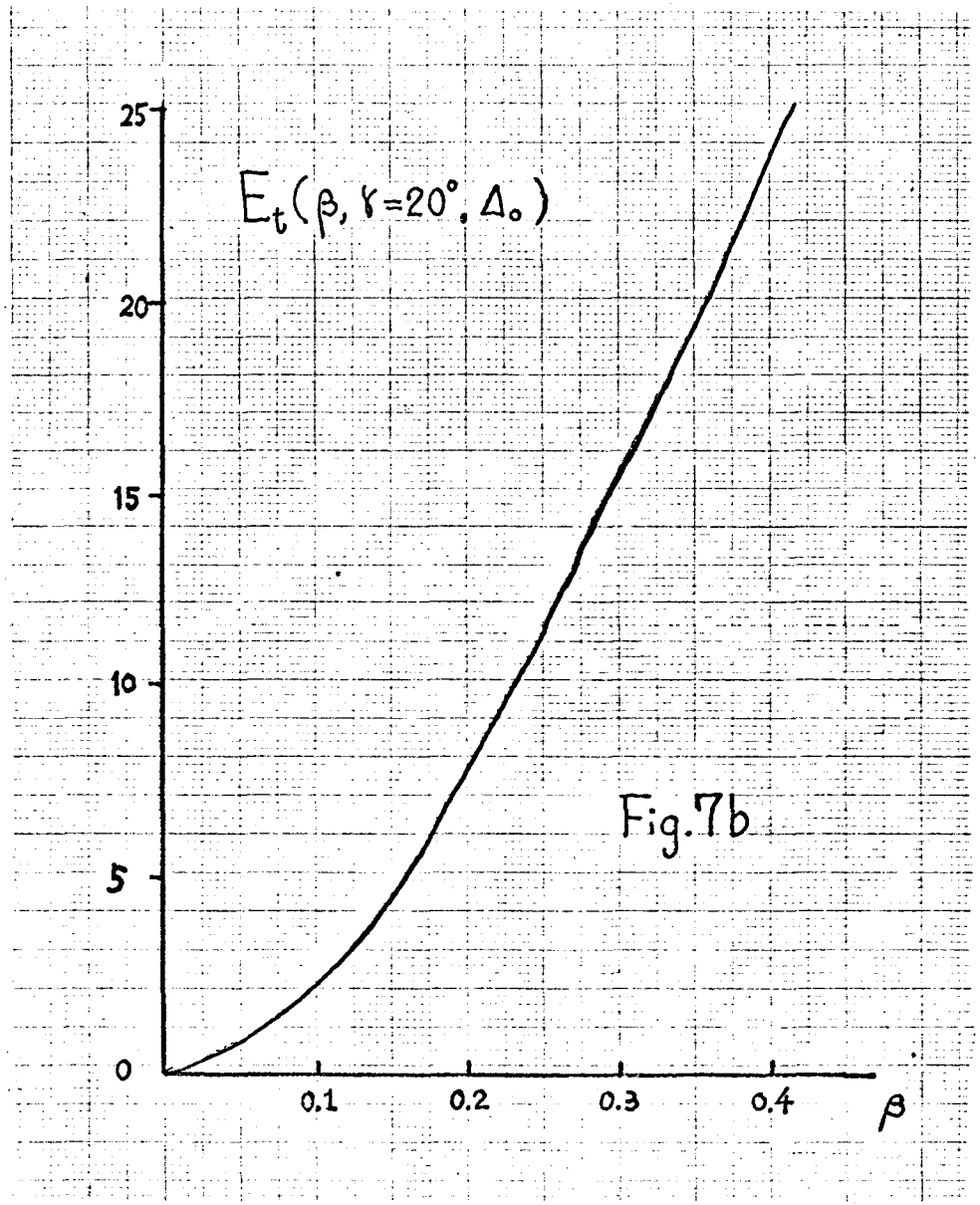

$E_t(\beta, \gamma=20^\circ, \Delta_o)$
25
20
15
10
5
0
0.1
0.2
0.3
0.4
β

Fig. 7b

