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Measuring Methods of Three-Dimensional Residual Stresses with Aid of Distribution Function of Inherent Strain (Report I) †

— A Function Method for Estimating Inherent Strain Distributions —

Yukio UEDA* and Ning Xu MA**

Abstract

When three-dimensional welding residual stresses are measured based on inherent strain using the Finite Element Method, too many inherent strain components in elements have to be specified to express its distribution. If these are to be determined experimentally, a large number of elastic strains must be measured. In order to obtain inherent strain distribution from a small number of measuring points, a function method is proposed.

To verify the function method, residual stresses and inherent strains in bead-on-plate welds are estimated. The residual stresses estimated by inherent strain described as functions show very good accuracy compared with those computed by thermal elastic-plastic FEM.

KEY WORDS: (Residual stresses) (Inherent strains) (Function method) (FEM) (Bead-on-plate welds)

1. Introduction

Inherent strain generated in welded joints is composed of the contraction of weld metal and plastic strain in the vicinity of the weld zone. It is considered as a source of welding residual stresses and has been used for stress measurement and prediction[1-3]. However, in the measurement and prediction of three-dimensional residual stresses, the following problems have to be solved :

(1) Until now, inherent strain distributions are described by the individual value in each finite element. This description of inherent strain distributions is called element description or element method. In this case, many unknown inherent strain components in elements have to be specified. If these are to be determined using measured elastic strains, the total components of elastic strains to be measured must be more than these of inherent strains, and a large quantity of the measurements are required.

(2) To estimate the local distribution of three-dimensional residual stresses, the finite element mesh has to be divided very fine. Because the local released elastic strains in these fine meshed elements are unmeasurable using strain gauges, the local distribution of inherent strains and residual stresses can not be determined by experiments.

(3) To predict welding residual stresses using inherent strain, the distribution patterns of inherent strains first have to be proposed. The distribution patterns are very difficult to derive from individual inherent strain component in elements.

To solve such problems, a function method for describing and estimating inherent strain distributions is proposed. With the aid of a parametric function, inherent strain distributions can be expressed only by a few unknown coefficients included in functions. When these coefficients are determined by a few measured elastic strains, inherent strain distributions and residual stress distributions can be computed by performing simple elastic analysis. Furthermore, if the effects of welding conditions and sizes of welded joints on inherent strain distributions are clarified by theoretical analysis such as thermal elastic plastic analysis, prediction for three-dimensional residual stresses will become possible using the function method.

In this paper, a general concept of the function

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method for estimating inherent strain distributions is described. Then, the elastic response relationship between unknown coefficients included in functions and residual stresses is derived. Further, to verify the validity and the accuracy of the function method, inherent strain distributions in a simple bead-on-plate weld are estimated. Then, residual stresses in this weld are computed by Inherent Strain Elastic Analysis (ISEA) and are compared with those obtained by Thermal Elastic Plastic Analysis (TEPA).

2. Estimation of inherent strain distributions described by element method

When element description for inherent strain distributions is used, a welded joint has to be meshed into elements. In the elements within inherent strain zones, if the total number of inherent strain components is q, its distributions can be expressed by inherent strain vector $\{\varepsilon^*\}_q$. Elastic strains $\{\varepsilon^e\}$ and residual stresses $\{\sigma\}$ at arbitrary positions produced by inherent strain $\{\varepsilon^*\}_q$ can be computed by the following equations:

$$\{\varepsilon^e\} = [H]\{\varepsilon^*\}_q \tag{1}$$

$$\{\sigma\} = [D]\{\varepsilon^e\} \tag{2}$$

where [H] is the elastic response matrix and [D] is the stress-strain relation matrix.

To determine q components of inherent strains $\{\varepsilon^*\}_q$, the $m(\geq q)$ components of elastic strains $\{\varepsilon^e\}_m$ have to be measured because various errors may be contained in experiments. According to equation (1), the errors $\{e\}_m$ can be expressed by the following equation:

$$\{e\}_m = [H]_{mq}\{\varepsilon^*\}_q - \{\varepsilon^e\}_m \tag{3}$$

The errors in Eq.(3) include measuring error of elastic strains $\{\varepsilon^e\}_m$, modeling error of matrix $[H]_{mq}$ and the error of inherent strains $\{\varepsilon^*\}_q$. In this paper, the elastic strains $\{\varepsilon^e\}_m$ are computed by thermal elastic plastic analysis, and the finite element division of the model used for analyzing the inherent strain is the same as the thermal elastic plastic analysis. Therefore, the measuring error of $\{\varepsilon^e\}_m$ and modeling error of $[H]_{mq}$ can be neglected; the main error is from the error of $\{\varepsilon^*\}_q$. The square sum of the errors is expressed by the following equation:

$$\begin{aligned} Er(\varepsilon^*) &= \{e\}^T\{e\} \\ &= ([H]_{mq}\{\varepsilon^*\}_q - \{\varepsilon^e\}_m)^T([H]_{mq}\{\varepsilon^*\}_q - \{\varepsilon^e\}_m) \end{aligned} \tag{4}$$

When the summation error $Er(\varepsilon^*)$ is minimum, i.e. the derivatives of the error $Er(\varepsilon^*)$ with respect to inherent strain components $\{\varepsilon^*\}_q$ are zero, the following equation can be obtained:

$$[H]^T_{qm}[H]_{mq}\{\varepsilon^*\}_q = [H]^T_{qm}\{\varepsilon^e\}_m \tag{5}$$

When measured elastic strains $\{\varepsilon^e\}_m$ are substituted into Eq.(5), inherent strains $\{\varepsilon^*\}_q$ can be obtained, and resulting residual stresses can be computed by Eqs.(1) and (2).

