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# Measuring Methods of Three-Dimensional Residual Stresses with Aid of Distribution Function of Inherent Strain (Report I) †

— A Function Method for Estimating Inherent Strain Distributions —

Yukio UEDA\* and Ning Xu MA\*\*

## Abstract

*When three-dimensional welding residual stresses are measured based on inherent strain using the Finite Element Method, too many inherent strain components in elements have to be specified to express its distribution. If these are to be determined experimentally, a large number of elastic strains must be measured. In order to obtain inherent strain distribution from a small number of measuring points, a function method is proposed.*

*To verify the function method, residual stresses and inherent strains in bead-on-plate welds are estimated. The residual stresses estimated by inherent strain described as functions show very good accuracy compared with those computed by thermal elastic-plastic FEM.*

**KEY WORDS:** (Residual stresses) (Inherent strains) (Function method) (FEM) (Bead-on-plate welds)

## 1. Introduction

Inherent strain generated in welded joints is composed of the contraction of weld metal and plastic strain in the vicinity of the weld zone. It is considered as a source of welding residual stresses and has been used for stress measurement and prediction<sup>1-3</sup>. However, in the measurement and prediction of three-dimensional residual stresses, the following problems have to be solved :

(1) Until now, inherent strain distributions are described by the individual value in each finite element. This description of inherent strain distributions is called element description or element method. In this case, many unknown inherent strain components in elements have to be specified. If these are to be determined using measured elastic strains, the total components of elastic strains to be measured must be more than these of inherent strains, and a large quantity of the measurements are required.

(2) To estimate the local distribution of three-dimensional residual stresses, the finite element mesh has to be divided very fine. Because the local released elastic strains in these fine meshed elements are

unmeasurable using strain gauges, the local distribution of inherent strains and residual stresses can not be determined by experiments.

(3) To predict welding residual stresses using inherent strain, the distribution patterns of inherent strains first have to be proposed. The distribution patterns are very difficult to derive from individual inherent strain component in elements.

To solve such problems, a function method for describing and estimating inherent strain distributions is proposed. With the aid of a parametric function, inherent strain distributions can be expressed only by a few unknown coefficients included in functions. When these coefficients are determined by a few measured elastic strains, inherent strain distributions and residual stress distributions can be computed by performing simple elastic analysis. Furthermore, if the effects of welding conditions and sizes of welded joints on inherent strain distributions are clarified by theoretical analysis such as thermal elastic plastic analysis, prediction for three-dimensional residual stresses will become possible using the function method.

In this paper, a general concept of the function

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method for estimating inherent strain distributions is described. Then, the elastic response relationship between unknown coefficients included in functions and residual stresses is derived. Further, to verify the validity and the accuracy of the function method, inherent strain distributions in a simple bead-on-plate weld are estimated. Then, residual stresses in this weld are computed by Inherent Strain Elastic Analysis (ISEA) and are compared with those obtained by Thermal Elastic Plastic Analysis (TEPA).

## 2. Estimation of inherent strain distributions described by element method

When element description for inherent strain distributions is used, a welded joint has to be meshed into elements. In the elements within inherent strain zones, if the total number of inherent strain components is  $q$ , its distributions can be expressed by inherent strain vector  $\{\varepsilon^*\}_q$ . Elastic strains  $\{\varepsilon^e\}$  and residual stresses  $\{\sigma\}$  at arbitrary positions produced by inherent strain  $\{\varepsilon^*\}_q$  can be computed by the following equations:

$$\{\varepsilon^e\} = [H]\{\varepsilon^*\}_q \quad (1)$$

$$\{\sigma\} = [D]\{\varepsilon^e\} \quad (2)$$

where  $[H]$  is the elastic response matrix and  $[D]$  is the stress-strain relation matrix.

To determine  $q$  components of inherent strains  $\{\varepsilon^*\}_q$ , the  $m(\geq q)$  components of elastic strains  $\{\varepsilon^e\}_m$  have to be measured because various errors may be contained in experiments. According to equation (1), the errors  $\{e\}_m$  can be expressed by the following equation:

$$\{e\}_m = [H]_{mq}\{\varepsilon^*\}_q - \{\varepsilon^e\}_m \quad (3)$$

The errors in Eq.(3) include measuring error of elastic strains  $\{\varepsilon^e\}_m$ , modeling error of matrix  $[H]_{mq}$  and the error of inherent strains  $\{\varepsilon^*\}_q$ . In this paper, the elastic strains  $\{\varepsilon^e\}_m$  are computed by thermal elastic plastic analysis, and the finite element division of the model used for analyzing the inherent strain is the same as the thermal elastic plastic analysis. Therefore, the measuring error of  $\{\varepsilon^e\}_m$  and modeling error of  $[H]_{mq}$  can be neglected; the main error is from the error of  $\{\varepsilon^*\}_q$ . The square sum of the errors is expressed by the following equation:

$$\begin{aligned} Er(\varepsilon^*) &= \{e\}^T\{e\} \\ &= ([H]_{mq}\{\varepsilon^*\}_q - \{\varepsilon^e\}_m)^T([H]_{mq}\{\varepsilon^*\}_q - \{\varepsilon^e\}_m) \end{aligned} \quad (4)$$

