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Essays on Human Capital Externalities and Population Dynamics

(人的資本の外部性と人口動態に関する研究)

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Ph.D. dissertation,
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Contents

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Chapter 1

Introduction

After the beginning of the Twentieth century, people began to aspire to acquire higher education more than than ever before. This tendency was due to people beginning to recognize that a higher education level would lead them larger financial and social benefits. Moreover, people also had a desire for higher education achievements, seeking the honor of the academic degree itself.

Higher levels of education also accelerate global technological development. Technological progress such as computers, the Internet, and telecommunications also improves the world wide human capital levels by reducing costs. As a result, higher education develops not only economic activity but also medical technology, which dramatically increases longevity.

In addition, people began to move away from their born country/region of birth to acquire a higher education level. Consequently, human capital externalities have an extremely close relationship between population dynamics, which is composed of net immigration, births and deaths.

Accordingly, in this dissertation, I investigate the relationship between human capital externalities and population dynamics. The composition is as follows: Chapter 2 analyzes the externalities for the education level itself. Chapter 3 investigates the migration decision on determining education and birth subsidy level between regions. Chapter 4 researches the longevity effect on demography issues, migration and the size of a country's economy.

Chapter 2 analyzes the effect of education externalities on schooling decisions. In some developed countries, such as Japan and Sweden, the number of years of education does not predict wage differences as opposed to in some countries such as the United States and Germany. To explain such seemingly contradictory observations, this study develops a simple model utilizing the ‘keeping up with Joneses’ effect regarding schooling decisions. A main result of this study is that the model can have multiple equilibria, which can explain the difference between the two groups of countries. Moreover, efficiency analysis reveals that changes in the strength of education reference and psychological cost parameters can alter the welfare ranking of multiple equilibria.

Chapter 3 considers problems associated with one's educational decisions and migration, considering the birth policy by local governments. In this chapter, I developed a bi-reginonal asymmetric economy model using heterogeneous individuals. In the model, governments spend their budgets on university education and birth support for individuals, yielding a fixed level of university quality; governments levy lump sum taxes from residents to pay for it. This paper shows that the existence of an asymmetric equilibrium with migration. Using the autarky equilibrium as a benchmark, higher university quality leads to higher welfare and a larger number of births in return for the low-skilled worker utility losses. At the immigration equilibrium, the university budget is larger than both the autarky and coordination cases. These results stem from the ratio between high-skilled and low-skilled workers, suggesting that restricting the number of immigrants to the agglomerated region is a welfare-improving policy.

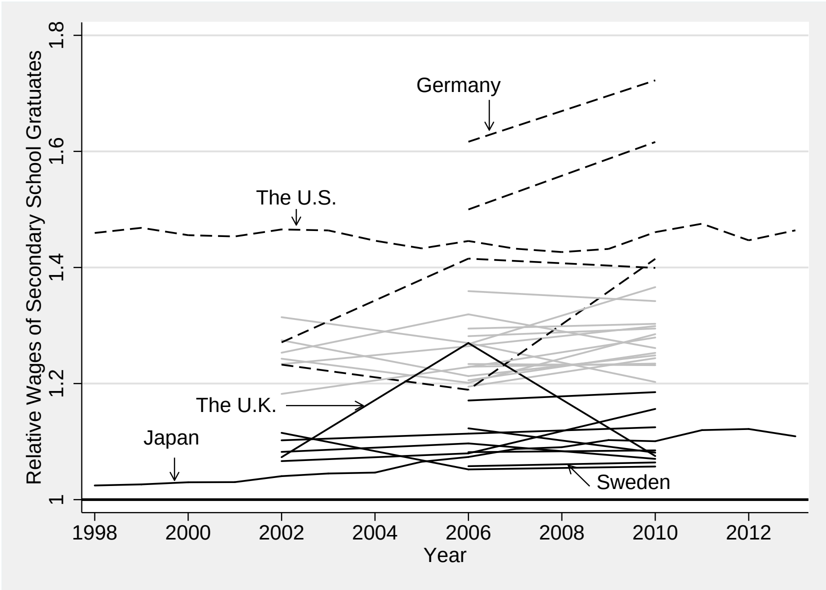
Chapter 4 develops an analysis of the relationship among demographics, immigration and market size in a country–considering the improvement in longevity due to technological progress. This chapter constructs an overlapping generations model wherein people decide their number of children and levels of consumption for differentiated goods. We further assume that immigration takes place according to the utility difference between inside and outside a country. We show that an improvement in longevity has three effects on the market size and welfare: First, it decreases the number of children. Second, it increases the per capita expenditure on consumption. Finally, it increases immigration. The first effect has negative impacts on the market size and welfare whereas the latter two effects have positive impacts. We then calibrate our model to match Japanese and U.S. data from 1955 to 2014 and find that the negative effects dominate the positive ones. Moreover, our counterfactual analyses show that accepting immigration in Japan can be useful in overcoming population and market shrinkage caused by an aging population.

Chapter 2

Effects of education externalities on schooling

2.1 Introduction

Education is widely considered to play a very important role in human society because it has contributed to both the growth of specific countries and overall global growth through technical innovations. For developed countries, primary education is provided to almost all children, and schooling rates of higher education have risen. Standard economic theory considers that the potential to earn a higher wage is a main factor for obtaining a higher education. It states that widening wage differences between educational levels will induce an increase in schooling length. However, some actual data contradict this theory. Figure 2.1 shows the relative wages of upper secondary school graduates to those of lower secondary school graduates. The higher wage paid to the former implies the presence of an economic incentive to attend upper secondary school. Standard economic theory states that the relative wages of graduates compared with non-graduates induces a rise in graduation rates. Figure 2.2 depicts student participation rate data from the Eurostat database. The data reveal trends in educational choice in different countries beyond compulsory education levels. The post-compulsory schooling rate was used to enable us to consider grade retention and differences between education systems.

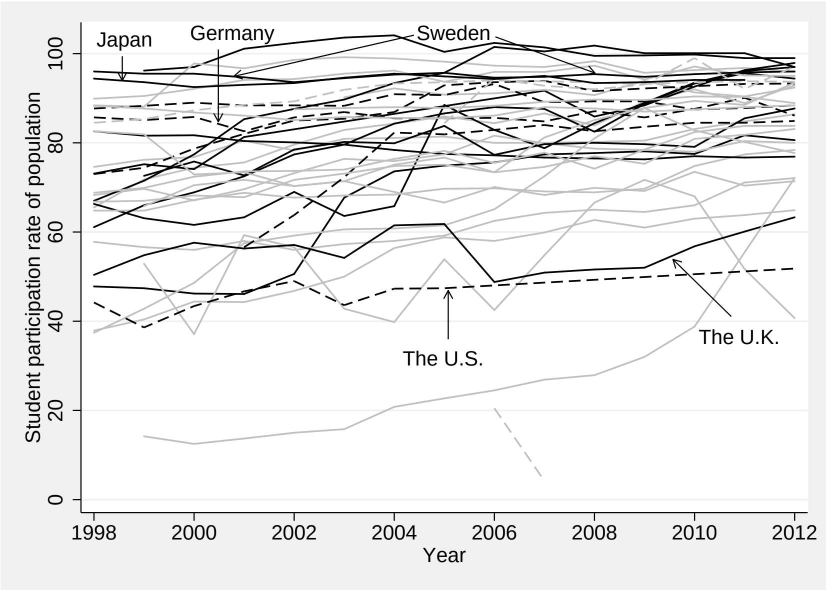


The data on U.S. relative wages are median data from weekly and hourly earnings data from the Current Population Survey(CPS). The data on Japan's relative wages are mean data from its Basic Survey on Wage Structure. The data on EU countries' relative wages are mean monthly earnings data from Structure of Earnings Survey 2002, 2006 and 2010 from the Eurostat database. The difference between median and mean is due to data availability. The U.S. data are the ratio of the median relative wages of males over 25 years old who are high school graduates compared with those who are not high school graduates. The Japanese data are the ratio of average relative wages of working males who are high school graduates compared with those who are not high school graduates. Dashed lines indicate countries where the relative wage is greater than 1.4 in 2010. Grey lines indicate countries where the relative wage is between 1.2 and 1.4 in 2010. Solid lines indicate countries where the relative wage is less than 1.2 in 2010. We used data on 27 EU countries obtained for more than two periods of the three surveys mentioned above. The included EU countries are listed in the Appendix.

Fig. 2.1 Relative average wages of secondary school graduates to non-graduates.

From the figures and the data, we conventionally divide these countries into two groups[1]. The first group is composed of Germany and the United States from the dashed-line group, and the United Kingdom from the grey-line group. In these countries, standard economic theory predicts the movement seen in the data; a high wage difference leads to a high schooling rate. For example, in Germany, the relative wage is 1.72 and the schooling rate 87.6 percent in 2010. These values indicate that high wage difference

[1] This division is a conventional one because of lacking definitive criterion.



Students' participation is the percentage of the population who aged two additional years from the end of compulsory age. The data are from Participation/ Enrolment in education (ISCED 0-4) (educ_ipart) in the Eurostat database. The lines imply the group used in Figure 2.1. The dashed grey lines are countries for which wage data is not available. The examined countries are listed in the Appendix.

Fig. 2.2 School participation rate of the same-age population cohort after two years from the end of compulsory education in EU countries, United States and Japan.

induces a high schooling rate. In the United States, the relative wage is 1.46 and the schooling rate is 51.8 percent in 2012, respectively. These values indicate that high wage differentials lead to a high schooling rate but that the presence of some educational or structural difficulties reduces the schooling rate. In the United Kingdom, the relative wage is 1.07 and schooling rates is 56.8 percent in 2010, respectively. The low ratio indicates that the low schooling rate is due to low wage differentials.

In contrast, the second group, which is composed of Japan and Sweden from the solid-line group, does not obey the standard economic theory. In this group, the relative wage is very low but most people nevertheless decide to attend upper secondary school. For instance, in Japan, the relative wage was 1.1 in 2010; however, in the same year, the schooling rate was 94.1 percent. In Sweden, the wage ratio is 1.06 and the schooling rate is 95.4 percent, respectively. Standard economic theory cannot explain the differences between these two groups. These figures suggest that people in these countries choose to obtain a higher education for reasons besides (or in addition to) the expectation of higher wages as a result. Therefore, schooling decisions in these countries can be viewed as being determined by non-monetary factors as well, such as the utilities obtained from education level, an argument that we develop in this chapter.

In this study, a simple model of the education externality is developed and investigated. The investigation result shows that two education equilibria exist in this model if the existence condition is satisfied. This presence of multiple equilibria can explain the differences between countries such as United States and Japan regarding the relationship between wages and schooling decisions. The model also indicates that an economy where education levels are determined solely by individuals cannot achieve a socially optimal level of education. Moreover, analysis from a social planner's perspective reveals that placing certain conditions on parameters can change individuals' social preferences regarding desired education level.

In the economics literature, Becker (1993) developed the widely-accepted theory of human capital formation. He investigated the effect of human capital, which is accumulated through education and training, upon the economy by considering the marginal benefits and marginal costs of education. Subsequently, many studies have examined the relationship between human capital investment and returns, the latter of which are primarily measured using wages or several other monetary measures. Therefore, wages are frequently viewed as being determined by education or human capital. Mincer (1974) theorized this relationship as the Mincerian wage equation, which is frequently used to determine factors that affect wages. Moretti (2004) used the Mincerian wage equation to empirically study the relationship between number of schooling years, place of residence and wages. The study showed that increasing the number of highly educated people in a Metropolitan Statistical Area (MSA) induces increased wages within that MSA, which is known as the spillover effect. And Calvó-Armengol et al. (2009) showed that after controlling for other factors, the school performance of children in the United States is significantly and positively affected by social networks. In a similar research, Giannini

(2003) considers a bargaining model of human capital involving the interaction between individual and aggregate human capital levels. However, that paper considers only an externality induced through wages. These results suggest the existence of a positive education externality, a finding this study builds upon. Many other empirical research studies support the existence of externalities. Therefore, this study views the existence of a human capital externality as a given.

Before describing the model, we review the so-called 'keeping up with the Joneses' consumption effect because the education effect is similar. Many researchers have studied 'keeping up with the Joneses' consumption effects. Abel (1990) developed an asset-pricing model that investigated the effect of aggregate consumption per capita on asset pricing and showed that the 'keeping up with the Joneses' effect is a source of equity premiums. Ljungqvist and Uhlig (2000) explored taxation effects on 'keeping up with the Joneses' consumption. Their study found that although several externalities exist, government taxation is a good method for controlling the economy. Their paper's implications are utilized in this study's discussion of policies. Dupor and Liu (2003) modified Abel (1990) by measuring the utility of the 'keeping up with the Joneses' effect from the marginal consumption level. Their finding on evaluating welfare is critical and is utilized here in constructing the proposed model. Liu and Turnovsky (2005) considered macroeconomic dynamics with consumption and production externalities as the 'keeping up with the Joneses' effect in consumption and production functions and capital stock externalities in production. In our study, the production function that determines wages is considered a human capital externality. This externality, described as the average education level of workers in an economy, is designed to increase personal income. Mino (2008) developed an overlapping generations (OLG) model with a consumption externality and demonstrated that this consumption externality fundamentally affects the characterization of both the equilibrium and steady state. Because parents primarily affect children's education, the OLG economy model is applicable when considering education. However, because the OLG economy model might be too complicated, this study does not consider generations. These previous studies carefully developed consumption externalities, but they only considered consumption externalities and excluded education externalities. In addition, classifying education as consumption is incorrect because education has capital characteristic. Similar to this study, Tournemaine and Tsoukis (2015) consider a growth model with human capital formation seeking educational status. Their study represents 'keeping up with Joneses' by choosing regime between public and private school. However, their model cannot explain the relationship between schooling decisions and wages that we want to explain in this study.

The remainder of this chapter is structured as follows. The second section describes the model, including the existence of an education externality. The third section analyses the discovered equilibria. The fourth section develops a social planner's solution and discusses policy implications. The final section sums up the findings and offers conclusions.

2.2 Model

In this section, we develop an education externality model. The economy comprises representative households and competitive firms. Households decide their own education and consumption levels, and the firms pay wages to households equivalent to their productivity.

2.2.1 Households

We assume that the representative households obtain utility from consumption and education. The representative households' utility function is as follows:

$$u(c,e) = c + v\left[(e-\alpha E)^2 - \varphi e^3\right], \tag{2.1}$$

where c represents consumption, e denotes household education level and E indicates the reference level of education, which can be interpreted as a society's average education level. The parameters are as follows: v is a constant that represents the importance of education in determining utility, α denotes the reference level of education and φ is a weight denoting the effort cost to obtain education.

We assume the parameter $v > 0$ to account for the 'keeping up with the Joneses' effect; this effect indicates that a household will increase its own education level if the education level of other households, i.e. the reference level, is high. To simplify the analysis of the model, we assume the parameter $\alpha \in (0, 1)$. This assumption implies that a person values other people's education levels less than his or her own education level. We assume the parameter $\varphi > 0$, indicating that the effort to obtain education is borne by the whole household, which we consider as comprising both psychological and monetary costs.

We consider the meaning of the educational term in the utility function. The term $(e - \alpha E)^2$ can be interpreted as the utility obtained from the relative education level, which reflects strictly increasing returns to scale. This term also implies a preference for diversity. The farther away someone is from the reference education level, the more utility is gained by increasing their education level. The $-\varphi e^3$ term indicates that disutility from education is the effort cost. This term reflects that the effort cost becomes greater than the utilities from education and consumption as education level grows.

For other externalities, education level is determined through the market. Education positively affects production, wages and so on. However, this chapter shows that education affects not only to wages but also individual utility levels. This is a major difference with the other externalities such as in Calvó-Armengol et al. (2009) and Giannini (2003).

Overall, this utility function assumes that peer group unilaterally affects the household and considers exogenous and fully correlated effects from the classification by Manski (1993), who divided the social effects operating within a group into three types: endogenous, exogenous, and correlated. In section three, we also consider the endogenous effects case.

Two points should be made regarding the utility function's specification. First, the utility function has a quasi-linear form, which is needed because the function representing the education externality requires a non-homothetic characteristic. Alonso-Carrera et al. (2008) developed the restricted homothetic (RH) property and showed that the equilibrium indeterminate case does not hold the RH property. The RH property implies that utility function holds homothetic property within individual variable (i.e. e) and aggregate variable (i.e.E), but it does not hold the property as a whole. In addition, we employ this specification to facilitate an evaluation of the education externality. Dupor and Liu (2003) suggested that the externality must be measured from the marginal utility of consumption. Therefore, I added the utility function given in equation (2.1) as a simple expression of these two factors. In addition, Ljungqvist and Uhlig (2000) used this type of quasi-linear utility function to examine consumption externalities and labor disutility.

The second point regarding the specification concerns the existence of an education externality. No empirical study has thus far estimated the level of the education externality. However, several empirical studies conjectured the presence of this externality, as noted in the introduction. Furthermore, although fewer empirical studies have been conducted in Japan due to data restrictions, Hashimoto and Heath (1995) estimated the income elasticities of educational expenditures in Japan. The average estimated elasticity of education is 1.72, which suggests the existence of an education externality in Japan because it implies that people's expenditure on education exceeds the increased income they earn from obtaining that education. Therefore, we can conjecture that the externality parameter v can be positive.

To gain utility from consumption and education, households spend money on them. Household members earn money only from wages, which they earn from a firm by supplying one unit of labor Thus, households face the following budget constraint:

$$c + pe = w(e) + \underline{w}, \tag{2.2}$$

where p represents the price of one unit of education cost, $w(e)$ represents the wage that the household earns from a firm, related to household education level and $\underline{w}$ represents the wage the household earns from a firm regardless of education level. Therefore, the household maximizes its utility according to the budget constraint (2.2).

2.2.2 Firms

Perfectly competitive firms pay wages to employees according to their productivity. Employee productivity is represented by the following equation:

$$A[\theta \ln e + (1-\theta)\ln E] + \underline{w} = w(e) + \underline{w}, \tag{2.3}$$

where A is total factor productivity and θ is a coefficient of the production function, which takes a value between 0 and 1.

Firms' final goods are used in consumption or in the education system. This production function indicates that the average level of education affects private wages; that is, if the social education level is high, households receive high wages, and vice versa. This wage function represents the way in which the social infrastructure of human capital affects wages. For example, if the average education level is high, people can communicate clearly yet succinctly. Thus, easy communication increases productivity, which induces higher wages. This relationship corresponds to secondary education rather than higher education, such as lectures at university.

The reason for this result is that, on one hand, a university education can be interpreted as a signal but it may not necessarily improve a person's ability. On the other hand, secondary education mainly comprises learning basic skills for work, i.e. reading, writing and arithmetic. Therefore, this specification is primarily considered in the relationship between secondary education and wages and is compatible with the results of empirical studies by Trostel (2004) and Moretti (2004).

In other externalities, the education level is determined through the market, and education positively affects production and/or wages (Calvó-Armengol et al. (2009) and Giannini (2003) are example). However, this chapter shows that education affects not only to wages but also individual utility level. This is a major difference to the other externalities.

2.3 Analysis of equilibria

In this section, we temporarily fix E to solve each household's decision regarding e. In a later subsection, E will be determined endogenously. Here we analyze the model by dividing it into three parts. The first part is the case with no education externality, the second part is the case with education externality where E is exogenously determined and the third part is the education externality case where E is endogenously determined. Hereafter, we concentrate on the variable e because it is the determinant variable of this model.

2.3.1 Optimization problem

The representative household maximizes its own utility when accounting for firm behavior regarding wages. Then, by combining equations (2.1), (2.2) and (2.3) and imposing non-negative conditions on e and c, the maximization problem is as follows:

$$\begin{aligned} \max_{c,e} \quad & u(c,e) = c + v\left[(e-\alpha E)^2 - \varphi e^3\right] \qquad (2.4)\\ s.t. \quad & c + pe = A[\theta \ln e + (1-\theta)\ln E] + \underline{w}, \\ & c \geq 0, \quad e \geq 0. \end{aligned}$$

The first-order condition of the problem is

$$v[2(e-\alpha E) - 3\varphi e^2] = p - \frac{A\theta}{e}. \tag{2.5}$$

Rearranging this condition yields

$$2ve + \frac{A\theta}{e} = p + 2v\alpha E + 3v\varphi e^2. \tag{2.6}$$

The non-negative condition is

$$\underline{w} \geq pe - A\theta \ln e - (1-\theta)\ln E, \quad \text{and} \quad e \geq 0.$$

Hereafter, we assume that $\underline{w}$ is sufficiently large to have interior solutions, upon which we concentrate for simplicity. The left-hand side of equation (2.6) indicates the individual's marginal benefits from education, and the right-hand side indicates the individual's marginal costs for education. The first and second terms of the left-hand side are marginal benefits from the education externality and wages, respectively; the first, second, and third terms of the right-hand side are marginal costs of education, 'keeping up with the Joneses,' and acquisition efforts, respectively. The budget constraint of (2.4) and second term of the right-hand side of (2.6) represent the externality in this model. Individuals ignore the effects of their education level upon production and reference level. However, this equation cannot be solved because of its non-linearity, and it will be analyzed later part in this section.

2.3.2 Benchmark case

First, we assume that no education externality exists, i.e. $v = 0$, as the benchmark case of the model. We derive the model equilibrium when regarding E as exogenous. From equation (2.6), we obtain:

$$\frac{A\theta}{e} = p. \tag{2.7}$$

Solving the equation by e yields the following equilibrium:

$$e = \frac{A\theta}{p} \equiv e_N, \quad c_N = A\theta\left(\ln A\theta - \ln p - \ln E - 1\right) + A\ln E + \underline{w}. \tag{2.8}$$

We compare this result with the education externality case discussed later in this section.

2.3.3 Case of exogenous reference level

Second, we assume that a positive education externality exists, i.e. $v > 0$. We derive the optimal solution when E is considered exogenous. Then, we conduct a comparative statistical analysis on the equilibria. First, we derive the condition for the two existing equilibria that exist if the following condition is satisfied:

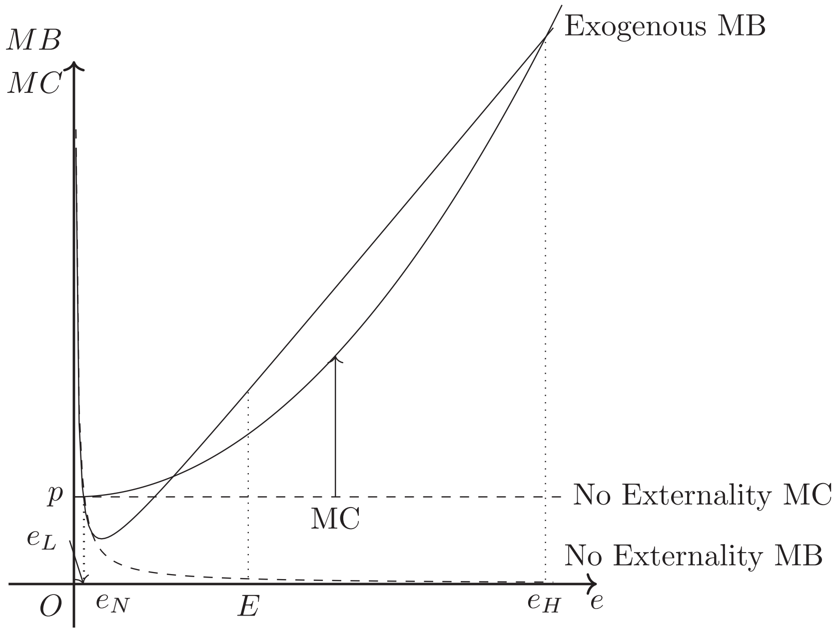
$$27S^2 - 4T^3 < 0. \tag{2.9}$$

A derivation of the condition and the definitions of S and T are presented in the Appendix, where we first derive the condition for the cubic equation to have three positive solutions. This inequality implies that the main parameters p, φ and v must be involved in some moderate relationship when multiple equilibria exist. We also exclude the second-largest solution of the cubic equation because the second-order condition is not satisfied in the second-largest solution. Then, we obtain the equilibria using the largest and smallest roots of the cubic equation. Figure 2.3 depicts this process of determining the existence of these equilibria. If condition (2.9) is not satisfied, the economy has one unique equilibrium, which is represented as a real solution of the cubic equation. Then, the optimal solution for the household is determined as e_U. In this economy, however, e_U is the same as e_L or e_H. Thus we concentrate on the multiple equilibria case.

We thus assume that inequality (2.9) is satisfied and define the two equilibria, e_L and e_H ($e_L < e_H$), for convenience. Hereafter, we analyze the properties of these equilibria. First, we check their stability. These equilibria are stable because the second-order condition of the optimization problem, given in the left-hand side of equation (2.6), is negative.

Second, we conduct a comparative statistical analysis on the reference level of education. We then have the next result. For each equilibrium e_L and e_H, the properties $\mathrm{d}e_i/\mathrm{d}E < 0$ and $\mathrm{d}e_i/\mathrm{d}\alpha < 0$ are satisfied for all $i \in \{H,\ L\}$ (see the Appendix for calculation). The comparative statistics state that if the reference level increases, the equilibria education levels decrease.

We can explain this dynamic in two ways. On the one hand, at the equilibrium e_H, an increase in the reference education level reduces the utility of education. Thus, individuals will try to compensate for reduced utility by increasing their personal levels of education. However, the psychological cost of obtaining additional education is greater than the increase in the educational utility. This leads to a decline in the education level at the equilibrium. On the other hand, at the equilibrium e_L, an increase in the



LHS and RHS mean the left-hand side and the right-hand side of (2.6), and LHS' and RHS' are those of (2.7).

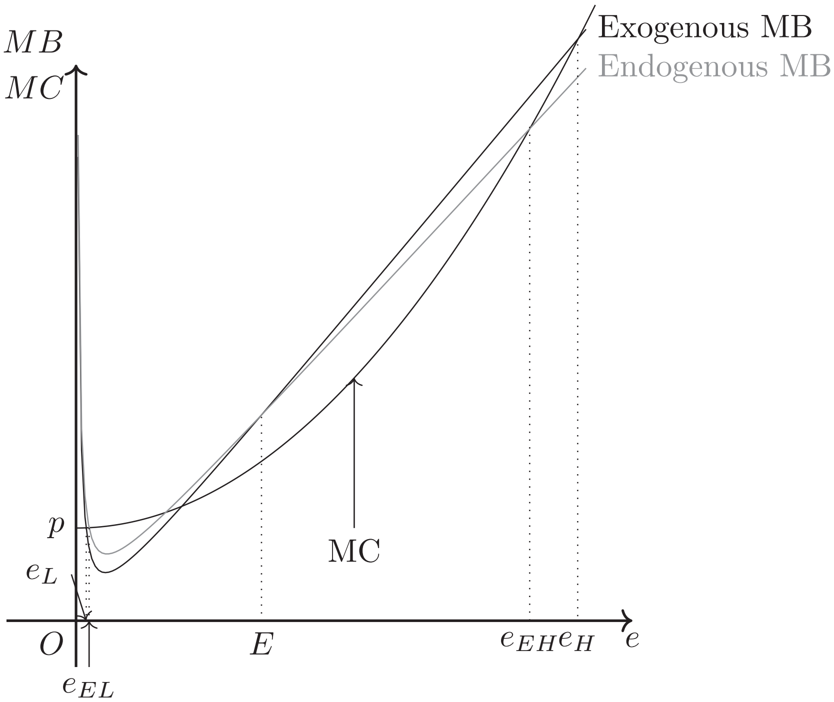
Fig. 2.3 Existence of equilibria: Case $A = 1,\ \theta = 0.6,\ E = 2,\ v = 3,\ \varphi = 0.1,\ p = 5,\ \alpha = 0.1$.

education reference level increases the utility from education. Thus, the education level is reduced to satisfy the first-order condition. Comparative statistical tests were conducted on other parameters, but these results are omitted here because the results are almost the same as those presented later.

In the economy used in this example, the reference level is exogenously determined; however, the reference level is endogenously determined in a normal economy. Thus, in the next subsection, we consider the case in which the education externality is endogenous; that is, the case in which the reference level is also determined in the economy.

2.3.4 Case of endogenous reference level

In this subsection, we consider the case in which the E term is endogenous and investigate whether the education externality defined by the utility function is realized by households' educational choices. When an endogenous equilibrium does not exist, the



The parameters are the same those in as Figure 2.3

Fig. 2.4 Existence of equilibria: Case of endogenous reference level

education externality that we first assumed by the utility function is viewed as a social norm, and the social norm is interpreted as a goal to be pursued through education. However, in the real world, the reference level has some factual foundation, such as average years of schooling. Thus, we analyze the endogenous economy case.

