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Automatic Control of Arc Welding (Report III)†
—Theoretical Consideration of the Kinetics of Heat Processing by a Travelling Heat Source—

Yoshiaki ARATA* and Katsunori INOUE**

Abstract

The kinetic equation is derived assuming a simple model for the various kinds of heat processings. Discussion is made on some characteristics of the derived equation. Behavior of feedback control system composed on the basis of these characteristics is also described.

1. Introduction

The heat sources travel relative to the work, rise its temperature and perform various kinds of heat processings, such as welding, building up, cutting, annealing and so on. The dynamic properties of such processings and, in consequence, their kinetic equations have particular forms owing to the heat source travelling. We must consider this fact sufficiently when designing the automatic feedback control system.

In this report, the kinetic equation is first derived by assuming a simple model for the above process. Discussion is made on some characteristics of the derived kinetic equation follows. Behavior of feedback control system composed on the basis of these characteristics is described.

2. Feedback Control of Heat Processings

The schema of the feedback control system for heat processings by a travelling heat source is shown in **Fig. 1.** The sensor receives information from the heat processing proceeding part in the base material and transmits it to the controller. The controller controls the travelling speed of the heat source (speed control) and/or controls the heat generating source on the basis of that information so as to maintain proper quality for the heat processing.

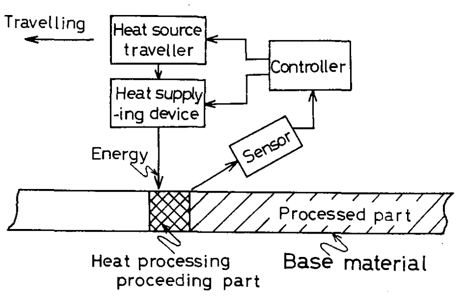

Fig. 1. Feedback control system for heat processing by a travelling heat source.

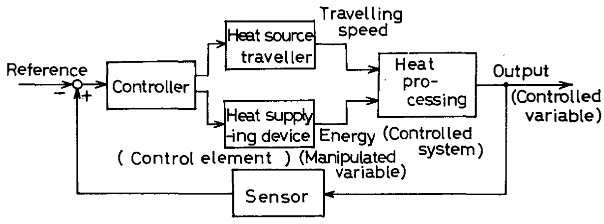

Fig. 2. Block diagram of feedback control system for heat processing.

The block diagram of this feedback system is shown in **Fig. 2.** The automatic control may be performed with one or both of two kinds of the manipulated variables, (the travelling speed and the input energy), so that the controlled variable may follow the reference in such a system.

3. Formularization of Heat Processings

The kinetic equation can be derived for a simple model of the heat processing for which the following three assumptions are made. The schematic explanation for the model is shown in **Fig. 3.**

Assumption 1 The heat processing proceeding part (part A, the meshed part) has the heat quantity Q, the heat capacity C and travels together with the heat source. The temperature of part A is raised uniformly by the heat quantity Q and the heat processings proceed.

Assumption 2 One or more of the variables which prescribe the quantity of the heat processings are sensed by the sensor. Hereafter, these variables are called "the output of the heat processing" or simply

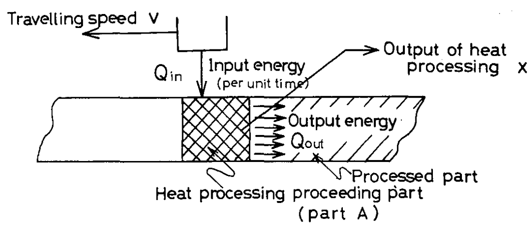

Fig. 3. Simplified model of heat processing.

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"the output". This output x is the value which is proportional to or corresponding to the temperature of part A.

Assumption 3 The flowing out energy from part A, Q_{out}, is proportional to the output x and the travelling speed v.

From Assumption 3,

$$Q_{out} = k_1 \cdot x \cdot v, \quad \text{------} (1)$$

where k_1 is constant.

The change of the heat quantity ΔQ in time Δt in part A is

$$\Delta Q = (Q_{in} - Q_{out}) \cdot \Delta t = (Q_{in} - k_1 \cdot x \cdot v) \cdot \Delta t.$$

From Assumption 1 and 2, the change of the output Δx due to the change of the heat quantity ΔQ is

$$x = k_2 \cdot \frac{\Delta Q}{C} = \frac{k_2}{C} (Q_{in} - k_1 \cdot x \cdot v) \cdot \Delta t, \quad \text{------} (2)$$

where k_2 is constant.

