<table>
<thead>
<tr>
<th>Title</th>
<th>On ribbon 2-knots. III. On the unknotting ribbon 2-knots in $S^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Yanagawa, Takaaki</td>
</tr>
<tr>
<td>Citation</td>
<td>Osaka Journal of Mathematics. 7(1) P.165-P.172</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1970</td>
</tr>
<tr>
<td>Text Version</td>
<td>publisher</td>
</tr>
<tr>
<td>URL</td>
<td><a href="https://doi.org/10.18910/7049">https://doi.org/10.18910/7049</a></td>
</tr>
<tr>
<td>DOI</td>
<td>10.18910/7049</td>
</tr>
<tr>
<td>Note</td>
<td></td>
</tr>
</tbody>
</table>

Osaka University Knowledge Archive : OUKA

https://ir.library.osaka-u.ac.jp/repo/ouka/all/

Osaka University
ON RIBBON 2-KNOTS III
ON THE UNKNOTTING RIBBON 2-KNOTS IN $S^1$

TAKAAKI YANAGAWA

(Received September 8, 1969)

1. Introduction

Concerning the unknotting theorem for the pair $(S^n, M^{n+2})$ with the codimension 2, there are several remarkable results; by T. Homma in the case $n=1$ and $M^3=S^3$, by C.D. Papakyriakopoulos in the case $n=1$ and any 3-manifold $M'$, by J. Stallings in the case $n \geq 3$ and $M^{n+2}=S^{n+2}$ in the topological sense and by J. Levine in the case $n \geq 3$ and $M^{n+2}=S^{n+2}$ in the combinatorial sense, see [1], [2], [3], [4] and [5]. Confining ourselves to the case $M^{n+2}=S^{n+2}$, the unknotting theorem has not been solved in the case $n=2$.

In this paper, we will prove the following theorem which is an answer under an additional condition to the unknotting theorem in the case $n=2$ and $M^4=S^4$:

Theorem (2, 2). For a ribbon 2-knot $K^2$ in $S^4$, $K^2$ is unknotted in $S^4$ if and only if $\pi_1(S^4-K^2)=\mathbb{Z}$.

In this paper, everything will be considered from the combinatorial point of view.

2. Proof of Theorem

Lemma (2, 1). Let $M^4$ be a combinatorial 4-manifold and let $\gamma$ be a simple closed curve in $\hat{M}^4$ which is contractible to a point in $\hat{M}^4$. Then, $\gamma$ bounds a non-singular, locally flat 2-ball in $\hat{M}^4$.

Proof. Since $\gamma$ is contractible to a point in $\hat{M}^4$, there is a PL-map $\varphi$ of a 2-ball $D^2$ into $\hat{M}^4$ satisfying the following (1), (2) and (3):

1) $\varphi(D^2) \subset \hat{M}^4$, $\varphi(\partial D^2) = \gamma$,
2) $\varphi(D^2)$ is in a general position in $\hat{M}^4$ so that the self-intersection consists of a finite number of double points,

2) Cf. the result in [7], the proof of Lemma (2, 7).
3) $X$ and $\partial X$ mean the interior and the boundary of $X$ respectively.
(3) there are at most a finite number of \textit{locally knotted points} on \(\varphi(D^2)\) which are different from the double points in (2).

Here, in (3), a point \(x\) of \(\varphi(D^2)\) is called a \textit{locally knotted point}\(^4\) if the pair \((\text{Lk}(x, \varphi(D^2)), \text{Lk}(x, M^4))\) is a knotted sphere-pair for the combinatorial triangulation of \(M^4\) for which \(\varphi(D^2)\) is a subcomplex and the point \(x\) is a vertex. If there is a locally knotted point \(x\) of \(\varphi(D^2)\), it is possible to exchange a non-singular 2-ball \(\text{St}(x, \varphi(D^2))\), which may be not locally flat, for an immersed 2-ball \(\rho(B^2)\) in a 4-ball \(B^4=\text{St}(x, M^4)\) by an immersion \(\rho\) of a 2-ball \(B^2\) such that \(\rho(\partial B^2) = \varphi(D^2) \cap \partial B^4\), \(\rho(B^2) \subset \hat{B}^4\) and that each pair \((\rho \text{Lk}(y, B^2), \text{Lk}(\rho(y), B^4))\) is unknotted for a fine subdivision of \(B^4\) and each vertex \(y\) of \(B^2\). Perform the exchange for all locally knotted points of \(\varphi(D^2)\). By making use of the general position theory, we have a PL-map \(\varphi'\) of \(D^2\) into \(M^4\) satisfying the following (1'), (2') and (3'):

