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Thermal Stress Analysis of Welds:
From Melting Point to Room Temperature

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Abstract

To compute stress, residual stress and distortion in welded structures, It is desirable that a stress analysis cover the entire temperature range from the melting point to room temperature. This paper presents a methodology that switches constitutive equations from rate independent or elasto-plasticity at temperatures below 0.5 of the melting point; to rate dependent or elasto-viscoplasticity for temperatures from 0.5 to 0.8 of the melting point and to linear viscoplastic model for temperatures from 0.8 of the melting point to the melting point and above. The issues in switching constitutive equations and transforming intenal variables from one constitutive equation to another are discussed. Numerical results for the thermal stress analysis of a weld are presented to demonstrate the performance of the methodology .

1. Introduction

To predict the stress, residual stress and distortion in a welded structure, one must solve for the transient temperature, microstructure evolution in the HAZ, stress and strain[1]. For a known weld pool shape, the transient temperature can be predicted accurately and efficiently. The evolution of microstructure in the HAZ and weld pool can be predicted with useful accuracy although much remains to be done to achieve higher resolution and accuracy. To solve for the stress and strain, the relationship between stress and strain as a function of strain rate, stress rate and temperature and microstructure is required. Almost all analyses of residual stress in welds published to date have used rate independent plasticity. This is not appropriate physics at temperatures above approximately 0.5 of the melting point. This paper will discuss a methodology that applies the appropriate physics to provide more accurate stress strain constitutive models for stress analysis of welded structures. Not only are these models more accurate, they promise to be affordable[2, 3].

Distortion and residual stress in welds are caused by non-linear material behavior. For sufficiently low stresses and sufficiently short times, the stress strain relationship for solids is linear elastic. For sufficiently high stresses and sufficiently long times, the stress strain relationship is nonlinear. What is a sufficiently low stress and a short time depends on the material type, its microstructure and the temperature. For Newtonian fluids the stress strain rate relationship is linear. The nonlinear behavior of a solid tends to be strain rate dependent at high temperatures and sufficiently long times. This means that the strain rate is a function of stress. The plastic deformation is often associated with the thermally activated motion of dislocations in the presence of a deviatoric stress field. This is called rate dependent plasticity. At low temperatures, the nonlinear behavior of a solid has strain rate a
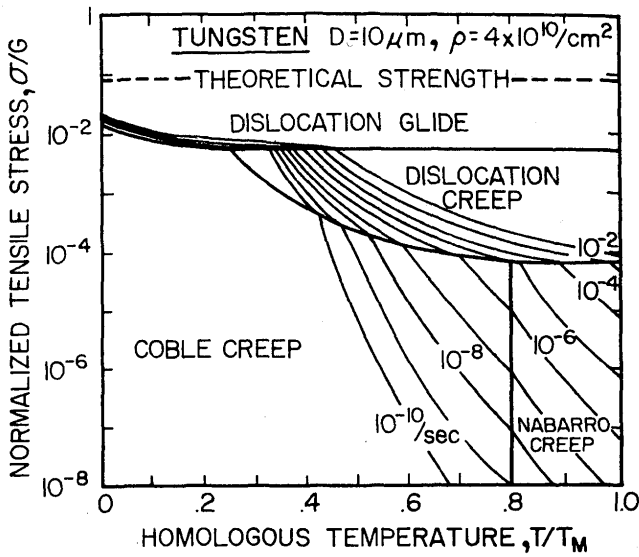
Figure 1. An Ashby diagram for tungsten illustrates the deformation mechanisms for a range of temperatures, stress levels and strain rates.

function of stress rate. To accurately predict stress, residual stress and distortion in welded structures, one must know how the material responds to a given stress or strain, i.e., which constitutive equation to use for a given state of the material and loading condition. Not only must one know which constitutive equation is most appropriate, one must know the material properties that describe this behavior such as yield stress or deformation resistance.

The best known representation of material behavior versus strain rate and temperature are Ashby diagrams such as **Fig. 1**. In welds, the strain rate is usually less than 0.01. A crack driven by elastic unloading could have much higher strain rates but that is not considered here. The Ashby diagram has several striking features. First there are many different mechanisms for deformation. Therefore one should not expect simple solutions to be accurate. Second the flow stress rises rapidly with decreasing temperature. Third the difference in flow stress for strain rates of say 0.0001 and 0.01 narrows rapidly as temperature is decreased.