Then, at $\Delta t \rightarrow 0$, we obtain the differentail equation as follows,

$$\frac{k_3}{v} \frac{dx}{dt} + x = k_4 \cdot \frac{Q_{in}}{v}, \quad \text{------} (3)$$

where $k_3 = \frac{C}{k_1}$ and $k_4 = \frac{k_2}{k_1}$. We call (3) as the kinetics equation of the heat processing.

In the steady state, $\frac{dx}{dt} = 0$, then,

$$x = k_4 \cdot \frac{Q_{in}}{v}, \quad \text{------} (4)$$

that is to say, the equation of the static characteristics is obtained.

4. Comparison of Kinetic Equation with Primary Lag Element and Derivation of Non-linear Equation

A typical primary lag element is shown in **Fig. 4**, which is formularized as follows,

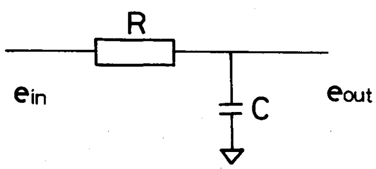

Fig. 4. Typical primary lag element.

$$CR \frac{de_{out}}{dt} + e_{out} = e_{in}, \quad \text{------} (5)$$

where $CR = \tau$ is time constant.

Equation (5) is rewritten as

$$(\text{time constant}) \cdot \frac{d(\text{output})}{dt} + (\text{output}) = (\text{input}) \quad \text{------} (6)$$

When we compare (3) with (6), we may put that time constant $= \frac{k_3}{v}$ and input $= k_4 \cdot \frac{Q_{in}}{v}$. If v= constant and Q_{in} is the manipulated variable, (namely, the heat processing is controlled by input energy only), time constant $\tau = \frac{k_3}{v}$ and (3) is the same form as (6).

This means that the controlled system is regarded as a primary lag element in case the input energy is the manipulated variable. In such case, we can apply the linear automatic control theory to the system analysis, but we must consider the fact that the time constant τ and the gain constant of the system are inverse to the travelling speed of the heat source v.

If Q_{in} = constant and v is the manipulated variable, inversely, canceling the denomination of (3) by multiplying v, we obtain,

$$k_3 \cdot \frac{dx}{dt} + x \cdot v = k_5, \quad \text{------} (7)$$

where $k_5 = k_4 \cdot Q_{in}$ is constant.

As (7) is a non-linear differential equation, the linear automatic control theory in which Laplacian transformation is used cannot be applied to the system analysis. Let us investigate the characteristics of (7) in the followings. In the steady state, corresponding to (4), we obtain,

$$x \cdot v = k_5. \quad \text{------} (8)$$

For a certain travelling speed v_0 and a steady-state output x_0 corresponding to v_0, (8) becomes

$$x_0 \cdot v_0 = k_5. \quad \text{------} (8)'$$

Dividing both members of (7) by (8)′, and introducing dimensionless variables of (10), the simplified dimensionless equation is obtained

$$\dot{x}^* + x^* \cdot v^* = 1, \quad \text{------} (9)$$

where $\dot{x}^* = \frac{dx^*}{dt^*}$

and

$$v^* = \frac{v}{v_0} : \text{dimentionless travelling speed (dimensionless input)},$$

$$x^* = \frac{x}{x_0} : \text{dimensionless output},$$

$$t^* = \frac{v_0 \cdot t}{k_3} : \text{dimensionless time.} \quad \text{------} (10)$$

If we choose the minimum value of the travelling speed as v_0, $v^*>1$ and $x^*<1$ should always hold. We call (9) the dimensionless non-linear equation of the controlled system.

5. Frequency Response for Non-linear Controlled System

The frequency response method is applied to the non-linear controlled system, which is formularized in (7), to investigate its characteristics.

