Restraint Intensity of Weld Joints in Structural Elements†

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Abstract

Weld cracking is one of the most important problems which influence the qualities of welds in steel constructions. To evaluate the possibility of weld cracking, actual restraint intensity in weld joints must be known.

In welding steel constructions, the intersection of butt weld joints and fillet weld joints is often experienced and is quite susceptible of weld cracking. At the intersection, it is usual practice to defer a certain length of fillet welds for preventing weld cracking.

This paper presents a practical method of estimation of local restraint intensity in the intersection of butt weld joint and fillet weld joint. In the method the authors introduce a concept of the effective width which gives a width, in the plate, affected by the restraint force acting at the intersection. The effective width is decided by the dimensions of the joint and the length of weld deferment.

The accuracy of the proposed method is verified for actual measurements of the restraint intensity in a butt weld joint of a deck longitudinal stiffener of an actual ship and the results obtained from the proposed method are very much close.

1. Introduction

Weld cracking is one of the most important problems which influence the qualities of welds in steel constructions. One of the authors has been long engaged in a fundamental study on the influence of restraint in weld joints on cold cracking in the case of a butt weld restrained between two restraints with a distance l as illustrated in **Fig. 1 (a).** He proposes two methods for estimating weld cracking susceptibility of weld joints; one is to compare the restraint intensity of the actual weld joint K_{ac} with the critical restraint intensity K_{cr}, which is obtained from the RRC test[1), 2), 3)] and the other is to compare the cooling time in actual welding $(tc)_{ac}$ with the critical cooling time $(tc)_{cr}$, which is obtained from the weldment cracking parameter Pw[4)]. For evaluating the possibility of weld cracking by these two methods, the actual restraint intensity must be known as it is a parameter in these methods.

In welding steel constructions such as ships, bridges, building frames, etc. restraint conditions as shown in **Fig. 1 (b)** and **(c)** are often experienced. When the welding of the butt joint is done after the fillet welds, high restraint will be encountered at the intersection of butt weld joint and fillet weld joint. The restraint is caused because the contraction of the weld is hindered by plate II through the fillet welds. In practice, a certain unwelded length of fillet welds Δ (called "weld deferment") is provided to release the high restraint force and thus to prevent weld cracking. This paper precents a practical method to estimate the

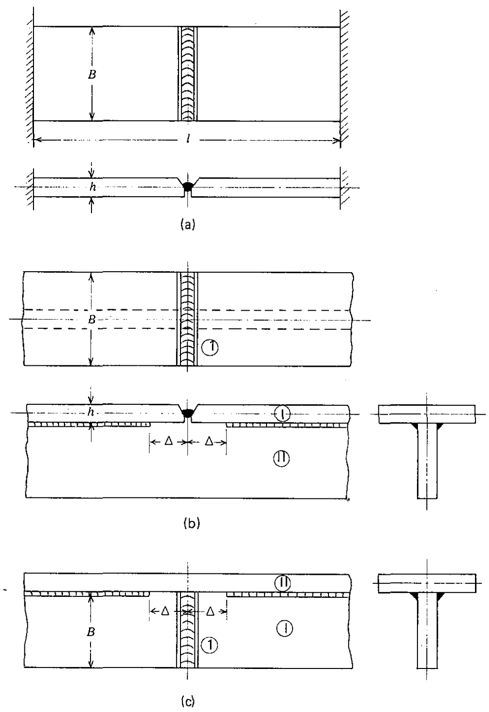

Fig. 1. Butt weld joint under restraint.

local restraint intensity K at the intersection of butt weld joint and fillet weld joint, having a weld deferment Δ.

The restraint intensity K of the butt weld joint in tensile restraint condition shown in **Fig. 1 (a)** is given by the following equation.[2)]

$$K \equiv \frac{P}{\lambda_b} \qquad (1)$$

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Where

P : restraint force per unit weld length (kg/mm)

λ_b : elongation of base plate caused by restraint force P, that is, contraction of root gap of weld joint (mm)

In other words, the restraint intensity K is defined as the force per unit weld length required to contract the root gap of the weld joint by a unit length.

The restraint intensity K of the butt weld joint in shear restraint condition as shown in **Fig. 1 (b)** and **(c)** can be determined by the equation (1).

2. Theory

The theoretical solution is derived by taking a weld joint in shear restraint condition shown in **Fig. 1 (b)** as an example. The condition in **Fig. 1 (c)** is identical with the condition in **Fig. 1 (b)** by taking the welding plate as plate I and the restraining plate as plate II.