This strain rate dependent plasticity can be described by many different equations. Primarily because of the experimental data was available[4], we chose the equation:

$$\dot{\varepsilon}^{p} = A \exp\left(-\frac{\Delta G}{kT}\right)\left[\sinh\left(\xi \frac{\bar{\sigma}}{s}\right)\right]^{(1/m)} \qquad (1)$$

where $A \exp\left(-\frac{\Delta G}{kT}\right)$ is the reference strain rate for a given temperature, m is the strain rate sensitivity, s is the internal variable; and $\sinh\left(\xi\frac{\bar{\sigma}}{s}\right)$ accounts for slip due to thermal activated dislocation motion. The evolution equation for the internal variable is taken to be:

$$\dot{s} = \left[h_0\left|1 - \frac{s}{s^*}\right|^{l} sign\left(1 - \frac{s}{s^*}\right)\right]\dot{\varepsilon}^{p} \qquad (2)$$

where ho is the reference hardening, s * is the saturation value of the deformation resistance.

Rate independent plasticity is usually characterized by a yield stress. For Von Mises or J2 plasticity, the yield surface is a cylinder in principal stress space or a sphere in deviatoric stress space. Feasible stress states are constrained to lie in the interior or on the constraint or yield surface. If the stress state is on the yield surface and a stress increment is applied with a component normal to the yield surface, then a plastic strain increment proportional to the stress increment is produced. The direction of the plastic strain increment is normal to the constraint surface in associated plasticity. With strain hardening, the yield strength increases and the constraint surface expands. The equations for rate independent plasticity are well known and will not be described here. The choice of material properties for rate independent plasticity is discussed in the literature on computational weld mechanics.

The rate dependent behavior in the Ashby diagram, strain rate, flow stress and temperature space can be related to the diagram in **Fig. 2**. The strain rate is computed with Equation 1. The dimensionless flow stress $\xi\bar{\sigma}/s$ and inverse rate sensitivity 1/m are taken as parameters. The strain rate is normalized for all values of rate sensitivity. A steep gradient of the strain rate is observed for lower rate sensitivity that corresponds to lower temperatures. In the limit of a very small rate sensitivity or a very large (1/m) the strain rate becomes a step function, i.e. , the rate independet yield behavior is observed.

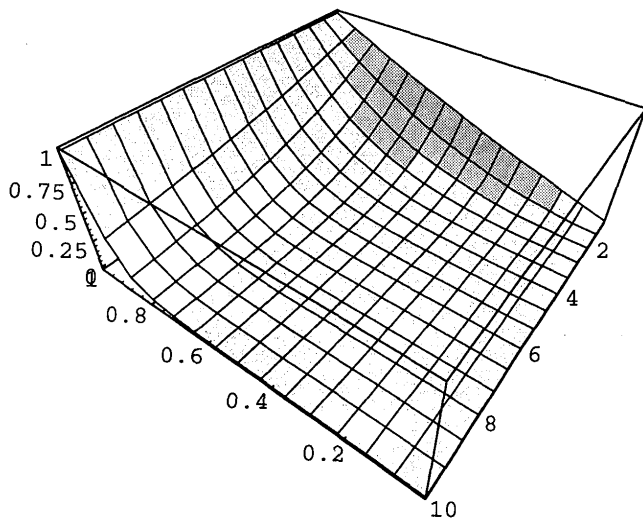At temperatures near the melting point,

Figure 2. The effective plastic strain rate is the vertical axis. The axis from 10 to 2 is $1/m$, that is the strain rate sensitivity in eq. 2. The axis labeled from 0.8 to 0.2 is a dimensionless flow stress. The figure shows that as $1/m$ increases from 2 to 10, the effective plastic strain rate becomes small except for dimensionless stresses near 1. This corresponds to lowering the temperature from say 0.8 of the melting point to say 0.5 of the melting point.