When the summation error  $Er(\varepsilon^*)$  is minimum, i.e. the derivatives of the error  $Er(\varepsilon^*)$  with respect to inherent strain components  $\{\varepsilon^*\}_q$  are zero, the following equation can be obtained:

$$[H]^T_{qm}[H]_{mq}\{\varepsilon^*\}_q = [H]^T_{qm}\{\varepsilon^e\}_m \quad (5)$$

When measured elastic strains  $\{\varepsilon^e\}_m$  are substituted into Eq.(5), inherent strains  $\{\varepsilon^*\}_q$  can be obtained, and resulting residual stresses can be computed by Eqs.(1) and (2).

## 3. Estimation of inherent strain distributions described by function method

### 3.1 Function description for inherent strain distributions

Inherent strain distributions can be expressed as functions of coordinates  $x,y,z$ . For a general three-dimensional problem, there are a total of six inherent strain components  $\{\varepsilon^*_x, \varepsilon^*_y, \varepsilon^*_z, \varepsilon^*_{xy}, \varepsilon^*_{yz}, \varepsilon^*_{zx}\}$ . These components are simply expressed by  $\varepsilon^*_s$  ( $s=x, y, z, xy, yz, zx$ ). The distribution of component  $\varepsilon^*_s$  can be described by the following series function:

$$\varepsilon^*_s(x,y,z) = \sum_{i=1}^L \sum_{j=1}^M \sum_{k=1}^N A^*_{sijk} h_{si}(x) f_{sj}(y) g_{sk}(z) \quad (6)$$

where  $h_{si}(x)$ ,  $f_{sj}(y)$ ,  $g_{sk}(z)$  are the distribution functions in  $x,y,z$  directions for component  $\varepsilon^*_s$ . These functions can be freely chosen from some basic functions such as power functions, exponential functions or trigonometric functions. However, these distribution functions have to be zero valued out of inherent strain zones.  $A^*_{sijk}$  is the coefficient of distribution functions  $h_{si}(x)$ ,  $f_{sj}(y)$  and  $g_{sk}(z)$ .  $L, M, N$  are the orders of series functions.

If Eq.(6) is written as a vector form, the following equation can be obtained:

$$\begin{aligned} \varepsilon^*_s(x,y,z) &= \{ A^*_{sijk} h_{si}(x) f_{sj}(y) g_{sk}(z) \} \\ &= \{P_s\}^T_{ps} \{A^*_s\}_{ps}, \quad (p_s=L \times M \times N) \end{aligned} \quad (7)$$

where  $\{P_s\}_{ps}$ ,  $\{A^*_s\}_{ps}$  and  $p_s$  are the vector related to the functions, the coefficients vector and the total number of unknown coefficients, respectively, for each inherent strain component  $\varepsilon^*_s$ .

For the six components  $\varepsilon^*_s$  ( $s=x, y, z, xy, yz, zx$ ) of inherent strains, the inherent strain distributions can be expressed as follows:

$$\begin{aligned} \{\varepsilon^*_s\} &= \{ \varepsilon^*_x, \varepsilon^*_y, \varepsilon^*_z, \varepsilon^*_{xy}, \varepsilon^*_{yz}, \varepsilon^*_{zx} \}^T \\ &= [P]\{A^*\}_p, \quad p=6p_s \end{aligned} \quad (8)$$

where  $p, p_s$  are the total number of unknown coefficients for each inherent strain component and for the total six components, respectively.  $[P], \{A^*\}_p$  are the function matrix and coefficient vector of inherent strains, and they can be expressed as follows:

$$\{A^*\}_p = \{ A^*_x, A^*_y, A^*_z, A^*_{xy}, A^*_{yz}, A^*_{zx} \}_p^T \quad (9)$$