(1') \(\varphi'(D^2) \subset M^4\), \(\varphi'(\partial D^2) = \gamma\),

(2') \(\varphi'(D^2)\) is in a general position in \(M^4\) so that the self-intersection consists of a finite number of double points,

(3') \(\varphi'(D^2)\) has no locally knotted point.

Let \(x\) be a double point of \(\varphi'(D^2)\) and \(x = \varphi'(y) = \varphi'(y')\) for just two points \(y, y'\) of \(D^2\). Then there is an arc \(\alpha\) spanning \(y\) and a point \(y^*\) on \(\partial D^2\) such that the image \(\varphi'(\alpha)\) does not cross any double point of \(\varphi'(D^2)\) except for \(x\). Let \(V^4\) be a regular neighborhood of \(\varphi'(\alpha)\) in \(M^4\). Since \(V^4\) is a 4-ball, there is a PL-homeomorphism \(\phi\) of \(V^4\) onto a standard 4-cube \(\Delta^4\) such that

(i) \(\Delta^4\); \(|x_1|, |x_2|, |x_3|, |x_4| \leq 2\)

(ii) \(\phi \varphi'(\alpha); 0 \leq x_1 \leq 1, x_2 = x_3 = x_4 = 0\)

(iii) \(\phi \varphi'(x); x_1 = x_2 = x_3 = x_4 = 0\)

(iv) \(\phi(V^4 \cap \varphi'(D^2)); \begin{cases} -2 \leq x_1 \leq 1, & x_2 = x_3 = 0, |x_4| \leq 2 \\ x_1 = 0, & |x_2|, |x_3| \leq 2, x_4 = 0 \end{cases} \tag{*}\)

Let \(N^2\) be the 2-ball in \(\phi \varphi'(D^2)\) defined by the equation (*) and let \(N'^2\) be the 2-ball in \(\Delta^4\) defined as follows:

(iii) \(N'^2; \begin{cases} 0 \leq x_1 < 2, & |x_2| \leq 2, |x_3| = 2, x_4 = 0, \\ x_1 = 2, & |x_2|, |x_3| \leq 2, x_4 = 0. \end{cases}\)

If we consider a singular 2-ball \(B'^2 = \phi^{-1}((\phi \varphi'(D^2) - N^2) \cup N'^2)\), then this 2-ball \(B'^2\) is not only locally flat but also has a number of the double points less

---

4) See [8] p. 34.
than the number of those of $\phi'(D^2)$. Moreover we have that $\partial B^2=\gamma$. Repeating this process, we have finally a locally flat, non-singular 2-ball $B^2$ such that $B^2 \subset \hat{M}^4$ and that $\partial B^2=\gamma$. The proof is thus complete.

Let $K^2$ be a ribbon 2-knot in $R^4$, then there is a 3-manifold $W^3$ satisfying the following properties:

(1) $W^3 \approx B^3$ or $W^3 \approx \#(S^1 \times S^2) - \hat{B}^3$,

(2) If $W^3 \approx B^3$, $W^3$ has a trivial system of 2-spheres $\{S^2_1, \ldots, S^2_{2n}\}$ satisfying that

(i) a 2-link $\{S^2_1, \ldots, S^2_{2n}\}$ is trivial in $R^4$,

(ii) $S^2_i \cup S^2_{n+i}$ bounds a spherical-shell $N^*_i$ in $W^3$ ($i=1, \ldots, n$),

(iii) $W^3 - \check{N}^*_1 \cup \cdots \cup \check{N}^*_{2n} \approx B^3 - \Delta^3_1 \cup \cdots \cup \Delta^3_{2n}$,

see (3, 5) and (3, 6) in [6].