say from 0.8 to 1.0 of the melting temperature, the deformation resistance approaches zero. Physically this means that in reasonably stress would decay to zero. This regime is well described by a linear viscous or Maxwell model. However, the viscosity is not that of the liquid, it is the viscosity of solid due to thermal activated diffusion of mobile dislocations in the presence of a stress field.

$$\dot{\varepsilon}^p = (1/v)\sigma; \qquad (3)$$

where $\dot{\varepsilon}^p$ is the effective plastic strain; v is the viscosity; σ is the stress. For the linear viscous model, the material property is viscosity of the solid near the melting point. We have defined the viscosity to be

$$v = \frac{kT}{v_D \rho |b| \varsigma v^*} \exp\left(\frac{\Delta G}{kT}\right)$$

where v_D the Debeye frequency of thermal fluctuations, ρ density of mobile dislocations, $|b|$ length of the Burger's vector, ς average jump

distance of one dislocation segment in one jump event, v^* activation volume, ΔG activation energy for self diffusion.

The point of this rather detailed exposition of rate dependent and rate independent plasticity is to show that a rather close relationship exists between them. In particular, equating deformation resistance of rate dependent plasticity and yield strength of rate dependent plasticity at a temperature of 0.5 of the melting point is not unreasonable. Details of the implementation can be found in[2, 3]. The theory of various constitutive equations is discussed in some depth in[5].

2. Changing Constitutive Equations in Time and in Space

As stress evolves with time, a new problem is solved for each time step. Suppose that in timestep n, rate dependent plasticity is used at Gauss point m. Then in time step n+1, suppose rate independent plasticity is used at Gauss point m. Does this discontinuity or switch of the constitutive equations cause problems? The answer is no. The reason is that the initial conditions required for each time step are the initial geometry, initial stress, initial strain, the boundary conditions from tn to tn+dt and the constitutive equations in the interior of the time step. If the time step is from tn to tn+dt, the constitutive equation must be defined only for time t such that tn < t <= tn+dt. There is no need to define the constitutive equation at time tn.