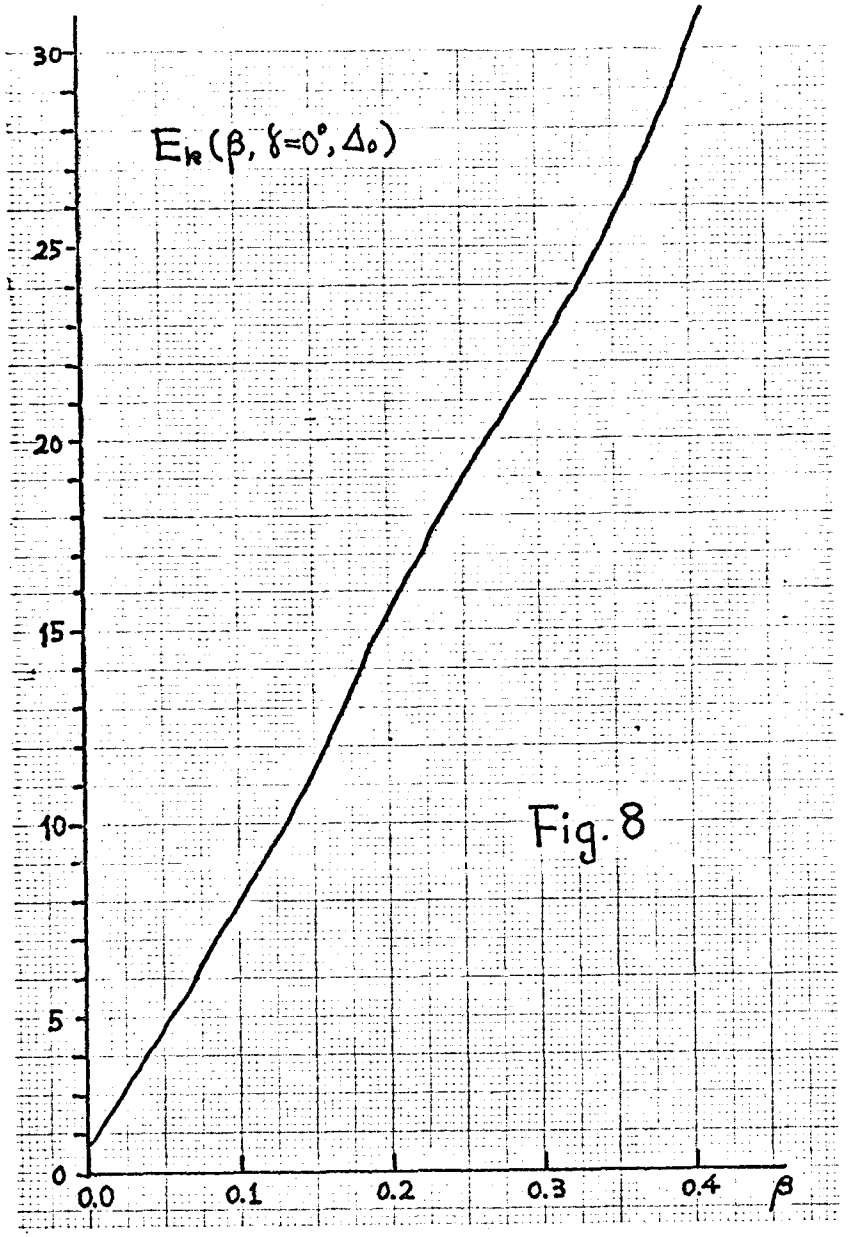

$E_k(\beta, \gamma=0°, \Delta_0)$
30
25
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
β

