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CORRIGENDUM TO "DEFORMATIONS OF SPECIAL LEGENDRIAN SUBMANIFOLDS WITH BOUNDARY" OSAKA J. MATH. 51 (2014), 673–693

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The lines 19-28 "Our first result is ...... boundary in [16]" on the page 675 of [2] are corrected into:

"Denote by g the Riemannian metric $\frac{1}{2}d\alpha(\cdot, J\cdot) + \alpha\otimes\alpha$ on M, see [2, (2.1)] for precise constructions. Let $N(L)$ be the normal bundle of L with respect to g, and let $\Gamma(N(L))_W$ be the set of all $V \in \Gamma(N(L))$ that are the deformation vector fields to normal deformation through special Legendrian submanifolds with boundary confined in W. Our first result is

Theorem 0.1. *Let (M, J, α, ϵ) be a contact Calabi-Yau manifold, and L be a connected compact special Legendrian submanifold with nonempty boundary ∂L inside a scaffold W of codimension two. Then the moduli space $\mathfrak{M}(L, W)$ has at most dimension* $\dim H^1(L;\mathbb{R}) + 1$ *near L; moreover $\Gamma(N(L))_W$ is a vector space of dimension at most* $\dim H^1(L;\mathbb{R}) + 1$.

This is similar to Butsher theorem [1]."

The original Theorem 1.1 in [2] is incorrect since Example 2.7 in [2] provided an counterexample to it as pointed out by Georgios Dimitroglou Rizell in his review MR3272612 in MathSciNet. The reason of the incorrectness of Theorem 1.1 was caused by incorrect Proposition 3.3 in [2]. In order to give a correct version of the latter, the following replacements are needed.

(iii) and (iv) in [2, Lemma 3.1] should be, respectively, changed into:

(iii) $(t, x, v, s_1, s_2) \in \phi(\mathscr{U}) \Rightarrow (t, x, v, 0, 0) \in \phi(\mathscr{U})$,

(iv) for any nowhere zero smooth section $V : W \to \xi'^{\perp}|_W$, ϕ can be required to satisfy $\phi_*(V(p)) = \frac{\partial}{\partial s_1}\Big|_{\phi(p)}$ for any $p \in \partial L$, where (s_1, s_2) are the coordinate functions of $\mathbb{R}^2$.

The metric "$\hat{g} := \rho g_1 + (1-\rho)g$" in line 20 of [2, page 684] is replaced by

$$\hat{g} := \rho g_1 + (1-\rho)g.$$

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The metric in (3.16) of [2, page 685] is replaced by

$$\hat{g} = \sum_{k,l=0}^{n-1} (g|_W)_{kl} dz_k \otimes dz_l + ds_1 \otimes ds_1 + ds_2 \otimes ds_2. \tag{3.16}$$

Proposition 3.3 in [2] should be changed into:

Proposition 3.3. *Let L be a compact Legendrian submanifold with boundary of the contact manifold (M, α), and let W be a codimension two scaffold for L. Denote by $\widehat{N}(L)$ the normal bundle of L with respect to $\hat{g}$. For $p \in \partial L$, suppose that $\hat{V} \in \widehat{N}_p(L)$ satisfies the boundary condition: $(d\alpha)_p(N(p), \hat{V}) = 0$. Then $\hat{V} \in T_pW$, and $\hat{V} - \alpha(\hat{V})R_\alpha(p)$ cannot be in T_pL if it is not zero.*

A detailed proof of this result was given in [3]. Correspondingly, the content of the original Remark 3.5 in [2] needs to be changed, see Remark 3.5 in [3]. Hence the related sentence below [2, Claim 2.6] "The Neumann boundary condition implies $\alpha(V|_{\partial L}) = 0$, see Remark 3.5." should be deleted.

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References

[1] A. Butsher: *Deformations of minimal Lagrangian submanifolds with boundary*, Proc. Amer. Math. Soc. **131** (2002), 1953–1964.
[2] G. Lu and X. Chen: *Deformations of special Legendrian submanifolds with boundary*, Osaka J. Math. **51** (2014), 673–693.
[3] G. Lu and X. Chen: *Deformations of special Legendrian submanifolds with boundary (corrected version)*, arXiv:1106.5086v2 [math.DG], 11 May 2015.

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