3. Estimation of inherent strain distributions described by function method

3.1 Function description for inherent strain distributions

Inherent strain distributions can be expressed as functions of coordinates x,y,z. For a general three-dimensional problem, there are a total of six inherent strain components $\{\varepsilon^*_x, \varepsilon^*_y, \varepsilon^*_z, \varepsilon^*_{xy}, \varepsilon^*_{yz}, \varepsilon^*_{zx})$. These components are simply expressed by ε^*_s (s=x, y, z, xy, yz, zx). The distribution of component ε^*_s can be described by the following series function:

$$\varepsilon^*_s(x,y,z) = \sum_{i=1}^{L}\sum_{j=1}^{M}\sum_{k=1}^{N} A^*_{sijk}\, h_{si}(x)\, f_{sj}(y)\, g_{sk}(z) \tag{6}$$

where $h_{si}(x)$, $f_{sj}(y)$, $g_{sk}(z)$ are the distribution functions in x,y,z directions for component ε^*_s. These functions can be freely chosen from some basic functions such as power functions, exponential functions or trigonometric functions. However, these distribution functions have to be zero valued out of inherent strain zones. A^*_{sijk} is the coefficient of distribution functions $h_{si}(x)$, $f_{sj}(y)$ and $g_{sk}(z)$. L, M, N are the orders of series functions.

If Eq.(6) is written as a vector form, the following equation can be obtained:

$$\begin{aligned} \varepsilon^*_s(x,y,z) &= \{ A^*_{sijk}\, h_{si}(x)\, f_{sj}(y)\, g_{sk}(z) \} \\ &= \{P_s\}^T_{ps}\{A^*_s\}_{ps}, \quad (p_s = L\times M\times N) \end{aligned} \tag{7}$$

where $\{P_s\}_{ps}$, $\{A^*_s\}$ and p_s are the vector related to the functions, the coefficients vector and the total number of unknown coefficients, respectively, for each inherent strain component ε^*_s.

For the six components ε^*_s(s=x, y, z, xy, yz, zx) of inherent strains, the inherent strain distributions can be expressed as follows:

$$\begin{aligned} \{\varepsilon^*_s\} &= \{ \varepsilon^*_x, \varepsilon^*_y, \varepsilon^*_z, \varepsilon^*_{xy}, \varepsilon^*_{yz}, \varepsilon^*_{zx}\}^T \\ &= [P]\{A^*\}_p, \quad p = 6p_s \end{aligned} \tag{8}$$

where p, p_s are the total number of unknown coefficients for each inherent strain component and for the total six components, respectively. [P], $\{A^*\}_p$ are the function matrix and coefficient vector of inherent strains, and they can be expressed as follows:

$$\{A^*\}_p = \{ A^*_x, A^*_y, A^*_z, A^*_{xy}, A^*_{yz}, A^*_{zx} \}_p^T \tag{9}$$

$$[P] = \begin{bmatrix} P_x & & & & & \\ & P_y & & 0 & & \\ & & P_z & & & \\ & & & P_{xy} & & \\ & 0 & & & P_{yz} & \\ & & & & & P_{zx} \end{bmatrix} \tag{10}$$

3.2 Estimation of coefficient vector of distribution functions

When inherent strain is described by functions, its distributions are determined only by the coefficients included in the functions. The elastic response relationship between coefficient vector $\{A^*\}_p$ and elastic strain $\{\varepsilon^e\}$ at arbitrary positions can be expressed by Eq.(11).

$$\{\varepsilon^e\} = [G]\{A^*\}_p \tag{11}$$

where [G] is the elastic response matrix between $\{A^*\}_p$ and $\{\varepsilon^e\}$. The component G_{ij} of matrix [G] is equal to the value of the i-th elastic strain ε^e_i, generated by an assumed inherent strain distribution corresponding to unit coefficient for j-th order (A^*_j=1) and zero for coefficients A^*_k(if k≠j) in $\{A^*\}_p$, shown as Eq.(12).

$$G_{ij} = \varepsilon^e_i \tag{12}$$

$$\text{when } \{A^*\}_p = \{0,...,0,\ A^*_j(=1),\ 0,...,0\}_p^T$$

According to Eq.(12), when elastic FEM analysis is performed p times, elastic response matrix [G] can be formed.