Suppose a rate independent constitutive equation is used at one Gauss point and at a neighboring Gauss point a rate dependent constitutive equation is used. Does this cause problems in the analysis? The answer is no. At each Gauss point, the analysis gives the Gauss point an initial stress, values of the internal variable such as yield stress or deformation resistance and strain rate and asks for the stress at that point at the end of the time step. Each Gauss point can have its own constitutive equations and it causes no problems and introduces no complexity into the analysis other than the capability of switching between constitutive equations and providing the mappings internal variables such as yield strength to deformation resistance.
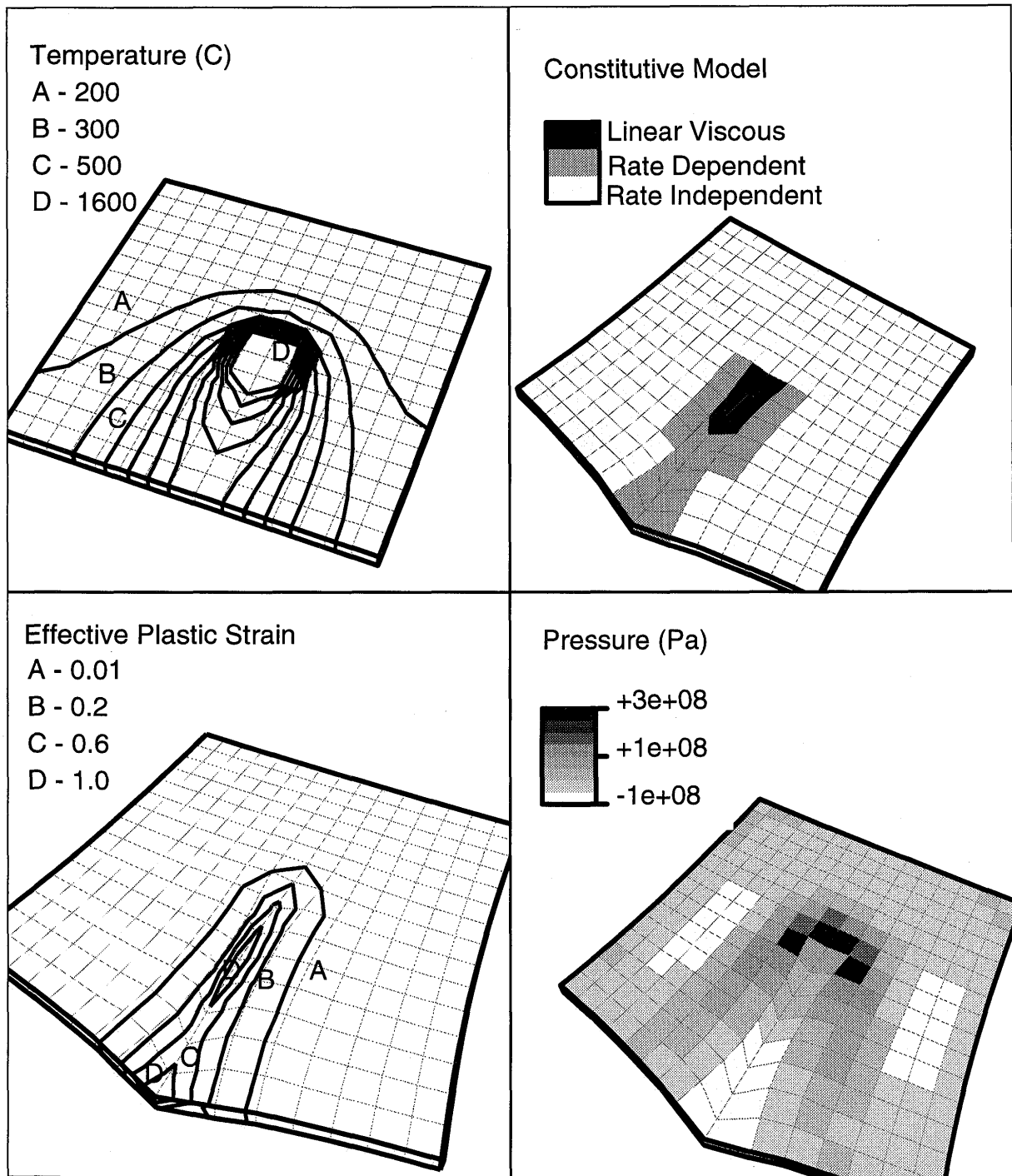
Figure 3. Three seconds after starting the weld, the transient temperature, constitutive equation type, effective plastic strain and pressure are shown.

3. An Example Weld Analysis

To demonstrate this method, results of a thermal stress analysis are presented. In this weld analysis a prescribed temperature weld heat source was used to model the arc. The maximum temperature was 2000C and the melting temperature was 1500C. The weld pool dimensions were 1.5cm long by 1.5cmwide and 0.5cm deep. The weld speed was 0.83cm/s. The materials properties are typical of a silicon steel. See [2] or [3] for details of the solution methods. The plate dimensions were 5x5x0.2cm. The mesh was 15X15X1 elements. Eight node bricks with

enhanced strain modes were used.

Time is discretized so that the weld pool travels about one element length per time step for the energy equation. For each time step for the energy equation, twenty time steps are taken to solve the momentum equation. Linear interpolation of temperature with in a time step is used to obtain stress related material properties. The stress time step is the stress relaxation time in solid around the melting point.

Results are shown in **Fig. 3**. Space does not permit more figures to be shown. See [2, 3] for related results. Note that the rate dependent plasticity constitutive equation is only applied on the sides of the HAZ. In front of the weld pool, the temperature rise is so rapid that this mesh is too coarse to capture the thin rate dependent plastic zone. Note that the pressure is highest in this zone. Also note that the cooling weldment pulls metal back from the weld pool. The effective plastic strain reaches a maximum some distance behind the weld pool.

4. Conclusions

A method for stress analysis of welded structures for the temperature range from absolute zero to the melting point and above has been presented. The issue of changing constitutive equations with time and with space has been discussed. The issue of estimating numerical values for parameters in constitutive equations has been discussed. Numerical results for thermal stress analysis of a weld have been presented to demonstrate the effectiveness of the method.

The authors believe that this methodology is a significant step towards computing stress and strain in welds from the weld pool to room temperature using physics as we know it.

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