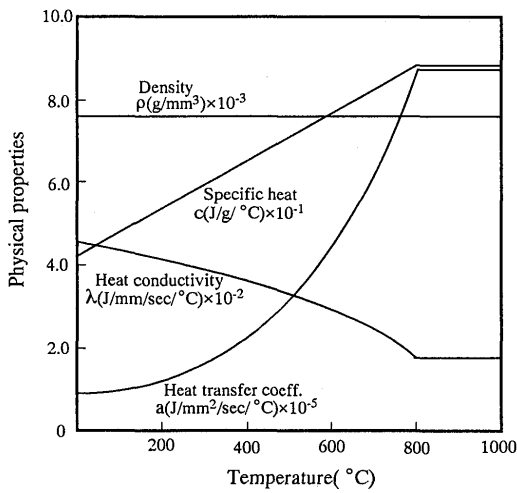
## A Function Method for Estimating Inherent Strain Distributions

plane strain. If the welding direction is taken as  $x$ , and the transverse, the thickness directions, are taken as  $y, z$ , respectively, the strain  $\epsilon_x$  distributed on the transverse sections can be expressed by the following equation:

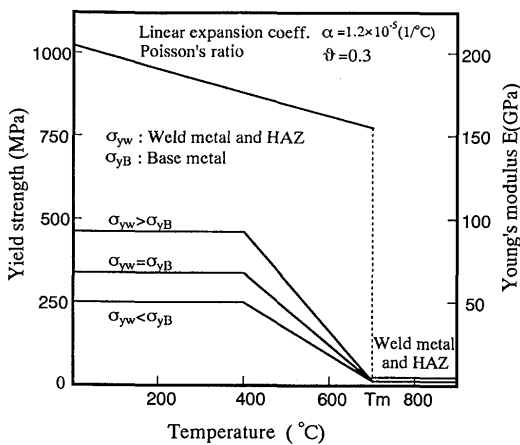
$$\epsilon_x = a_1 + a_2 y + a_3 z \quad (18)$$

where  $a_1, a_2, a_3$  are the coefficients of plane deformation and can be computed by FEM using equilibrium equation of force ( $F_x$ ) and moments ( $M_y$  and  $M_z$ ).

In this paper, residual stresses distributed on transverse sections are analyzed by thermal elastic plastic FEM and by inherent strain elastic analysis, respectively. In thermal elastic plastic analysis, the material of the welded plate is assumed to be steel, and its physical properties and mechanical properties are assumed as functions of temperature, as shown in Fig.2 and Fig.3.



**Fig. 2** Physical properties of mild steel used in heat conduction FEM analysis

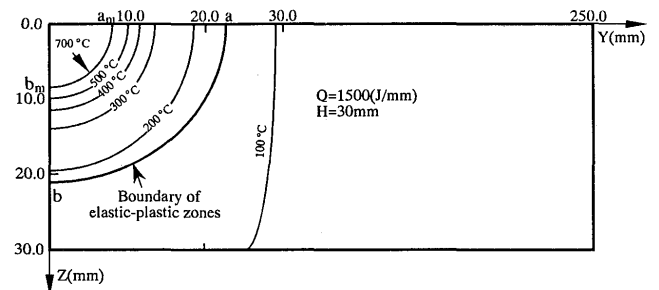


**Fig. 3** Mechanical properties of mild steel used in thermal elastic-plastic FEM analysis

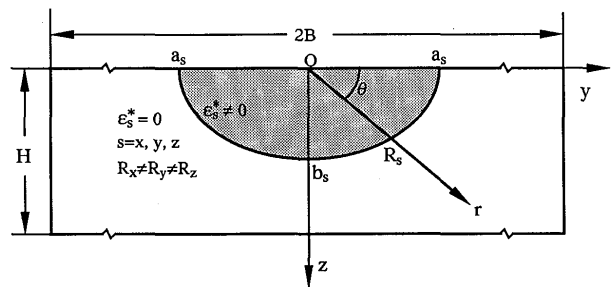
### 4.2 Inherent strain components and their distribution zones

On transverse sections of a long weld, there are four components of stresses. These components of stresses are  $\sigma_x, \sigma_y, \sigma_z, \sigma_{yz}$ . Corresponding to these stress components, there exist four inherent strain components  $\epsilon_x^*, \epsilon_y^*, \epsilon_z^*, \epsilon_{yz}^*$  on the transverse sections. Because the effect of shear inherent strain  $\epsilon_{yz}^*$  on residual stresses is small<sup>4,5</sup>, only the components  $\epsilon_x^*, \epsilon_y^*, \epsilon_z^*$  are considered in this paper.