Let $\Delta^3_\delta$ be a 3-ball in $W^3 - N^*_1 \cup \cdots \cup N^*_n$, let $S^3_0$ be a boundary 2-sphere of $\Delta^3_\delta$ and let $\beta_1, \ldots, \beta_{2n}$ be a collection of mutually disjoint arcs spanning $S^3_0$ and $S^2_i, \ldots, S^2_{2n}$ in $W^3 - N^*_1 \cup \cdots \cup N^*_n \cup \Delta^3_\delta$ respectively. Moreover, let $U^3_\lambda$ be a regular neighborhood of the arc $\beta_\lambda$ in $W^3 - \check{N}^*_1 \cup \cdots \cup \check{N}^*_n \cup \Delta^3_\delta$ where $U^3_\lambda \cap S^3_0 = e^3_\lambda$ and $U^3_\lambda \cap S^3_\delta = e^3_\delta$ are 2-balls such that $e^3_\lambda \cap e^3_\mu = 0$ ($\lambda \neq \mu$, $\lambda, \mu = 1, \ldots, 2n$). Since the 2-link $\{S^2_0, \ldots, S^2_{2n}\}$ is trivial in $R^4$, there is an isotopy $\xi$ of $R^4$ by which $\xi(S^3_\lambda)$ ($\lambda = 0, \ldots, 2n$) are moved into the position given by the equations below:

\[
\begin{align*}
\xi(S^3_0); & \quad x_1^2 + x_2^2 + x_3^2 = 1, \quad x_i = 0 \\
\xi(S^3_0); & \quad (x_i - 4i)^2 + x_2^2 + x_3^2 = 1, \quad x_i = 0 \\
\xi(S^3_{n+i}); & \quad (x_i - 4i)^2 + x_2^2 + x_3^2 = 2, \quad x_i = 0 \\
\xi(N^*_i); & \quad 1 \leq (x_i - 4i)^2 + x_2^2 + x_3^2 \leq 2, \quad x_i = 0 \quad (i = 1, \ldots, n).
\end{align*}
\]

Moreover, we may suppose that the center line $\xi(\beta_\lambda)$ of the tube $\xi(U^3_\lambda)$ is given by the equations below:

\[
\begin{align*}
x_i = 4i, & \quad x_2 = 0, \quad x_3 = 1 - x_i, \quad x_i \geq 0 \\
& \quad \text{in the neighborhood of } \xi(\beta_\lambda \cap S^3_i),
\end{align*}
\]

\[
\begin{align*}
x_i = 4i, & \quad x_2 = 0, \quad x_3 = \sqrt{2} - x_i, \quad x_i \leq 0 \\
& \quad \text{in the neighborhood of } \xi(\beta_{n+i} \cap S^3_{n+i})
\end{align*}
\]

(i = 1, 2, \ldots, n).

Theorem (2.2'). For a ribbon 2-knot $K^2$ in $R^4$, $K^2$ is unknotted in $R^4$, if and only if $\pi_1(R^4 - K^2) = Z$.

5) $B^3$ means a 3-ball and \approx means to be homeomorphic to.
6) $N^3_1 \approx S^2 \times [0, 1]$. 
The proof of this theorem is divided into two steps, and the second-step of
the proof will be given later after we have proved two lemmas (2, 3) and (2, 4).
Since \( K^2 \) is a ribbon 2–knot in \( R^4 \), it bounds a 3–manifold \( W^3 \) previously described,
therefore if \( W^3 \approx B^3 \), we have nothing particular to say. Hence, in the following
discussion we will consider the case that \( W^3 \approx B^3 \). Consider the trivial system
\( \{ S^2_1, \ldots, S^2_{2n} \} \) and the isotopy \( \xi \) of \( R^4 \) as before. Let \( \tilde{K}^2 \) be a 2–knot in \( R^4 \)
such that
\[
\tilde{K}^2 = \xi \left( \bigcup_{\lambda=1}^{2n} S^2_{\lambda} \right) \cup \xi \left( \bigcup_{\lambda=1}^{2n} \partial U^3_{\lambda} \right).
\]
Then, since two 2–spheres \( \xi(K^2) \) and \( \tilde{K}^2 \) bound a 3–manifold which is a sub-
complex of \( \xi(W^3) \) and which is homeomorphic to \( S^2 \times [0, 1] \) in \( R^4 \), \( \tilde{K}^2 \) belongs to
the 2–knot-type \( \{ \xi(K^2) \} \) which coincides with the 2–knot-type \( \{ K^2 \} \).