Fig. 8

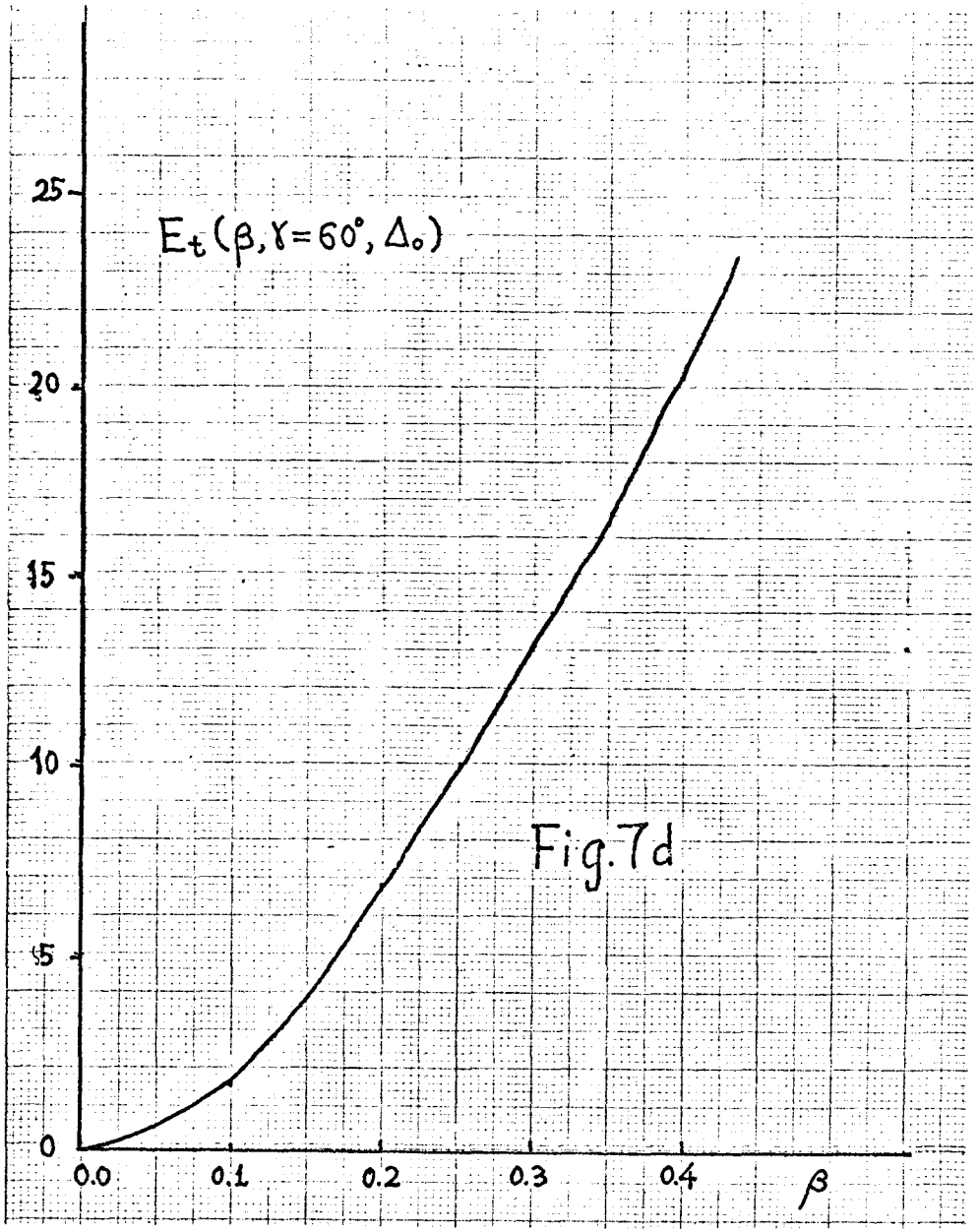

$E_t(\beta, \gamma=60°, \Delta_0)$
25
20
15
10
5
0
0.0
0.1
0.2
0.3
0.4
β

Fig. 7d

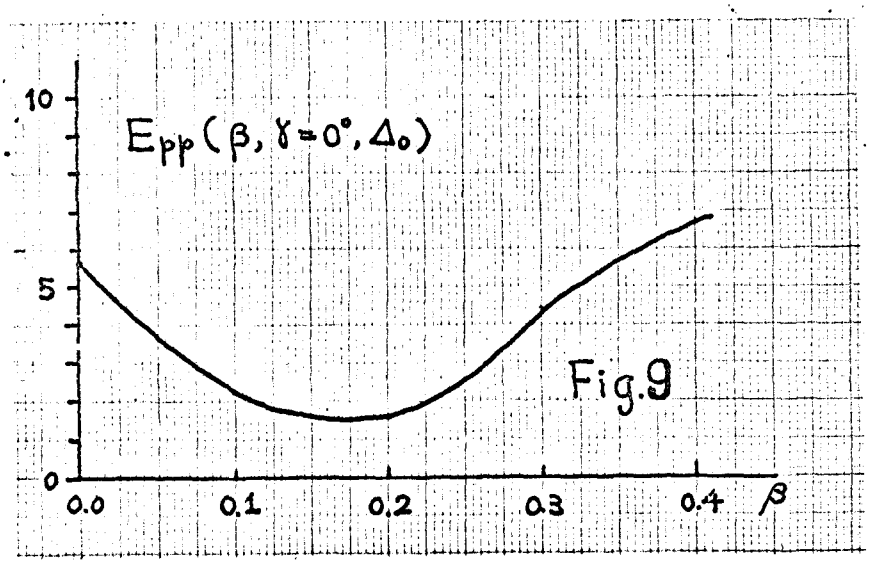

$E_{pp}(\beta, \gamma=0°, \Delta_0)$
10
5
0
0.0
0.1
0.2
0.3
0.4
β

Fig.9

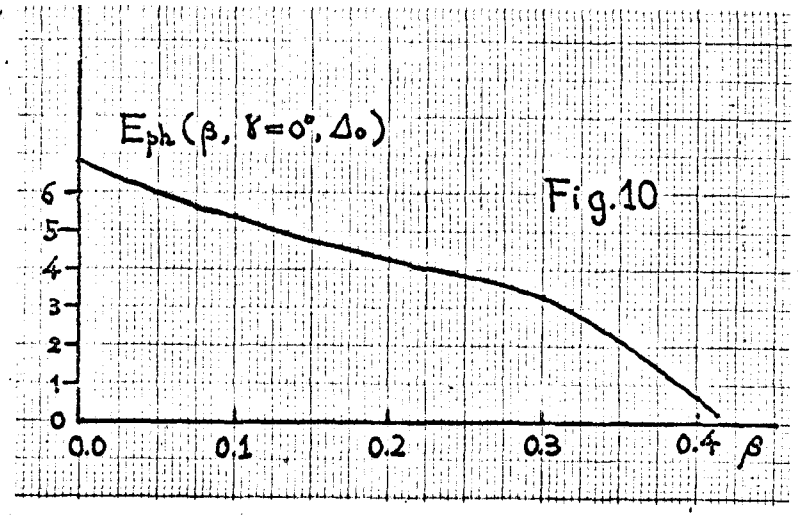

$E_{ph}(\beta, \gamma=0°, \Delta_0)$
6
5
4
3
2
1
0
0.0
0.1
0.2
0.3
0.4
β

Fig.10

Table I

in MeV

V_W	V_B	V_H	V_M	r_0
4.5	-15.93	9.03	-32.2	1.732fm

Table II

	neutrons		protons	
	N=4	N=5	N=3	N=4
κ	0.05	0.05	0.05	0.05
μ	0.45	0.45	0.45	0.45

Table III

		G_n	Δ_n	G_p	Δ_p	E_k	E_{ph}	E_{pp}	E_t
0.0	0°	26.5	2.616	32.2	1.709	1235.2000	29.3266	-18.5962	1245.9305
				28.0	1.228	1234.3391	29.5406	-16.8740	1247.0057
				24.0	0.858	1233.7961	29.6596	-15.6813	1247.7744
		23.0	2.054	32.2	1.709	1231.7830	29.3989	-16.3211	1245.3608
				28.0	1.228	1230.9221	30.1918	-14.5991	1246.5148
				24.0	0.858	1230.3792	30.3588	-13.4063	1247.3317
		20.0	1.587	32.2	1.709	1228.5593	30.4335	-14.1396	1244.8532
				28.0	1.228	1227.6985	30.8071	-12.4175	1246.0882