When the input v^* is given as

$$v^*=v_1^*+\Delta v^*\cdot e^{j\omega^* t^*}, \quad \text{(11)}$$

where $\Delta v^* \ll v_1^*$ and ω^* is the dimensionless angular frequency, it is assumed that the output x^* includes higher harmonic components

$$x^*=x_1^*+\sum_{m=1}^{\infty}\Delta x_m^*\cdot e^{j(m\cdot\omega^*\cdot t^*+\phi_m)} \quad \text{(12)}$$

Susbstituting (11) and (12) into (9), neglecting the second order of Δ term and rearranging,

$$\sum_{m=1}^{\infty}(j\cdot m\cdot\omega+v_1^*)\cdot\Delta x_m^*\cdot e^{j(m\cdot\omega^*\cdot t^*+\phi_m)}+x_1^*\cdot\Delta v^*\cdot e^{j\cdot\omega^*\cdot t^*}=0. \quad \text{(13)}$$

From the condition that all the coefficient of $e^{jm\cdot\omega\cdot t^*}$ (m=1, 2, 3...) are equal to zero,

$$\Delta x_m^*=0 \text{ for } m\geqq 2, \quad \text{(14)}$$

that is to say, x^* has no higher harmonic term. In case m=1, we obtain

$$\Delta x^*\cdot e^{j\phi}(j\omega+v_1^*)+x_1^*\cdot\Delta v^*=0,$$

$$\therefore e^{j\phi}\cdot\frac{\Delta x^*}{\Delta v^*}=\frac{-\dfrac{x_1^*}{v_1^*}}{\sqrt{1+\left(\dfrac{\omega^*}{v_1^*}\right)^2}}e^{j\tan^{-1}\left(\frac{\omega^*}{v_1^*}\right)}, \quad \text{(13)}$$

where we replace $\Delta x^*=\Delta x_1^*$ and $\phi=\phi_1$.

We can see from (15) as the log magnitude

$$\log\frac{\dfrac{x_1^*}{v_1^*}}{\sqrt{1+\left(\dfrac{\omega^*}{v_1^*}\right)^2}}=\log\frac{\dfrac{1}{v_1^{*2}}}{\sqrt{1+\left(\dfrac{\omega^*}{v_1^*}\right)^2}}, \quad \text{(16)}$$

and as the phase

$$\phi=-\tan^{-1}\left(\frac{\omega^*}{v_1^*}\right). \quad \text{(17)}$$

An example of Bode diagram is shown in **Fig. 5**, in which the log magnitude curves and the phase curves are drawn for both cases $v_1^*=v_2^*$ and $v_1^*=2v_2^*$. It is seen that the static characteristic depreciates by -12 decibel and the speed of response doubles at the input equal to twice its original value.

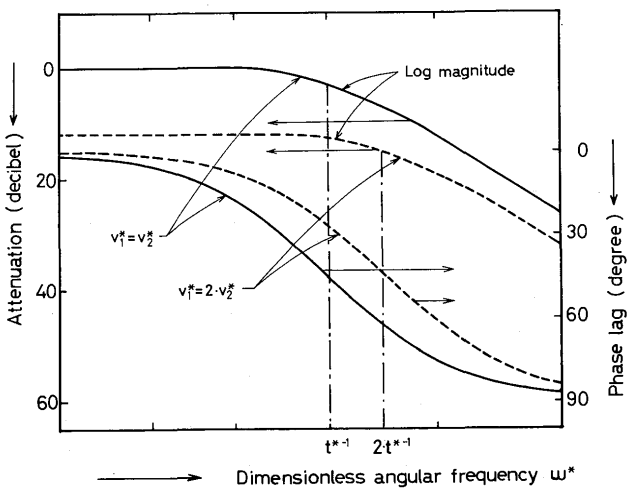

Fig. 5. Bode diagram for non-linear controlled system.

This means that the gain constant of the control element should be four times so that error may remain less than the definite value and that it may be eight times to assure the definite phase margin when the static component v_1^* of the input v^* becomes doubles.