To estimate inherent strains, the shape and the size of inherent strain zones have to be determined first. To propose the shape of inherent strain zones, contour lines of maximum temperature in the thermal cycle and the residual plastic strain zone are computed by thermal conduction analysis and thermal elastic plastic analysis, respectively. The computed results are shown by Fig.4. The shape of both contour lines of temperature and the plastic strain zone is roughly elliptical. Based on these results, the inherent strain zones are assumed to be elliptical as shown in Fig.5. The size of the ellipsis is expressed by radius  $R_s$  ( $s=x, y, z$ ), or by width  $a_s$  ( $s=x, y, z$ ) and  $b_s$  ( $s=x, y, z$ ), and it varies with each of inherent strain components  $\epsilon_x^*, \epsilon_y^*, \epsilon_z^*$ .



**Fig. 4** Contour lines of max. temperature in thermal cycle and plastic deformation zone in a bead-on-plate weld



**Fig. 5** Shape of inherent strain zone in bead-on-plate welds

### 4.3 Function description for inherent strain distributions

Inherent strain distribution for each component  $\epsilon^*_s$  ( $s=x, y, z$ ) can be expressed as functions of polar coordinates  $(r, \theta)$  shown in Fig.5.

$$\epsilon^*_s(x,y,z) = \sum_{i=1}^M \sum_{j=1}^N A^*_{sij} f_{si}(r) g_{sj}(\theta), \quad (s=x,y,z) \quad (19)$$

The polar coordinates  $(r, \theta)$  can be expressed in dimensionless form  $(\xi_s, \omega)$  defined by the following equation:

$$\xi_s = \frac{r}{R_s}, \quad \omega = \frac{\theta}{\pi/2}, \quad (s=x,y,z) \quad (20)$$

The range of  $(\xi_s, \omega)$  is from 0 to 1.  $\xi_s=0$  and  $\xi_s=1$  indicate the center of the ellipse and boundary line of inherent strain zones, respectively.  $\omega=0$  ( $\theta=0$ ) and  $\omega=1$  ( $\theta=\pi/2$ ) indicate the y axis and z axis, respectively. When dimensionless polar coordinates  $(\xi_s, \omega)$  are used, Eq.(19) can be rewritten as follows:

$$\epsilon^*_s(x,y,z) = \sum_{i=1}^M \sum_{j=1}^N A^*_{sij} f_{si}(\xi_s) g_{sj}(\omega), \quad (s=x,y,z) \quad (21)$$

For  $f_{si}(\xi_s)$  and  $g_{sj}(\omega)$ , many types of functions can be considered, such as power functions, exponential functions and trigonometric functions. In this paper, very simple power functions are used, and inherent strain distributions are expressed by Eq.(22).

$$\epsilon^*_s(x,y,z) = \sum_{i=1}^M \sum_{j=1}^N A^*_{sij} (1-\xi_s)^i \omega^{(j-1)}, \quad (s=x,y,z) \quad (22)$$

The values of inherent strain given by Eq.(22) are zero at the boundary line ( $\xi_s=1$ ) of inherent strain zones.

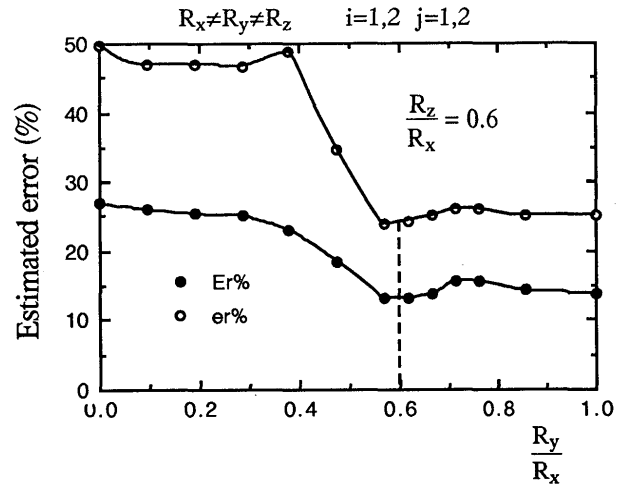
### 4.4 Estimation of inherent strain distributions by function method