				24.0	0.858	1227.1555	31.0231	-11.2248	1246.9538
0.1	0°	26.5	2.500	32.2	1.715	1236.4976	29.0618	-17.7596	1247.7998
				28.0	1.186	1235.6106	29.2998	-15.7884	1249.1220
				24.0	0.691	1234.7817	29.3759	-14.0517	1250.1090
		23.0	2.000	32.2	1.715	1233.8379	29.4533	-15.6725	1247.6187
				28.0	1.186	1232.9509	29.7531	-13.7013	1249.0027
				24.0	0.691	1232.1223	29.8714	-11.9646	1250.0291
		20.0	1.500	32.2	1.715	1231.3457	29.5808	-13.3185	1247.9080
				28.0	1.186	1230.4587	30.2506	-11.3472	1249.3621
				24.0	0.691	1229.6299	30.4178	-9.6107	1250.4300
0.2	0°	26.5	2.374	32.2	1.686	1241.6616	28.2768	-16.1826	1253.7558
				28.0	1.186	1240.1201	28.7122	-14.3618	1254.4705
				24.0	0.752	1239.2104	28.9708	-12.8374	1255.2438
		23.0	1.812	32.2	1.686	1239.7271	28.3195	-13.6833	1254.3633
				28.0	1.186	1238.1858	28.7818	-11.8624	1255.1052
				24.0	0.752	1237.2761	28.9536	-10.3382	1255.8915
		24.0	1.344	32.2	1.686	1238.7424	28.3588	-11.3810	1255.7202
				28.0	1.186	1237.2009	28.8469	- 9.5602	1256.4867
				24.0	0.752	1236.2913	29.0321	- 8.0360	1257.2874