To determine the coefficient vector $\{A^*\}_p$, elastic strains have to be measured. If m(≥p) components of elastic strains $\{\varepsilon^e\}_m$ are measured, the relationship between measured elastic strains $\{\varepsilon^e\}_m$ and unknown coefficients $\{A^*\}_p$ can be written as

$$[G]_{mp}\{A^*\}_p = \{\varepsilon^e\}_m \tag{13}$$

Because some errors are included in measured strains $\{\varepsilon^e\}_m$ and the distribution functions of inherent strains, two sides of Eq.(13) are not exactly equal. The errors between them can be written as the following equation:

$$\{e\}_m = [G]_{mp}\{A^*\}_p - \{\varepsilon^e\}_m \tag{14}$$

When the derivatives of square error summation with respect to each component of coefficient vector $\{A^*\}_q$ are zero, the minimum error will be obtained. Then, the following equation can be derived:

$$[G]^T_{pm}[G]_{mp}\{A^*\}_p = [G]^T_{pm}\{\varepsilon^e\}_m \tag{15}$$

When measured elastic strains $\{\varepsilon^e\}_m$ are substituted into Eq.(15), coefficients vector $\{A^*\}_q$, i.e. inherent strain distributions, can be estimated. Then elastic strains and residual stresses at arbitrary positions can be computed using elastic FEM analysis.

3.3 Accuracy of function method for inherent strain distributions

The accuracy of the function method can be evaluated by the error in residual stresses reproduced by inherent strains. The error in the reproduced residual stresses is shown by following equation:

$$E_r = \sqrt{\sum_{i=1}^{m}(\sigma_i^m - \sigma_i^e)^2} \Big/ \sqrt{\sum_{i=1}^{m}(\sigma_i^m)^2} \tag{16}$$

where $\sigma_i{}^m$ is the residual stress directly measured or computed by thermal elastic plastic FEM analysis, $\sigma_i{}^e$ is the residual stress reproduced by inherent strain whose distributions are described by functions, and m is the total number of residual stresses directly measured or computed by thermal elastic plastic FEM analysis

Er is called the normalized root mean square error[6]. It can express the total errors of m components of measured residual stresses. Therefore, it is called total error. Besides the total error, to evaluate the accuracy of local residual stresses, the ratio of maximum absolute error ($\max|\sigma_i{}^m - \sigma_i{}^e|$) of reproduced residual stresses to the yield stress (σ_{yw}) is introduced as the local maximum error e_r, i.e.

$$e_r = \max|\sigma_i{}^m - \sigma_i{}^e|/\sigma_{yw}\ , \quad (i=1,2,...,m) \tag{17}$$

4. Application of function method to bead-on-plate welds

4.1 Model to be analyzed

An example for estimation of residual stresses and inherent strains is a long bead-on-plate weld. The transverse section of the bead-on-plate weld is shown in Fig.1. The transverse section of a long weld except near the two ends of a weld keeps plane when it deforms during welding and after welding. This kind of deformation is called plane deformation or generalized

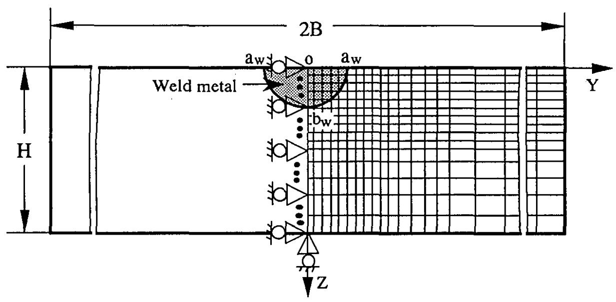


Fig.1 A bead-on-plate weld and mesh division used in FEM analysis

plane strain. If the welding direction is taken as x, and the transverse, the thickness directions, are taken as y,z, respectively, the strain ε_x distributed on the transverse sections can be expressed by the following equation:

$$\varepsilon_x = a_1 + a_2 y + a_3 z \tag{18}$$

where a_1, a_2, a_3 are the coefficients of plane deformation and can be computed by FEM using equilibrium equation of force (F_x) and moments (M_y and M_z).

In this paper, residual stresses distributed on transverse sections are analyzed by thermal elastic plastic FEM and by inherent strain elastic analysis, respectively. In thermal elastic plastic analysis, the material of the welded plate is assumed to be steel, and its physical properties and mechanical properties are assumed as functions of temperature, as shown in Fig.2 and Fig.3.