6. Proportional Control System and Disturbance Entering into

When the disturbance enters into the system, we need to change the constants $k_4\rightarrow k_4'$, $k_5\rightarrow k_5'$ in (3) and (7) according to the changing of the constant $k_2\rightarrow k_2'$ in (2). As the result, the right member of (9) is changed from 1 to d ($\neq 1$), then,

$$\dot{x}^*+v^*\cdot x^*=d. \quad \text{(18)}$$

We should solve (18) under the initial condition

$$\text{at } t^*=0,\ x^*(0)=d\cdot x_1^*. \quad \text{(19)}$$

In case d=constant, equations (18) and (19) express the step disturbance enters into the system. Such a step response of the proportional control system is investigated as follows. The equation of the control element in the proportional control system is

$$v^*=A\cdot(x^*-x_1^*)+v_1^*, \quad \text{(20)}$$

where

x_1^*: the reference value for the output x^*. (at the steady state before the disturbance enters, the output value is x_1^*)

v_1^*: the input value at the steady state corresponding to the input x_1^*, namely, $v_1^*\cdot x_1^*=1$.

A : the gain constant of the control element.

The block diagram of the proportional control element is shown in **Fig. 6**. Substituting (20) into (18), we obtain

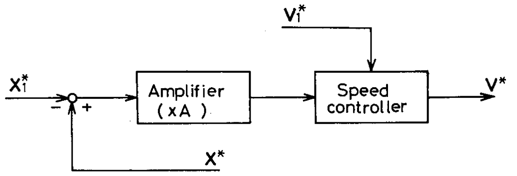

Fig. 6. Step response in proportional control system.

$$\dot{x}^*+A\cdot x^{*2}+B\cdot x^*-d=0\,, \qquad (21)$$

where $B=v_1^*-A\cdot x_1^*$.

As (21) is Riccatti's equation of constant coefficient, it can be solved analytically under the initial condition of (19) as

$$x^*(t^*)=\frac{(d\cdot A\cdot x_1^*-\alpha_2)\,\alpha_1\cdot e^{\alpha_1 t^*}+(\alpha_1-d\cdot A\cdot x_1^*)\,\alpha_2\cdot e^{\alpha_2 t^*}}{A\{(d\cdot A\cdot x_1^*-\alpha_2)e^{\alpha_1 t^*}+(\alpha_1-d\cdot A\cdot x_1^*)e^{\alpha_2 t^*}\}}\,, \qquad (22)$$

where α_1 and α_2 are $\dfrac{-B\pm\sqrt{B^2+4\cdot d\cdot A}}{2}$

Transforming (22),

$$x^*(t^*)=\frac{\alpha_1(d\cdot A\cdot x_1^*-\alpha_2)\,e^{\alpha_1 t^*}\left\{1+\dfrac{(\alpha_1-d\cdot A\cdot x_1^*)\,\alpha_2}{(d\cdot A\cdot x_1^*-\alpha_2)\,\alpha_1}\,e^{(\alpha_2-\alpha_1)t^*}\right\}}{A(d\cdot A\cdot x_1^*-\alpha_2)\,e^{\alpha_1 t^*}\left\{1+\dfrac{\alpha_1-d\cdot A\cdot x_1^*}{d\cdot A\cdot x_1^*-\alpha_2}\,e^{(\alpha_2-\alpha_1)t^*}\right\}}\,, \qquad (23)$$

and replacing the following terms

$$\left.\begin{aligned} A_1&=\frac{\alpha_1-d\cdot A\cdot x_1^*}{d\cdot A\cdot x_1^*-\alpha_2}\ (<1),\\ \alpha&=-\frac{\alpha_2}{\alpha_1}\ (>1)\,,\\ \Delta\alpha&=\alpha_1-\alpha_2\ (>0)\,. \end{aligned}\right\} \qquad (24)$$

we obtain the approximate equation

$$\begin{aligned} x^*(t^*)&=\frac{\alpha_1(1-A_1\cdot\alpha\cdot e^{-\Delta\alpha\cdot t^*})}{A(1+A_1\cdot e^{-\Delta\alpha\cdot t^*})}\\ &\doteqdot\frac{\alpha_1}{A}\,(1-A_1\cdot\alpha\cdot e^{-\Delta\alpha\cdot t^*}-A_1\cdot e^{-\Delta\alpha\cdot t^*}) \qquad (25)\\ &=\frac{\alpha_1}{A}\,\{1-A_1(1+\alpha)\,e^{-\Delta\alpha\cdot t^*}\}, \end{aligned}$$

then,

$$\lim_{t^*\to\infty}x^*(t^*)=\frac{\alpha_1}{A}\ . \qquad (26)$$

As (26) can also be introduced from (22) directly, it is seen that (22) and (26) are in accord with $t^*\to\infty$.