In the first-order condition, we regard E as e, which is a household's socially determined education level. We thus obtain a new first-order condition as follows:

$$2ve(1-\alpha) + \frac{A\theta}{e} = p + 3v\varphi e^2.$$

In this case, under almost the same conditions as in the previous case, this economy also contains two equilibria (the condition is described in the Appendix). We name these equilibria e_{EL} and e_{EH}, where $e_{EL} < e_{EH}$ is satisfied. These equilibria, whose existence are shown in Figure 2.4, satisfy the condition that marginal benefit of education equals to marginal cost of education. The marginal benefits are composed of two parts: marginal wages and marginal utility of education for comparison with education. In addition, the marginal costs also consist of two parts: marginal cost of education and marginal psychological cost of education. This yields the following proposition:

Proposition 2.1. *The equilibria e_{EL} and e_{EH} have the following properties:*

$$\begin{cases} e_{EL} < e_L < e_{EH} < e_H & \text{if} \quad E < e_L, \\ e_L < e_{EL} < e_{EH} < e_H & \text{if} \quad e_L < E < e_H, \\ e_L < e_{EL} < e_H < e_{EH} & \text{if} \quad e_H < E. \end{cases}$$

Proof. See Appendix. □

Proposition 2.1 demonstrates the relationship among the four equilibria when the endogenous economy exists. Figure 2.4 depicts a case in which $e_L < E < e_H$. Proposition 2.1 intuitively states that the equilibria of the endogenous economy are a weighted average of the reference level and the equilibria of the exogenous economy. Thus, the relationship between the four equilibria is determined by the reference level E. Furthermore, if we consider the reference level precisely at the equilibrium, both the endogenous equilibrium and exogenous equilibrium take the same value. The e_L equilibrium represents the U.S. and German group whereas the e_H equilibrium describes the Japan and Sweden group. This is because at the e_{EH} equilibrium, the level of education is mainly determined not by wages but by the utility gained from the externality. This implies that wages exert less influence on education levels; thus, the wage differential tends not to correspond to differences education levels. However, at the e_{EL} equilibrium, the level of education is mainly determined by wages. This suggests that wages have much greater effects on education levels; thus, the wage differential tends to correspond to the difference in education levels. The data discussed in the Introduction support this interpretation of the model.

Next, we conduct a comparative statistical analysis of equilibria e_{EL} and e_{EH}. For each equilibrium $e_{Ei} (i \in \{H, L\})$, the following properties are satisfied:

$$\frac{\mathrm{d}e_{Ei}}{\mathrm{d}\alpha} < 0, \quad \frac{\mathrm{d}e_{Ei}}{\mathrm{d}\varphi} < 0, \quad \begin{cases} \dfrac{\mathrm{d}e_{Ei}}{\mathrm{d}v} > 0 & \text{if} \quad e_{Ei} > e_N, \\ \dfrac{\mathrm{d}e_{Ei}}{\mathrm{d}v} < 0 & \text{if} \quad e_{Ei} < e_N. \end{cases}$$

The calculation is provided in the Appendix.

The first part of the above equation implies that in the endogenous economy, the same property holds as in the exogenous economy. That is, increasing the reference level of education reduces the actual level of education. The mechanism for this is the same as that in the exogenous economy. The second part of the proposition, meanwhile, says that increased psychological cost negatively affects the equilibria because such increases will reduce the marginal benefits of education. The last part of the proposition says that the relationship between the equilibrium in the no-externality case and that in the endogenous economy case affects the sign of $\mathrm{d}e/\mathrm{d}v$. In many cases, $e_N < e_{EL} < e_{EH}$ holds, as in the examples calculated above. In the first example depicted in the figures, namely where $A = 1,\ \theta = 0.6,\ v = 3,\ \varphi = 0.1,\ p = 5$ and $\alpha = 0.1$ holds, we call the resulting case as a weak educational comparison and large effort cost case, $e_N = 0.12,\ e_{EL} = 0.1410,$ and $e_{EH} = 4.8923$ at the equilibria. In the second example, when the parameters are $A = 1,\ \theta = 0.6,\ v = 3,\ \varphi = 0.02,\ p = 5$ and $\alpha = 1/3$, that is, the strong educational comparison and low effort cost case, $e_N = 0.12,\ e_{EL} = 0.1344,$ and $e_{EH} = 20.9008$ at the equilibria. Thus, $\mathrm{d}e/\mathrm{d}v > 0$ holds for both equilibria. The mechanism behind this result is that an increase in the externality parameter affects the first-order condition for both externality and non-externality terms. In the case where the former is greater, $e_{Ei} > e_N$ holds, and in the case where the latter is greater, $e_{Ei} < e_N$ holds.

Finally, we compare the indirect utilities of these equilibria to determine which is more beneficial for households. Thus, we have the following proposition:

Proposition 2.2. *When the following conditions are satisfied, the indirect utility of the equilibrium e_{EH} is greater than that of the equilibrium e_{EL}.*

$$A \ln\left(\frac{e_{EH}}{e_{EL}}\right) + \frac{v}{3}(e_{EH} - e_{EL})\left((e_{EH} + e_{EL})(1-\alpha)(1-3\alpha) - \frac{2p}{v}\right) > 0. \qquad (2.10)$$

Proof. See Appendix. □

Proposition 2.2 implies that the sign on the left-hand side of inequality (2.10) determines which equilibrium is better for households. Parameter α, strength in educational comparison is important for determining the sign of this inequality. In the case where $\alpha \in (0, 1/3)$ holds, the second term often takes a positive value and the inequality is satisfied. However, for the case where $\alpha \in [1/3, 1)$ holds, the second term takes a negative value and the inequality is likely to be reversed. This statement can be checked by considering some examples. When the weak educational comparison and large effort cost case, where the left-hand side of inequality (2.10) takes a positive value of 2.7754. Thus, the higher equilibrium e_{EH} is better than the lower equilibrium e_L in this case. However, when the strong educational comparison and low effort cost case, the left-hand side of the inequality (2.10) takes a negative value: -64.1744. Thus, the lower equilibrium e_{EL} is better than the higher equilibrium e_{EH}.

According to the standard economic theory, if an economy has an externality, the household's decision regarding its education level will differ from the socially optimal level. Thus, in the next section, we examine the preferred equilibrium from the social planner's perspective to compare it with the ones presented in this section.

2.4 Social planner's solution and policy implications

In this section, we consider the problem from the social planner's perspective to examine how education affects social welfare in the economy. Accordingly, we regard the reference level of education E as the personal level of education level e and solve the utility maximization problem.

2.4.1 Social planner's solution: No externality case

To examine how education affects social welfare of the economy, we solve another problem in which the reference level of education is considered as the personal education decision. FromManski (1993)'s classification of social effects in a group, this analysis can be classified as complete correlated effects, which considers the case where the members behave perfectly in the same manner. We maximize social welfare when $v = 0$ holds as follows:

$$\max_{c,e} \quad u(c,e) = c + v\left[(e - \alpha e)^2 - \varphi e^3\right] = c \tag{2.11}$$
$$s.t. \quad c + pe = A[\theta \ln e + (1-\theta)\ln e] + \underline{w} = A \ln e + \underline{w}.$$

Solving problem (2.11) yields

$$c_{SN} = A(\ln A - \ln p - 1) + \underline{w}, \quad e_{SN} = \frac{A}{p}$$

This solution implies that the social planner maximizes welfare in the economy in the case when only the monetary factor is considered. It seems that the planner often solves this problem in a real economy because it is difficult to quantify education's externality level. Comparing the individual's solution and planner's solution results in the following proposition.

Proposition 2.3. *When the externality does not exist, the education and consumption level determined by households are always lower than those determined by the social planner.*

Proof. The education level is obvious from comparison between e_N and e_{SN}. The consumption levels can be compared as follows:

$$\begin{aligned} c_{SN} - c_N &= [A(\ln A - \ln p - 1) + \underline{w}] - [A(\ln A - \ln p - 1) + \underline{w} + A\ln\theta - \theta] \\ &= -A\ln\theta + \theta > 0. \end{aligned}$$

Because $0 < \theta < 1$ holds, the last inequality is satisfied. $\square$

Proposition 2.3 means that if we ignore the externality—if we think only about the monetary factor in the economy—then the resulting economy is always under-educated because individuals do not fully consider the increase in wage by increasing their education level. In the following subsection, we analyze the education externality case referencing the above result.

2.4.2 Social planner's solution: Externality case

Next, we consider the social planner problem when the education externality exists. The planner solves the problem as follows:

$$\max_{c,e} \quad u(c,e) = c + v\left[(e - \alpha e)^2 - \varphi e^3\right] \tag{2.12}$$
$$s.t. \quad c + pe = A[\theta \ln e + (1-\theta)\ln e] + \underline{w} = A \ln e + \underline{w}.$$

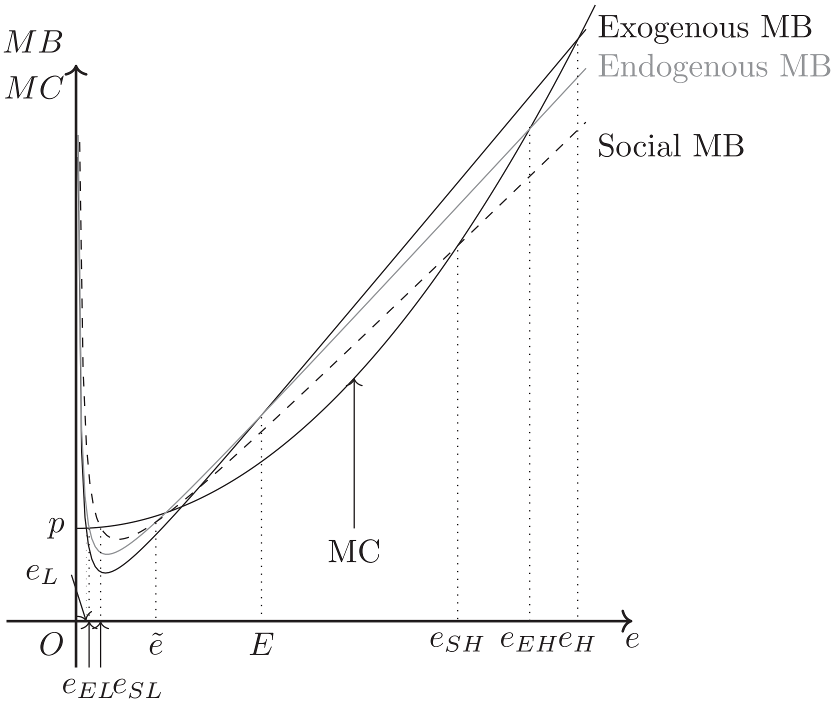
Solving the problem yields the first-order condition as follows:

$$2ve(1-\alpha)^2 + \frac{A}{e} = p + 3v\varphi e^2.$$

Under conditions similar to those in both the cases of exogenous and endogenous reference level economy, we have two equilibria for the social planner's problem, which is described in the Appendix. We concentrate on the case where both the endogenous economy and social planner's problem have two equilibria because the case includes one equilibrium case, as stated above.

First, we compare the education level of all equilibria with the level in the endogenous equilibria. Then, we obtain the following proposition.

Proposition 2.4. *For the equilibria e_{SL} and e_{SH}, the following properties are satisfied:*



Parameters are the same as in Figure 2.3.

Fig. 2.5 Existence of social planner's solution case

$$\begin{cases} e_{SL} < e_{EL} < e_{SH} < e_{EH} & \text{if} \quad \tilde{e} < e_{EL} \\ e_{EL} < e_{SL} < e_{SH} < e_{EH} & \text{if} \quad e_{EL} < \tilde{e} < e_{EH} \\ e_{EL} < e_{SL} < e_{EH} < e_{SH} & \text{if} \quad e_{EH} < \tilde{e} \end{cases}$$

where $\tilde{e}$ is determined as follows:

$$\tilde{e} = \sqrt{\frac{A(1-\theta)}{2v\alpha(1-\alpha)}}.$$

Proof. See the Appendix. □

Figure 2.5 indicates the case in which $e_{EL} < \tilde{e} < e_{EH}$. Proposition 2.4 says that the relationship among $\tilde{e}$, e_{EH} and e_{EL}determines the order of the equilibria. This result mainly affects the actual wage level, which is determined by education level. Individuals earn higher wages as their education levels rise.

Second, we conduct a comparative statistical analysis on the planner's equilibria e_{SH} and e_{SL}. For each equilibrium e_{SL} and s_{SH}, the following properties are satisfied:

$$\frac{\mathrm{d}e_{Si}}{\mathrm{d}\alpha} < 0, \quad \frac{\mathrm{d}e_{Si}}{\mathrm{d}\varphi} < 0, \quad \begin{cases} \dfrac{\mathrm{d}e_{Si}}{\mathrm{d}v} > 0 & \text{if} \quad e_{Si} > e_{SN}, \\ \dfrac{\mathrm{d}e_{Si}}{\mathrm{d}v} < 0 & \text{if} \quad e_{Si} < e_{SN}. \end{cases}$$

Calculations are given in the Appendix.

These comparative statistics are identical to result found earlier for the case of endogenous reference level. $\mathrm{d}e_{Si}/\mathrm{d}\alpha$ and $\mathrm{d}e_{Si}/\mathrm{d}\varphi$ have the same sign as in the case of endogenous reference level, but the magnitude is different, although non-linearity means that we cannot compare magnitudes. However, the threshold values of the cases also differ because the planner considers the full wage changes arising from changes in education level, which are not fully considered in an endogenous economy.

Third, we compare the indirect utilities between $e_{SL}$and e_{SH} to determine which is better for the planner.

Proposition 2.5. *When the following conditions are satisfied, the indirect utility of the equilibrium e_{SH} is greater than that of the equilibrium e_{SL}.*

$$A \ln\left(\frac{e_{SH}}{e_{SL}}\right) + \frac{v}{3}(e_{SH} - e_{SL})\left((e_{SH} + e_{SL})(1-\alpha)^2 - \frac{2p}{v}\right) > 0. \tag{2.13}$$

Proof. See the Appendix. □

Proposition 2.5 states that the relationship between equilibria solved by the social planner is Pareto optimal in this economy. If the formula is positive, e_{SH} is better than e_{SL}, but if the formula's sign is negative, e_{SL} is better than e_{SH}. Consider the examples discussed above. When $A = 1,\ \theta = 0.6,\ v = 3,\ \varphi = 0.1,\ p = 5$ and $\alpha = 0.1$ holds, that is, the low reference level and high effort cost case, the left-hand side of inequality (2.13) takes a positive value of 3.5713. Thus, the higher equilibrium e_{SH} is better than the lower equilibrium e_{SL} in but e_{EH} is better compared to thee_{EL} case. In the case where the parameters are $A = 1,\ \theta = 0.6,\ v = 3,\ \varphi = 0.02,\ p = 5$ and $\alpha = 1/3$, i.e. high reference level and low effort cost case, the left-hand side of the inequality (2.13) takes a positive value of 33.7437. Thus, the higher equilibrium e_{SH} is also better than the lower equilibrium e_{SL} even in the case where e_{EL} is better than e_{EH} . This preference change reflects the fact that the parameter α always positively affects the left-hand side of inequality (2.13). As the sign of inequality (2.10) is mainly determined by α, as discussed above, the higher level of education is better in most cases.

Finally, we conduct a preference comparison between two education levels that are determined from the social welfare perspective. To compare these education levels, we consider social welfare as follows:

$$V(e_i, E(e_i)) = A\left[\theta \ln e_i + (1-\theta) \ln E(e_i)\right] + \underline{w} - pe + v\left[(e_i - \alpha E(e_i))^2 - \varphi e_i^3\right].$$

Differentiating by e_i around the endogenous equilibria e_i ($i = \{EH, EL\}$) yields the following proposition:

Proposition 2.6. *From the social planner's view, the education level achieved by individuals is evaluated as follows: when the equilibrium is greater than $\tilde{e}$, the equilibrium is over-educated, and when the equilibrium is less than $\tilde{e}$, the equilibrium is under-educated.*

Proof. See Appendix. □

Proposition 2.6 states that except in a case where the endogenous equilibrium corresponds to the planner's equilibrium, the endogenous equilibrium cannot achieve the social optimum. It also states that the relationship between the equilibrium and $\tilde{e}$ determines whether the equilibrium is over-educated or under-educated. The difference between the social optimal and equilibrium is due to ignoring the effect of the reference level of education upon wages.

Combining the results of Propositions 2.2, 2.5 and 2.6, some cases determine the Pareto rank of these equilibria. When considering the weak comparison and high effort cost case, we obtain the following Pareto ranking by summarizing the discussion above: e_{SH}, e_{SL}, and e_{EL}. We know e_{EH} is better than e_{EL} and worse than e_{SH}; however, we cannot rank $e_{SL}$and e_{EH} from these propositions. Then, utilizing formula (2.15) from the Appendix, we find the sign of formula (2.15) to have a negative value of -6.81128

in this case. Thus, we obtain the following Pareto ranks : e_{SH}, $e_{EH,}$ e_{SL}, and e_{EL}. However, when examining the strong comparison and low effort cost case, summarizing the discussions above results in the following Pareto rankings: e_{SH}, e_{SL}, e_{EL}, and e_{EH}. From these examples, we know that changes in parameters may change the Pareto ranks of these equilibria. Therefore, we must consider this point when considering policy implications in the next subsection.

2.4.3 Discussion and policy implications

The discussion of the results from the model and the subsequent policy implications are divided into three parts. The first part focuses on the implication of multiple equilibria. The second part compares the social planner's solutions. The third part[2] considers the subsidy and tax policy of the governments.

First, we develop another interpretation of the equilibria in addition to the discussion of Proposition 2.1 regarding the difference in the equilibria between the two country groups. The level of education tends to be higher compared with that in the no-externality case, especially at the e_{EH} equilibrium, due to a lack of consideration of the externality's utility. This suggests that education has both an investment and consumption role. Standard economic theory has not considered the effect of education as consumption. However, the data and the discussion above suggest that this effect must also be considered in addition to the monetary effect when conducting an economic analysis of the decision to pursue additional education. If we do not consider the externality effect, we may over-estimate the marginal productivity of education.

Second, we compare the endogenous reference level equilibria and the social planner's equilibria. Standard economic theory states that the firms determine the social-welfare-maximizing education level, as developed in the no-externality case. However, according to the above result, the social-welfare-maximizing education level is determined not only by firms but also individuals.

Next, in this model, by imposing existence conditions, we obtain the equilibria e_{EL},

[2] This part is suggested by an anonymous referee.

e_{EH}, e_{SL} and e_{SH}. In this case, the endogenous equilibria are under-educated or over-educated from the social planner's perspective, as compared to $\tilde{e}$. When politicians and bureaucrats (as opposed to an omniscient social planner) executes the wrong policy, social welfare will drop due to a rise in either over-education or under-education. For example, at the e_{EH} equilibrium, if the politicians and bureaucrats subsidizes the individual's cost of education, students will choose to become over-educated. Therefore, the politicians and bureaucrats must also consider the 'keeping up with Joneses' effect when subsidization or taxing education. We closely discuss this subsidization and taxing education problem later.

In addition, the possibility arises that the equilibrium that would be chose through marginal analysis may achieve less net social welfare than the maximum possible –even when the equilibrium is derived by the social planner– because the planner's problem has two equilibria. The condition for determining the welfare-maximizing equilibrium is discussed in Proposition 2.5. In many cases, the inequality in Proposition 2.5 is satisfied because the consumption of education brings more utility than the utility of consumption of goods through earning wages. Thus, the planner needs to consider the effect of educational consumption when considering policies to maximize social welfare.

Third, we consider the subsidisation and taxing policy of the governments. According to Chapter 13 of Hartog and van den Brink (2007), the governments have three roles in subsidization and taxing education and skill formation: external effects; capital and insurance market failures; and merit or public good arguments. The above discussion depends mainly on the role of internalizing externalities.

In capital and insurance market failures, subsidies on education may restore the desired level of educational attainment when underinvestment in human capital has occurred due to a failure in the capital and insurance markets. Several researchers have investigated this role in several articles although this chapter does not consider it due to the requirement of using a quasi-linear utility function. Perroni (1995) explores the endogenous human capital formation when intertemporal leisure substitution mechanisms exist. Caucutt and Kumar (2003) develops an analysis on education policies when individuals are heterogeneous in ability. Beladi et al. (2016) researches education policies in developing countries when risk aversion exists. These researches reveal the optimal subsidization and taxing scheme by numerical calculation because the models are analytically unsolvable. Inclusion of these substitutional effects or imperfect information structure in the model will complicate the model. Making it more difficult to make a comparison between market equilibrium and the optimal scheme.

Third, merit or public good arguments, or the non-monetary values of education (such as promoting citizenship, contributing to culture, etc.) by subsidization and taxing education. In this chapter, the role can affect the reference parameter v . We can construct a model where v is endogenously determined, however, the model would be too complicated to analyze. In addition, the values are extremely difficult to measure because values are also formed by a politically determined subsidy which is pointed out in Chapter 13 of Hartog and van den Brink (2007).

Therefore, as discussed above, we consider the educational externality from both the perspective of the household and planner, while remembering the effects of subsidization and taxing when we discuss educational policies.

2.5 Conclusion

Our study investigated the economic impact arising from the existence of an education externality. Two education equilibria occur in this model if the existence condition is satisfied. This presence of multiple equilibria can explain the differences in wages and schooling decisions between countries such as the United States and Japan. The model also indicates that an economy determined by individuals cannot achieve a socially optimal level of education. Moreover, analysis from a social planner's perspective reveals that placing certain conditions on reference parameters can change social preferences regarding desired level of education.

Two limitations of note are present in our analysis. First, we cannot interpret the model explicitly because of its complexity, although we have attempted to remedy this problem by offering numerical examples. Second, the household's demand function does not have an income effect because the utility function is quasi-linear in form. However, we do consider income effects that correspond to increased wages caused by increasing non-educational productivity.

These results also raise the possibility of extending this model. The first possible extension is to consider the full income effects of education, which were not fully considered here due to the quasi-linear utility function. For example, increasing $\underline{w}$ does not affect education but rather consumption. However, in the real world, spending on education has a much stronger relationship with income. Thus, we must research the relationship between income and education, including the effect of scholarships and subsidization and taxing. Such future research may help us completely understand the education externality.

The second possible extension is to consider the addition of dynamic variables. Time-series graphs are shown in the introduction; however, the model does not consider the dynamics of economic variables. In addition, parents often intervene in children's education decisions by placing on their financial supports of their children's educational endeavors and the shaping of their children's values. Therefore, introducing dynamic variables that consider generational influences in this model may change the results.

Third, estimating the parameters of v , α and φ, is necessary. If v is not significant, the theory collapses. Furthermore, if $\alpha > 1$, the model does not have two equilibria. Parameters α and φ are also important for determining the Pareto ranks among the equilibria. Therefore, determining parameters is necessary for evaluating education from a social perspective. Although it is important to estimate the parameters, estimating parameters such as α, φ and v is quite difficult because of structural restrictions. Krueger and Lindahl (2001) and Ciccone and Peri (2006) would be helpful when considering the estimation method.

Fourth,[3] in this chapter, we ignore cultural differences to explain the suspicious movements of education and wages. If we incorporate culture into the model, the result might be changed in an insightful manner. For example, we can consider a model wherein cultural differences endogenously determine parameter α and/or v, which would explain

[3] This beneficial point is suggested by a reviewer.

why equilibria differ in the countries such as Japan, Sweden and the US. However, the model would be too complicated to analyze

Fifth, considering the role of government would be beneficial as we discussed in the third part of Discussion and policy implications. We may thus have many other extensions of this model, and research on these extensions promises to be insightful.

Appendix

Derivation and existence of the equilibria

We derive the equilibria using the formula of cubic equations. First, we define the left-hand side of (2.6) as $f(e)$:

$$f(e) = e^3 - \frac{2}{3\varphi}e^2 + \left(\frac{2\alpha E}{3\varphi} + \frac{p}{3v\varphi}\right)e - \frac{A\theta}{3v\varphi} = 0. \tag{2.14}$$

Then we define $x = e - 2/(9\varphi)$, rearrange the equation, and obtain the following equation:

$$x^3 - \frac{1}{3\varphi}\left(\frac{4}{9\varphi} - 2\alpha E - \frac{p}{v}\right)x - \frac{A\theta}{3v\varphi} + \frac{2}{27\varphi^2}\left(2\alpha E + \frac{p}{v} - \frac{8}{27}\right) = 0.$$

Here, we define the first-order term and the constant of the equation as S and T, respectively, as follows:

$$S = \frac{1}{3\varphi}\left(\frac{4}{9\varphi} - 2\alpha E - \frac{p}{v}\right) > 0,$$

$$T = -\frac{A\theta}{3v\varphi} + \frac{2}{27\varphi^2}\left(2\alpha E + \frac{p}{v} - \frac{8}{27\varphi}\right).$$

Suppose we can write the solution as $x = a + b \neq 0$ using the complex number a and b. Then, the equation becomes:

$$(a+b)^3 - S(a+b) + T = 0.$$

Developing the equation, we obtain

$$a^3 + b^3 + T + (3ab - S)(a + b) = 0.$$

This implies that $a^3 + b^3 + T = 0$ and $3ab - S = 0$ must hold. $3ab = S$ implies $27a^3b^3 = S^3$. Then, a^3 and b^3 are solutions to the following quadratic equation:

$$t^2 + Tt + \frac{S^3}{27} = 0.$$

We obtain the solution to the quadratic equation as

$$t = \frac{-T \pm \sqrt{T^2 - \dfrac{4S^3}{27}}}{2}.$$

Thus, we acquire the solution to the cubic equation by using $\sqrt{-1} = i$ and $\omega = (-1 + \sqrt{3}i)/2$ as follows:

$$e_1 = \frac{2}{9\varphi} + \left(\frac{-T + \sqrt{T^2 - \dfrac{4S^3}{27}}}{2} \right)^{\frac{1}{3}} + \left(\frac{-T - \sqrt{T^2 - \dfrac{4S^3}{27}}}{2} \right)^{\frac{1}{3}},$$

$$e_2 = \frac{2}{9\varphi} + \omega \left(\frac{-T + \sqrt{T^2 - \dfrac{4S^3}{27}}}{2} \right)^{\frac{1}{3}} + \omega^2 \left(\frac{-T - \sqrt{T^2 - \dfrac{4S^3}{27}}}{2} \right)^{\frac{1}{3}},$$

$$e_3 = \frac{2}{9\varphi} + \omega^2 \left(\frac{-T + \sqrt{T^2 - \dfrac{4S^3}{27}}}{2} \right)^{\frac{1}{3}} + \omega \left(\frac{-T - \sqrt{T^2 - \dfrac{4S^3}{27}}}{2} \right)^{\frac{1}{3}}.$$

The condition for the existence of three different solutions is

$$27T^2 - 4S^3 < 0.$$

If the inequality is averse, the equation has complex solutions.

Next, we prove that two of these three solutions are the equilibria. Suppose $e_1 < e_2 < e_3$. Then, we prove that solutions $e_1$and e_3 are the equilibria. To have three real number solutions to the cubic equation, the function $f(e)$ must have a local minimum and a local maximum. The values that take the local minimum and local maximum are obtained by solving the following equation:

$$f'(e) = 3e^2 - \frac{4}{3\varphi}e + \left(\frac{2\alpha E}{3\varphi} + \frac{p}{3v\varphi}\right) = 0.$$

The solution is as follows:

$$e = \frac{2 \pm \sqrt{4 - 9\varphi\left(2\alpha E + \frac{p}{v}\right)}}{9\varphi}.$$

The solutions are both positive because the constant term of $f'(e)$ is positive. Utilizing the fact that the constant term of $f(e)$ is negative, we proved that if there are three solutions to the cubic equation, they are all positive. Note that the following inequalities are satisfied from the nature of the cubic equation:

$$\frac{2 - 2\sqrt{4 - 9\varphi\left(2\alpha E + \frac{p}{v}\right)}}{9\varphi} < e_1 < \frac{2 - \sqrt{4 - 9\varphi\left(2\alpha E + \frac{p}{v}\right)}}{9\varphi},$$
$$\frac{2 + \sqrt{4 - 9\varphi\left(2\alpha E + \frac{p}{v}\right)}}{9\varphi} < e_3 < \frac{2 + 2\sqrt{4 - 9\varphi\left(2\alpha E + \frac{p}{v}\right)}}{9\varphi}.$$

To show that the largest and smallest solutions, e_1 and e_3, are the equilibria, we consider the second-condition for the optimization problem as follows:

$$-\frac{A\theta}{e^2} + 2v\left(1 - 3\varphi e\right) < 0.$$

Using the first-order condition, we obtain the following inequality:

$$3e^2 - \frac{4}{3\varphi}e + \left(\frac{2\alpha E}{3\varphi} + \frac{p}{3v\varphi}\right) > 0.$$

Therefore, e_1 and e_3 both satisfy the second-order condition.