#### 4.4.1 Estimation of the sizes of inherent strain zones

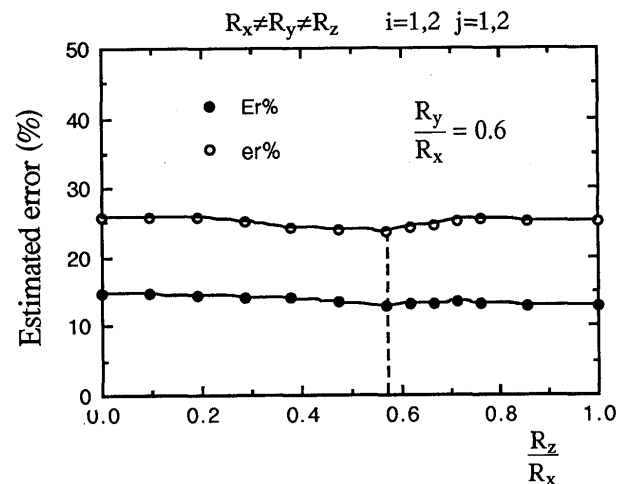
As shown in Fig.5, the sizes of inherent strain zones of components  $\epsilon^*_x, \epsilon^*_y$  and  $\epsilon^*_z$  are expressed by their radii  $R_x, R_y$  and  $R_z$ . The values of  $R_x, R_y$  and  $R_z$  can be determined by numerical computation so that the total error  $E_r$  and local maximum error  $e_r$  of estimated residual stresses take the minimum, i.e.

$$\frac{\partial E_r}{\partial R_s} = 0, \quad \frac{\partial e_r}{\partial R_s} = 0, \quad (s=x,y,z) \quad (23)$$

If all of the three parameters  $R_x, R_y$  and  $R_z$  are taken as unknowns, much numerical computation has to be tried in order to determine their values. Here, inherent strain zone  $R_x$  of component  $\epsilon^*_x$  corresponding to the largest stress component  $\sigma_x$  due to welding is assumed to be the same as that of the residual plastic strain zone



(a) Estimation of  $R_y$



(b) Estimation of  $R_z$

Fig.6 Estimation of the sizes  $R_y, R_z$  of inherent strain zones from the errors of reproduced stresses

shown in Fig.4.  $R_y$  and  $R_z$  of components  $\epsilon^*_y, \epsilon^*_z$  corresponding to stress components  $\sigma_y, \sigma_z$  are assumed to be smaller than  $R_x$ . When  $R_y/R_x$  and  $R_z/R_x$  vary from 0 to 1, the changes of total error  $E_r$  and local error  $e_r$  with  $R_y/R_x$  and  $R_z/R_x$  are shown in Fig.6. When  $R_y/R_x$  and  $R_z/R_x$  are about 0.6 and 0.55 respectively, the smallest errors will be obtained, and these  $R_y$  and  $R_z$  can be considered the sizes of inherent strain zones of components  $\epsilon^*_y, \epsilon^*_z$ . Because the sizes of inherent strain zones of components  $\epsilon^*_y, \epsilon^*_z$  show very little difference, they can be assumed to be the same.

#### 4.4.2 Accuracy of function method

When inherent strain distributions are expressed by series function, the accuracy is governed by the order  $M$

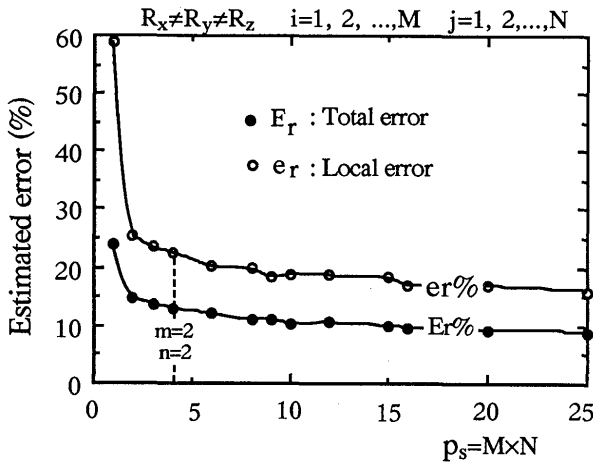


Fig. 7 Effect of total unknown coefficients in series function on accuracy of reproduced residual stresses