Let \( B^3_i \) be a 3–ball bounded by the 2–sphere \( \xi(S^2_i) \):

\[
B^3_i: (x_1-4)^2+x_2^2+x_3^2 \leq 1, \quad x_4 = 0 \quad (i = 0, 1, 2, \cdots, n).
\]

**First-step of the proof of (2, 2)**: Each 3–ball \( B^3_i \) bounded by the 2–sphere
\( \xi(S^2_i) \) \( (i=1, \cdots, n) \) in \( R^3 \) does not meet any arc \( \xi(\beta_\lambda) \) except for the end points
\( (\lambda=1, \cdots, 2n) \).

Since we can find a regular neighborhood \( U^3_\lambda \) of \( \beta_\lambda \) so fine that \( \xi(U^3_\lambda) \cap B^3_i = 0 \) because \( \xi(\beta_\lambda) \cap B^3_i = 0 \) for all \( i \) and \( \lambda \) \( (i=1, \cdots, n, \lambda=1, \cdots, 2n) \), the 2–knot
\( \tilde{K}^2 \) bounds a 3–ball \( B^3_0 \cup B^3_1 \cup \cdots \cup B^3_n \cup B^3_{n+1} \cup \cdots \cup B^3_{2n} \cup \xi(U^3_1) \cup \cdots \cup \xi(U^3_{2n}) \),
where the 3–ball \( B^3_{n+i} \) is bounded by \( \xi(S^2_{n+i}) \) in the neighborhood of \( \xi(N^3_i) \cup B^3_i \) in \( R^4 \) so that \( B^3_{n+i} \cap B^3_i = 0 \), \( B^3_{n+i} \cap \xi(U^3_\lambda) = 0 \) \( (\lambda=\lambda+i) \) and \( \sim \xi(\varepsilon^3) \) \( (\lambda=n+i) \) \( (i, j=1, \cdots, n, \lambda=1, \cdots, 2n) \): for a sufficiently small \( \varepsilon(\varepsilon<0) \),

\[
B^3_{n+i}: \begin{cases}
(x_1-4)^2+x_2^2+x_3^2 = 2, \\
(x_1-4)^2+x_2^2+x_3^2 \leq 2, \\
\lambda=\lambda+i \quad (i=1, 2, \cdots, n).
\end{cases}
\]

If there is a 3–ball \( B^3_i \) which meets some arcs \( \xi(\beta_\lambda) \) \( (1 \leq i \leq n, 1 \leq \lambda \leq 2n) \),
we will consider how to remove the intersection of the 3–ball \( B^3_i \) and the arcs
\( \xi(\beta_\lambda) \) without changing the 2–knot-type of \( \tilde{K}^2 \). We need following two lemmas
(2, 3) and (2, 4) to remove the intersection.

**Lemma (2, 3)**. If there are an arc \( b \) in \( R^4-\tilde{K}^2 \) and a subarc \( \beta' \) of the arc
\( \beta_\lambda \) \( (1 \leq \lambda \leq 2n) \) such that the simple closed curve \( \gamma=b \cup \xi(\beta') \) is contractible in
\( R^4-\tilde{K}^2 \), then there exists an isotopy \( \eta \) of \( R^4 \) by which \( \eta(\tilde{K}^2) = \tilde{K}^2 \) and \( \eta \xi(\beta_\lambda) = \xi(\beta_\lambda-\beta') \cup b \).

Proof. Since \( \beta_\lambda \) is contained in \( \tilde{U}^3_\lambda \) except two end points, we can triangulate
\( R^4 \) so that the regular neighborhood \( N(\tilde{K}^2) \) of \( \tilde{K}^2 \) in \( R^4 \) does not meet \( \xi(\beta_\lambda) \).
If we apply (2, 1) to the 4–manifold \( M^4=R^4-\tilde{N}(\tilde{K}^2) \) and the simple closed
curve \( \gamma = b \cup \xi(\beta') \), the simple closed curve \( \gamma \) bounds a locally flat 2-ball \( B^2 \) in \( \tilde{M}^4 \). Therefore there exists a combinatorial 4-ball \( B^4 \) containing \( B^2 \) in its interior and contained in \( R^4 - K^2 \). Now, we have easily an isotopy which is identical on \( \partial B^4 \) and transfers the subarc \( \xi(\beta') \) onto the arc \( b \). Hence, the proof is complete.