Calculations for comparative statistics

We differentiate $f(e)$ by E and α, respectively, to obtain

$$\frac{\mathrm{d}e}{\mathrm{d}\alpha} = -\frac{\dfrac{2eE}{3\varphi}}{3e^2 - \dfrac{4}{3\varphi}e + \left(\dfrac{2\alpha E}{3\varphi} + \dfrac{p}{3v\varphi}\right)} < 0, \quad \frac{\mathrm{d}e}{\mathrm{d}E} = -\frac{\dfrac{2e\alpha}{3\varphi}}{3e^2 - \dfrac{4}{3\varphi}e + \left(\dfrac{2\alpha E}{3\varphi} + \dfrac{p}{3v\varphi}\right)} < 0.$$

Thus, the proposition is proved.

Existence condition of equilibria in the endogenous economy

Almost the same as with the exogenous economy, we consider the cubic equation $g(e)$ as follows:

$$g(e) = e^3 - \frac{2(1-\alpha)}{3\varphi}e^2 + \frac{p}{3v\varphi}e - \frac{A\theta}{3v\varphi} = 0.$$

The same transformation gives us the existence condition as follows:

$$27S'^2 - 4T'^3 < 0,$$

where S' and T' are as follows:

$$S' = \frac{1}{3\varphi}\left(\frac{4(1-\alpha)^2}{9\varphi} - \frac{p}{v}\right), \quad T' = -\frac{A\theta}{3v\varphi} + \frac{2}{27\varphi^2}\left(\frac{(1-\alpha)p}{v} - \frac{8(1-\alpha)^3}{27\varphi}\right).$$

Using the same logic as above, the second-order condition is as follows:

$$3e^2 - \frac{4(1-\alpha)}{3\varphi}e + \frac{p}{3v\varphi} > 0.$$

The second-order condition is satisfied around the equilibria for the same reason given above.

Proof of proposition 2.1

Subtracting the first-order condition of the endogenous economy from that of exogenous economy yields

$$\left(p + 2v\alpha E + 3v\varphi e^2 - 2ve - \frac{A\theta}{e}\right) - \left[p + 3v\varphi e^2 - 2ve(1-\alpha) - \frac{A\theta}{e}\right] = 2v\alpha(E - e).$$

If the formula is positive, the first-order condition of exogenous economy is larger than that of the endogenous economy. If the formula is negative, the relationship is reversed. Thus, if $e < E$ holds, the former is above the latter, and if $e > E$ holds, the latter is above the former. Note that these two curves have only one intersection at $e = E$. Therefore, the equilibria must be the order of the proposition.

Proof of proposition 2.2

We compare the indirect utility functions at e_{EH} and e_{EL}. Subtracting the indirect utility function of e_{EL} from that of e_{EH}, we obtain the following equation:

$$V(e_{EH}) - V(e_{EL}) = A\ln\left(\frac{e_{EH}}{e_{EL}}\right) + \frac{v}{3}(e_{EH} - e_{EL})\left((e_{EH} + e_{EL})(1-\alpha)(1-3\alpha) - \frac{2p}{v}\right).$$

Because the equation is positive, the indirect utility at e_{EH} is greater than that at e_{EL}.

Calculation of comparative statistics in section 3

We differentiate $g(e)$ by e, α and v, respectively, to obtain

$$\frac{\mathrm{d}e}{\mathrm{d}\alpha} = -\frac{\dfrac{2e^2}{3\varphi}}{3e^2 - \dfrac{4(1-\alpha)}{3\varphi}e + \dfrac{p}{3v\varphi}} < 0, \quad \frac{\mathrm{d}e}{\mathrm{d}v} = -\frac{\dfrac{1}{3v^2\varphi}(A\theta - pe)}{3e^2 - \dfrac{4(1-\alpha)}{3\varphi}e + \dfrac{p}{3v\varphi}}.$$

Thus, the same discussion applies as in the $\mathrm{d}e/\mathrm{d}\alpha$ case. However, in the $\mathrm{d}e/\mathrm{d}v$ case, if $e > e_N$ holds, the numerator takes a negative value and $\mathrm{d}e/\mathrm{d}v > 0$ holds, and if $e < e_N$ holds, the numerator takes a positive value and $\mathrm{d}e/\mathrm{d}v < 0$ holds. Finally, we obtain the following formula:

$$\frac{\mathrm{d}e}{\mathrm{d}\varphi} = -\frac{\dfrac{1}{3\varphi^2}\left[2(1-\alpha)e^2 - \dfrac{p}{v}e + \dfrac{A\theta}{v}\right]}{3e^2 - \dfrac{4}{3\varphi}e + \dfrac{p}{3v\varphi}} = -\frac{e^3}{3e^2 - \dfrac{4}{3\varphi}e + \dfrac{p}{3v\varphi}} < 0.$$

The second equality is from the first-order condition.

Existence of equilibria in the social planner's economy

Keeping conditions mostly the same as in the cases above, we consider the cubic equation $h(e)$ as follows:

$$h(e) = e^3 - \frac{2(1-\alpha)^2}{3\varphi}e^2 + \frac{p}{3v\varphi}e - \frac{A}{3v\varphi} = 0.$$

The same transformation gives us the following existence condition:

$$27S''^2 - 4T''^3 < 0,$$

where S' and T' are as follows:

$$S'' = \frac{1}{3\varphi}\left(\frac{4(1-\alpha)^4}{9\varphi} - \frac{p}{v}\right), \quad T'' = -\frac{A\theta}{3v\varphi} + \frac{2}{27\varphi^2}\left(\frac{(1-\alpha)^2 p}{v} - \frac{8(1-\alpha)^6}{27\varphi}\right).$$

Using the same logic as above, the second-order condition is as follows:

$$3e^2 - \frac{4(1-\alpha)^2}{3\varphi}e + \frac{p}{3v\varphi} > 0.$$

The second-order condition is satisfied around the equilibria for the same reason given above.

Proof of proposition 2.4

Similar to the method used compare the endogenous and exogenous reference levels, subtracting the first-order condition of the social planner's problem from that of the endogenous economy yields the following formula:

$$\begin{aligned}\left[3v\varphi e^3 - 2ve^2(1-\alpha) + pe - A\theta\right] - \left[3v\varphi e^3 - 2ve^2(1-\alpha)^2 + pe - A\right]\\ = A(1-\theta) - 2ve^2\alpha(1-\alpha)\end{aligned}$$

The sign of the formula determines the relationship among the equilibria. If $e > \tilde{e} \equiv \sqrt{A(1-\theta)/2v\alpha(1-\alpha)}$ holds, the endogenous condition is larger, and if $e < \tilde{e}$ holds, the social planner's condition is larger. In addition, the first-order conditions have only one intersection.

Calculation of comparative statistics in section 4

By using the same operations conducted in the endogenous economy example, we obtain the following formulas from differentiating $h(e)$:

$$\frac{\mathrm{d}e}{\mathrm{d}\alpha} = -\frac{\dfrac{4e^2}{3\varphi}(1-\alpha)}{3e^2 - \dfrac{4(1-\alpha)^2}{3\varphi}e + \dfrac{p}{3v\varphi}} < 0, \quad \frac{\mathrm{d}e}{\mathrm{d}v} = -\frac{\dfrac{1}{3v^2\varphi}(A-pe)}{3e^2 - \dfrac{4(1-\alpha)^2}{3\varphi}e + \dfrac{p}{3v\varphi}}.$$

$\mathrm{d}e/\mathrm{d}\alpha < 0$ holds for all $\alpha \in (0,1)$. Because $pe > A$ holds, $\mathrm{d}e/\mathrm{d}v > 0$ is satisfied. Similar to the above discussions, by taking the difference of $h(e)$ by e and φ, we obtain

$$\frac{\mathrm{d}e}{\mathrm{d}\varphi} = -\frac{\frac{1}{3\varphi^2}\left[2(1-\alpha)^2 e^2 - \frac{p}{v}e + \frac{A}{v}\right]}{3e^2 - \frac{4(1-\alpha)^2}{3\varphi}e + \frac{p}{3v\varphi}} = -\frac{e^3}{3e^2 - \frac{4(1-\alpha)^2}{3\varphi}e + \frac{p}{3v\varphi}} < 0.$$

Using the same logic as above, the inequality holds.

Proof of proposition 2.5

Similar to the comparison of indirect utilities, by substituting the equilibrium and utilizing the first-order condition, we obtain

$$V(e_{SH}) - V(e_{SL}) = A\ln\left(\frac{e_{SH}}{e_{SL}}\right) + \frac{v}{3}(e_{SH} - e_{SL})\left[(e_{SH} + e_{SL})(1-\alpha)^2 - 2\frac{p}{v}\right].$$

If the formula is positive, $V(e_{SH}) > V(e_{SL})$ holds, and if the formula is negative, $V(e_{SH}) < V(e_{SL})$ holds.

Comparison of social welfare in endogenous and social planner cases

The indirect utility of each economy is as follows:

$$\begin{aligned}
V(e_{Ei}) = &\quad A\ln e_{Ei} + \underline{w} + \frac{v}{3}\left[e_{Ei}^2(1-\alpha)(1-3\alpha) - \frac{2p}{v}e_{Ei} - \frac{A\theta}{v}\right], \\
V(e_{Si}) = &\quad A\ln e_{Si} + \underline{w} + \frac{v}{3}\left[e_{Si}^2(1-\alpha)^2 - \frac{2p}{v}e_{Si} - \frac{A}{v}\right],
\end{aligned}$$

where $i \in \{H, L\}$. Subtracting the endogenous case from the social planner case yields

$$V(e_{Si}) - V(e_{Ei}) = A \ln\left(\frac{e_{Si}}{e_{Ei}}\right) + \frac{1}{3}(e_{Si} - e_{Ei})\left[-2p + v(1-\alpha)^2(e_{Si} + e_{Ei})\right]$$
$$+\frac{1}{3}\left[2v\alpha(1-\alpha)e_{Ei}^2 - A(1-\theta)\right]. \qquad (2.15)$$

Using the equation above, we can determine the Pareto rankings among the equilibria.

Proof of proposition 2.6

Substituting the equilibrium value in the utility function, we obtain the indirect utility function as follows:

$$V(e_i, E(e_i)) = A\theta \ln e_i + A(1-\theta)\ln E(e_i) + \underline{w} - pe + v\left[(e_i - \alpha E(e_i))^2 - \varphi e_i^3\right].$$

We differentiate around the equilibrium of endogenous economy by e_i, and use the first-order condition to obtain

$$\begin{aligned}
\frac{\partial V(e_i, E(e_i))}{\partial e_i} &= \frac{A\theta}{e_i} + \frac{A(1-\theta)}{E(e_i)}\frac{\partial E}{\partial e_i} - p + v\left[2\left(e_i - \alpha E(e_i)\right) - 3\varphi e_i^2\right] \\
&\quad -2v\alpha(e_i - \alpha E(e_i))\frac{\partial E}{\partial e_i}, \\
&= \frac{A\theta}{e_i} - p + v\left[2\left(e_i - \alpha E(e_i)\right) - 3\varphi e_i^2\right] \\
&\quad +\frac{\partial E(e_i)}{\partial e_i}\left[\frac{A(1-\theta)}{E(e_i)} - 2\alpha v(e_i - \alpha E(e_i))\right] \\
&= \frac{\partial E(e_i)}{\partial e_i}\left[\frac{A(1-\theta)}{E(e_i)} - 2\alpha v(e_i - \alpha E(e_i))\right].
\end{aligned}$$

If we impose $E(e_i) = e_i$, we obtain

$$\frac{\partial V(e_i, E(e_i))}{\partial e_i} = \frac{A(1-\theta)}{e_i} - 2ve_i\alpha(1-\alpha).$$

Thus, if $e_i > \tilde{e}$ holds, the endogenous equilibrium is over-educated, and if $e_i < \tilde{e}$ holds, the endogenous equilibrium is under-educated.

Countries in Figure 2.1

The below countries are described in Figure 2.1 because we obtained more than two periods of data from the 2002, 2006 and 2010 surveys.

Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Turkey, United Kingdom.

Countries in Figure 2.2

Albania, Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, the Former Yugoslav Republic of Macedonia, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Latvia, Liechtenstein, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

Chapter 3

Quality or Quantity: Problems of Local Governments under Asymmetric Economies

3.1 Introduction

Attracting young people to live and work in regional urban areas is getting more important for developed country municipalities. Those local governments recognize the importance of young people for local governments for two reasons. For one thing, reproduction of children makes regional markets larger–at least once children reach adulthood. Moreover, government can also expect the replenishment of population by the children. For another thing, agglomerating talented young people makes a market larger. Recently, the benefits of human capital agglomeration can hardly be doubted: peer effects, industrial specification, and so forth, all contribute to the effect. In addition, scale economies co-exist in such agglomerated economies, too.

To attract young people, given the kinds of jobs available, governments must consider two policies: birth subsidies and education subsidies. Better birth subsidy policies attract more young people to a region. Better education policies do the same thing, however, the benefits from them are mainly restricted to the skilled workers. Thus, both policies are beneficial for young people but the range of beneficiaries will differ according to class. In addition, local governments have to levy taxes from individuals to conduct the policies. Considering these structures, local governments are forced to compete with each other while seeking benefits for residents, which is called (and well-investigated as) tax competition.

Tax competition has a rich history. Zodrow and Mieszkowski (1986) and Wilson (1986) were the first articles in the tax competition literature. After them, numerous papers were published within the tax competition literature concerning the mobility of human capital. Rey (2001) developed university competition that universities compete for a certain the level of education and research, depending on the substitution between the two things. Stark and Wang (2002) considered the relationship between human capital formation subsidies and migration. Recently, due to the beginnings of the Bologna process in the European Union, human capital and mobility are getting more focused. Bucovetsky (2003) considered the migration and tax competition relationship with quadratic dead weight loss. This paper assumes a significant productivity level and that the ratio of skills is given. Mechtenberg and Strausz (2008) analyzed the relationship among multicultural skills, education quality and student mobility. Haupt and Janeba (2009) investigated the problem with government time-consistent problems. Delpierre and Verheyden (2014) develop general form discussion considering the location preference of individuals although their model was restricted to a symmetric case.

These articles mainly focused on the investigation of the effects of the mobility of students and workers. They handled free ride problem, which refer to those human capital benefits that a receiver does not correspond to the payer of the expenses. In contrast to the existing papers, this chapter mainly focuses on the relationships among demography, mobility and education. By restricting the mobility of workers after acquiring the human capital, we can consider the asymmetric situation in terms of university efficiency. This difference supposes the existence of a rural-urban relationship that attracts more attention within developed countries. In addition, this chapter describes the relationship between the quality and the quantity of the children in the regions. This important element is often overlooked–especially in the tax competition literature. Yet, such specifications allow us to obtain richer implications and a better discussion of the topic.

The model utilized shows that the existence of an asymmetric equilibrium with migration. Using autarky equilibrium as a benchmark, higher university quality leads to greater welfare and a larger number of births, in return for low skilled worker utility. At the point of immigration equilibrium, the university budget is larger than that of the autarky case and the coordination case. These results stem from the ratio between highly-skilled and low-skilled workers. This outcome suggests that restricting the number of immigrants to the agglomerated region is a welfare-improving policy.

The rest of the chapter is composed as follows: Section 2 provides the model's description. Sections 3 and 4 supply the tax competition results and the tax coordination results calculated by the backward induction. Finally, section 5 describes conclusions and policy implications, along with possible future extensions of the model.

3.2 Model

Individuals, governments and universities exist in this two-region model.

3.2.1 Individuals

Individuals born in region $i \in \{1, 2\}$ choose their universities, and their location to raise their children. Individuals are born with ability $a \in [0, \bar{a}]$, uniformly distributed. We assume that $2\bar{a} - \beta_i^2 > 0$ represents the upper bound of the distribution and that it is sufficiently large. According to their ability, individuals decide whether to go to university or not. We assume that the low-skilled laborers cannot move from their region of birth. However, highly-skilled labor can do so, with zero moving costs. The number of people in the region i is denoted as N_i.

Households living in region i are thus determined by the following utility maximization problem.

$$\max_{c_i, n_i} \quad u(c_i, n_i; a) = n_i^{\alpha} c_i^{(1-\alpha)} \tag{3.1}$$

subject to

$$c_i + \left(k - \frac{\theta\sqrt{b_i}}{n_i}\right) n_i = w(a) - t_i,$$

where c_i is consumption,n_i is number of children, $\left(k - \theta\sqrt{b_i}/n_i\right) > 0$ represents the cost to raise a child, $w(a)$ is wage. $\theta\sqrt{b_i}/n_i$ is the per child effective subsidy for parents. k represents the cost of raising a child without subsidies. $\theta > 0$ represents a parameter for the efficiency of local government.

Rearranging the budget constraint, we obtain:

$$c_i + kn_i = w\,(a) + \theta\sqrt{b_i} - t_i.$$

This utility maximization problem is, therefore:

$$\begin{aligned} c_i &= (1-\alpha)\left[w(a) + \theta\sqrt{b_i} - t_i\right], \\ n_i &= \frac{\alpha}{k}\left[w(a) + \theta\sqrt{b_i} - t_i\right]. \end{aligned}$$

giving the following individual indirect utility function for living in region i with innate abilities a:

$$V(a) = \alpha^{\alpha}\,(1-\alpha)^{(1-\alpha)}\,k^{-\alpha}\left[w(a) + \theta\sqrt{b_i} - t_i\right].$$

Subsequently, we substitute $\Phi(\alpha) = \alpha^{\alpha}\,(1-\alpha)^{(1-\alpha)}$ for simplicity. At the equilibrium, there is no difference in the number of children between cases with zero subsidy and income $w(a) + \theta\sqrt{b_i} - t_i$ and cases with subsidized income $w(a) - t_i$. If the subsidy is implemented, the payments by government offset the individual payments to children. Thus, the subsidy has the same effect as an increase in income for an individual.

Wages

Individuals earn wages according to their human capital levels. Individual wage rates are normalized to one for simplicity. They choose skill levels that determines the human capital level and the wages of individual workers. Ability is the determinant of individuals' human capital choice. Human capital is the level and individual has with innate abilities a, given as:

$$w_i(a) = \begin{cases} w_L & \text{if choose low skill} \\ w_H - a + \beta_i\sqrt{g_i} & \text{if choose high skill} \end{cases}$$

where g_i is university spending for students, w_H and w_L are exogenous wages of individuals representing highly-skilled wages and low-skilled wages. If individuals choose low-skilled labor, the amount of human capital becomes w_L. But, if he chooses highly-skilled labor, it becomes $w_H - a + \beta_i\sqrt{g_i}$ in region i. The first term represents money, and the second a non-monetary one, where $\beta_i > 0$ is a parameter for the efficiency of university education. To ensure the existence of the equilibrium, we assume that $w_H > w_L$ and $w_H + \beta_i\sqrt{g_i} - t_i > w_L - t_i > 0$.

3.2.2 Local Government and University

Local Government

Governments levy lump sum taxes on individuals and use the revenues to improve a university's quality and to support raising children. We define L_i as the number of residents after migration in region i for convenience. These numbers vary by the case of the parameters. Of course, $N_1 + N_2 = L_1 + L_2$is satisfied. Then the budget constraint of local government i thus becomes:

$$b_iL_i + g_iL_i = t_iL_i,$$

where g_iL_i is the total transfer to a university. The first term of the left hand side describes total budget for supporting raising children. The second term of the left hand side means total expenditure for university. The right hand side is the total revenue of the local government i.

Considering the budget constraint, governments maximize total indirect utility of their residents. This is often called as Benthamian social welfare function.

University

In this chapter, we assume that the university spends its whole budget on students. We assume that a university with efficiency parameter β_i and per capita budget g_i makes students to acquire human capital $\beta_i \sqrt{g_i}$ measured by wages.

Most previous studies concentrate on the substitution between education and research in the university[1]. This assumption is somewhat questionable especially with respect to the central question of this chapter. Since the most of the research was done under the sponsorship of organizations such as the National Science Foundation (NSF), Agence Nationale de la Recherche(ANR), Deutsche Forschungsgemeinschaft(DFG), Japan Society for the Promotion of Science (JSPS), and so on, the studies are competitively distributed by central governments or by firms. Local governments use their budgets destined for a university to attract students, e.g., scholarships, subsidies, etc. Of course, there is relationship between research and education in the university. However, to focus on student mobility, we disregard the function of research.

3.3 Tax game structure

The tax game structure is as follows: first, the governments i and j decide tax rate t_i and t_j simultaneously. Second, these governments determine the expenditure ratio between g and b. Third, observing the governments' decision, individuals choose among going to the university in region i, going to the university in region j and working in the region where they were born without going to university. The individuals' decisions are made for given levels of t, b and g.

[1] See literatures consequence of Rey (2001).

3.3.1 Third Stage:

In this stage, individuals compare the indirect utilities considering their own innate ability a and decide whether to go to university or not. We assume that parameter condition $w_H - w_L + \beta_i^2/2 < \bar{a}$ for both regions $i \in 1, 2$ to ensure the interior solution.

In this case, the indirect utility of low-skilled people in region i is always larger than that of region j. For convenience, we define $\Delta V = \left(t_j - \theta\sqrt{b_j} - t_i + \theta\sqrt{b_i}\right) \geq 0$. Then, i-born individuals with abilities a compare to the following utilities:

- Not going to university:

$$V_i^L = \Phi(\alpha)k^{-\alpha}\left[w_L + \theta\sqrt{b_i} - t_i\right].$$

- Going to university i:

$$V_i^H = \Phi(\alpha)k^{-\alpha}\left[w_H - a + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i\right]$$

- Going to university j:

$$V_j^H = \Phi(\alpha)k^{-\alpha}\left[w_H - a + \beta_j\sqrt{g_j} + \theta\sqrt{b_j} - t_j\right].$$

Then we define threshold the value for convenience. First, we define the threshold ability that determines whether or not go to university i, $\hat{a}_{ii}$, by comparing V_i^L and V_i^H as follows:

$$\widehat{a}_{ii} = [w_H - w_L + \beta_i\sqrt{g_i}] > 0.$$

The inequality comes from the assumption above. Second, we define the threshold ability $\hat{a}_{ij}$ by comparing V_i^L and V_j^H as follows:

$$\widehat{a}_{ij} = \left[w_H - w_L + \beta_j\sqrt{g_j} + \theta\sqrt{b_j} - t_j + t_i - \theta\sqrt{b_i}\right].$$

On the other hand, j-born individuals with abilities a compare to the following utilities:

- Not going to university:

$$V_j^L = \Phi(\alpha)k^{-\alpha}\left[w_L + \theta\sqrt{b_j} - t_j\right].$$

- Going to university i:

$$V_i^H = \Phi(\alpha)k^{-\alpha}\left[w_H - a + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i\right]$$

- Going to university j:

$$V_j^H = \Phi(\alpha)k^{-\alpha}\left[w_H - a + \beta_j\sqrt{g_j} + \theta\sqrt{b_j} - t_j\right].$$

Similarly, we define threshold value. First, we define $\hat{a}_{ji}$ by comparing V_j^L and V_i^Has follows:

$$\widehat{a}_{ji} = \left[w_H - w_L + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i + t_j - \theta\sqrt{b_j}\right].$$

Second, we define $\hat{a}_{jj}$ by comparing V_j^L and V_j^H as follows:

$$\widehat{a}_{jj} = \left[w_H - w_L + \beta_j\sqrt{g_j}\right] > 0.$$

In this model, we define that the brain drain from the country is equal to lower-skilled labor utility in one country compared to the same thing in another country.

Lemma 3.1. *In the asymmetric situation, the region with lower utility for low-skilled laborers has greater ability criteria on going to universities than that of higher utility regions.*

Proof. Suppose all highly-skilled people go to university i. Then by comparing $\widehat{a}_{ii}$ and $\widehat{a}_{ji}$, we obtain $\widehat{a}_{ii} \leq \widehat{a}_{ji}$. Suppose all highly-skilled individuals go to university j. Then by comparing the two, $\widehat{a}_{ij}$ and $\widehat{a}_{jj}$, we obtain $\widehat{a}_{ji} \leq \widehat{a}_{jj}$. Suppose each individual goes to university in each region. Then from the comparison of $\widehat{a}_{ii}$ and $\widehat{a}_{jj}$and $\left(\beta_j\sqrt{g_j} - \beta_i\sqrt{g_i}\right) = \Delta V \geq 0$, we have $\widehat{a}_{ii} \leq \widehat{a}_{jj}$. □

The intuition of this lemma is used to avoid such disutility, wherein people go to university and thus promote a brain drain from the region. We concentrate on the situation where somebody goes to university.

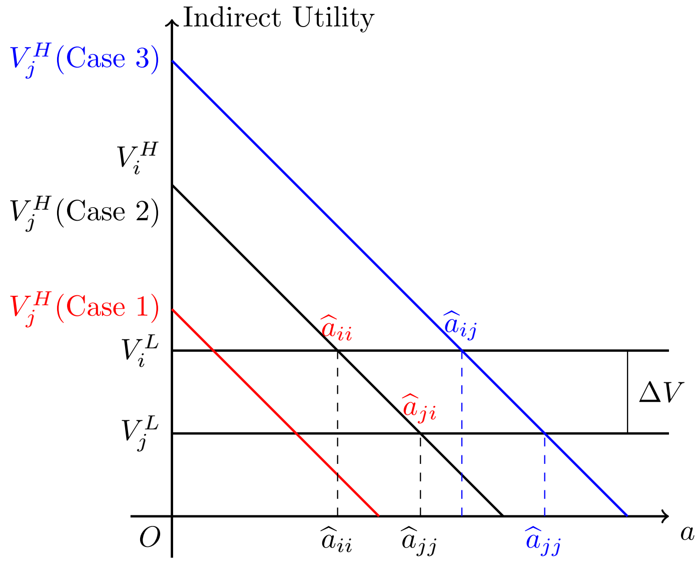


Fig. 3.1 Individual decision

Then we have to consider three cases by certain parameters. Suppose that $\Delta V \geq 0$, that is, region j is the region of lower utility for low-skilled workers and region i offers them higher utility. This is depicted as Figure 3.1. For i- and j-born individuals,

1. $\left(\beta_j \sqrt{g_j} - \beta_i \sqrt{g_i}\right) < \Delta V$ hold,
   - i individual: with ability $a \in [0, \widehat{a}_{ii}]$ go to university i and with ability $a \in [\widehat{a}_{ii}, \bar{a}]$ do not go, remaining in the region,
   - j individual: with ability $a \in [0, \widehat{a}_{ji}]$ go to university i, with ability $a \in [\widehat{a}_{ji}, \bar{a}]$ do not go, remaining in the region,
2. $\left(\beta_j \sqrt{g_j} - \beta_i \sqrt{g_i}\right) = \Delta V$ hold,
   - i individual: with ability $a \in [0, \widehat{a}_{ii}]$ go to university i, with ability $a \in [\widehat{a}_{ii}, \bar{a}]$ do not go, remaining in the region,
   - j individual: with ability $a \in [0, \widehat{a}_{jj}]$ go to university j, with ability $a \in [\widehat{a}_{jj}, \bar{a}]$ do not go, remaining in the region,
3. $\left(\beta_j \sqrt{g_j} - \beta_i \sqrt{g_i}\right) > \Delta V$, hold,
   - i individual: with ability $a \in [0, \widehat{a}_{ij}]$ go to university j, and with ability

$a \in [\widehat{a}_{ij}, \bar{a}]$ do not go, remaining in the region,

- j individual: with ability $a \in [0, \widehat{a}_{jj}]$ go to university j and with ability $a \in [\widehat{a}_{jj}, \bar{a}]$ do not go, remaining in the region,

Here after, we consider the aforementioned three cases.

3.4 Competitive results with different population

3.4.1 Second and first stage

In the second stage, governments simultaneously determine g and b for given t. In the first stage, governments simultaneously determine t considering the effects for after stages. We examine the competitive results and later, compare them with the coordination results. We consider the case that initial population is also an asymmetric case. We assume that $N_1 \neq N_2$. However, the ensuing discussions are also applied to the $N_1 = N_2$ case.