Table III continued 1

		G_n	Δ_n	G_p	Δ_p	E_k	E_{ph}	E_{pp}	E_t
0.3	0°	26.5	2.123	32.2	1.588	1248.3145	27.3394	-13.6125	1262.0414
				28.0	1.093	1246.5669	28.0364	-11.8607	1262.7426
				24.0	0.566	1244.6641	28.8080	-10.0122	1263.4599
		23.0	1.593	32.2	1.588	1246.8635	27.4030	-11.2604	1263.0061
				28.0	1.093	1245.1160	28.1479	- 9.5086	1263.7553
				24.0	0.566	1243.2131	28.9744	- 7.6602	1264.5273
		20.0	1.125	32.2	1.588	1245.4626	27.5504	- 9.0399	1263.9731
				28.0	1.093	1243.7151	28.3538	- 7.2880	1264.7809
				24.0	0.566	1241.8123	29.2524	- 5.4395	1265.6252
0.4	0°	26.5	1.875	32.2	1.498	1256.3801	24.6603	-11.3077	1269.7327
				28.0	1.125	1255.2097	25.1531	-10.1442	1270.2186
				24.0	0.750	1253.2986	25.7718	- 8.9242	1270.1462
		23.0	1.375	32.2	1.498	1255.0234	24.5941	- 9.3285	1270.2890
				28.0	1.125	1253.8530	25.1189	- 8.1650	1270.8069
				24.0	0.750	1251.9419	25.7800	- 6.9450	1270.7769
		20.0	1.000	32.2	1.498	1254.0835	24.7683	- 7.7914	1271.0600
				28.0	1.125	1252.9131	25.3379	- 6.6280	1271.6229
				24.0	0.750	1251.0020	26.0677	- 5.4080	1271.6617
0.1	20°	26.5	2.499	32.2	1.715	1236.2283	29.0840	-17.7730	1247.5393
				28.0	1.186	1235.3413	29.3257	-15.8018	1248.8652
				24.0	0.691	1234.5125	29.4042	-14.0651	1249.8516
		23.0	2.000	32.2	1.715	1233.8103	29.4882	-15.6875	1247.6110
				28.0	1.186	1232.9233	29.7930	-13.7164	1248.9999
				24.0	0.691	1232.0945	29.9150	-11.9796	1250.0299
		20.0	1.500	32.2	1.715	1231.0151	29.9161	-13.3340	1247.5972
				28.0	1.186	1230.1282	30.2930	-11.3628	1249.0584
				24.0	0.691	1229.2996	30.4653	- 9.6262	1250.1387