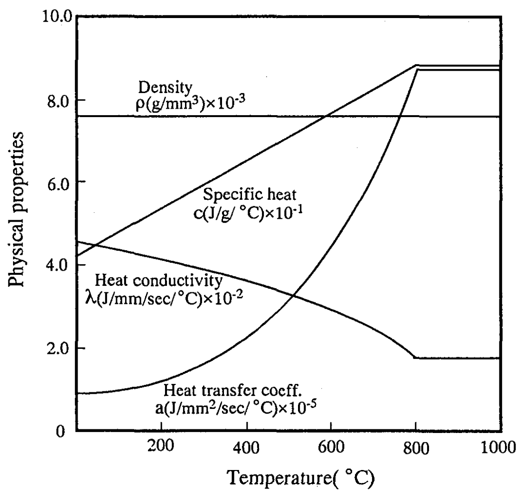


Fig.2 Physical properties of mild steel used in heat conduction FEM analysis

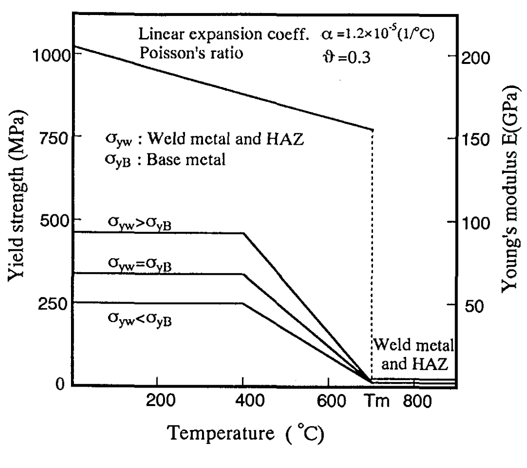


Fig.3 Mechanical properties of mild steel used in thermal elastic-plastic FEM analysis

4.2 Inherent strain components and their distribution zones

On transverse sections of a long weld, there are four components of stresses. These components of stresses are σ_x, σ_y, σ_z, σ_{yz}. Corresponding to these stress components, there exist four inherent strain components ε^*_x, ε^*_y, ε^*_z, ε^*_{yz} on the transverse sections. Because the effect of shear inherent strain ε^*_{yz} on residual stresses is small[4,5], only the components ε^*_x, ε^*_y, ε^*_z are considered in this paper.

To estimate inherent strains, the shape and the size of inherent strain zones have to be determined first. To propose the shape of inherent strain zones, contour lines of maximum temperature in the thermal cycle and the residual plastic strain zone are computed by thermal conduction analysis and thermal elastic plastic analysis, respectively. The computed results are shown by Fig.4. The shape of both contour lines of temperature and the plastic strain zone is roughly elliptical. Based on these results, the inherent strain zones are assumed to be elliptical as shown in Fig.5. The size of the ellipsis is expressed by radius R_s(s=x, y, z), or by width a_s(s=x,y,z) and b_s(s=x,y,z), and it varies with each of inherent strain components ε^*_x, ε^*_y, ε^*_z.

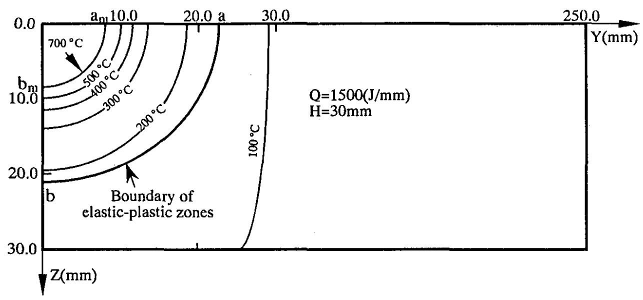


Fig.4 Contour lines of max. temperature in thermal cycle and plastic deformation zone in a bead-on-plate weld

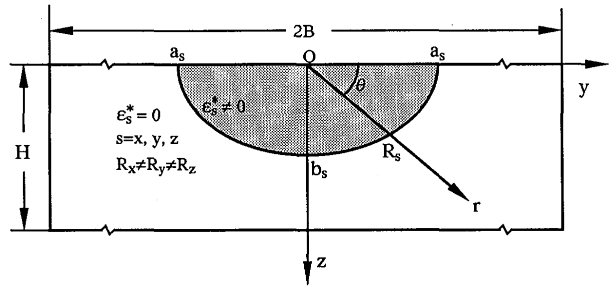


Fig.5 Shape of inherent strain zone in bead-on-plate welds

4.3 Function description for inherent strain distributions

Inherent strain distribution for each component ε^*_s(s=x, y, z) can be expressed as functions of polar coordinates (r, θ) shown in Fig.5.

$$\varepsilon^*_s(x,y,z)=\sum_{i=1}^{M}\sum_{j=1}^{N} A^*_{sij}\, f_{si}(r)\, g_{sj}(\theta), \quad (s=x,y,z) \tag{19}$$

The polar coordinates (r, θ) can be expressed in dimensionless form (ξ_s, ω) defined by the following equation:

$$\xi_s=\frac{r}{R_s}, \quad \omega=\frac{\theta}{\pi/2}, \quad (s=x,y,z) \tag{20}$$