Case1:

We consider the total welfare in region i as follows:

$$
\begin{aligned}
W_i &= N_i \int_{\widehat{a}_{ii}}^{\bar{a}} V_i^L(a) \frac{1}{\bar{a}} da + N_i \int_0^{\widehat{a}_{ii}} V_i^H(a) \frac{1}{\bar{a}} da + N_j \int_0^{\widehat{a}_{ji}} V_i^H(a) \frac{1}{\bar{a}} da \\
&= \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \Big\{ N_i \left[w_L + \theta\sqrt{b_i} - t_i \right] (\bar{a} - \widehat{a}_{ii}) \\
&\quad + N_i \left[\left(w_H + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i \right) \widehat{a}_{ii} - \frac{\widehat{a}_{ii}^2}{2} \right] + N_j \left[\left(w_H + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i \right) \widehat{a}_{ji} - \frac{\widehat{a}_{ji}^2}{2} \right] \Big\} \\
&= \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \Big\{ N_i \left(w_L + \theta\sqrt{b_i} - t_i \right) [\bar{a} - (w_H - w_L + \beta_i\sqrt{g_i})] \\
&\quad + N_i (w_H - w_L + \beta_i\sqrt{g_i}) \left[\frac{w_H + w_L + \beta_i\sqrt{g_i} + 2\left(\theta\sqrt{b_i} - t_i\right)}{2} \right] \\
&\quad + N_j \left[w_H - w_L + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i + t_j - \theta\sqrt{b_j} \right] \\
&\quad \left[\frac{w_H + w_L + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i + \theta\sqrt{b_j} - t_j}{2} \right] \Big\}
\end{aligned}
$$

and maximize welfare according to the following budget constraint:

$$b_i + g_i = t_i.$$

Then we obtain the following simultaneous equations in the second stage:

$$\begin{gathered} b_i + g_i = t_i, \\ \frac{\partial W_i}{\partial b_i} = \frac{\partial W_i}{\partial g_i}, \end{gathered}$$

where the partial differentials are:

$$\begin{aligned} \frac{\partial W_i}{\partial b_i} &= \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \frac{\theta}{2\sqrt{b_i}} \left[N_i \bar{a} + N_j \left(w_H + \beta_i \sqrt{g_i} + \theta \sqrt{b_i} - t_i \right) \right], \\ \frac{\partial W_i}{\partial g_i} &= \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \frac{\beta_i}{2\sqrt{g_i}} \left[N_i \left(w_H + \beta_i \sqrt{g_i} - w_L \right) + N_j \left(w_H + \beta_i \sqrt{g_i} + \theta \sqrt{b_i} - t_i \right) \right]. \end{aligned}$$

These simultaneous equations are hard to solve. Consequently, we put the solutions as $g_i(t_i)$ and $b_i(t_i)$ and proceed to the first stage.

Building on the first stage, and substituting the result to the objective function and budget constraint, the best tax rate response of government i is determined by

$$\begin{aligned} &\frac{\partial W_i}{\partial b_i}\frac{\partial b_i}{\partial t_i} + \frac{\partial W_i}{\partial g_i}\frac{\partial g_i}{\partial t_i} + \frac{\partial W_i}{\partial t_i} = \frac{\partial W_i}{\partial b_i} + \frac{\partial W_i}{\partial t_i} \\ =&\Phi(\alpha)\frac{k^{-\alpha}}{\bar{a}} \left(\frac{\theta}{2\sqrt{b_i(t_i)}} - 1 \right) \left[\bar{a} N_i + \left(w_H + \beta_i \sqrt{g_i(t_i)} + \theta \sqrt{b_i(t_i)} - t_i \right) N_j \right] = 0. \end{aligned}$$

Then we have

$$b_i = \left(\frac{\theta}{2} \right)^2.$$

By substituting the budget constraint, we obtain the equation of $\sqrt{g_i}$ as follows:

$$\begin{aligned} &\beta_i \left[\left(w_H + \beta_i \sqrt{g_i} - w_L \right) N_i + \left(w_H + \beta_i \sqrt{g_i} + \frac{\theta^2}{4} - g_i \right) N_j \right] \\ =&2\sqrt{g_i} \left[\bar{a} N_i + \left(w_H + \beta_i \sqrt{g_i} + \frac{\theta^2}{4} - g_i \right) N_j \right]. \end{aligned} \tag{3.2}$$

Then we obtain:

$$t_i = \left(\frac{\theta}{2}\right)^2 + \widehat{g}_i,$$

where $\widehat{g}_i$ is the solution of equation (3.2).

Similarly, we consider region j. The total welfare of region j is,

$$\begin{aligned} W_j &= N_j \int_{\widehat{a}_{ji}}^{\bar{a}} \Phi(\alpha) k^{-\alpha} \left[w_L + \theta\sqrt{b_j} - t_j\right] \frac{1}{\bar{a}} da, \\ &= \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} N_j \left\{ \left[w_L + \theta\sqrt{b_j} - t_j\right] \left[\bar{a} - \left(w_H - w_L + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i + t_j - \theta\sqrt{b_j}\right)\right] \right\} \end{aligned}$$

And we obtain the following solution from $\partial W_j / \partial g_j = 0$, $g_j = 0$ and $b_j = t_j$. The first order condition is

$$\begin{aligned} \frac{\partial W_j}{\partial t_j} &= \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} N_j \{ \left(\frac{\theta}{2\sqrt{t_j}} - 1\right) \left[\bar{a} - \left(w_H - w_L + \beta_i\sqrt{g_i} + \theta\sqrt{b_i} - t_i + t_j - \theta\sqrt{t_j}\right)\right] \\ &\quad + \left(\frac{\theta}{2\sqrt{t_j}} - 1\right) \left[w_L + \theta\sqrt{t_j} - t_j\right] \} = 0. \end{aligned}$$

Then we obtain

$$b_j = t_j = \frac{\theta^2}{4}, \quad g_j = 0.$$

The prerequisite conditions of the equilibrium thus become,

$$\begin{aligned} -\beta_i\sqrt{g_i^*} < \theta\sqrt{b_i} - \theta\sqrt{b_j} - t_i + t_j, \quad &\Leftrightarrow \quad \sqrt{g_i^*} < \beta_i, \\ \Delta V = \theta b_i - \theta b_j - t_i + t_j = -g_i^* \geq 0. \end{aligned}$$

Obviously, $\widehat{g}_i \geq 0$ and we must hold $\widehat{g}_i = 0$. However, left hand side of equation (3.2) at $\widehat{g}_i = 0$ is strictly positive and the right hand side of equation (3.2) is zero, which is contradictory. Thus, there is no equilibrium in the case 1.

Case 2:

Second, we consider the autarky situation, that is, there is no migration between the regions. Government i problem must maximize:

$$
\begin{aligned}
W_i &= N_i \int_{\widehat{a}_{ii}}^{\bar{a}} V_i^L(a)\frac{1}{\bar{a}}da + N_i \int_0^{\widehat{a}_{ii}} V_i^H(a)\frac{1}{\bar{a}}da \\
&= \Phi(\alpha)\frac{k^{-\alpha}}{\bar{a}}N_i\left\{\left[w_L+\theta\sqrt{b_i}-t_i\right](\bar{a}-\widehat{a}_{ii})+\left[\left(w_H+\beta_i\sqrt{g_i}+\theta\sqrt{b_i}-t_i\right)\widehat{a}_{ii}-\frac{\widehat{a}_{ii}^2}{2}\right]\right\} \\
&= \Phi(\alpha)\frac{k^{-\alpha}N_i}{\bar{a}}\{\left(w_L+\theta\sqrt{b_i}-t_i\right)\left[\bar{a}-(w_H+\beta_i\sqrt{g_i}-w_L)\right] \\
&\quad +(w_H+\beta_i\sqrt{g_i}-w_L)\left[\frac{\left(w_H+\beta_i\sqrt{g_i}+w_L\right)+2\left(\theta\sqrt{b_i}-t_i\right)}{2}\right]\}
\end{aligned}
$$

subject to

$$g_i + b_i = t_i.$$

Accordingly, we obtain the following simultaneous equations:

$$
\begin{aligned}
b_i + g_i &= t_i, \\
\frac{\partial W_i}{\partial b_i} &= \frac{\partial W_i}{\partial g_i}.
\end{aligned}
$$

where the partial derivatives are as follows:

$$
\begin{aligned}
\frac{\partial W_i}{\partial b_i} &= \Phi(\alpha)\frac{k^{-\alpha}N_i}{\bar{a}}\frac{\bar{a}\theta}{2\sqrt{b_i}} \\
\frac{\partial W_i}{\partial g_i} &= \Phi(\alpha)\frac{k^{-\alpha}N_i}{\bar{a}}\frac{\beta_i}{2\sqrt{g_i}}(w_H+\beta_i\sqrt{g_i}-w_L)
\end{aligned}
$$

However, these simultaneous equations are hard to solve. Thus, we substitute $b_i(t_i)$ and $g_i(t_i)$ for them. The the tax rate is then determined by:

$$\frac{\partial W_i}{\partial b_i}\frac{\partial b_i}{\partial t_i}+\frac{\partial W_i}{\partial g_i}\frac{\partial g_i}{\partial t_i}+\frac{\partial W_i}{\partial t_i}=\frac{\partial W_i}{\partial b_i}+\frac{\partial W_i}{\partial t_i}$$
$$=\Phi\left(\alpha\right)\frac{k^{-\alpha}N_i}{\bar{a}}\left(\frac{\bar{a}\theta}{2\sqrt{b_i\left(t_i\right)}}-\bar{a}\right)=0.$$

Solving these equations, we obtain the following solution:

$$b_i^a=\left(\frac{\theta}{2}\right)^2,\quad g_i^a=\left[\frac{\beta_i\left(w_H-w_L\right)}{2\bar{a}-\beta_i^2}\right]^2,\quad t_i^a=\left(\frac{\theta}{2}\right)^2+\left[\frac{\beta_i\left(w_H-w_L\right)}{2\bar{a}-\beta_i^2}\right]^2,$$

This equation also applies to the region j.

By substituting the equilibrium value to the prerequisite conditions, we obtain the following condition:

$$\left[\frac{\beta_i\left(w_H-w_L\right)}{2\bar{a}-\beta_i^2}\right]^2-\left[\frac{\beta_i^2\left(w_H-w_L\right)}{2\bar{a}-\beta_i^2}\right]=\left[\frac{\beta_j\left(w_H-w_L\right)}{2\bar{a}-\beta_j^2}\right]^2-\left[\frac{\beta_j^2\left(w_H-w_L\right)}{2\bar{a}-\beta_j^2}\right].$$

If$\beta_i=\beta_j$ holds, the condition is always satisfied and we obtain a symmetric equilibrium. The $\beta_1\neq\beta_2$ case is not consistent with the condition $2\bar{a}-\beta_i^2>0$. Therefore, we obtain the following proposition.

Proposition 3.1. *There is an autarky equilibrium when the parameters are $\beta_1=\beta_2$.*

This proposition implies that if $\beta_1=\beta_2$ is satisfied, there is an equilibrium with no immigration and the policy variables will have the same value regardless the size of the ex-ante population. The birth subsidy level is determined by the net marginal benefits of the subsidy. The level of university spendings is determined by the ratio between highly- and lower-skilled workers. Highly-skilled workers receive benefits from education and pay part of their education cost by lump sum taxes. In contrast, lower-skilled workers receive no such benefit and still pay part of the educational cost by means of a lump sum tax. The equilibrium spending level of education is determined by the point where the net total benefit from education is zero. Thus, each region reaches the its welfare maximizing point at the autarky equilibrium.

Next, we examine the relationship between parameters– especially university levels β and the number of births pertaining to regional welfare. The regional numbers of births are a linear transformation of the total indirect utility of the region. Thus, checking the number of births to examine the welfare level is sufficient. The number of births in region i, B_i is determined as follows:

$$
\begin{aligned}
B_i^a =& N_i \int_{\widehat{a}_{ii}}^{\bar{a}} \alpha k^{-\alpha} \left[w_L + \theta\sqrt{b_i} - t_i \right] \frac{1}{\bar{a}} da + N_i \int_0^{\widehat{a}_{ii}} \alpha k^{-\alpha} \left[w_H - a + \beta_i \sqrt{g_i} + \theta\sqrt{b_i} - t_i \right] \frac{1}{\bar{a}} da \\
=& N_i \alpha \frac{k^{-\alpha}}{\bar{a}} \left\{ \left(w_L + \frac{\theta^2}{4} - g_i \right) (\bar{a} - \widehat{a}_{ii}) + \left[\left(w_H + \beta_i \sqrt{g_i} + \frac{\theta^2}{4} - g_i \right) \widehat{a}_{ii} - \frac{\widehat{a}_{ii}^2}{2} \right] \right\} \\
=& N_i \alpha \frac{k^{-\alpha}}{\bar{a}} \left[\bar{a} \left(w_L + \frac{\theta^2}{4} \right) - \bar{a} \frac{\beta_i^2 (w_H - w_L)^2}{(2\bar{a} - \beta_i^2)^2} + \frac{1}{2} \left(\frac{\beta_i^2 (w_H - w_L)}{2\bar{a} - \beta_i^2} + (w_H - w_L) \right)^2 \right] \\
=& N_i \alpha k^{-\alpha} \left[\frac{(w_H - w_L)^2}{(2\bar{a} - \beta_i^2)} + w_L + \frac{\theta^2}{4} \right].
\end{aligned}
$$

Then we obtain partial derivatives as follows:

$$
\frac{\partial B_i^a}{\partial \beta_i} > 0, \quad \frac{\partial B_i^a}{\partial w_H} > 0, \quad \frac{\partial B_i^a}{\partial w_L} > 0, \quad \frac{\partial B_i^a}{\partial \theta} > 0, \quad \frac{\partial B_i^a}{\partial \bar{a}} < 0.
$$

The intuitions of these inequalities are as follows. Increasing in parameters β_i and w_H relating to highly-skilled workers increases the number of highly-skilled workers, which increases the total income in the region. This results in an increasing the number of births and, thus, total welfare. Increasing parameter w_L decreases the number of highly-skilled workers. However, it also increase the number of births and, thus, the welfare of lower-skilled workers. In sum, the positive effect always overwhelms the negative one. An increase in parameter θ increases the effective income of an individual, which in turn increases the number of children and social welfare. Finally, an increase parameter $\bar{a}$ implies an increase in the variance of abilities and thus reduces the relative number of highly-skilled workers. Doing so decreases total income, the number of children reproduced, and the total welfare of the region.

Case 3:

To derive the best response for region i, we consider the total welfare in region i as follows:

$$
\begin{aligned}
W_i &= N_i \int_{\widehat{a}_{ij}}^{\bar{a}} V_i^L(a)\frac{1}{\bar{a}}da \\
&= \Phi\left(\alpha\right)\frac{k^{-\alpha}}{\bar{a}}N_i\left[w_L+\theta\sqrt{b_i}-t_i\right]\left[\bar{a}-\left(w_H-w_L+\beta_j\sqrt{g_j}-\left(t_j-\theta\sqrt{b_j}-t_i+\theta\sqrt{b_i}\right)\right)\right]
\end{aligned}
$$

Differentiating this equation by t_i and considering the budget constraint $b_i = t_i$ from the fact that $\partial W_i/\partial g_i = 0$, we obtain:

$$
\frac{\partial W_i}{\partial t_i} = \Phi\left(\alpha\right)\frac{k^{-\alpha}}{\bar{a}}N_i\left(\frac{\theta}{2\sqrt{t_i}}-1\right)\left[\bar{a}-w_H-\beta_j\sqrt{g_j}+2w_L+t_j-\theta\sqrt{b_j}+2\left(\theta\sqrt{t_i}-t_i\right)\right]=0.
$$

Rearranging this equation, we get:

$$
b_i^* = t_i^* = \left(\frac{\theta}{2}\right)^2, \quad g_i^* = 0.
$$

Similarly, to obtain the best response for region j, we consider its total welfare in region j as follows:

$$
\begin{aligned}
W_j &= N_j \int_{\widehat{a}_{jj}}^{\bar{a}} V_j^L\left(a\right)\frac{1}{\bar{a}}da + N_j\int_0^{\widehat{a}_{jj}} V_j^H\left(a\right)\frac{1}{\bar{a}}da + N_i\int_0^{\widehat{a}_{ij}} V_j^H\left(a\right)\frac{1}{\bar{a}}da \\
&= \Phi\left(\alpha\right)\frac{k^{-\alpha}}{\bar{a}}\{N_j\left(w_L+\theta\sqrt{b_j}-t_j\right)\left[\bar{a}-\left(w_H+\beta_j\sqrt{g_j}-w_L\right)\right] \\
&\quad + N_j\left(w_H+\beta_j\sqrt{g_j}-w_L\right)\left[\frac{\left(w_H+\beta_j\sqrt{g_j}+w_L\right)}{2}+\theta\sqrt{b_j}-t_j\right] \\
&\quad + N_i\left[w_H-w_L+\beta_j\sqrt{g_j}-\theta\sqrt{b_i}+t_i+\theta\sqrt{b_j}-t_j\right] \\
&\qquad \left[\frac{\left(w_H+w_L+\beta_j\sqrt{g_j}+\theta\sqrt{b_j}-t_j+\theta\sqrt{b_i}-t_i\right)}{2}\right],
\end{aligned} \tag{3.3}
$$

subject to the following budget constraint:

$$
b_j + g_j = t_j.
$$

The derivatives of the resulting social welfare (3.3) are as follows:

$$\frac{\partial W_j}{\partial b_j} = \Phi(\alpha)\frac{k^{-\alpha}}{\bar{a}}\frac{\theta}{2\sqrt{b_j}}\left[N_j\bar{a} + N_i\left(w_H + \beta_j\sqrt{g_j} + \theta\sqrt{b_j} - t_j\right)\right]$$
$$\frac{\partial W_j}{\partial g_j} = \Phi(\alpha)\frac{k^{-\alpha}}{\bar{a}}\frac{\beta_j}{2\sqrt{g_j}}\left[N_j\left(w_H - w_L + \beta_j\sqrt{g_j}\right) + N_i\left(w_H + \beta_j\sqrt{g_j} + \theta\sqrt{b_j} - t_j\right)\right]$$

Using these results, we consider the first order conditions of the tax rate t_j:

$$\begin{aligned}&\frac{\partial W_j}{\partial b_j}\frac{\partial b_j}{\partial t_j} + \frac{\partial W_j}{\partial g_j}\frac{\partial g_j}{\partial t_j} + \frac{\partial W_j}{\partial t_j} = \frac{\partial W_j}{\partial b_j} + \frac{\partial W_j}{\partial t_j}\\ =&\Phi(\alpha)\frac{k^{-\alpha}}{\bar{a}}\left(\frac{\theta}{2\sqrt{b_j(t_j)}} - 1\right)\left[N_j\bar{a} + N_i\left(w_H + \beta_j\sqrt{g_j(t_j)} + \theta\sqrt{b_j(t_j)} - t_j\right)\right] = 0\end{aligned}$$

Then we obtain

$$b_j^* = \left(\frac{\theta}{2}\right)^2 \quad \text{and} \quad t_j^* = \left(\frac{\theta}{2}\right)^2 + g_j^*$$

By substituting the budget constraint, we obtain the equation which determine g_j^* as follows:

$$\begin{aligned}f(g_j) =& \frac{\beta_j}{2\sqrt{g_j}}\left[N_j\left(w_H - w_L + \beta_j\sqrt{g_j}\right) + N_i\left(w_H + \beta_j\sqrt{g_j} + \frac{\theta^2}{4} - g_j\right)\right]\\ &- \left[N_j\bar{a} + N_i\left(w_H + \beta_j\sqrt{g_j} + \frac{\theta^2}{4} - g_j\right)\right].\end{aligned} \tag{3.4}$$

By solving the above equation, $f(g_j) = 0$, we obtain the optimal g_j^*. The numerical example of function (3.4) is depicted in Figure 3.2.

The prerequisite conditions of the equilibrium become,

$$\begin{aligned}\beta_j\sqrt{g_j^*} > \theta\sqrt{b_i^*} - \theta\sqrt{b_j^*} - t_i^* + t_j^* \quad &\Leftrightarrow \quad \beta_j > \sqrt{g_j^*}\\ \Delta V = \theta\sqrt{b_i^*} - \theta\sqrt{b_j^*} - t_i^* + t_j^* \geq 0 \quad &\Leftrightarrow \quad g_j^* \geq 0.\end{aligned}$$

Thus, the equilibrium must be in the range between 0 and β_j^2. Furthermore, to examine the shape of f, we must first consider the derivative of f:

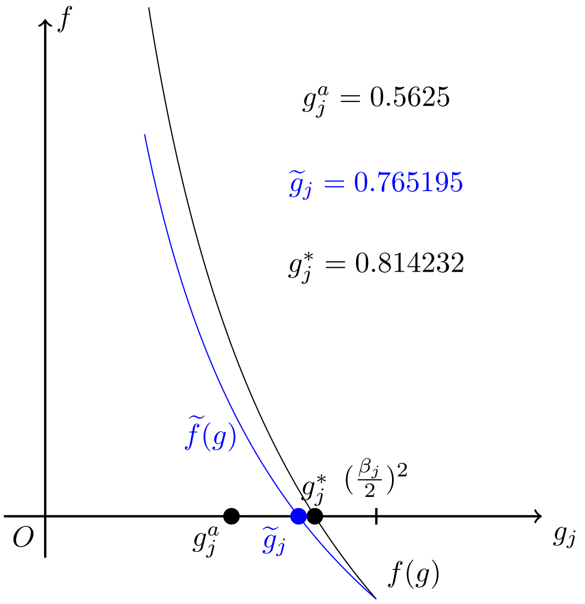


Fig. 3.2 g_j determination: Case $\beta_j = 2$, $w_H = 5$, $w_L = 2$, $N_i = N_j = 1$, $\theta = 1$, $\bar{a} = 6$

$$f'(g_j) = N_i - \frac{3\beta_j}{4\sqrt{g_j}} N_i - \frac{\beta_j}{4\sqrt{g_j}^3}\left[N_j(w_H - w_L) + N_i\left(w_H + \frac{\theta^2}{4}\right)\right].$$

$f' < 0$ is satisfied in the range between 0 and $(\beta_j/2)^2$. If the following conditions hold, there is a unique equilibrium g_j^* between the range 0 and $(\beta_j/2)^2$:

$$f\left(\left(\frac{\beta_j}{2}\right)^2\right) < 0 \quad \Leftrightarrow \quad \left(w_H - w_L + \frac{\beta_j^2}{2}\right) < \bar{a}.$$

To compare the university spending level with the autarky case, we evaluate f at the $g_j^a = \left[\beta_j(w_H - w_L) / \left(2\bar{a} - \beta_j^2\right)\right]^2$. The value is

$$f\left(g_j^a\right) = N_i\left[\frac{2\bar{a} - \beta_j^2}{2(w_H - w_L)} - 1\right]\left[w_H + \frac{\theta^2}{4} + \frac{\beta_j^2(w_H - w_L)}{2\bar{a} - \beta_j^2}\left(1 - \frac{w_H - w_L}{2\bar{a} - \beta_j^2}\right)\right] > 0.$$

The first parenthesis is known to be positive from the equilibrium condition. The second parenthesis indicates after-tax income without education costs considered for the immigrants from region i at the autarky university level. In total, $f(g_j^a) > 0$ is satisfied. Then we obtain the following proposition:

Proposition 3.2. *In the immigration case, the agglomerated region has a larger university budget than that in the autarky case.*

The proposition implies that the natives of the agglomerated region pay costs of immigrants' university education and the additional native highly-skilled workers by paying higher lump sum tax compared to the autarky case. This proposition is also explained by the marginal benefit and cost of education. At the immigration equilibrium, highly-skilled workers with positive net benefits from education migrate to the agglomerated region. This increases the total marginal benefits of a university, and the level of university spending becomes g_j^*. The cost is borne by the native workers in the agglomerated region. However, the native highly-skilled workers in agglomerated regions receive the benefits of highly-skilled workers' migration by increasing university spending level g_j^*. Thus, the native lower-skilled workers lose both income and welfare.

Next, we consider the welfare and the number of children in each region by calculating the number of births. At the equilibrium, the number of births under the immigration case in region i, B_i^f, is calculated as follows:

$$\begin{aligned}
B_i^* &= N_i \int_{\widehat{a}_{ij}}^{\bar{a}} \alpha k^{\frac{1}{\alpha}} \left(w_L + \frac{\theta^2}{4} \right) \frac{1}{\bar{a}} da \\
&= N_i \alpha \frac{k^{-\alpha}}{\bar{a}} \left[w_L + \frac{\theta^2}{4} \right] \left[\bar{a} - \left(w_H - w_L + \beta_j \sqrt{g_j^*} - g_j^* \right) \right].
\end{aligned}$$

Similarly, the number of births for the immigration case in region j, B_j^f, is calculated as follows:

$$
\begin{aligned}
B_j^* =& N_j \int_{\widehat{a}_{jj}}^{\bar{a}} \alpha k^{-\alpha} \left(w_L + \frac{\theta^2}{4} - g_j^*\right) \frac{1}{\bar{a}} da + N_j \int_0^{\widehat{a}_{jj}} \alpha k^{-\alpha} \left[w_H - a + \beta_j \sqrt{g_j^*} + \frac{\theta^2}{4} - g_j^*\right] \frac{1}{\bar{a}} da \\
&+ N_i \int_0^{\widehat{a}_{ij}} \left[w_H - a + \beta_j \sqrt{g_j^*} + \frac{\theta^2}{4} - g_j^*\right] \frac{1}{\bar{a}} da \\
=& \alpha \frac{k^{-\alpha}}{\bar{a}} \{ N_j \left(w_L + \frac{\theta^2}{4} - g_j^*\right) \left[\bar{a} - \left(w_H - w_L + \beta_j \sqrt{g_j^*}\right)\right] \\
&+ N_j \left(w_H - w_L + \beta_j \sqrt{g_j^*}\right) \left[\frac{\left(w_H - w_L + \beta_j \sqrt{g_j^*}\right)}{2} + w_L + \frac{\theta^2}{4} - g_j^*\right] \\
&+ N_i \left[w_H - w_L + \beta_j \sqrt{g_j^*} - g_j^*\right] \left[\frac{\left(w_H - w_L + \beta_j \sqrt{g_j^*} - g_j^*\right)}{2} + w_L + \frac{\theta^2}{4}\right] \} \\
=& \alpha \frac{k^{-\alpha}}{\bar{a}} \{ N_j \left[\bar{a} \left(w_L + \frac{\theta^2}{4} - g_j^*\right) + \frac{1}{2} \left(w_H - w_L + \beta_j \sqrt{g_j^*}\right)^2\right] \\
&+ \frac{N_i}{2} \left(w_H - w_L + \beta_j \sqrt{g_j^*} - g_j^*\right)^2 + N_i \left(w_L + \frac{\theta^2}{4}\right) \left(w_H - w_L + \beta_j \sqrt{g_j^*} - g_j^*\right) \}.
\end{aligned}
$$

The total number of births in regions i and j are:

$$
\begin{aligned}
B_i^* + B_j^* =& \alpha k^{-\alpha} (N_i + N_j) \left[w_L + \frac{\theta^2}{4}\right] \\
&+ \alpha k^{-\alpha} \{ N_j \left[-g_j^* + \frac{1}{2\bar{a}} \left(w_H - w_L + \beta_j \sqrt{g_j^*}\right)^2\right] + \frac{N_i}{2\bar{a}} \left(w_H - w_L + \beta_j \sqrt{g_j^*} - g_j^*\right)^2 \}.
\end{aligned}
$$

At the equilibrium, partial derivatives of the number of births in each region i and j by g_j^* become:

$$
\frac{\partial B_i}{\partial g_j^*} = N_i \alpha \frac{k^{-\alpha}}{\bar{a}} \left[w_L + \frac{\theta^2}{4}\right] \left(1 - \frac{\beta_j}{2\sqrt{g_j^*}}\right),
$$

$$
\begin{aligned}
\frac{\partial B_j}{\partial g_j^*} =& \alpha \frac{k^{-\alpha}}{\bar{a}} \left\{ N_j \left[-\bar{a} + \frac{\beta_j}{2\sqrt{g_j^*}} \left(w_H - w_L + \beta_j \sqrt{g_j^*}\right)\right] \right. \\
&\left. + N_i \left(w_H + \frac{\theta^2}{4} + \beta_j \sqrt{g_j^*} - g_j^*\right) \left(\frac{\beta_j}{2\sqrt{g_j^*}} - 1\right) \right\} = 0. \qquad (3.5)
\end{aligned}
$$

The last equality comes from $f\left(g_j^*\right) = 0$. Then, the relationship between $\left(\beta_j/2\right)^2$ and g_j^* determines the sign of $\partial B_i/\partial g_j^*$. Evaluating f at $\left(\beta_j/2\right)^2$ results in:

$$f\left(\left(\frac{\beta_j}{2}\right)^2\right) = N_j\left(w_H - w_L + \frac{\beta_j^2}{2} - \bar{a}\right) < 0.$$

The last inequality comes from the interior solution condition. At the equilibrium, similar comparative statistics with the autarky case is satisfied. Therefore, we obtain the following proposition.