The relative steady-state error of the step response is

$$\begin{aligned} E_{step}&=\lim_{t^*\to\infty}\left\{\frac{x^*(t^*)-x_1^*}{x_1^*(d-1)}\right\}\\ &=\frac{-\dfrac{1}{x_1^*}-A\cdot x_1^*+\sqrt{\left(\dfrac{1}{x_1^*}-A\cdot x_1^*\right)^2+4\cdot d\cdot A}}{2\cdot A\cdot x_1^*(d-1)}\,, \end{aligned}$$

and it does not only depend on the gain constant A of the control element, but also on the value of the disturbance d and the static values of the input v_1 and the output x_1^*. This is the essential feature of a non-linear control system. Examples of the step response which are calculated from (23) are shown in **Figs. 7** and **8.** The step response curves are drawn for several values of the gain constant A in Fig. 7 and of the static component v_1^* of the input in Fig. 8.

A example of the relative steady-state error calculated from (27) is shown in **Fig. 9.** It is natural

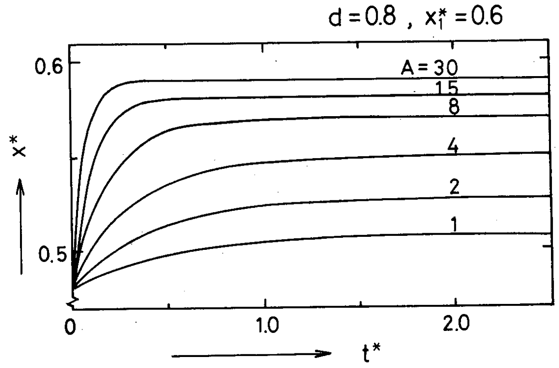

Fig. 7. Step response in proportional control system.

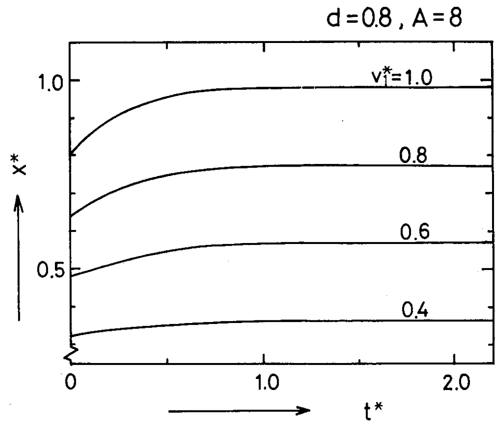

Fig. 8. Step response in proportional control system.

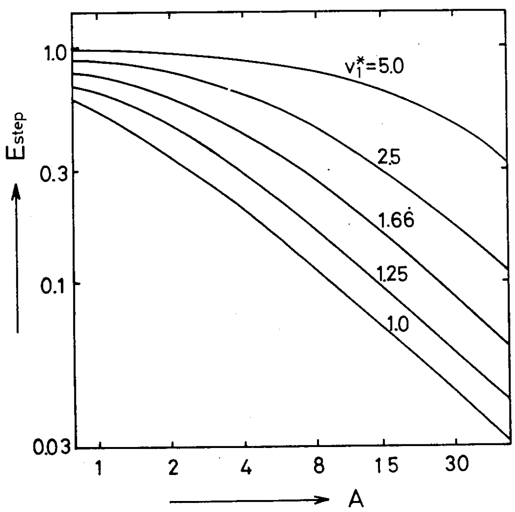

Fig. 9. Relative steady-state error of step response in proportional control system.

that the relative steady-state error should decrease as the gain constant A increases, but they also decrease as the values of v_1^* increase. The latter is due to the fact that the magnitude curve in Fig. 5, Bode diagram, goes down as v_1^* increases.