The range of (ξ_s, ω) is from 0 to 1. ξ_s=0 and ξ_s=1 indicate the center of the ellipse and boundary line of inherent strain zones, respectively. ω=0 (θ=0) and ω=1 (θ=π/2) indicate the y axis and z axis, respectively. When dimensionless polar coordinates (ξ_s, ω) are used, Eq.(19) can be rewritten as follows:

$$\varepsilon^*_s(x,y,z)=\sum_{i=1}^{M}\sum_{j=1}^{N} A^*_{sij}\, f_{si}(\xi_s)\, g_{sj}(\omega), \quad (s=x,y,z) \tag{21}$$

For $f_{si}(\xi_s)$ and $g_{sj}(\omega)$, many types of functions can be considered, such as power functions, exponential functions and trigonometric functions. In this paper, very simple power functions are used, and inherent strain distributions are expressed by Eq.(22).

$$\varepsilon^*_s(x,y,z)=\sum_{i=1}^{M}\sum_{j=1}^{N} A^*_{sij}\,(1-\xi_s)^i\,\omega^{(j-1)}, \quad (s=x,y,z) \tag{22}$$

The values of inherent strain given by Eq.(22) are zero at the boundary line (ξ_s=1) of inherent strain zones.

4.4 Estimation of inherent strain distributions by function method

4.4.1 Estimation of the sizes of inherent strain zones

As shown in Fig.5, the sizes of inherent strain zones of components ε^*_x, ε^*_y and ε^*_z are expressed by their radii R_x, R_y and R_z. The values of R_x, R_y and R_z can be determined by numerical computation so that the total error E_r and local maximum error e_r of estimated residual stresses take the minimum, i.e.

$$\frac{\partial Er}{\partial Rs}=0, \quad \frac{\partial e_r}{\partial Rs}=0, \quad (s=x,y,z) \tag{23}$$

If all of the three parameters R_x, R_y and R_z. are taken as unknowns, much numerical computation has to be tried in order to determine their values. Here, inherent strain zone R_x of component ε^*_x corresponding to the largest stress component σ_x due to welding is assumed to be the same as that of the residual plastic strain zone shown in Fig.4. R_y and R_z of components ε^*_y, ε^*_z corresponding to stress components σ_y, σ_z are assumed to be smaller than R_x. When R_y/R_x and R_z/R_x vary from 0 to 1, the changes of total error E_r and local error e_r with R_y/R_x and R_z/R_x are shown in Fig.6. When R_y/R_x and R_z/R_x are about 0.6 and 0.55 respectively, the smallest errors will be obtained, and these R_y and R_z can be considered the sizes of inherent strain zones of components ε^*_y, ε^*_z. Because the sizes of inherent strain zones of components ε^*_y, ε^*_z show very little difference, they can be assumed to be the same.

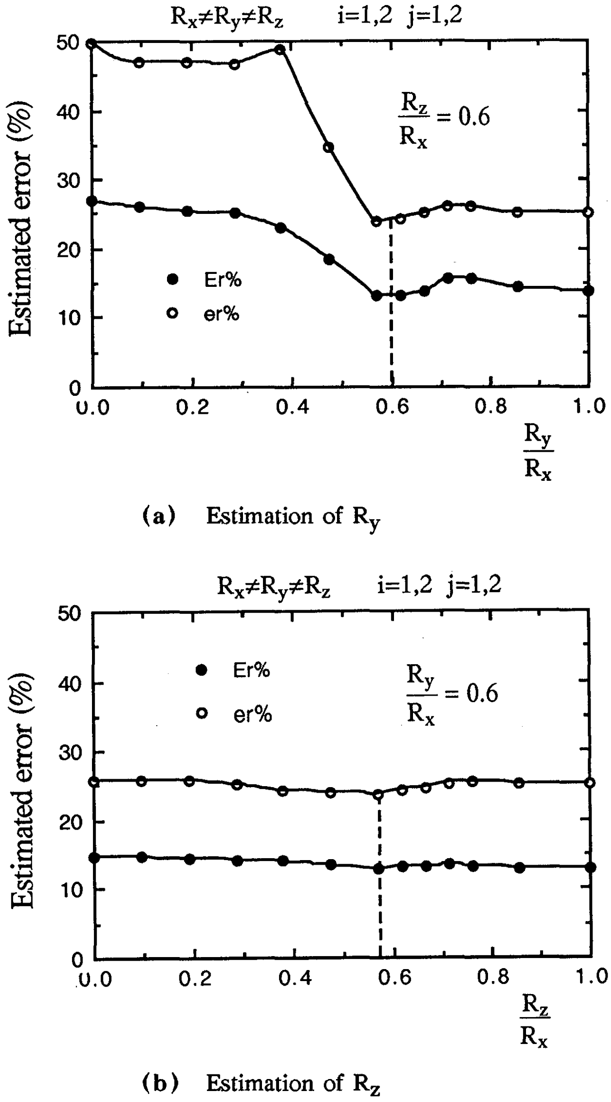


Fig.6 Estimation of the sizes R_y, R_z of inherent strain zones from the errors of reproduced stresses

4.4.2 Accuracy of function method

When inherent strain distributions are expressed by series function, the accuracy is governed by the order M