Proposition 3.3. *In the immigration case, the total number of births decrease at g_j^*.*

The proposition says that increasing g_j^* reduces the lower-skilled residents in region i that in turn reduces the total number of births in region i. However, the residents in region j still do not increase the number of births, since the benefits of such an increment is offset by higher taxes and costs. In other words, government j determines the university spending level g_j^* considering only the welfare of region j. This result is obviously related to the ensuring tax coordination results, which we discuss in the next subsection.

We also note that at the equilibrium, the number of births of lower-skilled workers in region i is larger than in region j. This is explained as follows. In region j, lower-skilled workers are beleaguered by higher tax rates than in region i since they bear the cost of university education. In addition, the number of lower-skilled workers in region j is smaller than in region i (from Lemma 3.1), $\widehat{a}_{ij} < \widehat{a}_{jj}$. This implies that highly-skilled workers in region j have more children in region j.

3.4.2 Coordination results

We investigated whether the tax rates determined by tax competition are higher or lower than the tax coordination case. We considered only the case 3 for comparison because social welfare in case 2 is perfectly separated between region i and region j.

The total welfare of region i and j in the case 3 is as follows:

$$
\begin{aligned}
W_i + W_j =& N_i \int_{\widehat{a}_{ij}}^{\bar{a}} V_i^L(a) \frac{1}{\bar{a}} da + N_j \int_{\widehat{a}_{jj}}^{\bar{a}} V_j^L(a) \frac{1}{\bar{a}} da + N_j \int_0^{\widehat{a}_{jj}} V_j^H(a) \frac{1}{\bar{a}} da + N_i \int_0^{\widehat{a}_{ij}} V_j^H(a) \frac{1}{\bar{a}} da \\
=& \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \{ N_j \left(w_L + \theta \sqrt{b_j} - t_j \right) \left[\bar{a} - \left(w_H + \beta_j \sqrt{g_j} - w_L \right) \right] \\
&+ N_j \left(w_H + \beta_j \sqrt{g_j} - w_L \right) \left[\frac{\left(w_H + \beta_j \sqrt{g_j} + w_L \right)}{2} + \theta \sqrt{b_j} - t_j \right] \\
&+ N_i \left[w_H - w_L + \beta_j \sqrt{g_j} - \theta \sqrt{b_i} + t_i + \theta \sqrt{b_j} - t_j \right] \\
&\left[\frac{\left(w_H + w_L + \beta_j \sqrt{g_j} + \theta \sqrt{b_j} - t_j + \theta \sqrt{b_i} - t_i \right)}{2} \right] \\
&+ N_i \left[w_L + \theta \sqrt{b_i} - t_i \right] \left[\bar{a} - \left(w_H - w_L + \beta_j \sqrt{g_j} - \left(t_j - \theta \sqrt{b_j} - t_i + \theta \sqrt{b_i} \right) \right) \right] \}.
\end{aligned}
$$

In the tax coordination situation, budget constraints are given by the following equations:

$$
b_i + g_i = t_i, \quad b_j + g_j = t_j.
$$

From $\partial (W_i + W_j) / \partial g_i = 0$, we obtain $b_i = t_i$. Then, the first order conditions and partial derivatives are:

$$
\begin{aligned}
\frac{\partial (W_i + W_j)}{\partial t_i} =& \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} N_i \left(\frac{\theta}{2\sqrt{t_i}} - 1 \right) \left[\bar{a} - w_H - \beta_j \sqrt{g_j} + w_L + t_j - \theta \sqrt{b_j} + \left(\theta \sqrt{t_i} - t_i \right) \right] = 0, \\
\frac{\partial (W_i + W_j)}{\partial b_j} =& \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \frac{\theta}{2\sqrt{b_j}} \left[N_j \bar{a} + N_i \left(w_H - w_L + \beta_j \sqrt{g_j} + \theta \sqrt{b_j} - t_j + t_i - \theta \sqrt{t_i} \right) \right], \\
\frac{\partial (W_i + W_j)}{\partial g_j} =& \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \frac{\beta_j}{2\sqrt{g_j}} \left[N_j \left(w_H - w_L + \beta_j \sqrt{g_j} \right) \right. \\
&\left. + N_i \left(w_H - w_L + \beta_j \sqrt{g_j} + \theta \sqrt{b_j} - t_j - \theta \sqrt{t_i} + t_i \right) \right].
\end{aligned}
$$

Then the tax rate is determined by $\partial (W_i + W_j) / \partial b_j = \partial (W_i + W_j) / \partial g_j$ and the following equation:

$$
\begin{aligned}
&\frac{\partial (W_i + W_j)}{\partial b_j} \frac{\partial b_j}{\partial t_j} + \frac{\partial (W_i + W_j)}{\partial g_j} \frac{\partial g_j}{\partial t_j} + \frac{\partial (W_i + W_j)}{\partial t_j} = \frac{\partial (W_i + W_j)}{\partial b_j} + \frac{\partial (W_i + W_j)}{\partial t_j} \\
=& \Phi(\alpha) \frac{k^{-\alpha}}{\bar{a}} \left(\frac{\theta}{2\sqrt{b_j}} - 1 \right) \left[N_j \bar{a} + N_i \left(w_H - w_L + \beta_j \sqrt{g_j} + \theta \sqrt{b_j} - t_j + t_i - \theta \sqrt{t_i} \right) \right] = 0.
\end{aligned}
$$

By solving these equations, we obtain

$$b_i = t_i = b_j = \left(\frac{\theta}{2}\right)^2, \quad t_j = \left(\frac{\theta}{2}\right)^2 + \widetilde{g}_j, \quad g_i = 0,$$

where $\widetilde{g_j}$is a solution for the following equation $\widetilde{f}(g_j) = 0$:

$$\begin{aligned}\widetilde{f}(g_j) =& \frac{\beta_j}{2\sqrt{g_j}}\left[N_j\left(w_H - w_L + \beta_j\sqrt{g_j}\right) + N_i\left(w_H - w_L + \beta_j\sqrt{g_j} - g_j\right)\right] \\ & - N_j\bar{a} - N_i\left(w_H - w_L + \beta_j\sqrt{g_j} - g_j\right).\end{aligned} \tag{3.6}$$

An numerical example of this function is depicted in Figure 3.2. The prerequisite conditions for the equilibrium become,

$$\begin{aligned}\beta_j\sqrt{g_j} > \theta\sqrt{b_i} - \theta\sqrt{b_j} - t_i + t_j \quad &\Leftrightarrow \quad \beta_j > \sqrt{\widetilde{g}_j} \\ \Delta V = \theta\sqrt{b_i} - \theta\sqrt{b_j} - t_i + t_j \geq 0 \quad &\Leftrightarrow \quad \widetilde{g}_j \geq 0.\end{aligned}$$

Similar to the tax competition case, the solution $\widetilde{g}_j$ must be in the range between 0 and β_j^2. To examine the shape of $\widetilde{f}$, we consider first derivatives of $\widetilde{f}$:

$$\widetilde{f}'(g_j) = N_i - \frac{3\beta_j}{4\sqrt{g_j}}N_i - \frac{\beta_j}{4\sqrt{g_j}^3}(w_H - w_L)(N_j + N_i) < 0,$$

The inequality is satisfied in a range between 0 and $(\beta_j/2)^2$. Then if the following conditions hold, there is unique coordination equilibrium $\widetilde{g}_j$ in the range of 0 and $(\beta_j/2)^2$:

$$\widetilde{f}\left(\left(\frac{\beta_j}{2}\right)^2\right) < 0 \quad \Leftrightarrow \quad N_j\left(w_H - w_L + \frac{\beta_j^2}{2} - \bar{a}\right) < 0.$$

This outcome is the same as the interior solution condition. To compare it with the tax competition case, we evaluate equation (3.4) at $\widetilde{g}_j$, generating the following equation:

$$f(\widetilde{g}_j) = N_i\frac{\theta^2}{4}\left(\frac{\beta_j}{2\sqrt{\widetilde{g}_j}} - 1\right) > 0.$$

This equality implies that the relationship between $(\beta_j/2)^2$ and $\widetilde{g}_j$ determines the tax rate relationship between the tax competition case and the tax coordination case. The last inequality comes from the interior solution condition. Then we have the following proposition:

Proposition 3.4. *In the immigration case, the competition tax rate is higher than it is in the coordination case.*

As we have mentioned in propositions above, at the immigration equilibrium, the government of region j determine tax rate t_j^* and university spending g_j^* without considering the welfare of region i. This negative externality reduces the total welfare of the region by over-immigration from i to j. Thus, if both regions coordinate the policy, the government j will set a lower tax rate than the competitive rate to compensate for the welfare loss in region i. At the tax coordination equilibrium, the governments set g_j to balance the marginal gains from increasing g_j in region j with the marginal losses from increasing g_j in region i. The difference is seen in whether governments recognize the welfare from birth subsidies as gains or not.

Next, we discuss the effect of θ. The tax coordination level of university spending $\widetilde{f}$ has no θ term while tax competition f has it. The difference comes from whether the governments consider lower-skilled worker's utility or not. An increase in θ has two effects on the welfare in region i. The first effect is to increase the welfare of lower-skilled workers in both regions. This effect increases the number of births and welfare in both regions. The second is to increase the number of immigrants to region j. This is explained as follows: the increase in θ does not affect to the threshold value $\widehat{a}_{jj}$ and $\widehat{a}_{ij}$. However, the increase in θ increases the value of migrating highly-skilled workers to region j. The incremental change improves the value of total highly-skilled workers in region j, implying that there is an increase in the marginal benefit of human capital investment in the university. The power affect for the threshold value $\widehat{a}_{jj}$ and $\widehat{a}_{ij}$ is to increase the number of highly-skilled workers. Thus, the number of immigrants increases. This effect reduces the number of births and welfare in region i. Therefore, the effect of increasing θ for region i is determined by the composition of the two effects.

3.5 Conclusion

In this chapter, I developed a bi-regional asymmetric economies model with heterogeneous individuals. In the model, local governments spend their budgets on university education and reproductive (birth) support of the individuals, maintaining fixed university quality, while the governments levy lump sum tax on the residents. The existence of asymmetric equilibrium occurs with migration. In the autarky equilibrium, higher university quality leads to higher welfare and larger numbers of births at the expense of lower-skilled workers' utility. In the immigration equilibrium, the university spending is larger than in the autarky case and the coordination case. These results stem from the ratio between the highly-skilled and lower-skilled workers. The outcome suggests that restricting the number of immigrants to an agglomerated region is a welfare-improving policy.

The agglomerated equilibrium may have some policy implications in real economies. In Japan, the Ministry of Education, Culture, Sports, Science and Technology (MEXT) announced that it would limit the student quota for universities located in the Tokyo metropolitan area due to the over-agglomeration of the students in Tokyo. The governor of Tokyo opposes to the policy. Considering this model, the MEXT policy may be evaluated as rational coordination through restricting the number of immigrants. Thus, we can interpret the policy as being welfare-enhancing overall, even though the policy reduces social welfare in the agglomerated region. This interpretation is not contradicted by the governor of Tokyo's opposition to the policy.

Finally, I would like to recommend some possible future extensions of this research. First, different θ levels should be considered. From the above, the effect of θ on the eroding region is ambiguous. Considering different θ levels will better determine and define the effects. In addition, by considering different θ levels, the threshold value will also be affected. Thus, further investigation would need to be conducted. Second, it is necessary to consider the effort's cost difference between the regions. In this model, I did not investigate policy interaction between local governments. Furthermore, other situations such as choice of students from both regions also emerge by integrating differences. Thus, that situation also must be researched. Third, demography must be studied with respect to this case. This model omits demography to concentrate on the investigation of the relationship between worker mobility and university subsidies and spending on education. In addition, the elderly also exist in the real world. Thus, the interaction between younger people and the elderly must be checked. Fourth, aspects of political economy should be investigated. From the forgoing research, the ratio between lower-skilled and highly-skilled workers plays an important role in the model. At the competitive equilibrium, highly-skilled workers take the initiative of determining the university expenditure level. Thus, the median voter differentiation by skill level must be examined. These future elaborations would be fruitful, and best efforts should be made to do them.

Chapter 4

Demographics, immigration, and market size

4.1 Introduction

Changes in a country's population consist of natural changes, which are determined by fertility and mortality, and social changes, which are determined by immigration. For the former change, we have observed very similar trends in many countries during the past half century, that is, consecutive improvements in longevity and declines in fertility. For instance, the life expectancy of age 15 males from 1955 to 2014 has steadily increased from 53.09 to 65.81 years in Japan and from 54.8 to 62.1 years in the United States.[1] And from 1960 to 2015, the total fertility rate has declined from 2.00 to 1.46 in

[1] The sources of data used in this section are as follows. We obtained life expectancy data from the Life Table (Ministry of Health, Labour and Welfare) for Japan and the National Vital Statistics System (CDC/National Center for Health Statistics) for the United States. The total population size comes from the Vital Statistics (Ministry of Health, Labour and Welfare) for Japan and the Annual Estimates of the Resident Population for Selected Age Groups by Sex (Population Division, U.S. Census Bureau) for the United States. The total fertility rate for both countries is taken from http://data.worldbank.org/indicator/SP.DYN.TFRT.IN? on April 18, 2017. The immigration data comes from the Statistical Survey on Legal Migrants (Ministry of Justice) for Japan and the 2014 Yearbook of immigration Statistics (Office of Immigration Statistics, Department of

Japan and from 3.65 to 1.84 in the United States. In contrast to such similar natural changes, we have observed distinct differences in the social changes, that is, immigration between these countries. The number of total cumulative net immigrants from 1955 to 2014 was about 1.2 million in Japan and 40.3 million in the United States. Because Japan had a population of 127 million and the United States had a population of 319 million in 2014, the population of the United States is 2.5 times that of Japan, whereas immigration to the United States is 33.5 times that to Japan, implying that the United States has absorbed immigrants much more intensively than Japan. Such a difference in social changes inevitably results in a difference in population growth, which has become visible in recent years. The average annual population growth rate from 1955 to 1989 was 0.93% in Japan and 1.12% in the United States, while the figures from 1990 to 2014 were 0.12% in Japan and 1.02% in the United States.[2] What can we uncover from such similarity in the natural changes and such difference in the social changes?

This chapter aims to investigate the linkages among life expectancy, fertility, immigration, and population, and to uncover the possible impacts of increases in longevity on social welfare through changes in population. For this purpose, we focus on the role of market size. The population undoubtedly affects a country's market size, which in turn, is known to be a major engine that attracts firm activities in a global economy (Fujita et al. (1999); Baldwin et al. (2003); Combes et al. (2008)). Hence, if a change in demographics increases the market size, firm activities will subsequently rush into the country, resulting in a rise in the country's welfare. If a change decreases the market size, the opposite holds true and there is a decline in the country's welfare. This is not the end of the story. If immigration takes place in response to the change in demographics, then it will also affect the country's market size and welfare.

In this chapter, we develop an overlapping generations model wherein people decide their number of children, i.e., fertility, and their consumption levels of differentiated goods. Differentiated goods are produced under monopolistic competition, implying that

Homeland Security) for the United States.

[2] Very recently, the Japanese population had already started to decrease. The 2015 Population Census reported that the Japanese population had decreased by 0.96 million from 2010 to 2015.

a larger market size induces more firms to enter the market and increases the variety of differentiated goods, which increases the people's utility. Moreover, we assume a small open country and immigration occurs when the utility becomes higher inside the country than outside the country.

By using this framework, we examine the effects of improvements in longevity on population size, market size, and welfare. Our theoretical analysis shows that improvements in longevity affect the market size through three effects: First, it decreases fertility because parents need to prepare for consumption in their old period. Second, it increases the per capita lifetime consumption. Finally, it increases immigration, since the improved longevity raises the individual utility. The first effect has negative impacts on the market size and welfare whereas the latter two effects have positive impacts.

We then calibrate our model to match the Japanese and U.S. data from 1955 to 2014 and conduct counterfactual analyses. Our first counterfactual analysis examines the effects of improvements in longevity and shows that a higher value for the survival rate results in a smaller market size. This implies that the negative impacts of improvements in longevity dominate the positive ones both in Japan and the United States.

Our second counterfactual analysis considers the scenario wherein Japan is as open towards immigration as the United States and the United States is as closed towards immigration as Japan. We then show that under this scenario Japan would have experienced a much higher growth in population and market size whereas the United States would have experienced much lower growth. This result implies that the United States has enjoyed gains from immigration whereas Japan can overcome shrinkages in its market size caused by aging if it accepts more immigrants.

Here, we present the related literature. Many existing studies including Acemoglu and Johnson (2007), Lorentzen et al. (2008), and Cervellati and Sunde (2005) provided empirical evidences that longer life expectancy reduces the fertility rate. Several studies have developed frameworks that can explain this stylized fact: Kalemli-Ozcan (2002, 2003) and Kalemli-Ozcan et al. (2000) investigated the impact of uncertainty about the number of surviving children on fertility and population growth, and showed that if parents are risk averse, they reduce the number of children as the child's survival rate improves. Ehrlich and Lui (1991), Soares (2005), and Bar and Leukhina (2010) presented models wherein longer life expectancy leads parents to invest more in their children's education and to have fewer children. Yakita (2001), Zhang and Zhang (2001a,b) and Miyazawa (2006) extended the model of accidental bequest a la Abel (1985) by endogenizing fertility, and showed that longer life expectancy induces people to save more when they are young in preparation for consumption when they are old, which decreases the number of children. In this chapter, we also employ the model of the accidental bequest with endogenous fertility and further extend it by incorporating immigration and the market size effect on welfare.

Our analysis is also related to the literature on the impact of immigration on the labor markets of host countries that includes Card (2001, 2009), Borjas (2003), Ottaviano and Peri (2012), and Ottaviano et al. (2013), among others. These studies empirically investigated the impact of immigration on wage and employment in the host countries. We examine the impact of immigration using a larger scale by focusing on the market size, and theoretically investigating the welfare impacts. In this sense, our analysis is more closely related to the literature on trade and geography models a la Fujita et al. (1999), Baldwin et al. (2003) and Combes et al. (2008). In a standard trade and geography model, mobile workers are attracted to countries that offer a large variety of goods. Such immigration enlarges the host countries' market size and induces the entry of firms, which increases the variety of goods and welfare there. We depart from trade and geography models by incorporating the overlapping generations structure, longevity, and fertility.

Demographics consist of both natural changes and social changes. The first strand of the related literature focused only on the former and the second strand focused only on the latter. Our analysis bridges a gap between the two strands by considering the interlinkages among life expectancy, fertility, immigration, and market size in order to examine the possible impacts of improvements in longevity on the social welfare.

The chapter is structured as follows. Section 2 provides the baseline model. Section 3 characterizes equilibrium and Section 4 examines the effects of improvements in longevity on the market size and welfare. Section 5 conducts calibration analyses. Section 6 concludes the chapter.

4.2 The model

4.2.1 Individuals

Consider a discrete time overlapping generations model wherein an individual resides in a small open country and lives for three periods: childhood, young (working), and old (retirement) periods. During childhood, the individual does nothing. While young, she works to obtain a wage income, consumes goods, and has children. When old, her children grows up to become young individuals and she spends her savings on consumption. We employ the individual's utility function as follows:

$$U_t = \ln c_{yt} + \phi\beta \ln c_{ot+1} + \gamma \ln n_t. \tag{4.1}$$

where c_{yt} is the young individual's consumption in period t, and c_{ot+1} is the old individual's consumption in period $t+1$. The subscripts y and o represents that the individual is young and old, respectively. Following the literature of endogenous fertility models, such as Eckstein and Wolpin (1985), we assume that individuals obtain utility from having children and that the level of utility depends on the number of children, n_t. β, γ, and ϕ are positive constants: β represents the discount factor satisfying that $0 < \beta < 1$, and ϕ is the survival rate of a young individual living into the old period and satisfies that $0 < \phi < 1$. In this chapter, the value of ϕ represents the degree of a society's longevity, and a rise in ϕ implies an improvement in longevity. We focus on changes in this parameter to investigate the impacts of increases in longevity on market structure and welfare.

We assume that consumption goods are differentiated and produced under monopolistic competition *a la* Dixit and Stiglitz (1977). Letting M_{jt} denote the consumption of differentiated goods, j individual's consumption ($j = y, o$), c_{jt}, is given by

$$c_{yt} = M_{yt}, \qquad c_{ot+1} = M_{ot+1}. \tag{4.2}$$

Moreover, M_{jt} is nested by a CES function as

$$M_{jt} = \left(\int_0^{m_t} x_{jt}(i)^{(\sigma-1)/\sigma} di + \int_0^{m_w} x_{wjt}(i)^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)}, \tag{4.3}$$

where σ is the elasticity of substitution satisfying that $\sigma > 1$. $x(i)$ is the consumption level of a particular differentiated good i. Here, m_t and m_w represent the number of differentiated goods produced in the country and that of differentiated goods imported from abroad (the rest of the world), respectively. The subscript w represents the variable is related to the imported goods. Because we assume the country is small open, the number of imported differentiated goods, m_w, is exogenous whereas the number of domestically produced goods, m_t, is endogenous. As is well known in the literature on trade and geography models, the existence of such differentiated goods results in the backward-linkage effect Fujita et al. (1999), which means that a larger market size encourages a greater number of firms to enter the market. This increases the number of available differentiated goods, and makes it possible for an individual to enjoy higher utility for a given nominal income.[3] This effect plays a key role in understanding the relationship between the market structure and demographics.[4]

We assume a global capital market, so the assumption of a small open country implies that the interest, R_t, in a country is fixed at the exogenous world interest: $R_t = R$. To abstract from the risk associated with uncertain lifespans, we follow Blanchard (1985) and Yaari (1965) in assuming a perfect annuities market, that is, all savings are intermediated through mutual funds. At the end of her young period, each individual deposits her savings with a mutual fund. The mutual fund invests these savings in the global capital market and guarantees a gross return of $\widehat{R}$ to the survivors entering the old period. If a fund earns a gross return $\widehat{R}$ on its investment, then perfect competition yields

[3] In a multi-country setting involving trade of differentiated goods, this causes the home market effect, under which a country with a larger market size hosts a more than proportionate share of firms and production activities.

[4] We ignore the population distribution within a country, which can potentially affect the degree of backward linkage through responses of households' location choices. If we fully incorporate the multiple regions and location choices of firms and households, our model would explode and become intractable.

$\widehat{R} = R/\phi$ in equilibrium. Having this in mind, the budget constraints are given as

$$w_t = \int_0^{m_t} p_t(i)x_{yt}(i)di + \int_0^{m_w} \tau p_w x_{wyt}(i)di + bn_t + s_t, \tag{4.4}$$

$$\frac{R}{\phi}s_t = \int_0^{m_{t+1}} p_{t+1}(i)x_{ot+1}(i)di + \int_0^{m_w} \tau p_w x_{wot+1}(i)di. \tag{4.5}$$

We describe the price of differentiated good i by $p(i)$. To simplify the notation, we assume that the prices of differentiated goods imported from abroad are the same, that is, $p_w(i) = \tau p_w$, $\forall i \in [0, m_w]$, where p_w is the price of a foreign differentiated good sold in the country of production, which is given and constant, because of an assumption of a small open economy. In this chapter, we assume the iceberg transport cost, that is, to consume one unit of a foreign good, τ units of the good must be transported, where $\tau > 1$.[5] Equation (4.4) represents the young individual's constraint, where s_t and w_t are savings and wage income, respectively, and b is a positive constant representing the child rearing cost. A young individual inelastically supplies her labor endowments, which are normalized to one, spends her wage income on the consumption of differentiated goods, child rearing, and savings. Equation (4.5) describes the old individual's constraint, wherein she uses her savings for consumption. We treat labor in the country as the numéraire. This implies that the wage income of a young individual is equal to one: $w_t = 1$.

Plugging (4.2) and (4.3) into (4.1), and maximizing it under (4.4) and (4.5), we obtain the following demand functions for the differentiated goods:[6]

$$x_{yt}(i) = \frac{1}{(1+\gamma+\phi\beta)p_t(i)^\sigma P_t^{1-\sigma}}, \qquad x_{ot+1}(i) = \frac{\beta R}{(1+\gamma+\phi\beta)p_{t+1}(i)^\sigma P_{t+1}^{1-\sigma}}, \tag{4.6}$$

where P_t is the price index defined as

$$P_t = \left(\int_0^{m_t} p_t(i)^{1-\sigma}di + \int_0^{m_w} \tau^{1-\sigma}p_w^{1-\sigma}di\right)^{1/(1-\sigma)}. \tag{4.7}$$

The number of children is given by

$$n_t = \frac{\gamma}{b(1+\gamma+\phi\beta)}, \tag{4.8}$$

[5] As we see later, under the iceberg transport cost, τ, a profit maximizing firm sets its export price as the domestic price multiplied by τ.

[6] $x_{wyt}(i)$ and $x_{wot+1}(i)$ are obtained by replacing $p_t(i)$ and $p_{t+1}(i)$ with τp_w in (4.6), respectively.

and the level of savings , s_t, is determined as

$$s_t = \frac{\phi\beta}{1+\gamma+\phi\beta}. \tag{4.9}$$

By using (4.3) and (4.6), the young and old individuals' consumption (4.2) becomes as follows:

$$c_{yt} = \frac{1}{P_t(1+\gamma+\phi\beta)}, \qquad c_{ot+1} = \frac{\beta R}{P_{t+1}(1+\gamma+\phi\beta)}. \tag{4.10}$$

We can observe that $\partial c_{yt}/\partial\phi < 0$, $\partial n_t/\partial\phi < 0$, $\partial s_t/\partial\phi > 0$ and $\partial c_{ot+1}/\partial\phi < 0$. When the survival rate rises, an individual has an incentive to increase her savings for old period consumption by decreasing her young period consumption and number of children. Despite such incentive, the old period consumption also decreases with the survival rate because of reductions in the real interest rate in this economy, $RP_t/\phi P_{t+1}$. Moreover, as is standard in trade and geography models, consumption and utility depend on the price index, P_t, which, in turn, depends on the market size as will be shown later.

4.2.2 Firms

Now we move to a description of the production structure. The differentiated goods are produced under monopolistic competition. To produce a differentiated good, f units of labor are required as fixed inputs, and producing one unit of a differentiated good requires c units of labor as variable inputs. Hence, letting L_t denote the number of young individuals in period t, the profit of a firm producing differentiated good i in a country is given as[7]

$$\pi_t(i) = (p_t(i)-c)\left(x_{ot}(i)\phi L_{t-1} + x_{yt}(i)L_t\right) + (p_{wt}(i)-\tau c)\left(x_{wot}(i)\phi_w L_{wt-1} + x_{wyt}(i)L_{wt}\right) - f.$$

The first term represents the profit from domestic sales whereas the second term describes the profit from foreign sales. Here, we assume that the foreign demand structure is similar

[7] This implies that the number of surviving old individuals in period t becomes as ϕL_{t-1}, and the number of children in period t is given by $n_t L_t$, which is the number of young individuals in period $t+1$.

to the domestic demand structure.[8] The profit function can be written as

$$\pi_t(i) = (p_t(i) - c)\frac{MS_t}{p_t(i)^{\sigma} P_t^{1-\sigma}} + (p_{wt}(i) - \tau c)\frac{MS_w}{p_{wt}(i)^{\sigma} P_w^{1-\sigma}} - f, \tag{4.11}$$

where MS_t is the country's market size and defined as

$$MS_t = \frac{\phi\beta R}{1+\gamma+\phi\beta} L_{t-1} + \frac{1}{1+\gamma+\phi\beta} L_t. \tag{4.12}$$

We define

$$P_w = \left(\int_0^{m_t} \tau^{1-\sigma} p_t(i)^{1-\sigma} di + \int_0^{m_w} p_w^{1-\sigma} di \right)^{1/(1-\sigma)}.$$

The market size represents the aggregate income spent on consumption. The foreign market size, MS_w, is defined in a similar way and we assume it is exogenous. A firm's profit maximization with respect to price, $p_t(i)$, yields

$$p_t(i) = p \equiv \frac{\sigma c}{\sigma - 1}, \qquad p_{wt}(i) = \tau p. \tag{4.13}$$

From this, we readily know that the price index (4.7) and firm's profit (4.11) become

$$P_t = \left(m_t p^{1-\sigma} + \delta m_w p_w^{1-\sigma} \right)^{1/(1-\sigma)}, \tag{4.14}$$

$$\pi_t = \left(\frac{1}{\sigma}\right) \left[\frac{MS_t}{m_t + \delta m_w (c_w/c)^{1-\sigma}} + \frac{\delta MS_w}{\delta m_t + m_w (c_w/c)^{1-\sigma}} \right] - f. \tag{4.15}$$

where δ is defined as $\delta \equiv \tau^{1-\sigma} \in (0,1)$ and denotes the degree of trade freedom. c_w is defined as $c_w \equiv (\sigma-1)p_w/\sigma$, which represents the foreign marginal cost of production. As is standard in trade and geography models, (4.14) and (4.15) imply that a larger number of firms decreases the price index and the firm's profit:

$$\frac{\partial P_t}{\partial m_t} < 0, \qquad \frac{\partial \pi_t}{\partial m_t} < 0. \tag{4.16}$$

We assume free entry and exit of firms. Hence, new firms enter until the profit is driven to zero. In equilibrium, the number of firms, m_t, is determined by the following free-entry condition:

$$\frac{MS_t}{m_t + \delta m_w (c_w/c)^{1-\sigma}} + \frac{\delta MS_w}{\delta m_t + m_w (c_w/c)^{1-\sigma}} = f\sigma. \tag{4.17}$$

[8] Note here that the foreign wage rate is not necessarily equal to one. Labor productivity can differ between countries.