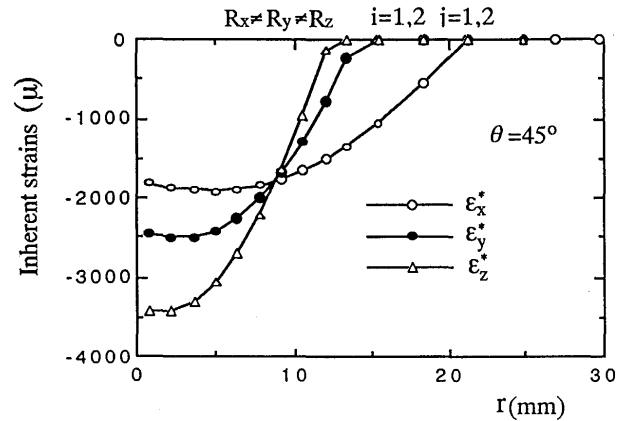


Fig. 8 Estimated inherent strain distributions

and  $N$  of series function. The effects of  $M$  and  $N$ , or the total number  $p_s (=M \times N)$  of unknown coefficients for each inherent strain component  $\epsilon_s^*$  ( $s=x, y, z$ ) on the errors  $E_r$ ,  $e_r$  of reproduced residual stresses, are shown by Fig. 7. By increasing the order  $M$  and  $N$  of series function, or by increasing the total number  $p_s (=M \times N)$  of unknown coefficients, the total error  $E_r$  and local error  $e_r$  decrease. In actual measurements, according to the expected accuracy,  $M$  and  $N$  of series function, or measuring points corresponding to the total number  $p_s (=M \times N)$  of unknown coefficients, can be freely selected. As shown in Fig. 7, when  $M$  and  $N$  are more than 2, or the total number  $p_s$  of unknown coefficients are more than 4, the change of errors becomes smaller and practically reasonable accuracy can be obtained. In this case, if elastic strains are measured at 4 points for each component, inherent strain distributions can be estimated. Then residual stress distributions can be computed. Therefore, when the function method is employed to describe inherent strain distributions, the required measurements for elastic strains will be greatly reduced compared with the element method.

#### 4.4.3 Distributions of inherent strains and residual stresses

When  $M$  and  $N$  of series function are taken as 2, inherent strain distributions estimated by the function method are shown in Fig. 8. In the radial direction, the inherent strain distributions show a trapezoidal pattern. Using the inherent strains given in Fig. 8, residual stresses shown in Fig. 9 are estimated by performing simple elastic analysis. As shown in Fig. 9, very good accuracy of residual stresses estimated by elastic analysis using inherent strain (ISEA) can be obtained compared

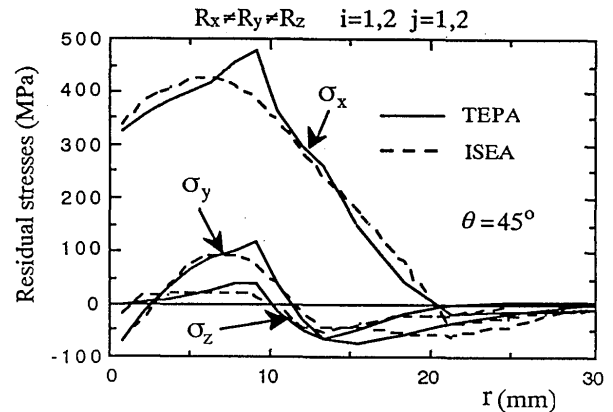


Fig. 9 Residual stresses by TEPA and ISEA (TEPA : Thermal Elastic-Plastic Analysis) (ISEA : Inherent Strain Elastic Analysis)

with those computed by thermal elastic plastic analysis (TEPA).

### 5. Application of function method to overmatching and undermatching welds

Compared with the strength of base metal, the same or higher or lower strength of weld metals are used in welding. These options are called evenmatching, overmatching and undermatching, respectively. For evenmatching welded joints, the inherent strain distributions can be described by series function as shown in chapter 4. In this chapter, the function method is applied to overmatching and undermatching welded joints. For simplicity, the strength of HAZ is assumed to be different from that of base metal but the same as that of weld metal.