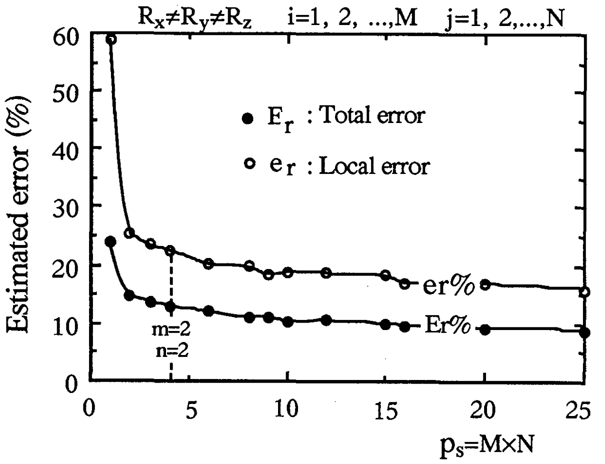


Fig.7 Effect of total unknown coefficients in series function on accuracy of reproduced residual stresses

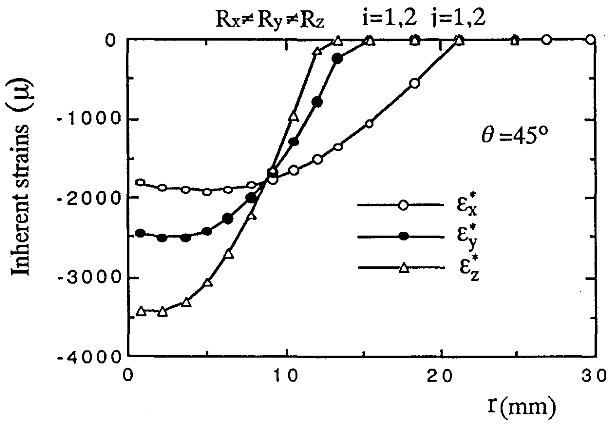


Fig.8 Estimated inherent strain distributions

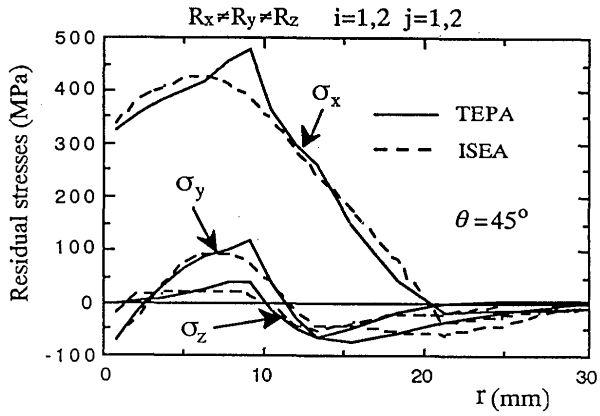


Fig.9 Residual stresses by TEPA and ISEA (TEPA : Thermal Elastic-Plastic Analysis) (ISEA : Inherent Strain Elastic Analysis)

and N of series function. The effects of M and N, or the total number p_s(=M×N) of unknown coefficients for each inherent strain component ε^*_s(s=x, y, z) on the errors Er, e_r of reproduced residual stresses, are shown by Fig.7. By increasing the order M and N of series function, or by increasing the total number p_s(=M×N) of unknown coefficients, the total error E_r and local error e_r decrease. In actual measurements, according to the expected accuracy, M and N of series function, or measuring points corresponding to the total number p_s(=M×N) of unknown coefficients, can be freely selected. As shown in Fig.7, when M and N are more than 2, or the total number p_s of unknown coefficients are more than 4, the change of errors becomes smaller and practically reasonable accuracy can be obtained. In this case, if elastic strains are measured at 4 points for each component, inherent strain distributions can be estimated. Then residual stress distributions can be computed. Therefore, when the function method is employed to describe inherent strain distributions, the required measurements for elastic strains will be greatly reduced compared with the element method.

4.4.3 Distributions of inherent strains and residual stresses

When M and N of series function are taken as 2, inherent strain distributions estimated by the function method are shown in Fig.8. In the radial direction, the inherent strain distributions show a trapezoidal pattern. Using the inherent strains given in Fig.8, residual stresses shown in Fig.9 are estimated by performing simple elastic analysis. As shown in Fig.9, very good accuracy of residual stresses estimated by elastic analysis usingi inherent strain (ISEA) can be obtained compared with those computed by thermal elastic plastic analysis (TEPA).

5. Application of function method to overmatching and undermatching welds

Compared with the strength of base metal, the same or higher or lower strength of weld metals are used in welding. These options are called evenmatching, overmatching and undermatching, respectively. For evenmatching welded joints, the inherent strain distributions can be described by series function as shown in chapter 4. In this chapter, the function method is applied to overmatching and undermatching welded joints. For simplicity, the strength of HAZ is assumed to be different from that of base metal but the same as that of weld metal.