We can solve it for the equilibrium number of firms:

$$\begin{aligned} m_t &= \frac{1}{2f\delta\sigma}\left[\delta\left(MS_t+MS_w\right)-fk\sigma m_w\left(1+\delta^2\right)\right] \\ &+\frac{1}{2f\delta\sigma}\sqrt{\left[\delta\left(MS_t+MS_w\right)-fk\sigma m_w\left(1+\delta^2\right)\right]^2+4fk\delta\sigma m_w\left[MS_t+\delta(\delta MS_w-fk\sigma m_w)\right]} \\ &= \frac{1}{2f\sigma}(MS_t+MS_w)-\frac{km_w}{2\delta}(1+\delta^2) \\ &+\sqrt{\left[\frac{1}{2f\sigma}(MS_t+MS_w)-\frac{km_w}{2\delta}(1+\delta^2)\right]^2+\frac{km_w}{f\sigma}\left(\frac{MS_t}{\delta}+\delta MS_w\right)-(km_w)^2} \end{aligned} \tag{4.18}$$

where k is the relative marginal cost and defined as $k \equiv (c_w/c)^{1-\sigma}$. We know from (4.18) that a growth in market size induces further firm entry: $\partial m_t/\partial MS_t > 0$, which, combined with (4.16), yields

$$\frac{\partial P_t}{\partial MS_t} < 0. \tag{4.19}$$

4.2.3 Demographic structure

Given the demand and supply sides structures described so far, we obtain the level of an individual's (indirect) utility, V_t. We assume that if V_t is sufficiently large to exceed the exogenous level of a foreign individual's utility, V_w, immigrants will enter the country, and if V_t is smaller than V_w, some individuals in the country will emigrate. We assume that only young individuals will enter and exit the country and that such migration will take place at the beginning of each period.[9] Let λ_t denote the number of (young) immigrants, which we explain in detail later. From (4.8), the law of motion of the youth population can be described as

$$L_{t+1} = \frac{\gamma}{b(1+\gamma+\phi\beta)}L_t + \lambda_{t+1}. \tag{4.20}$$

The first term represents the number of young individuals born in the country whereas the second term represents the number of young individuals immigrating (emigrating)

[9] We assume this for analytical tractability. Issues related to the timing of migration are beyond the scope of this chapter.

from abroad (to the country). The total number of young individuals in the next period is the sum of these two numbers. In this chapter, we focus on the steady-state, which requires that the population size does not change over time.

4.3 Equilibrium

We characterize the steady-state equilibrium. First, we pin down the relationship between the youth population size, L_t, and the number of immigrants, λ_t, which is given by (4.26). For this, we need to know the means of dependence of the individual's indirect utility, V_t, on L_t. Plugging (4.8) and (4.10) into (4.1), an individual's Indirect utility in the country, V_t, can be written as

$$\begin{aligned} V_t &= \ln \frac{1}{(1+\gamma+\phi\beta)P_t} + \phi\beta \ln \frac{\beta R}{(1+\gamma+\phi\beta)P_{t+1}} + \gamma \ln \frac{\gamma}{(1+\gamma+\phi\beta)b} \\ &= \Psi(\phi) - \ln P_t - \phi\beta \ln P_{t+1}, \end{aligned} \tag{4.21}$$

where $\Psi(\phi)$ is defined as$\Psi(\phi) \equiv \gamma(\ln\gamma - \ln b) + \phi\beta \ln(\beta R) - (1+\gamma+\phi\beta)\ln(1+\gamma+\phi\beta)$. Equation (4.21) implies that it is sufficient to examine the level of the price index to know the level of indirect utility. Moreover, (4.14) and (4.18) show that the market size, via the number of firms, determines the price index, and as shown in (4.19), a larger market size results in a lower price index. Hence, all we need to know is the market size in order to examine the level of indirect utility.

From (4.21), we obtain

$$V_t|_{\lambda_t=\lambda_{t+1}=0} = \Psi(\phi) - \ln P_t|_{\lambda_t=0} - \phi\beta \ln P_{t+1}|_{\lambda_{t+1}=0}. \tag{4.22}$$

From (4.12) and (4.20), the market size of the country can be written as

$$\begin{aligned} MS_t &= \left(\frac{1}{1+\gamma+\phi\beta} + \frac{b\phi\beta R}{\gamma}\right)(L_t - \lambda_t) + \frac{1}{1+\gamma+\phi\beta}\lambda_t, \\ MS_{t+1} &= \frac{\phi\beta R}{1+\gamma+\phi\beta}L_t + \frac{1}{1+\gamma+\phi\beta}\left[\frac{\gamma}{b(1+\gamma+\phi\beta)}L_t + \lambda_{t+1}\right]. \end{aligned} \tag{4.23}$$

In the absence of migration ($\lambda_t = \lambda_{t+1} = 0$), the market size then becomes as

$$MS_t|_{\lambda_t=0} = \left(\frac{1}{1+\gamma+\phi\beta} + \frac{b\phi\beta R}{\gamma}\right) L_t, \tag{4.24}$$
$$MS_{t+1}|_{\lambda_{t+1}=0} = \left(\frac{1}{1+\gamma+\phi\beta} + \frac{b\phi\beta R}{\gamma}\right) \frac{\gamma}{b(1+\gamma+\phi\beta)} L_t,$$

which implies that a larger youth population results in a larger market size. This, combined with (4.19), leads to a lower price index:

$$\frac{\partial\, P_t|_{\lambda_t=0}}{\partial L_t} = \frac{\partial\, P_t|_{\lambda_t=0}}{\partial\, MS_t|_{\lambda_t=0}} \frac{\partial\, MS_t|_{\lambda_t=0}}{\partial L_t} < 0,$$
$$\frac{\partial\, P_{t+1}|_{\lambda_{t+1}=0}}{\partial L_t} = \frac{\partial\, P_{t+1}|_{\lambda_{t+1}=0}}{\partial\, MS_{t+1}|_{\lambda_{t+1}=0}} \frac{\partial\, MS_{t+1}|_{\lambda_{t+1}=0}}{\partial L_t} < 0.$$

From (4.22), we readily know that the indirect utility rises along with an increase in the young population size:

$$\frac{\partial\, V_t|_{\lambda_t=\lambda_{t+1}=0}}{\partial L_t} > 0 \tag{4.25}$$

Hereafter, we make the following assumption:[10]

Assumption 4.1.

$$\lim_{L_t\to 0} V_t|_{\lambda_t=\lambda_{t+1}=0} < V_w < \lim_{L_t\to\infty} V_t|_{\lambda_t=\lambda_{t+1}=0}.$$

This assumption requires that the indirect utility without young population is lower than a foreign individual's utility, V_w, whereas the indirect utility with an infinitely large young population is greater than V_w. Under Assumption 4.1, (4.25) ensures that there exists a certain threshold value of the youth population size, L_t, that satisfies $V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w$. We define such L_t as $\widehat{L}$.[11]

[10] This assumption can be written in parameters as $\Psi(\phi) - (1+\phi\beta)\ln p_w\,(\delta m_w)^{1/(1-\sigma)} < V_w < \Psi(\phi)$.

[11] Note that $\widehat{L}$ is time-invariant.

To derive equilibrium, we assume that λ_t is determined as follows:

$$\lambda_{t+1} = \begin{cases} \varepsilon + \mu(L_t - \widehat{L}) & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} > V_w, \\ 0 & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w, \\ -\varepsilon + \mu(L_t - \widehat{L}) & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} < V_w, \end{cases} \tag{4.26}$$

where ε and μ are positive constants.[12] This specification implies that if the utility in the absence of migration is higher in the country than abroad in period t (i.e., $L_t > \widehat{L}$), a certain number of immigrants ($\varepsilon + \mu(L_t - \widehat{L})$ young individuals) enter the country in the next period, and if the opposite holds true (i.e., $L_t < \widehat{L}$), a certain number of emigrants ($\varepsilon - \mu(L_t - \widehat{L})$ young individuals) exit the country. The size of immigrant flows depends on the difference between the current population and the population that equalizes the domestic utility and foreign utility. This immigration process is similar to the replicator dynamics often used in trade and geography models (Fujita et al. 1999). The replicator dynamics assumes that immigration size depends on the difference between the domestic utility and foreign utility. Our specification is the first-order approximation of the replicator dynamics. We assume that a certain mass, ε, of immigrants will move irrespective of the degree of utility difference to ensure the existence of a steady-state with a positive population size.

Given the relationship between L_t and V_t in hand, the law of motion of the youth population (4.20) can be depicted as in Figure 4.1.

From Figure 4.1, we can readily see that no stable steady-state equilibrium with positive population size exists if $\mu+\gamma/\left[b(1+\gamma+\phi\beta)\right] \geq 1$. And if this inequality holds true, even a small perturbation makes L_t eventually converge to 0 or diverge to infinity, implying that the steady-state is unstable. Because our focus is on a stable steady-state, we assume that $\mu + \gamma/\left[b(1+\gamma+\phi\beta)\right] < 1$, which requires the survival rate to be sufficiently high.

Assumption 4.2.

$$\mu + \frac{\gamma}{b(1+\gamma+\phi\beta)} < 1$$

[12] Strictly speaking, (4.26) is defined for $L_t \geq \varepsilon$. For $L_t < \varepsilon$, we define $\lambda_{t+1} = -L_t$. In this chapter, we focus on a case with sufficiently large L_t where only (4.26) is relevant.

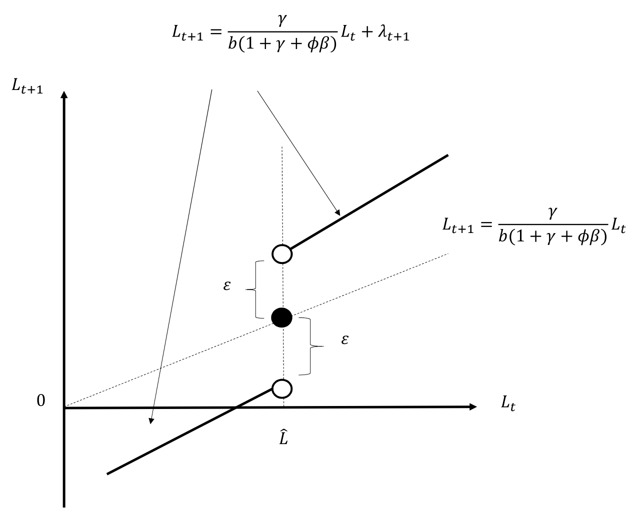


Fig. 4.1 The law of motion of youth population

Under this assumption, a possible steady-state equilibrium is associated with a population size of L^* determined by (4.20) and $L_{t+1} = L_t$, which is given by,

$$L^* = \frac{\varepsilon - \mu \widehat{L}}{1 - \mu - \gamma/[b(1+\gamma+\phi\beta)]}. \tag{4.27}$$

For this to be attained, we need to impose that $L^* > \widehat{L}$. Under Assumption 4.2, imposing this inequality is equivalent to impose the following assumption.

Assumption 4.3.

$$\varepsilon > \left[1 - \frac{\gamma}{b(1+\gamma+\phi\beta)}\right] \widehat{L}.$$

In the remaining parts of the chapter, we assume Assumptions 4.1 to 4.3. In Figure 4.2, we depict L^*.

Proposition 4.1. *Under Assumptions 4.1 to 4.3, the model has a unique steady-state equilibrium with positive population size, L^*.*

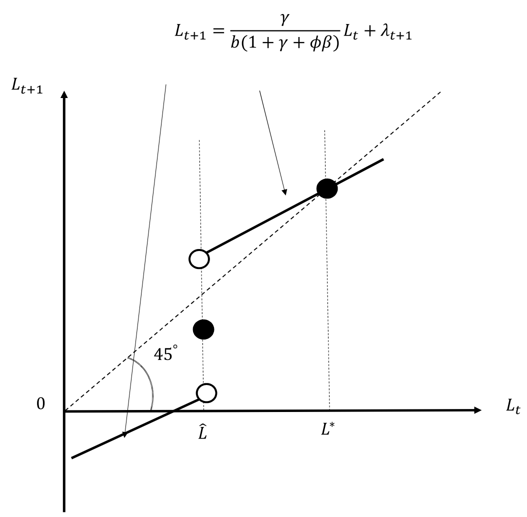


Fig. 4.2 Steady-state equilibrium

4.4 Effects of improvements in longevity

Now we are ready to investigate the impacts of increases in longevity. From (4.27), we can see that

$$\frac{\partial L^*}{\partial \phi} = I_{mm} - F_{er}, \tag{4.28}$$

where I_{mm} and F_{er} are defined as

$$I_{mm} \equiv -\frac{\mu}{1-\mu-\gamma/[b(1+\gamma+\phi\beta)]}\frac{\partial \widehat{L}}{\partial \phi} > 0,$$

$$F_{er} \equiv \frac{b\beta\left(\varepsilon-\mu\widehat{L}\right)\gamma/[b(1+\gamma+\phi\beta)]^2}{\{1-\mu-\gamma/[b(1+\gamma+\phi\beta)]\}^2} > 0.$$

I_{mm} represents the effect that improved longevity induces more immigrants. In fact, $\partial\widehat{L}/\partial\phi$ describes the responsiveness of immigration to improvements in longevity. $-F_{er}$ represents the effect that improved longevity decreases the number of children. Thus, our model includes two channels through which longevity affects population size.

From (4.12) and (4.27), the market size in the steady-state equilibrium is written as

$$MS^* = \frac{1+\phi\beta R}{1+\gamma+\phi\beta} L^*.$$

Differentiating this with respect to ϕ, we obtain

$$\frac{\partial MS^*}{\partial \phi} = C_{on} + \Gamma \frac{\partial L^*}{\partial \phi}, \tag{4.29}$$

where C_{on} and Γ are defined as

$$C_{on} \equiv \frac{\beta\left[R(1+\gamma)-1\right]}{(1+\gamma+\phi\beta)^2} L^*,$$
$$\Gamma \equiv \frac{1+\phi\beta R}{1+\gamma+\phi\beta}.$$

C_{on} captures the effect of changes in per capita expenditure on consumption whereas $\Gamma \partial L^*/\partial\phi$ represents the effect of population changes. From (4.28) and (4.29), we know that

$$\frac{\partial MS^*}{\partial \phi} > 0 \Leftrightarrow I_{mm} + \frac{C_{on}}{\Gamma} > F_{er}. \tag{4.30}$$

Proposition 4.2. *An increase in longevity increases the market size if and only if* $I_{mm} + C_{on}/\Gamma > F_{er}$*.*

Furthermore, from (4.19), (4.22), (4.30), and Assumption 4.3, we obtain the following proposition:

Proposition 4.3. *An increase in longevity increases the individual's utility if* $I_{mm} + C_{on}/\Gamma > F_{er}$*.*

Thus, we know from Propositions 4.2 and 4.3 that the improved longevity has a positive immigration effect, a positive consumption effect, and a negative fertility effect, and can enlarge the market size and result in higher utility if and only if the positive effects dominate the negative one.

Thus far, we have examined the characteristics of steady-state equilibrium. However, in calibrating our model to match the real data, there is no guarantee that the economy is in a steady-state. To understand the calibration results from the theoretical viewpoint, we present a short discussion about the longevity effects on population and market size dynamics, starting with the effects on population dynamics. Combining (4.20) and (4.26), we obtain

$$L_{t+1} = \begin{cases} \frac{\gamma}{b(1+\gamma+\phi\beta)} L_t + \varepsilon + \mu(L_t - \widehat{L}) & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} > V_w \\ \frac{\gamma}{b(1+\gamma+\phi\beta)} L_t & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w \\ \frac{\gamma}{b(1+\gamma+\phi\beta)} L_t - \varepsilon + \mu(L_t - \widehat{L}) & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} < V_w \end{cases}.$$

Differentiating the equation with respect to ϕ, we obtain

$$\frac{\partial L_{t+1}}{\partial \phi} = \begin{cases} -\frac{\gamma\beta}{b(1+\gamma+\phi\beta)^2} L_t - \mu \frac{\partial \widehat{L}}{\partial \phi} & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} \neq V_w \\ -\frac{\gamma\beta}{b(1+\gamma+\phi\beta)^2} L_t & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w \end{cases}.$$

The first term on the right hand side, $-\gamma\beta L_t/[b(1+\gamma+\phi\beta)^2]$, implies the negative fertility effect on population. The second term, $-\mu\partial\widehat{L}/\partial\phi$, positively affects population if $\partial\widehat{L}/\partial\phi < 0$, which holds true under Assumption 4.2 and the definition of I_{mm} (> 0).

Next, we consider the effects on market size dynamics. By differentiating MS_{t+1} with respect to ϕ, from (4.12), (4.20) and (4.26), we obtain

$$\begin{aligned} \frac{\partial MS_{t+1}}{\partial \phi} &= \frac{\beta R(1+\gamma)}{(1+\gamma+\phi\beta)^2} L_t - \frac{\beta}{(1+\gamma+\phi\beta)^2} L_{t+1} + \frac{1}{1+\gamma+\phi\beta} \frac{\partial L_{t+1}}{\partial \phi} \\ &= \begin{cases} \frac{\beta R(1+\gamma)}{(1+\gamma+\phi\beta)^2} L_t - \frac{\beta}{(1+\gamma+\phi\beta)^2} L_{t+1} - \frac{\gamma\beta}{b(1+\gamma+\phi\beta)^3} L_t - \frac{\mu}{1+\gamma+\phi\beta} \frac{\partial \widehat{L}}{\partial \phi} & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} \neq V_w \\ \frac{\beta R(1+\gamma)}{(1+\gamma+\phi\beta)^2} L_t - \frac{\beta}{(1+\gamma+\phi\beta)^2} L_{t+1} - \frac{\gamma\beta}{b(1+\gamma+\phi\beta)^3} L_t & \text{if} \quad V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w \end{cases}. \end{aligned}$$

The first term on the right hand side represents the positive consumption effect. The second and third terms show the negative fertility effect. The last term describes the positive immigration effect. Thus, again, an increase in longevity influences the market size via the three channels.

4.5 Calibration

In this section, we calibrate the model to match the Japanese and U.S. data from 1955 to 2014.[13] Because we focus on demographics and market size, we calibrate the equations that determine population dynamics (4.20) and market-size dynamics (4.23). For this

[13] The matlab codes for calibration are available upon request.

purpose, we extend our baseline model in two ways. First, in our baseline model, all individuals live for two periods, young and old. In our calibration, we assume that each period consists of 35 years, and there is one cohort in each year. This implies that a total of 70 cohorts exist in the country. The number of members in a cohort in a period is an endogenous variable which is determined by the fertility rates in the previous period. We set the initial value of a cohort's population size by age from ages 15 to 84 in 1955. That is, we use the population size of age 15 in 1955 as the initial value of a cohort's population size, and the population size of age 16 in 1955 as that of another cohort's population size, and so on. We assume that the individuals whose ages are from 15 to 49 belong to the young period, and the individuals whose ages are from 50 to 84 are in the old period. Then, the size of the youth population in 1955 is the sum of the population sizes from ages 15 to 49, and the size of the old population is the sum of the population sizes from ages 50 to 84. Each year, a cohort gets one year older. The oldest cohort in the young period survives with a probability of ϕ and enters into the old period. The oldest cohort in the old period exits the economy the next year. We assume that all immigrants belong to the youngest cohort.

Second, because labor is treated as the sole production input in our baseline model, the market size in the baseline model does not match the actual market size (natural logarithm of nominal GDP), which includes the output produced by other production factors. To fill the gap between the model's and the actual market sizes, we linearly transform the model's market size as $a_0 + a_1 MS_t$, and choose a_0 and a_1 to minimize the mean squared error (MSE). In conducting minimization, we employ the Adaptive Mesh Refinement method (AMR).[14]

As we stated earlier, in calibrating our model to match the real data, there is no guarantee that the economy is in a steady-state. Hence, in our calibration, we do not

[14] AMR first divides the admissible intervals of relevant parameters to create meshes, and picks one point from each mesh. Then it calculates the MSE for each point to find the point that minimizes the MSE. Next, it divides the neighborhood of the point with the minimum MSE to create finer meshes, and again picks one point from each mesh. It repeats this process until the chosen points converge. See Berger and Oliger (1984).

impose the steady state condition. The equilibrium conditions here are the agent's utility maximization (4.10), firm's profit maximization (4.13), population dynamics (4.20), and immigration process (4.26).

4.5.1 Data

In calibrating our model, we use data for population by age, number of immigrants, nominal GDP, life expectancy, and interest rates from 1955 to 2014 . Here, we summarize the sources of the Japanese and U.S. data.

Japanese data

Japanese population size (in million persons) is taken from the Vital Statistics (Ministry of Health, Labour and Welfare). In each year, the population size of cohorts in the young period and that in the old period are calculated as the sum of population sizes from ages 15 to 49 and that from ages 50 to 84. We assume that the population size for the age 14 represents the number of birth, which becomes the population size of the youngest cohort of the young period in the next year. Net immigration size (in million persons) comes from the Statistical Survey on Legal Migrants (Ministry of Justice). We calculate the number by subtracting the people departing Japan from the people entering Japan (including the Japanese). We use the nominal GDP (in billion yen) published in the Annual Report on National Accounts, Department of National Accounts, Economic and Social Research Institute, Cabinet Office. Life expectancy is from the Life table (Ministry of Health, Labour and Welfare). Total average life expectancy at age 15 Av is calculated as follows: $Av = w_m a_m + w_f a_f$ where $a_i (i = m, f)$ is the life expectancy at age15 of each sex and $w_i (i = m, f)$ is the sex ratio of the 15 years old population. Yearly average nominal interest rates (Basic Discount Rate and Basic Loan Rate) are available in the Bank of Japan database (accessed at https://www.stat-search.boj.or.jp/index on January 13, 2016).

U.S. data

The U.S. population size (in million persons) is taken from the Annual Estimates of the Resident Population for Selected Age Groups by Sex for the United States (Population Division, United States Census Bureau). Net immigration size (in million persons, and only including persons with lawful permanent resident status) is taken from the 2014 Yearbook of immigration Statistics (Office of Immigration Statistics, Department of Homeland Security). We ignore illegal immigrants in the United States due to data restriction. Nominal GDP (in billion U.S. dollars) comes from the Bureau of Economic Analysis, United States Department of Commerce database (accessed at http://www.bea.gov/iTable/iTable.cfm?ReqID=9 on January 13, 2016). Life expectancy available in National Vital Statistics System (CDC/National Center for Health Statistics). Total average life expectancy at age 15 is obtained from calculations using this data source. Yearly average nominal interest rates (Federal funds effective rate) are available in the Federal Reserve Bank database (accessed at https://fred.stlouisfed.org/series/FEDFUNDS on January 13, 2016).

4.5.2 Parameters and calibration method

In order to calibrate (4.20) and (4.23), we need to determine β, γ, ε, ϕ, μ, b, and R. We follow Eckstein et al. (1999) and choose the discount factor between the young and old periods, $\beta = 2/3$, and the costs of child rearing, $b = 0.11$.[15]

The preference for children, γ, is set to minimize MSE between the model's number of children and the actual population size of the age 14 in each year. We again use AMR to minimize MSE and set $\gamma_{JP} = 0.00487$ and $\gamma_{US} = 0.00524$, where the subscripts JP and US describes that the parameters are associated with Japan and the United States, respectively.

Immigration parameters, ε and μ, are determined to minimize MSE between the model's net immigration size and the actual net immigration size by using AMR, result-

[15] Because there are 35 years in each period, $\beta = 2/3$ implies that the annual discount rate is approximately 0.0117. This value is close to recent annual interest rates in Japan.

ing in $\varepsilon_{JP} = 0.03569$, $\mu_{JP} = 0.00112$, $\varepsilon_{US} = 0.19639$, and $\mu_{US} = 0.01002$. Thus, we know that both parameters, ε and μ are much higher in the United States than in Japan, reflecting the fact that the United States has been more open towards immigrants than Japan.

We assume that parameters β, b, γ, ε, and μ are constant over time. In contrast, we assume parameters ϕ and R, and hence, $\widehat{L}$, can be different over time. We allow ϕ to take different values for different years because we focus on the effects of improvements in longevity, implying the need to consider consecutive increases in longevity during the last half century. We use different Rs for different years because we have observed drastic declines in the interest rate in recent years. Because $\widehat{L}$ depends on ϕ and R, we need to set $\widehat{L}$ for each year.[16]

We can calculate a parameter of longevity, ϕ, for each year to match the model's expected longevity with the average lifespan at age 15. Because the model's expected longevity is given by $35 \times (1-\phi) + 70 \times \phi$, ϕ is determined by $Av = 35 \times (1-\phi) + 70 \times \phi$ if we denote the average lifespan at age 15 by Av. Rearranging this equation, we obtain $\phi = (Av - 35)/35$. Figure 4.3 represents the obtained values of ϕ, from which we confirm that both Japan and the United States experienced improvements in longevity over the past half century.

We determine the interest, R, by using yearly average nominal interest rates.

Finally, $\widehat{L}$ is determined by $V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w$ for different values of ϕ and R. Unfortunately, we have no clue to fix the indirect utility outside of the country, V_w. Moreover, $V_t|_{\lambda_t=\lambda_{t+1}=0}$ includes parameters not specified so far, and they are difficult to be pinned down. Hence, we employ a heuristic method. We assume that V_w is constant over time. First, we linearly approximate $V_t|_{\lambda_t=\lambda_{t+1}=0}$ as $V_t|_{\lambda_t=\lambda_{t+1}=0} = K_0 + K_1\phi + K_2 L_t + K_3 R$. Note here that K_0 represents all parts not related to ϕ, L_t, and R, and K_0 is constant over time. Then, substituting $V_t|_{\lambda_t=\lambda_{t+1}=0} = V_w$, we can write $K_1\phi + K_2\widehat{L} = V_w - K_0 - K_3 R$. We obtain $k_1\phi + k_2\widehat{L} = 1$, where $k_i \equiv K_i/\left(V_w - K_0 - K_3 R\right)$. Since R takes the same value as that for the world and V_w also involves the term $K_3 R$, $V_w - K_0 - K_3 R$ must

[16] We assume that the agents are aware of these changes.

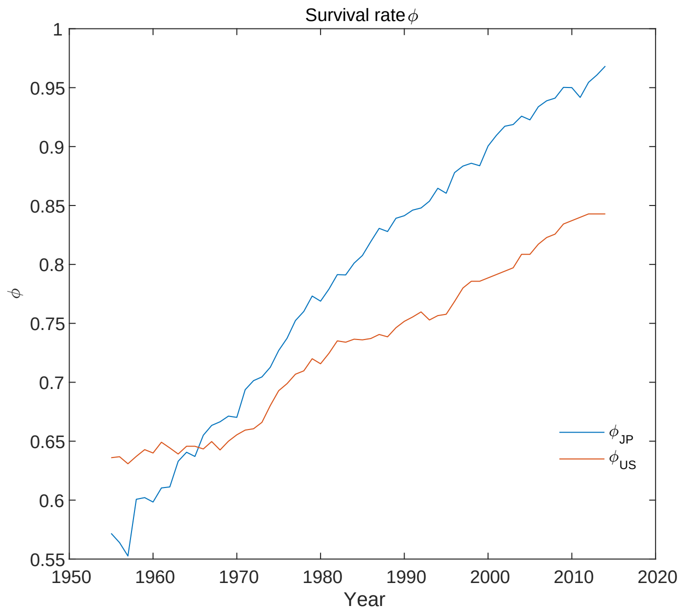


Fig. 4.3 Survival rate ϕ

be constant over time. We then obtain k_1 and k_2 as follows: We develop simultaneous equations $k_1\phi_{1950} + k_2L_{1950} = 1$ and $k_1\phi_{1955} + k_2L_{1955} = 1$ by using ϕ for 1950 and 1955, ϕ_{1950} and ϕ_{1955}, and by setting $\widehat{L}$ to the actual population size of the young cohort in 1950 and 1955, L_{1950} and L_{1955}. We choose 1950 and 1955 because their immigration size were smaller in these years than in all other years of calibration, and hence L_{1950} and L_{1955} are considered to be reasonable approximations of $\widehat{L}$.[17][18] Solving the two equations, we obtain k_1 and k_2. Then, for the years of calibration, we can obtain $\widehat{L}$ by $(1 - k_1\phi)/k_2$ for each year. We summarize the determination of these parameters in

[17] The numbers are 0.000939 (1950) and -0.006601(1955) in Japan and 0.24919 (1950) and 0.23779 (1955) in the United States.