7. Integral Control for Non-linear Controlled System

It is necessary to compose the integral control element for the controlled system, which is expressed by (9), in order to avoid the steady error as is produced in the proportional control system on a step disturbance entering into the system. Considering the discussion made in section 5, it is desirable for the control element in the system to be modified by its own output. The integral control system having such a function is realized in the block diagram as shown in **Fig. 10.**

The gain of the integrator in the control element is proportional to its output to the n th power in this system. The equation of the control element is given as

$$\dot{v}^* = A \cdot v^{*n} (x^* - x_1^*) \quad \text{(28)}$$

If we solve (18) and (28) under the condition of (19), we can see the behavior of the system, but these non-linear equations cannot be solved analytically. Then, we solve them numerically. An example of the numerical solution by R. K. G. method is shown in **Fig. 11.** The numerical calculation was conducted with NEAC—2000 series model 700 at Computation

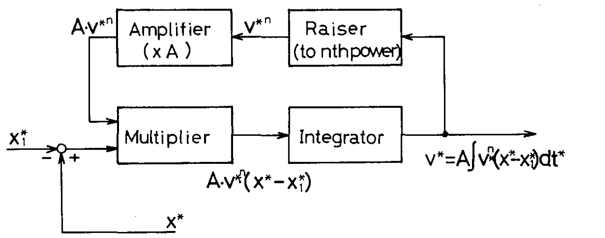

Fig. 10. Block diagram of integral control element.

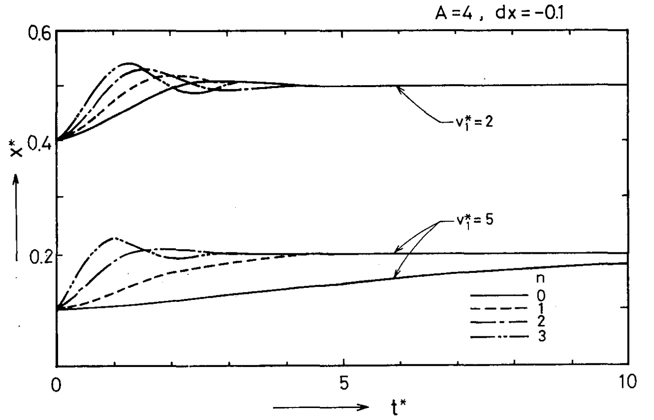

Fig. 11. Step response in integral control system.

Center Osaka University. The square integral of error $I_{E2} = \int_0^\infty \{x_1^* - x^*(t^*)\}^2 dt^*$ is also caluculated and plotted by power n for a few values of v_1^* in **Fig. 12.**

When the power n is equal to 2, the value I_{E2} is nearly constant, independently of the value v_1^*.

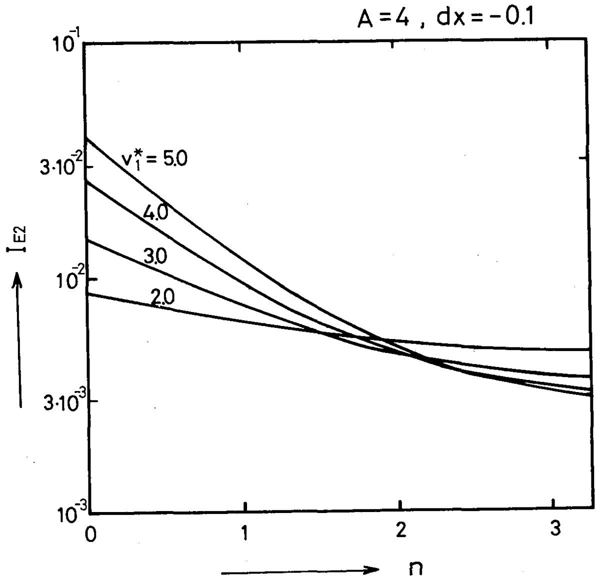

Fig. 12. Square integral of error in integral control system.

8. Conclusion

1. The kinetic equation of the heat processings by travelling heat source in derived by assuming a simplified model for it.
2. The derived kinetic equation is a first order linear differential equation in the case that the input energy is the manipulated variable to the controlled system.
3. The derived kinetic equation is a non-linear differential equation in the case that the heat source travelling speed is the manipulated variable.
4. The frequency response method is applied to the non-linear controlled system and Bode diagram is obtained. Both the magnitude curve and the phase curve in Bode diagram shift their position according as the static component of the input.
5. The proportional feedback control for the non-linear controlled system is described in Riccatti's equation which can be solved analytically. This feedback control shows characteristics peculiar to a non-linear system.
6. On performing an automatic control for the non-linear system, the system whose control element can be modified by its own output is desirable. Such a system can be realized in the special integral control element.