Table III continued 2

		G_n	Δ_n	G_p	Δ_p	E_k	E_{ph}	E_{pp}	E_t
0.2	20°	26.5	2.374	32.2	1.686	1241.2900	28.1883	−16.2448	1253.2335
				28.0	1.186	1239.7485	28.6194	−14.4240	1253.9439
				24.0	0.752	1238.8389	28.7753	−12.8996	1254.7146
		23.0	1.844	32.2	1.686	1239.3906	28.3097	−13.9011	1253.7992
				28.0	1.186	1237.8491	28.7816	−12.0802	1254.5505
				24.0	0.752	1236.9395	28.9607	−10.5560	1255.3442
		20.0	1.344	32.2	1.686	1238.2639	28.3538	−11.4589	1255.1588
				28.0	1.186	1236.7227	28.8550	− 9.6381	1255.9396
				24.0	0.752	1235.8130	29.0495	− 8.1139	1256.4860
0.3	20°	26.5	2.215	32.2	1.588	1248.0181	27.1515	−14.0884	1261.0812
				28.0	1.093	1246.2705	27.8581	−12.3366	1261.7920
				24.0	0.566	1244.3677	28.6434	−10.4881	1262.5230
		23.0	1.623	32.2	1.588	1246.3535	27.2049	−11.4428	1262.1156
				28.0	1.093	1244.6060	27.9933	− 9.6910	1262.9083
				24.0	0.566	1242.7031	28.8145	− 7.8425	1263.6751
		20.0	1.125	32.2	1.588	1244.8931	27.2230	− 9.0256	1263.0905
				28.0	1.093	1243.1455	28.0316	− 7.2737	1263.9034
				24.0	0.566	1241.2427	28.9328	− 5.4253	1264.7502
0.4	20°	26.5	2.000	32.2	1.498	1255.8916	25.0455	−11.9272	1269.0099
				28.0	1.125	1254.7214	25.5455	−10.7637	1269.5032
				24.0	0.750	1252.8101	26.1689	− 9.5437	1269.4353
		23.0	1.468	32.2	1.498	1254.8596	25.0226	− 9.8181	1270.0641
				28.0	1.125	1253.6892	25.5563	− 8.6546	1270.5909
				24.0	0.750	1251.7781	26.2246	− 7.4346	1270.5681
		20.0	1.062	32.2	1.498	1253.9360	25.0307	− 8.1561	1270.8106
				28.0	1.125	1252.7656	25.6536	− 6.9926	1271.4266
				24.0	0.750	1250.8545	26.3514	− 5.7726	1271.4333

Table III continued 3

β	γ	G_n	Δ_n	G_p	Δ_p	E_k	E_{ph}	E_{pp}	E_t
0.1	40°	26.5	2.498	32.2	1.715	1235.8672	29.1354	−17.8111	1247.1915
				28.0	1.186	1234.9802	29.4022	−15.8399	1248.5425
				24.0	0.691	1234.1514	29.4726	−14.3653	1249.2587
		23.0	2.000	32.2	1.715	1233.1462	29.5525	−15.7292	1246.9695
				28.0	1.186	1232.2593	29.8712	−13.7580	1248.3725
				24.0	0.691	1231.4307	30.0032	−12.0213	1249.4126
		20.0	1.562	32.2	1.715	1230.6560	29.9494	−13.6856	1246.9198
				28.0	1.186	1229.7690	30.3360	−11.7144	1248.3906
				24.0	0.691	1228.9404	30.5160	−9.9778	1249.4786
0.2	40°	26.5	2.432	32.2	1.686	1240.7922	28.3011	−16.6316	1252.4617
				28.0	1.186	1239.2510	28.7581	−14.8108	1253.1983
				24.0	0.752	1238.3413	28.9355	−13.2864	1253.9904
		23.0	1.872	32.2	1.686	1238.5994	28.5016	−14.2200	1252.8810
				28.0	1.186	1237.0579	29.0137	−12.3999	1253.6717
				24.0	0.752	1236.1482	29.2898	−10.8749	1254.5631
		20.0	1.405	32.2	1.686	1236.8616	28.6762	−12.0046	1253.5332
				28.0	1.186	1235.3201	29.2416	−10.1838	1254.3779
				24.0	0.752	1234.4104	29.4169	−8.6596	1255.1677
0.3	40°	26.5	2.247	32.2	1.688	1247.1003	27.2134	−14.6271	1259.6866
				28.0	1.093	1245.3528	27.9538	−12.8754	1260.4312
				24.0	0.566	1243.4500	28.7730	−11.0268	1261.1962
		23.0	1.718	32.2	1.588	1245.4641	27.2230	−12.2706	1260.4165
				28.0	1.093	1243.7166	28.0135	−10.5188	1261.2112
				24.0	0.566	1241.8137	28.8915	− 8.6703	1262.0349
		20.0	1.250	32.2	1.588	1244.2925	27.1691	− 9.9744	1261.4872
				28.0	1.093	1242.5449	27.9981	− 8.2225	1262.3205
				24.0	0.566	1240.6421	28.9208	− 6.3741	1263.1888