[18] Because Japanese data on population size by age are not available from 1951 to 1954, we use data for 1950 and 1955.

Parameters	Values		Description
	Japan	United States	
β	2/3		discount rate
γ	0.00487	0.00524	preference for children
ε	0.03569	0.19639	fixed component of immigration
ϕ	set to match average life-year at age 15		survival rate
μ	0.00112	0.01002	immigration responsiveness
b	0.11		costs of child rearing
$\widehat{L}$	calculated by using ϕ and L		threshold population size
R	yearly average nominal interest rates		return on savings

Table 4.1 Parameter values

Table 1, and describe the results of our calibration in Figure 4.4.

As we can see from Figure 4.4, our calibrated model exhibits a good match with the actual data. In particular, it can successfully replicate the trends observed in the actual data although it fails to capture the effects of temporary shocks such as baby booms.

The Japanese population has increased for several decades after the Second World War, but it started to decrease in recent years. In contrast, the U.S. population has increased monotonically during the past sixty years. Our theoretical analysis implies that such difference between two countries might arise for three reasons. First, if the preference for children is higher in the United States than in Japan, then the United States would have higher fertility and higher population growth. However, the obtained values of γ are similar for both two countries, and hence, we can not employ this possibility. Second, differences in the survival rate, ϕ, can be a source of differences in population growth because a higher survival rate induces an individual to increase her savings for old period consumption by decreasing her young period consumption and number of children. Because the obtained values of ϕ for Japan are higher than those for the United States, the differences in ϕ can possibly explain the lower population growth rate (and recent negative population growth rate) in Japan. Finally, differences

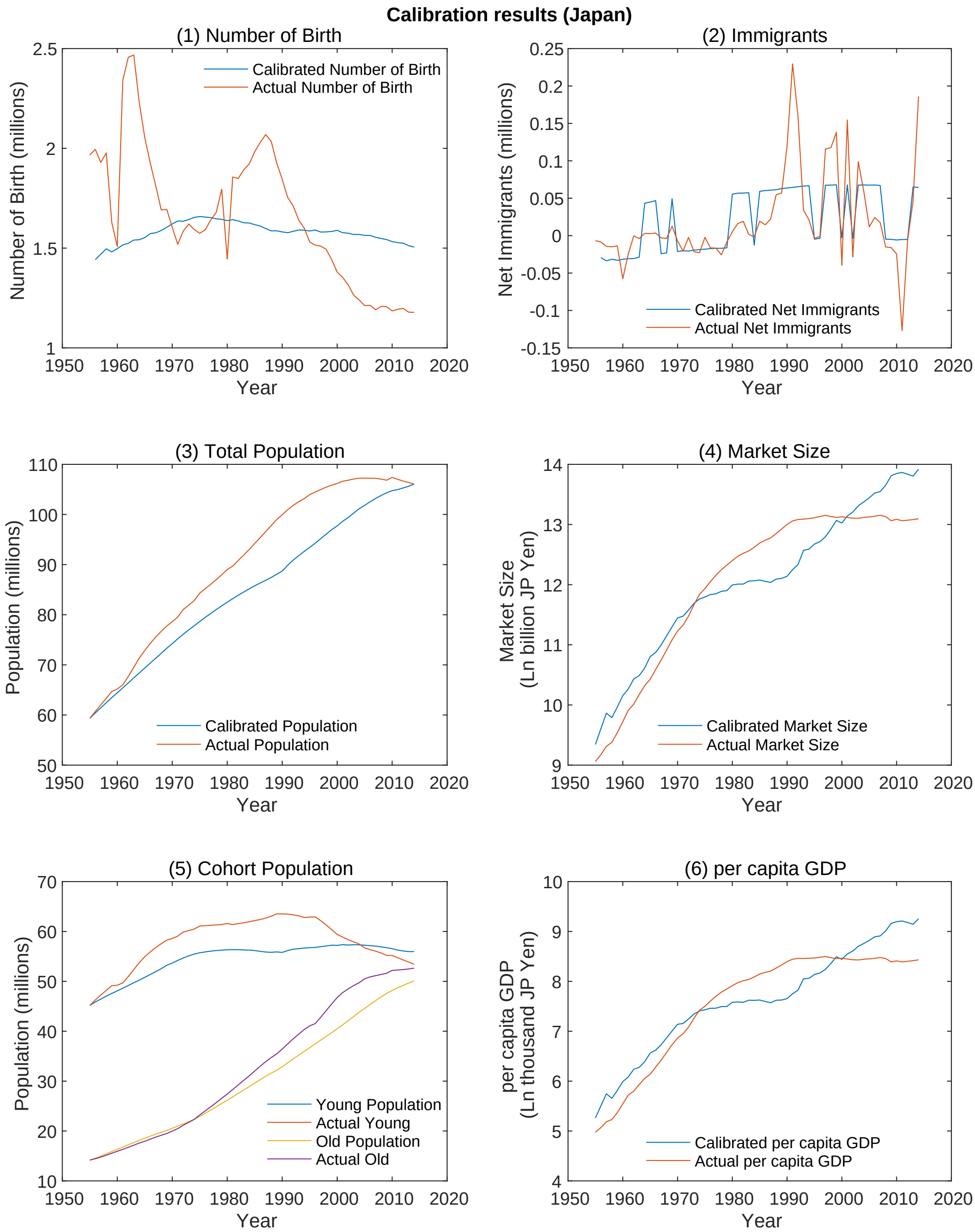


Fig. 4.4 (a) Calibration results (Japan)

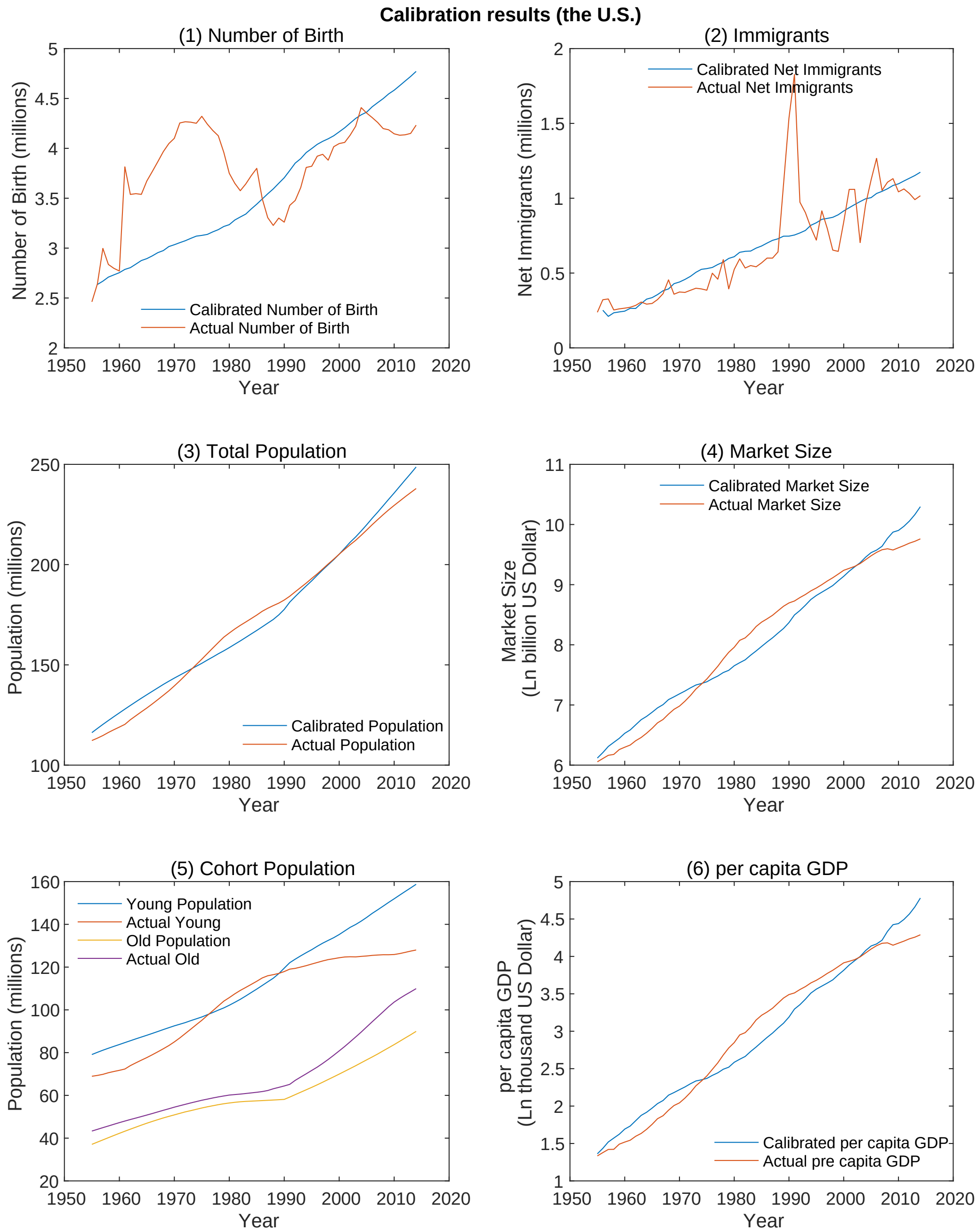


Fig. 4.4 (b) Calibration results (U.S.)

in immigration size might be a source of differences in population growth. Because the United States has higher immigration parameters, ε and μ, than Japan, a larger immigration size might have supported the United States consecutive population growth. Thus, we can consider the differences in ϕ and/or those in ε and μ as the causes of differences in population growth between the two countries. However, our analysis so far can not tell us about the degree of contribution from these factors. To uncover this information, we conduct several counterfactual analyses in the next subsection.

4.5.3 Counterfactual analysis

In this section, we study the quantitative effects of changing longevity and immigration on population and market sizes by counterfactual analyses. First, we examine the effects of improvements in longevity. For this purpose, we consider the following two counterfactual scenarios: (i) the survival rate, ϕ, takes the initial value (i.e., the value in 1955) for all years, and (ii) ϕ takes the value in 2014 for all years.[19] As we can see from Figure 4.3, the values of ϕ have risen in the two countries over the past sixty years. Hence, by scenarios (i) and (ii) we can examine what the population and market sizes would look like if we observed no improvements in longevity and if we experienced improvements in longevity at the beginning of the years under consideration, respectively. Figures 4.5 and 4.6 show the results of our counterfactual analyses.

Figure 4.5 represents the analysis under scenario (i). In both Japan (Figure 4.5-a) and the United States (Figure 4.5-b), setting ϕ at its initial, low level does not significantly change the total population (Figures 4.5-a-(3) and 4.5-b-(3)). However, setting a low ϕ drastically affects the population distribution by increasing the number of births (young population) (Figures 4.5-a-(1) and 4.5-b-(1)) and decreasing the old population (Figures 4.5-a-(5) and 4.5-b-(5)). In addition, immigration size is larger for a lower ϕ (Figures 4.5-a-(2) and 4.5-b-(2)). Thus, both the number of births and immigration size positively affect the market size. This is because $\widehat{L}$ does not change over time due to a constant ϕ. Hence, from (4.26), immigration size grows as the population grows. Moreover, the

[19] For parameters other than ϕ, we use the same values as those specified in the previous section.

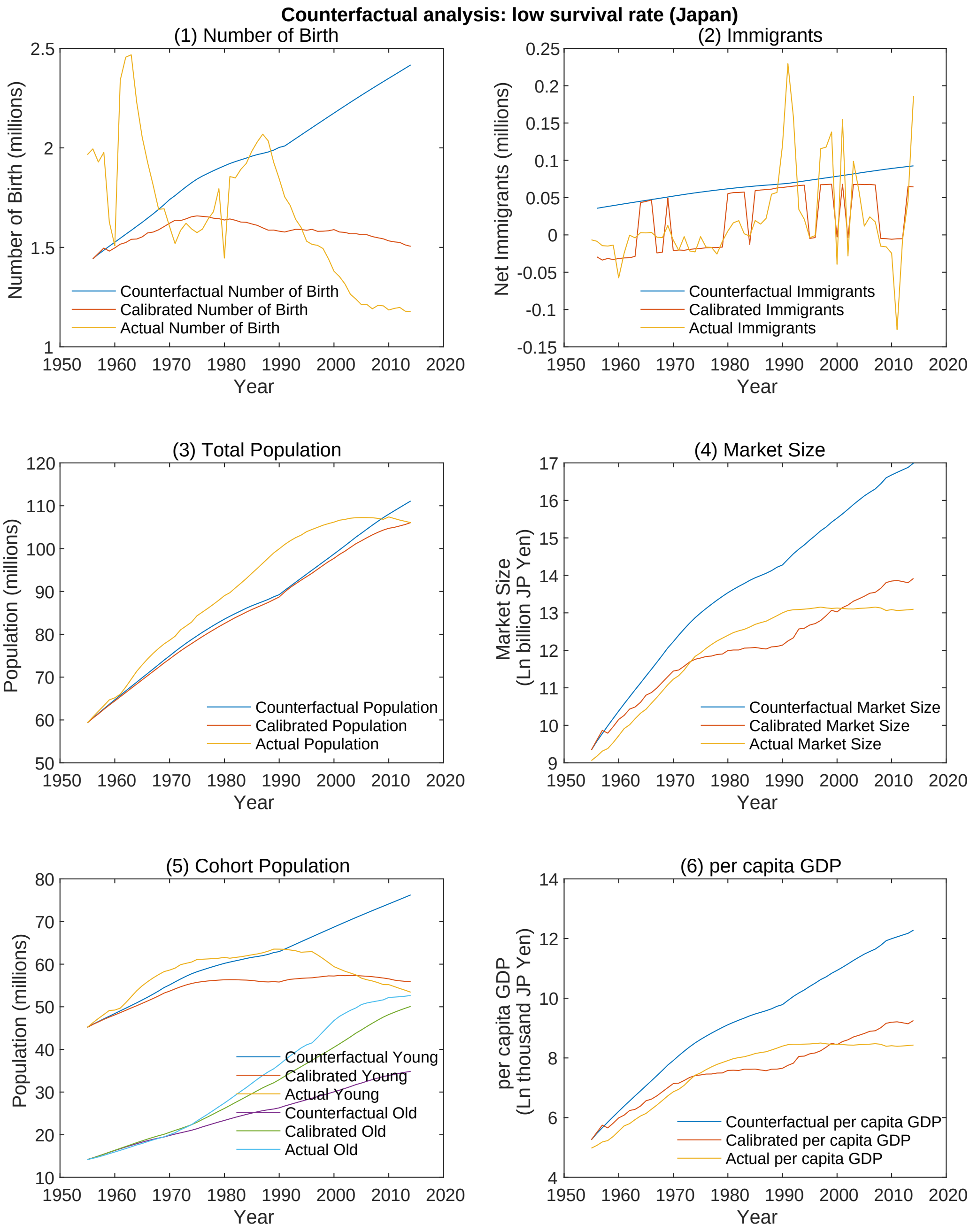


Fig. 4.5 (a) Counterfactual analysis: low survival rate (Japan)

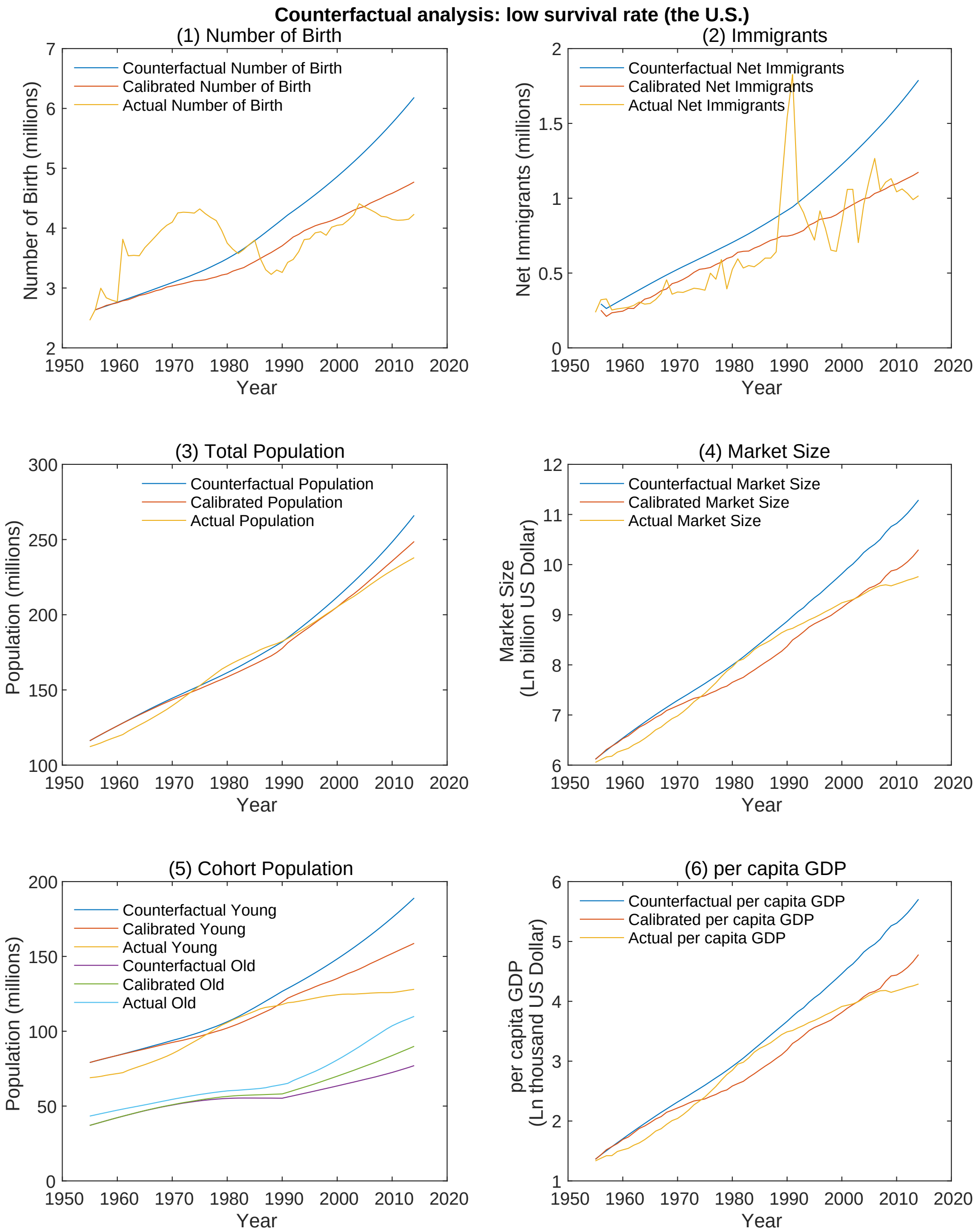


Fig. 4.5 (b) Counterfactual analysis: low survival rate (U.S.)

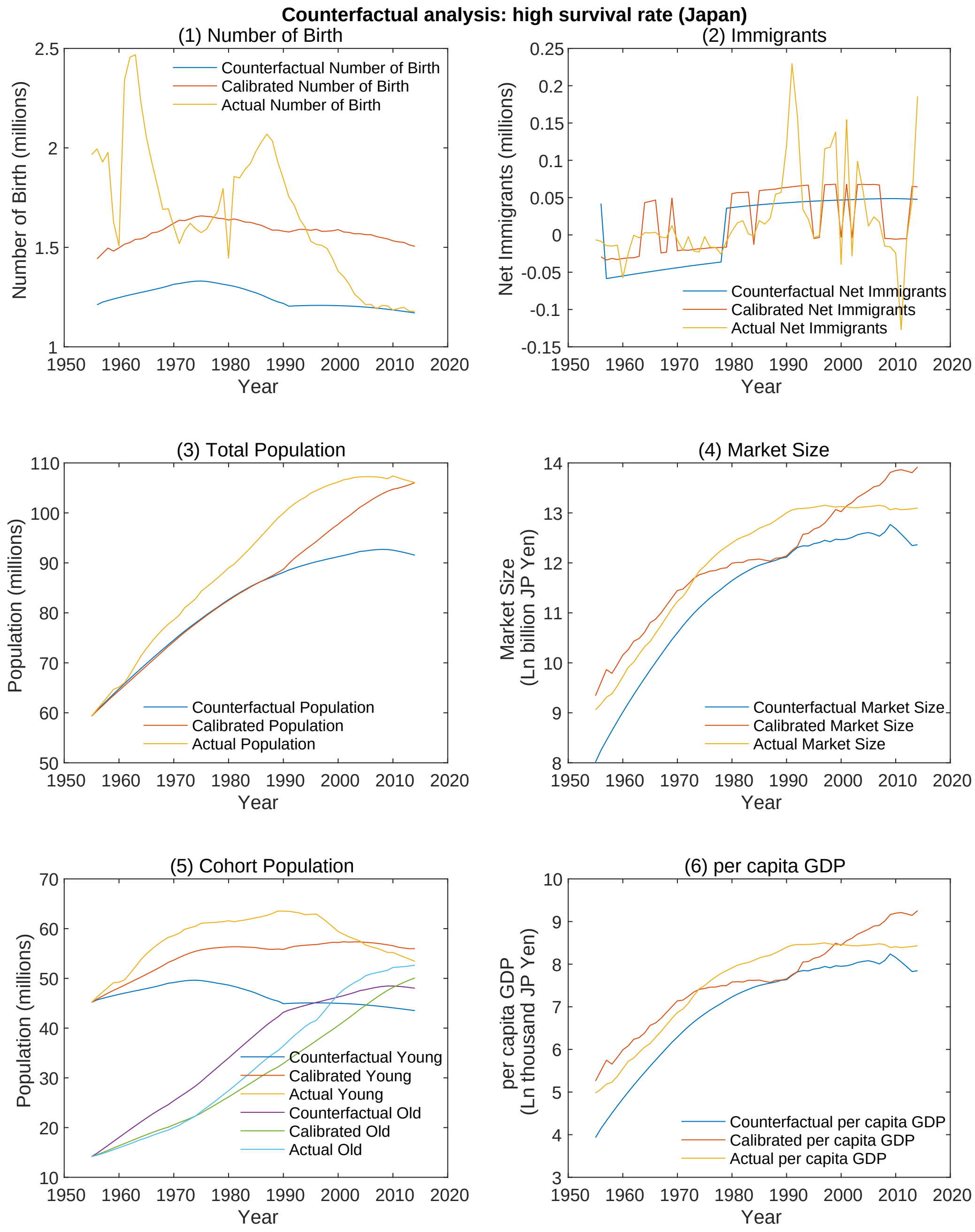


Fig. 4.6 (a) Counterfactual analysis: high survival rate (Japan)

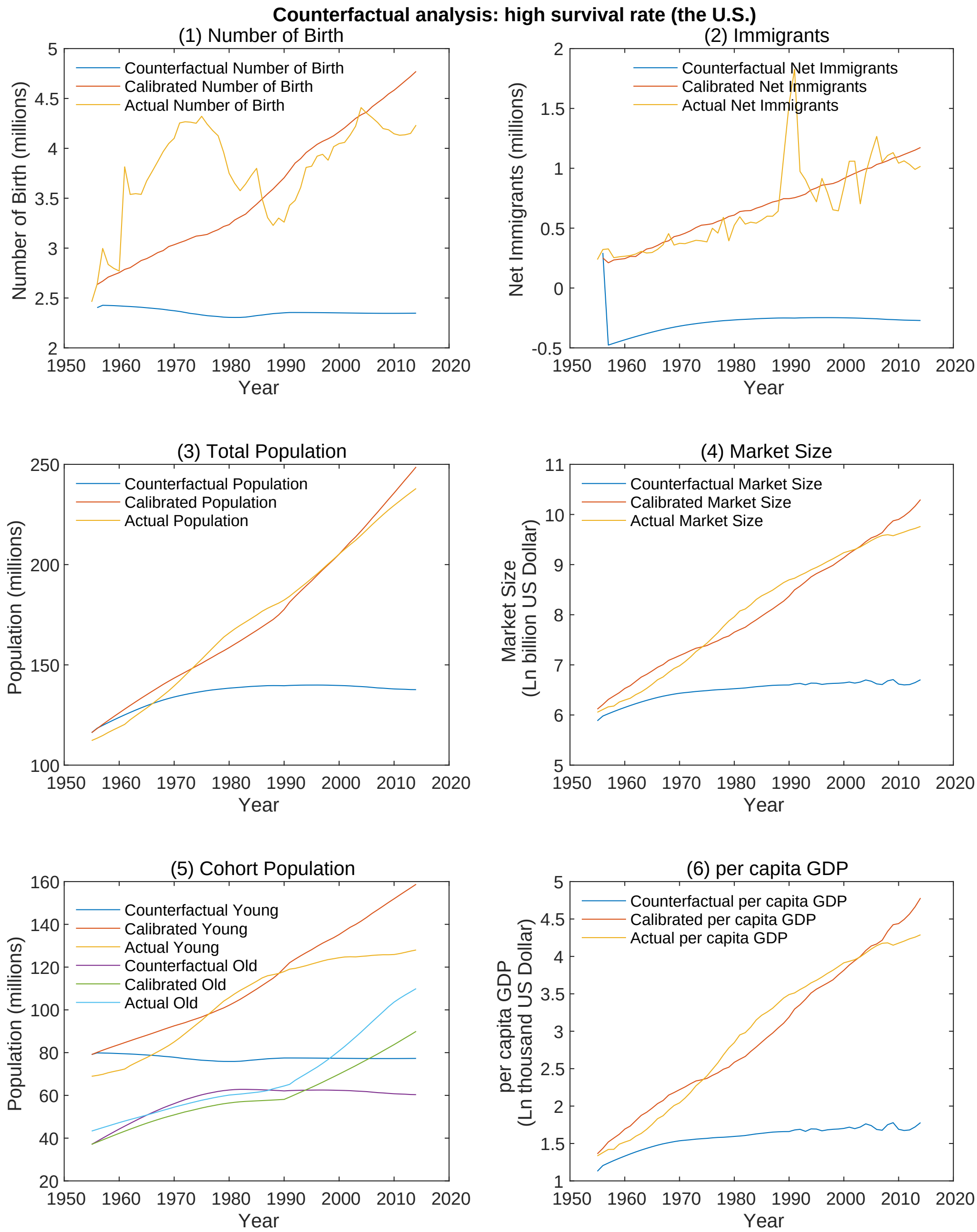


Fig. 4.6 (b) Counterfactual analysis: high survival rate (U.S.)

effects on the number of births are sufficiently large to dominate the negative effects on the per capita expenditure on consumption, which corresponds to the per capita GDP here (Figures 4.5-a-(6) and 4.5-b-(6)). In such a case, as shown in Proposition 4.2, the market size becomes larger when we use a lower ϕ, which we confirm in Figures 4.5-a-(4) and 4.5-b-(4).[20]

Figure 4.6 describes the analysis under scenario (ii). If we set ϕ to its latest, high value and keep it constant over time, then we observe opposite changes to those observed under a low ϕ, that is, the number of births decreases (Figures 4.6-a-(1) and 4.6-b-(1)), which leads to decreases in immigration size (Figures 4.6-a-(2) and 4.6-b-(2)) and per capita expenditure on consumption (Figures 4.6-a-(6) and 4.6-b-(6)) to decrease the market size (Figures 4.6-a-(4) and 4.6-b-(4)). Thus, we know that over the past sixty years, the negative effects of increases in the survival rate have dominated the positive ones, implying that improvements in longevity have decreased the market size in Japan and the United States.