**Lemma (2, 4).** If \( \pi_1(R^4 - K^2) = \mathbb{Z} \) and an arc \( \xi(\beta_\lambda) \) \((1 \leq \lambda \leq 2n)\) pierces through a 3-ball \( B^3_\lambda \) \((1 \leq i \leq n)\) at a point \( A \), there are an arc \( b \) in \( R^4 - K^2 \) and a subarc \( \beta_i \) on the arc \( \beta_\lambda \) containing \( A \) such that the simple closed curve \( b \cup \xi(\beta'_i) \) is contractible in \( R^4 - K^2 \).

**Proof.** For convenience's sake, we may suppose that \( \xi(\beta_\lambda) \) is given in the neighborhood of the point \( A \) as follows:

\[
\xi(\beta_\lambda); \; x_1 = 4i, \; x_2 = x_3 = 0, \; -1 \leq x_4 \leq 1.
\]

Consider the cross-sections of \( \xi(S^3) \), \( \xi(S^2_{n+1}) \) and \( \xi(N^3) \) by the hyperplane \( P; \; x_3 = 0 \). Then, we have the following figure Fig. (1).

Place \( \xi(U^3_\lambda) \) in a general position with respect to the hyperplane \( P \), then the cross-section \( \xi(U^3_\lambda) \cap P \) is at most 2-dimensional, and we can find an arc \( b \) spanning two points \( A_+ \) and \( A_- \) in \( P - P \cap K^2 \) as follows:

\[
\begin{align*}
&\begin{cases}
\lambda = 4i, & 0 \leq x_2 \leq 2, \; x_3 = 0, \; x_4 = \varepsilon \\
b; & \begin{cases} x_1 = 4i, & x_2 = 2, \; x_3 = 0, \; -\varepsilon \leq x_4 \leq \varepsilon \\
 x_1 = 4i, & 0 \leq x_2 \leq 2, \; x_3 = 0, \; x_4 = -\varepsilon,
\end{cases}
\end{cases}
\end{align*}
\]

see Fig. (2).
Since the 2-knot $K^2$ bounds the orientable 3-manifold $\tilde{W}^3 = \xi(N_1^3 \cup \cdots \cup N_n^3 \cup U_1^3 \cup \cdots \cup U_m^3 \cup \Delta_3)$ in $R^4$, we will give an orientation induced from the orientation of $\tilde{W}^3$ for $K^2$. Then, the trivial link $\xi(S_1^3 \cup S_2^3 \cup \cdots \cup S_{n+1}^3 \cup \Delta_3)$ in $P$, which bounds an annulus $\xi(N_1^3 \cap P)$ in $P$, can be given the orientation induced from that of $\xi(N_1^3 \cap P)$, see Fig. (1) again. Since $\tilde{K}^2 \cap P$ is a cross-section of a 2-knot $\tilde{K}^2$, the simple closed curves $c_i$ and $c_{n+i}$ represent the generator of $H^1(R^4 - K^2)$, see Fig. (2) again. Therefore, the loop $\gamma w \gamma^{-1}$ represents an element of the commutator subgroup of $\pi_1(R^4 - K^2)$ for any arc $w$, because $\pi_1(R^4 - K^2) = \mathbb{Z}$; that is, $\gamma$ is contractible in $R^4 - K^2$.

Second-step of the proof of (2, 2)' : There is a 3-ball $B_i^3$ which meets some arcs $\xi(\beta_\lambda)$. Since the 2-knot $\tilde{K}^2$ constructed by making use of $\tilde{W}^3$, $S_\lambda^3$ and $U_\lambda^3$ and bounding the 3-manifold $\tilde{W}^3 = \xi(N_1^3 \cup \cdots \cup N_n^3 \cup U_1^3 \cup \cdots \cup U_m^3 \cup \Delta_3)$ in $R^4$ belongs to the 2-knot-type $\{K^2\}$, it is sufficient to prove that $\tilde{K}^2$ is unknotted. On the other hand, by making use of (2, 4) and (2, 5), there exists an isotopy $\eta$ of $\eta(K^2) = \tilde{K}^2$ such that $\eta(\xi(\beta_\lambda)) = \xi(\beta_\lambda)$ (for the segment $\xi(\beta_\lambda)$ between $A_+$ and $A_-$ on $\xi(\beta_\lambda)$). Now, $\gamma w \gamma^{-1} \sim 0$ for any arc $w$, because $\pi_1(R^4 - K^2) = \mathbb{Z}$; that is, $\gamma$ is contractible in $R^4 - K^2$.