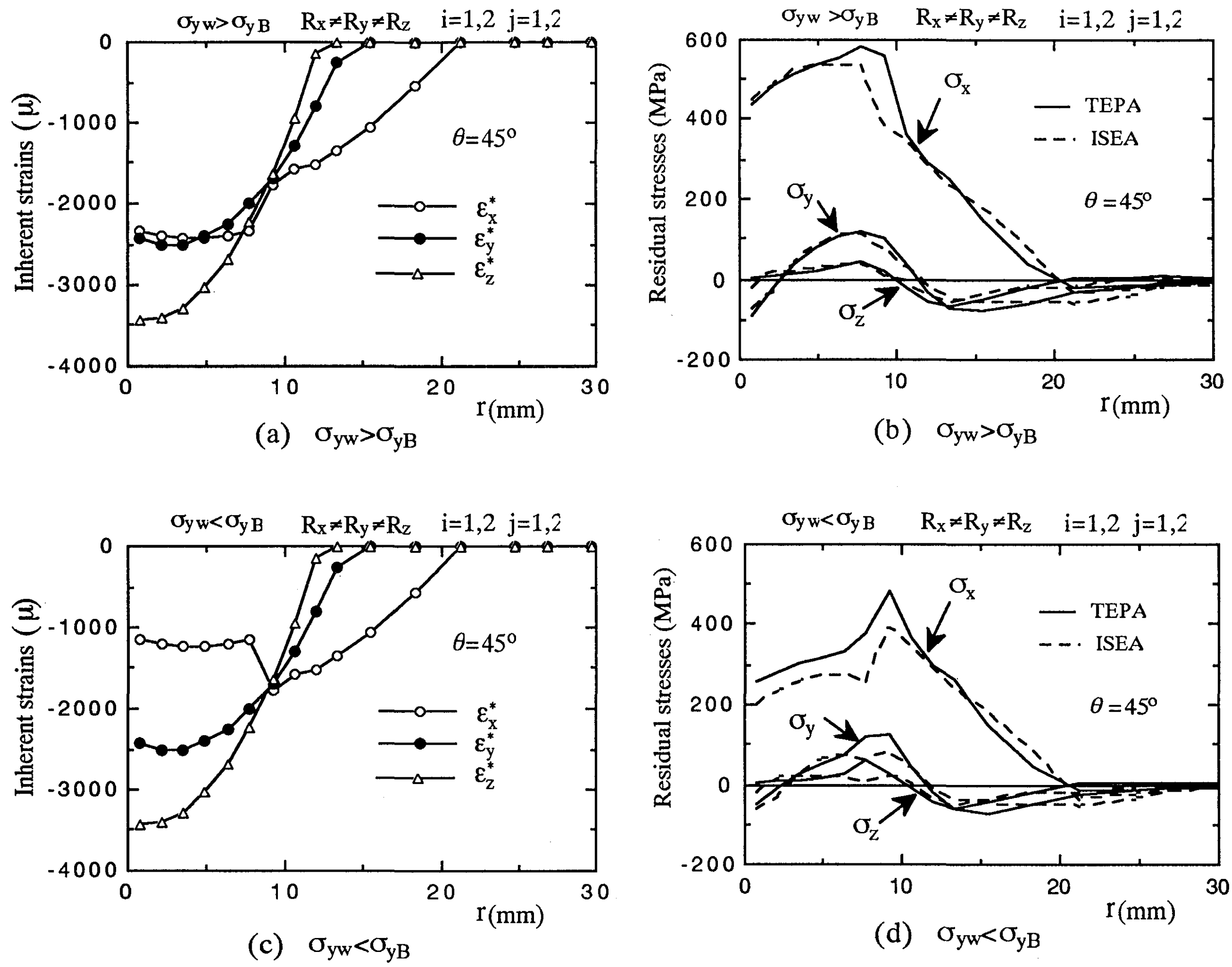


Fig.10 Inherent strain distributions estimated with aid of functions including additional strains $\Delta\varepsilon^*_s$ for weld metal and HAZ in overmatching and undermatching welds, and residual stresses estimated by TEPA and ISEA (TEPA : Thermal Elastic-Plastic Analysis, ISEA : Inherent Strain Elastic Analysis)

5.1 Application of series function to overmatching and undermatching welds

Theoretically, a continuous series function shown in Eq.(22) can also describe inherent strain distributions even in overmatching and undermatching welded joints. However, to obtain high accuracy, the higher order M and N of series function have to be used. To describe such discontinuous characteristics of inherent strain distributions due to overmatching and undermatching using the same order M and N as evenmatching welded joints, a supplemental inherent strain $\Delta\varepsilon^*_s$(s=x,y,z) for weld metal and HAZ is introduced into series function as follows:

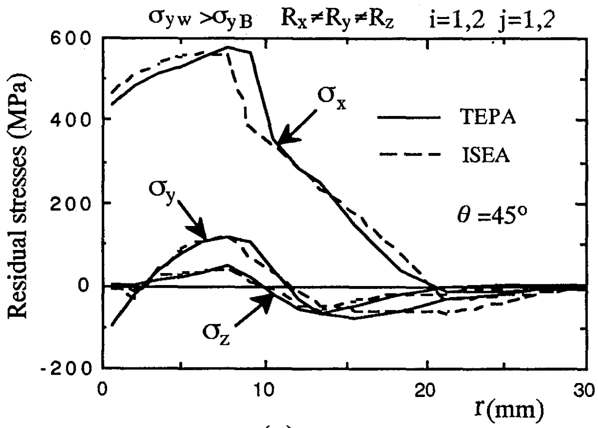
$$\varepsilon^*_s(x,y,z)=\sum_{i=1}^{M}\sum_{j=1}^{N}A^*_{sij}\,(1-\xi_s)^i\,\omega^{(j-1)}+\Delta\varepsilon^*_s \tag{24}$$

The supplemental strain $\Delta\varepsilon^*_s$ does exist in weld metal and HAZ, but it is zero in base metal.

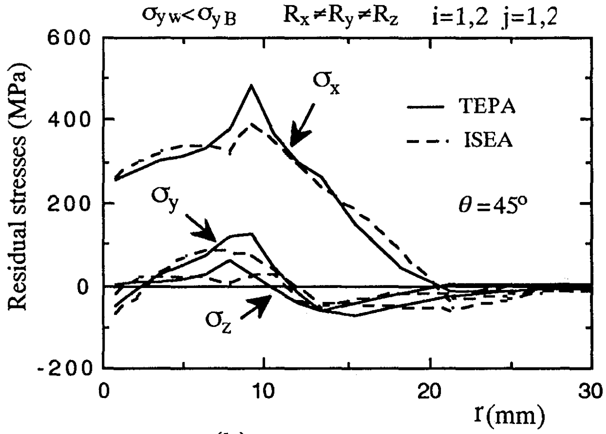
Fig.10 shows inherent strain distributions described by Eq.(24) with M=N=2 and residual stress distributions when the yield stress of weld metal is 450MPa and 250MPa, and the yield stress of base metal is 330MPa, respectively. Comparing Fig.10 with Fig.9, the difference of yield stress of weld metal and base metal has a significant effect on inherent strain component ε^*_x and less effect on the other two components ε^*_y, ε^*_z.

5.2 Prediction of supplemental inherent strain $\Delta\varepsilon^*_s$

Generally, residual stress component σ_x in weld metal and HAZ attains their yield stresses. The other two components σ_y, σ_z are less than the yield stresses. For this reason, by changing the yield stresses of weld metal and HAZ, nearly the same change can be observed in residual stress component σ_x, but little change is produced on residual stress components σ_y, σ_z, and



(a) $\sigma_{yw} > \sigma_{yB}$



(b) $\sigma_{yw} < \sigma_{yB}$

Fig.11 Residual stresses, estimated by inherent strain, whose distributions are described by series function and approximate value $\Delta\varepsilon^*_x \approx (\varepsilon_{yw} - \varepsilon_{yB})$ for weld metal and HAZ in overmatching and undermatching welds (TEPA : Thermal Elastic-Plastic Analysis, ISEA : Inherent Strain Elastic Analysis)

inherent strain components ε^*_y, ε^*_z. Therefore, the supplemental inherent strain $\Delta\varepsilon^*_x$ is approximately equal to the difference of yield strains between weld metal and base metal shown by the following equation, and the supplemental strains $\Delta\varepsilon^*_y$, $\Delta\varepsilon^*_z$ can be neglected.

$$\Delta\varepsilon^*_x \approx (\varepsilon_{yw} - \varepsilon_{yB}) = (\sigma_{yw} - \sigma_{yB})/E \tag{25a}$$

$$\Delta\varepsilon^*_y \approx \Delta\varepsilon^*_z \approx 0 \tag{25b}$$

where ε_{yw}, ε_{yB} are the yield strains of weld metal and base metal respectively. σ_{yw}, σ_{yB} and E are the yield stresses of weld metal, base metal and Young's modulus of the steel, respectively.

To demonstrate the accuracy of the prediction based on Eq.(25) for $\Delta\varepsilon^*_s$(s=x, y, z), the residual stresses are estimated by inherent strain elastic analysis (ISEA) and the results are shown by Fig.11(a), (b), respectively, for overmatching and undermatching welded joints. Compared with the results computed by thermal elastic plastic analysis (TEPA), very good accuracy is obtained.

6. Conclusions

(1) A function method for estimating inherent strain distributions is proposed.

(2) Inherent strain zones show elliptical shape on the transverse sections. The size of elliptical zones varies with each of inherent strain components.

(3) With the aid of series function, inherent strain distributions in bead-on-plate welds are expressed by a few unknown coefficients, residual stresses are reproduced with good accuracy compared with those computed by thermal elastic plastic analysis.

(4) The function method for estimating inherent strains is also efficient for overmatching and undermatching welded joints without increasing unknown coefficients.

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