Although improvements in longevity have negatively affected the market size in both countries, the magnitude is smaller in the United States than Japan. Compared to the baseline calibration, scenario (i) increases the market size by 9.64 % for the United States and 22.10 % for Japan. And scenario (ii) decreases the market size by 10.52 % for the United States and 11.17 % for Japan. Where do such differences come from? As we can see in Table 1, we obtained very different values for the immigration parameters between the two countries, and thus, inducing us to focus on them as the potential causes of the differences in magnitude. To grasp the importance of immigration in determining the market size, we conduct the following counterfactual analyses: what do the population and market sizes look like if Japan (resp. the United States) has the immigration parameters of the United States (resp. those of Japan)? In so doing, we replace ε_{JP} and μ_{JP} with ε_{US} and μ_{US} to rerun our simulations. Given that both parameters are higher in the United States than Japan, such an exercise uncovers the effects of making Japan as open towards immigration as the United States and those of

[20] Note here that Proposition 4.2 deals with the steady-state whereas our numerical analyses do not because the total population grows over time.

making the United States as closed towards immigration as Japan.

The results of our counterfactual analyses are given in Figures 4.7.[21]

In Figure 4.7, Japan experiences increases in the number of birth, immigrants, total and young cohort population sizes, market size, and per capita expenditure on consumption if it becomes as open towards immigration as the United States, and the United States experiences decreases in them if it becomes as closed towards immigration as Japan. This implies that immigration affects not only the current population size but also the population size of the next generation and their expenditures, resulting in large impacts on the market size.

A few comments are in order. First, our results indicate that large immigration inflows into the United States were a significant engine of economic growth over the past half century. If the United States had been as closed towards immigration as Japan, its market size would have been much smaller than that observed today. Second, our results also imply that Japan could avoid shrinkages in population and market sizes caused by aging if it becomes more open towards immigration. Given that the Japanese population has already started to decrease, it would be worthwhile for Japan to consider accepting immigrants as a possible option for overcoming its population and market size declines.

4.5.4 Robustness

In this section, we conduct a few robustness checks. First, we change the child rearing cost parameter b by utilizing the results of Eckstein et al. (1999), who obtained money and time cost parameters for child rearing as 0.11 and 0.29, respectively. Thus, we check the case that $b = 0.29$, which represents the case where child rearing costs only consist of time costs. We also check the case of $b = 0.11 + 0.29 = 0.4$. In addition, we check the

[21] In Figure 4.7, we replace both ε and μ between the two countries. If we replace only ε or only μ, we obtain very similar results to those shown in Figure 4.7. By comparing the case wherein we replace only ε to that wherein we replace μ, we can see that the effects on the number of birth and the cohort population distribution are larger in the former than in the latter. These results are available upon request.

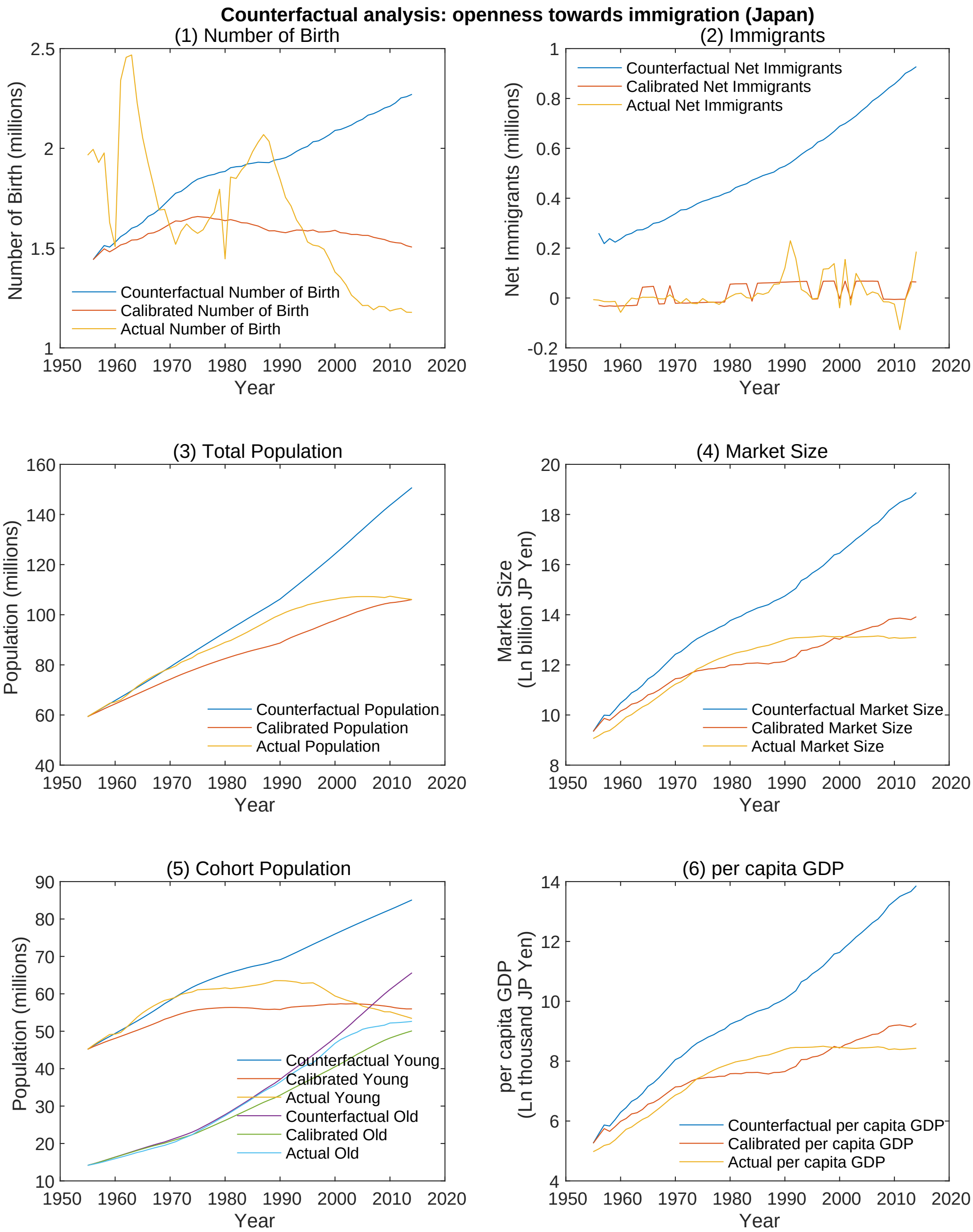


Fig. 4.7 (a) Counterfactual analysis: openness towards immigration (Japan)

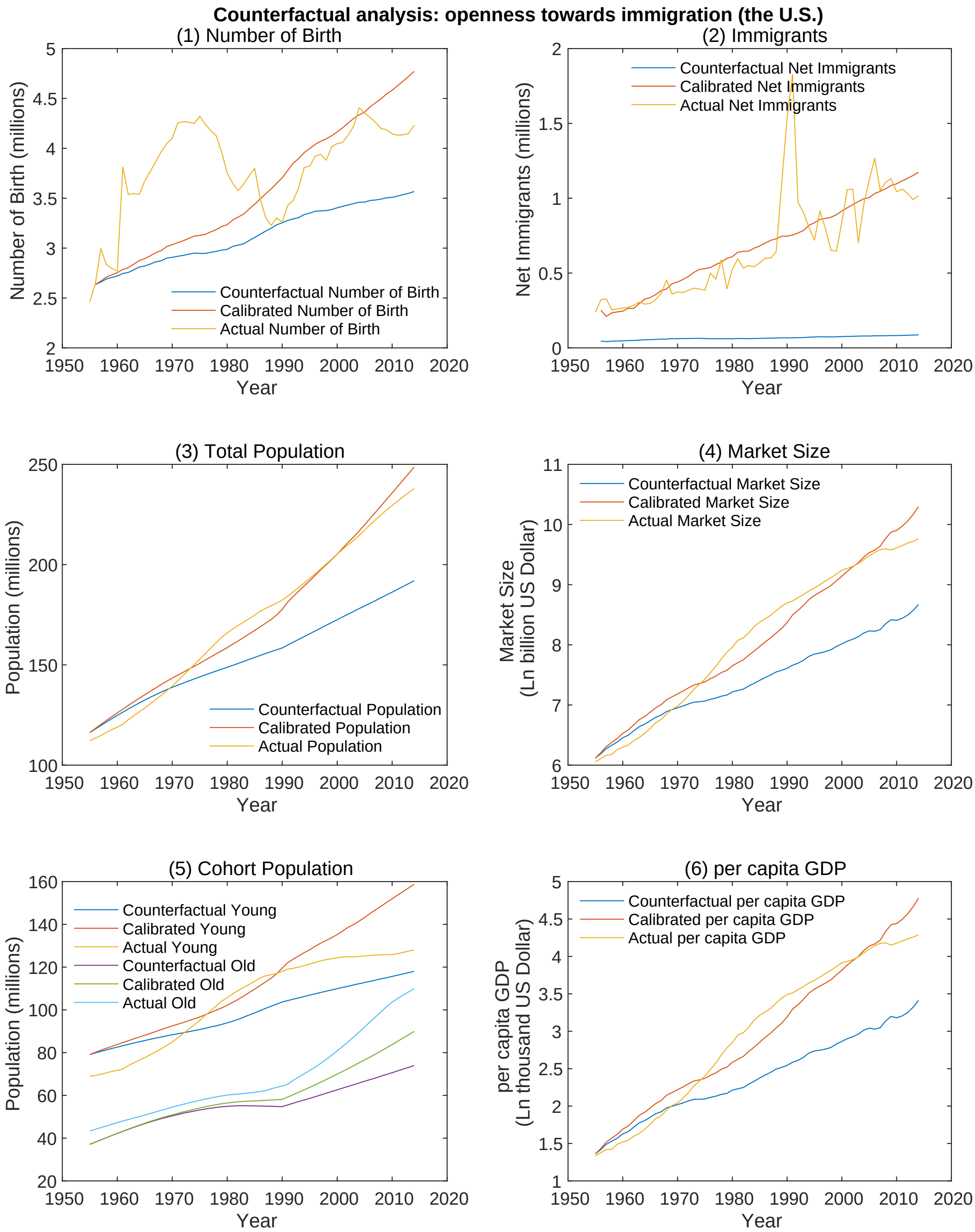


Fig. 4.7 (b) Counterfactual analysis: openness towards immigration (U.S.)

b	γ_{JP}	γ_{US}
0.06	0.002651	0.002852
0.11	0.004867	0.005237
0.17	0.007535	0.008110
0.23	0.010213	0.010993
0.29	0.012900	0.013888
0.40	0.017851	0.019224

Table 4.2 Robustness check: estimated γ under different values of b

case wherein b is 0.06, 0.17, or 0.23 in order to adjust the relationship between length of periods and individual's income in Eckstein et al. (1999) to those in our chapter. Under all different values of b, we obtain very similar calibration and counterfactual results to those obtained under $b = 0.11$. Hence, we here show the case of $b = 0.4$ as a representative case in Figure 4.8.

Table 4.2 shows the estimated γ under different values of b. From the table, we can see that a higher b increases the estimated γ to sustain the number of births, which indicates that changes in b are absorbed by changes in γ.

Second, we change the discount factor to $\beta = (1.03)^{-35} = 0.3554$ to achieve 3% depreciation per year, following Eckstein et al. (1999). Our calibration results are similar to those in the case of $\beta = 2/3$. However, the counterfactual results look somewhat different from the the case of $\beta = 2/3$ especially in the counterfactual that assumes a high survival rate. Figure 4.9 shows the counterfactual result of a high survival rate in the case of $\beta = 0.3554$, which is comparable to Figure 4.6. By comparing Figure 4.9 with Figure 4.6, we find that the number of birth and population are larger in Figure 4.9 than in Figure 4.6. This is because a lower β relatively increases the demand for children and decreases the demand for future consumption. Note also that the magnitude of the increases in births and population is larger in Japan than in the United States. This may reflect the difference in the main source of population dynamics between Japan and the United States. In Japan, population dynamics are mainly driven by births. Thus, the

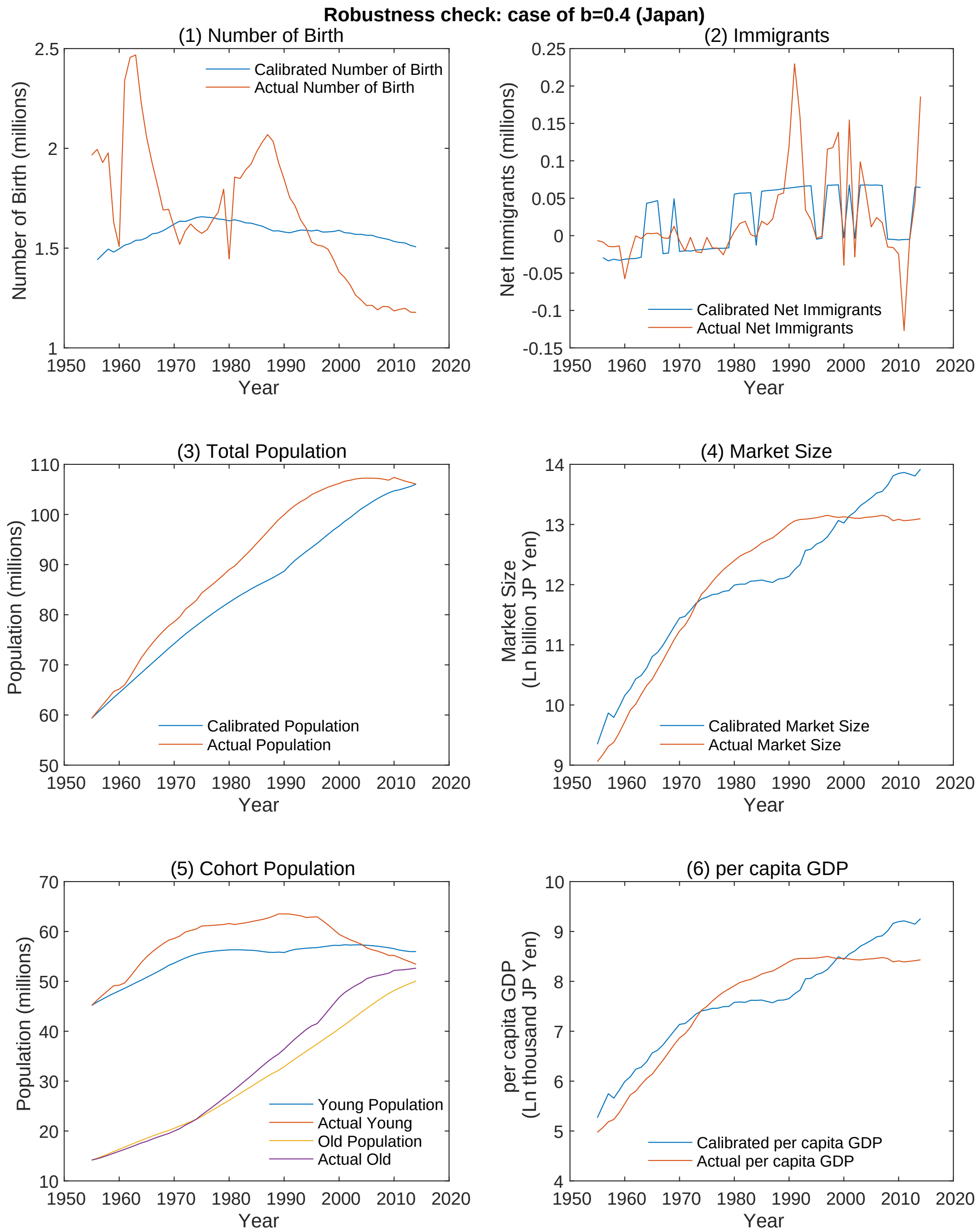


Fig. 4.8 (a) Robustness check: case of $b = 0.4$ (Japan)

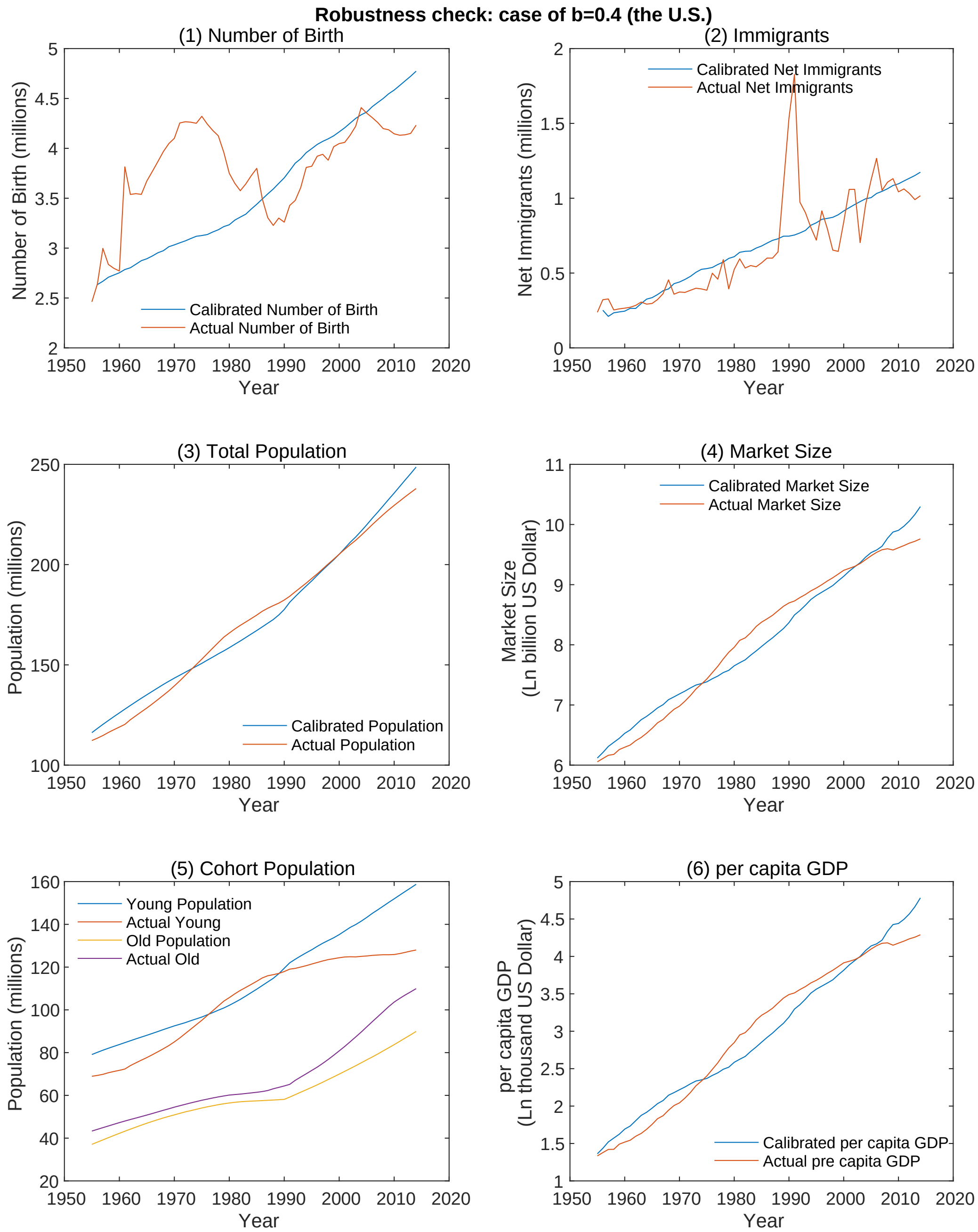


Fig. 4.8 (b) Robustness check: case of $b = 0.4$ (U.S.)

population dynamics of Japan are sensitive to ϕ that determines the number of births. In contrast, population dynamics in the United States are mainly driven by immigrants. Thus, the population dynamics of the United States are not sensitive to ϕ.

4.6 Concluding remarks

This chapter developed an overlapping generations model with endogenous fertility and immigration. Because we employ monopolistic competition wherein firms produce differentiated goods, population size and hence, market size matter for welfare in our framework. We then investigated the effects of improvements in longevity on population size, market size, and welfare. Our theoretical analysis showed that improvements in longevity affect the market size through three effects: First, it decreases the number of children because parents need to prepare for consumption in the old period. Second, it increases the per capita lifetime consumption. Finally, it increases the immigration size. The first effect has negative impacts on the market size whereas the latter two effects have positive impacts. We then calibrated our model using Japanese and U.S. data from 1955 to 2014 and conducted counterfactual analyses. Our first counterfactual analysis examined the effects of improvements in longevity and showed that a lower survival rate results in a larger market size. This implies that the negative impacts dominate the positive ones, and that the improvements in longevity can be a major source of shrinkage in market size. Our second counterfactual analysis considered the scenario wherein Japan is as open towards immigration as the United States and the United States is as closed towards immigration as Japan. Under this scenario, we showed that Japan experienced much higher growth in terms of population and market size whereas the United States experienced much lower growth, implying that the United States enjoyed gains from immigration and that Japan can overcome the shrinkage of its market size, which was caused by aging, if it accepts more immigrants.

One of the most important future extensions would be to incorporate human capital. On the one hand, human capital accumulation increases the market size by raising wages as shown by Prettner et al. (2013), Strulik et al. (2013), and Prettner and Strulik

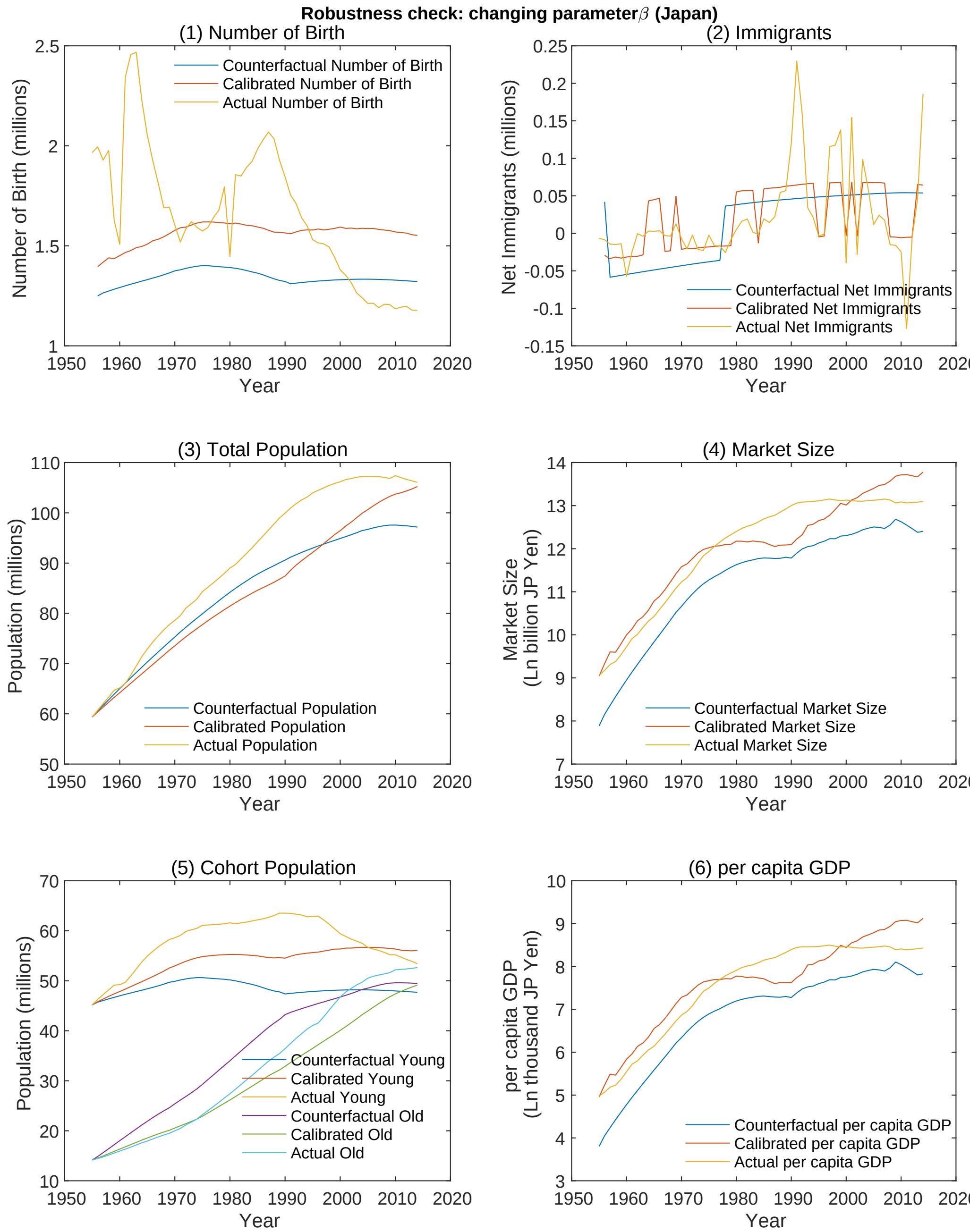


Fig. 4.9 (a) Robustness check: changing parameter β (Japan)

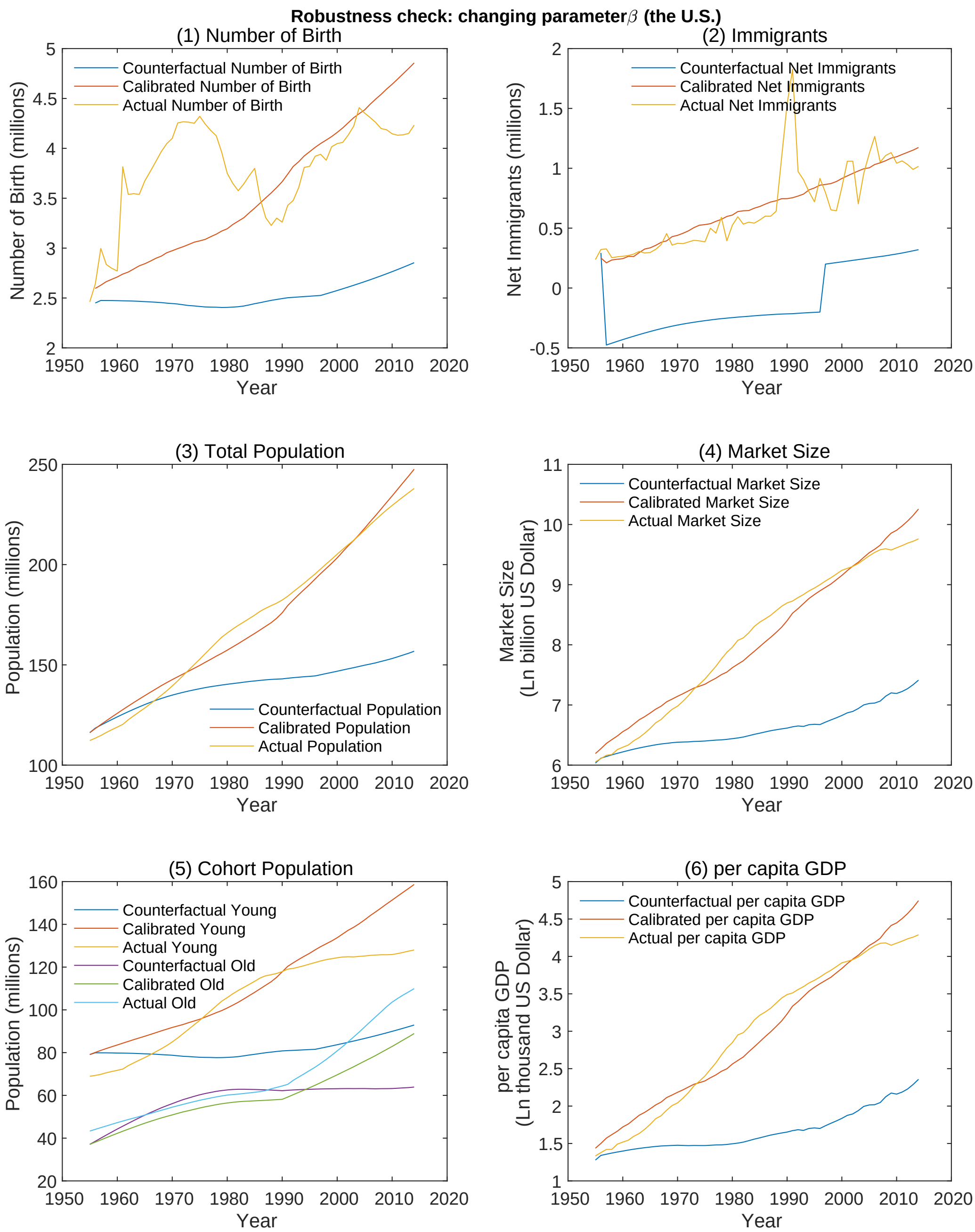


Fig. 4.9 (b) Robustness check: changing parameter β (U.S.)

(2016). On the other hand, it lowers fertility by increasing the opportunity costs of having children, which make the market size shrink. Moreover, Cervellati and Sunde (2005) pointed out that human capital accumulation can potentially improve longevity. Hence, human capital accumulation might be significantly related to our analysis and hence would be worth incorporating.

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