Theorem (2, 2). For a ribbon 2-knot $K^2$ in $S^4$, $K^2$ is unknotted in $S^4$, if and only if $\pi_1(S^4 - K^2) = \mathbb{Z}$.

Corollary (2, 3). Let $K^2$ be a 2-knot in $R^4$ satisfying the following (1), (2) and (3). Then $K^2$ is unknotted in $R^4$;

1. A 2-node $K^2 \cap H_+^4$ contains no minimum,
2. The 2-nodes $K^2 \cap H_+^4$ and $K^2 \cap H_-^4$ are symmetric each other with respect to the hyperplane $R_0^3$,
3. The knot $k = K^2 \cap R_0^3$ is unknotted in $R_0^3$.

Proof. This follows from (2, 2)'. Since $K^2$ satisfies (1) and (2), $K^2$ is a
ribbon 2–knot, see [6]. Moreover there is a homomorphism of \( \pi_1(R^3 - K^3) \) onto \( \pi_1(R^4 - K^4) \), cf. p. 132–6 in [9]. Then, it is easy to see that \( \pi_1(R^4 - K^4) = \mathbb{Z} \) as \( \pi_1(R^3 - K^3) = \mathbb{Z} \) by the condition (3). (2, 3) is a proposition analogous to the theorem in [11].

The converse of (2, 3) is not always true, see the remark below:

**Remark.** There is an unknotted 2-knot \( K^2 \) which satisfies (1) and (2) in (2, 3) but does not satisfy (3) in (2, 3), see the following example.

The knot \( k \) in \( R^3_0 \), described in Fig. (3), is knotted in \( R^3_0 \), although its Alexander polynomial \( \Delta(t) = 1 \), see Fig. (13) on p. 151 in [10].

![Fig. 3](image)

**3. A generalization to the higher dimensional case**

Let \( K^m \) be a locally flat \( m \)-sphere in \( R^{m+2} \) and let \( W^{m+1} \) be a \((m+1)\)-manifold satisfying the following (1), (2) and (3):

1. \( W^{m+1} \subset R^{m+2} \), \( \partial W^{m+1} = K^m \),
2. \( W^{m+1} \approx B^{m+1} \) or \( W^{m+1} \approx \#(S^1 \times S^m) - \hat{B}^{m+1} \),
3. if \( W^{m+1} \approx B^{m+1} \), \( W^{m+1} \) has a trivial system of \( m \)-spheres \( \{ S^n_1, \ldots, S^n_{2n} \} \) such that
   - the locally flat \( m \)-link \( \{ S^n_1, \ldots, S^n_{2n} \} \) is combinatorially trivial in \( R^{m+2} \),
   - \( S^n_1 \cup S^n_{m+1} \) bounds a spherical-shell \( N^{m+1}_1 \) in \( W^{m+1} \),
   - \( W^{m+1} - \hat{N}^{m+1}_1 \cup \cdots \cup \hat{N}^{m+1}_{2n} \approx B^{m+1} - \Delta^{m+1}_1 \cup \cdots \cup \Delta^{m+1}_{2n} \).

Then, we have the following theorem in the same way as (2, 2).

**Theorem (3, 1).** Let \( K^m \) be a locally flat \( m \)-sphere in \( S^{m+2} \) and \((m+1)\)-manifold \( W^{m+1} \) satisfying the above conditions. Then, \( K^m \) is unknotted \(^{10}\) in \( S^{m+2} \), if and only if \( \pi_1(S^{m+2} - K^m) = \mathbb{Z} \).

**Kobe University**

---

7) Prof. R.H. Fox named this 2-knot *Terasaka-Kinoshita 2-sphere*.
8) \( N^{m+1}_1 = S^m \times [0, 1] \).
9) \( \Delta^{m+1}_1, \ldots, \Delta^{m+1}_{2n} \) are disjoint \((m+1)\)-simplices in \( a(m+1)\)-ball \( B^{m+1} \).
10) At least topologically unknotted.
References