Table III continued 4

β	γ	G_n	Δ_n	G_p	Δ_p	E_R	E_{ph}	E_{pp}	E_t
0.4	40°	26.5	2.123	32.2	1.498	1254.7319	25.8292	−12.9987	1267.5624
				28.0	1.125	1253.5615	26.3184	−11.8353	1268.0446
				24.0	0.750	1251.6504	26.9752	−10.6153	1268.0103
		23.0	1.594	32.2	1.498	1253.4856	25.8762	−10.8089	1268.5529
				28.0	1.125	1252.3154	26.4794	− 9.6454	1269.1494
				24.0	0.750	1250.4041	27.2041	− 8.4254	1269.1828
		20.0	1.125	32.2	1.498	1251.4546	25.9511	− 8.7349	1268.6708
				28.0	1.125	1250.2842	26.5420	− 7.5714	1269.2548
				24.0	0.750	1248.3730	27.3121	− 6.3513	1269.3338
0.1	60°	26.5	2.558	32.2	1.715	1235.9561	29.0723	−18.0695	1246.9589
				28.0	1.186	1235.0691	29.3322	−16.0983	1248.3030
				24.0	0.691	1234.2402	29.4077	−14.3616	1249.2863
		23.0	1.997	32.2	1.715	1232.8616	29.5539	−15.7394	1246.6761
				28.0	1.186	1231.9746	29.8828	−13.7682	1248.0892
				24.0	0.691	1231.1458	30.0226	−12.0315	1249.1369
		20.0	1.560	32.2	1.715	1230.3223	29.9665	−13.6814	1246.6074
				28.0	1.186	1229.4353	30.3677	−11.7101	1248.0929
				24.0	0.691	1228.6064	30.5769	− 9.9736	1249.2097
0.2	60°	26.5	2.456	32.2	1.686	1240.3484	28.3202	−16.8340	1251.8346
				28.0	1.186	1238.8069	28.8356	−15.0132	1252.6293
				24.0	0.752	1237.8972	29.0372	−13.4888	1253.4456
		23.0	1.867	32.2	1.686	1237.9631	28.5896	−14.3093	1252.2434
				28.0	1.186	1236.4219	29.1371	−12.4885	1253.0705
				24.0	0.752	1235.5122	29.3775	−10.9642	1253.9255
		20.0	1.433	32.2	1.686	1236.1970	28.7706	−12.2520	1252.7156
				28.0	1.186	1234.6492	29.3708	−10.4312	1253.5888
				24.0	0.752	1233.7395	29.6450	− 8.9070	1254.4775

Table III continued 5

β	γ	G_n	Δ_n	G_p	Δ_p	E_k	E_{ph}	E_{pp}	E_t
0.3	60°	26.5	2.358	32.2	1.588	1246.4077	27.1018	-15.3224	1258.1871
				28.0	1.093	1244.6602	27.8585	-13.5707	1258.9480
				24.0	0.566	1242.7573	28.6892	-11.7221	1259.7244
		23.0	1.741	32.2	1.588	1244.3916	27.1940	-12.6370	1258.9486
				28.0	1.093	1242.6440	28.0240	-10.8852	1259.7828
				24.0	0.566	1240.7412	28.9363	- 9.0367	1260.6408
		20.0	1.309	32.2	1.588	1242.9934	27.1925	-10.5124	1259.6735
				28.0	1.093	1241.2458	28.0705	- 8.7606	1260.5557
				24.0	0.566	1239.3430	29.0363	- 6.9121	1261.4672
0.4	60°	26.5	2.245	32.2	1.498	1254.4736	24.9720	-13.9516	1265.4940
				28.0	1.125	1253.3035	25.4968	-12.7883	1266.0120
				24.0	0.750	1251.3921	26.1647	-11.5683	1265.9885
		23.0	1.716	32.2	1.498	1252.7998	25.0632	-11.8105	1266.0525
				28.0	1.125	1251.6296	25.6325	-10.6470	1266.6151
				24.0	0.750	1249.7183	26.3587	- 9.4270	1266.6500
		20.0	1.249	32.2	1.498	1251.4646	25.1912	- 9.7094	1266.9464
				28.0	1.125	1250.2944	25.8102	- 8.5459	1267.5587
				24.0	0.750	1248.3831	26.6016	- 7.3259	1267.6589