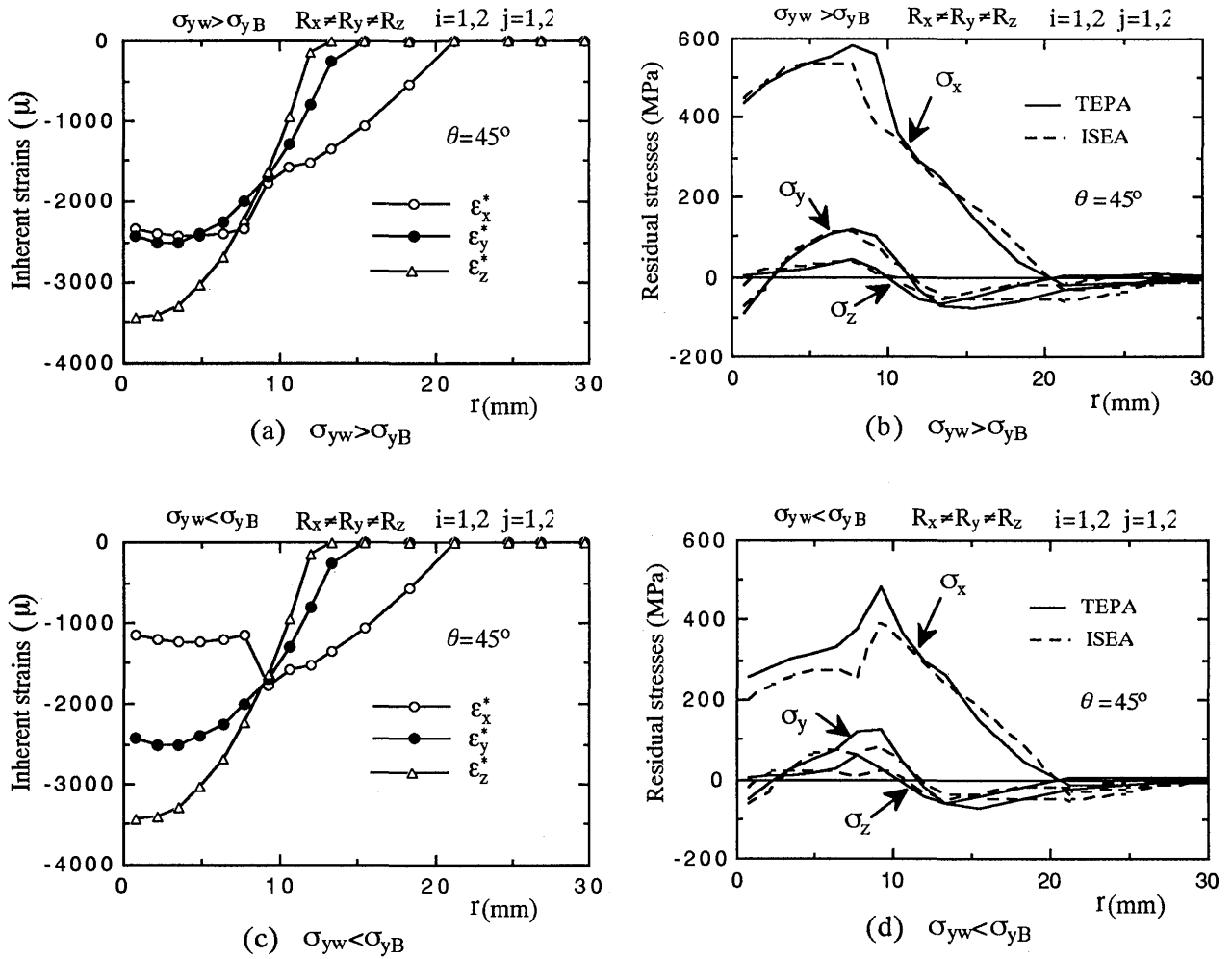


Fig.10 Inherent strain distributions estimated with aid of functions including additional strains  $\Delta \epsilon^*_s$  for weld metal and HAZ in overmatching and undermatching welds, and residual stresses estimated by TEPA and ISEA (TEPA : Thermal Elastic-Plastic Analysis, ISEA : Inherent Strain Elastic Analysis)

5.1 Application of series function to overmatching and undermatching welds

Theoretically, a continuous series function shown in Eq.(22) can also describe inherent strain distributions even in overmatching and undermatching welded joints. However, to obtain high accuracy, the higher order M and N of series function have to be used. To describe such discontinuous characteristics of inherent strain distributions due to overmatching and undermatching using the same order M and N as evenmatching welded joints, a supplemental inherent strain  $\Delta \epsilon^*_s$  ( $s=x,y,z$ ) for weld metal and HAZ is introduced into series function as follows:

$$\epsilon^*_s(x,y,z) = \sum_{i=1}^M \sum_{j=1}^N A^*_{sij} (1-\xi_s)^i \omega^{(j-1)} + \Delta \epsilon^*_s \quad (24)$$

The supplemental strain  $\Delta \epsilon^*_s$  does exist in weld metal and HAZ, but it is zero in base metal.

Fig.10 shows inherent strain distributions described by Eq.(24) with  $M=N=2$  and residual stress distributions when the yield stress of weld metal is 450MPa and 250MPa, and the yield stress of base metal is 330MPa, respectively. Comparing Fig.10 with Fig.9, the difference of yield stress of weld metal and base metal has a significant effect on inherent strain component  $\epsilon^*_x$  and less effect on the other two components  $\epsilon^*_y, \epsilon^*_z$ .

5.2 Prediction of supplemental inherent strain  $\Delta \epsilon^*_s$

Generally, residual stress component  $\sigma_x$  in weld metal and HAZ attains their yield stresses. The other two components  $\sigma_y, \sigma_z$  are less than the yield stresses. For this reason, by changing the yield stresses of weld metal and HAZ, nearly the same change can be observed in residual stress component  $\sigma_x$ , but little change is produced on residual stress components  $\sigma_y, \sigma_z$ , and

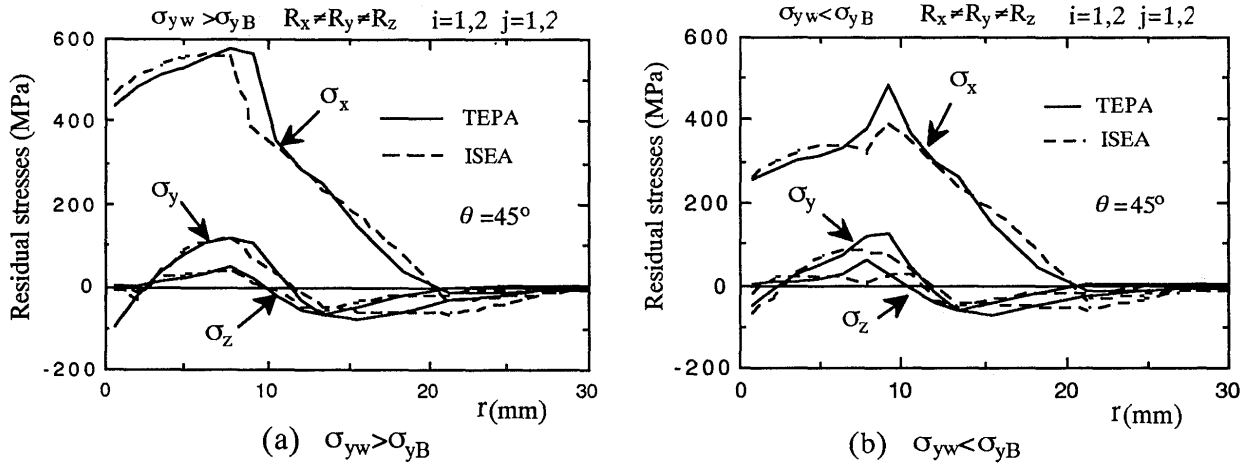


Fig. 11 Residual stresses, estimated by inherent strain, whose distributions are described by series function and approximate value  $\Delta \epsilon_x^* \approx (\epsilon_{yw} - \epsilon_{yB})$  for weld metal and HAZ in overmatching and undermatching welds (TEPA : Thermal Elastic-Plastic Analysis, ISEA : Inherent Strain Elastic Analysis)

inherent strain components  $\epsilon_y^*$ ,  $\epsilon_z^*$ . Therefore, the supplemental inherent strain  $\Delta \epsilon_x^*$  is approximately equal to the difference of yield strains between weld metal and base metal shown by the following equation, and the supplemental strains  $\Delta \epsilon_y^*$ ,  $\Delta \epsilon_z^*$  can be neglected.

$$\Delta \epsilon_x^* \approx (\epsilon_{yw} - \epsilon_{yB}) = (\sigma_{yw} - \sigma_{yB})/E \quad (25a)$$

$$\Delta \epsilon_y^* \approx \Delta \epsilon_z^* \approx 0 \quad (25b)$$

where  $\epsilon_{yw}$ ,  $\epsilon_{yB}$  are the yield strains of weld metal and base metal respectively.  $\sigma_{yw}$ ,  $\sigma_{yB}$  and  $E$  are the yield stresses of weld metal, base metal and Young's modulus of the steel, respectively.

To demonstrate the accuracy of the prediction based on Eq.(25) for  $\Delta \epsilon_s^*$  (s=x, y, z), the residual stresses are estimated by inherent strain elastic analysis (ISEA) and the results are shown by Fig.11(a), (b), respectively, for overmatching and undermatching welded joints. Compared with the results computed by thermal elastic plastic analysis (TEPA), very good accuracy is obtained.

### 6. Conclusions

- (1) A function method for estimating inherent strain distributions is proposed.
- (2) Inherent strain zones show elliptical shape on the transverse sections. The size of elliptical zones varies with each of inherent strain components.
- (3) With the aid of series function, inherent strain distributions in bead-on-plate welds are expressed by a few unknown coefficients, residual stresses are reproduced with good accuracy compared with those computed by thermal elastic plastic analysis.

- (4) The function method for estimating inherent strains is also efficient for overmatching and undermatching welded joints without increasing unknown coefficients.

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