At first, let us consider a semi-infinite beam consisting of plates I and II as shown in **Fig. 2 (a)**, the plates being fillet welded together. A tensile force P is applied at plate I (cross-sectional area A') and a compressive force P at plate II (cross-sectional area A'') at the end as shown in **Fig. 2 (a)**. The displacement between plates I and II will be derived with the following assumptions:

(1) the stresses in both plates I and II are elastic,
(2) at a section **S—S**, uniform force is acting in plates I and II,
(3) only shearing force is acting in fillet welds,
(4) fillet welds are in elastic condition and shearing displacement is directly proportional to shearing force:

$$T = D \cdot \delta \tag{2}$$

where

T : shearing force acting on unit length of fillet welds (kg-mm)

δ : shearing displacement (mm)

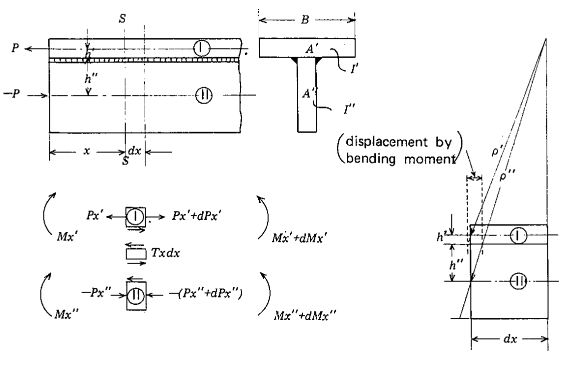


Fig. 2.

D : displacement coefficient of fillet welds (kg/mm·mm)

We now consider the equilibrium of a small element of length dx at the cross section S—S, a distance x from the end as shown in **Fig. 2 (a).** The equation of equilibrium of forces on the element:

$$T_x = -\frac{dP_x'}{dx} = \frac{dP_x''}{dx} \tag{3}$$

and the equation of equilibrium of forces and bending moments at the cross section **S—S:**

$$P_x' = -P_x'' \tag{4}$$

$$M_x' + M_x'' = P_x'(h' + h'') \tag{5}$$

where

P_x', P_x'' : forces acting on plates I and II at the cross section **S—S** (kg)

M_x', M_x'' : bending moments acting on plates I and II at the cross section **S—S** (kg-mm)

T_x : shearing force per unit length action on fillet welds at the cross section **S—S** (kg/mm)

h', h'' : distances from fillet welds to neutral axes of plates I and II (mm)

And the relation between the radius of courvature and bending moment is given by the following equations:

$$\frac{1}{\rho'} = \frac{M_x'}{EI'} \tag{6}$$

$$\frac{1}{\rho''} = \frac{M_x''}{EI''} \tag{7}$$

where

ρ', ρ'': radii of curvature of plates I and II (mm)

I', I'' : moments of inertia of the cross sections of plates I and II (mm^4)

E : Young's modulus of material (kg/mm^2)

Since at the location of fillet welds, the radii of curvature of plates I and II are the same as shown in **Fig. 2 (b),** $\frac{1}{\rho' + h'} = \frac{1}{\rho'' - h''}$. Since $\rho', \rho'' \gg h', h''$ $\frac{1}{\rho'} = \frac{1}{\rho''}$ apploximately. By substituting the equations (6) and (7) in (5) we obtain the following equation:

$$M_x' = \frac{I'}{I' + I''} \cdot P_x' \cdot (h' + h'') \tag{8}$$

As the shearing displacement of the fillet welds at the cross section **S—S** is equal to the displacement between plates I and II at the section, the following equation is given:

$$\delta_x = \int_x^\infty \frac{P_x'}{EA'}dx - \int_x^\infty \frac{P_x''}{EA''}dx + \int_x^\infty \frac{h'}{\rho'}dx - \int_x^\infty \frac{-h''}{\rho''}dx \quad \text{-------------} \quad (9)$$

where

δ_x : shearing displacement at the section **S—S** (mm)

By differentiating the equation (9) we get the following equation:

$$\frac{d\delta_x}{dx} = -(\frac{P_x'}{EA'} - \frac{P_x''}{EA''}) - (\frac{h'}{\rho'} + \frac{h''}{\rho''}) \quad \text{------} \quad (10)$$

By substituting the equation (3) in (2), the left side of the equation (10) becomes:

$$\frac{d\delta_x}{dx} = -\frac{1}{D}\frac{d^2P_x'}{dx^2}$$

and the right side of equation (10) becomes by employing the equations (4)∼(8):

$$-(\frac{P_x'}{EA'} - \frac{P_x''}{EA''}) - (\frac{h'}{\rho'} + \frac{h''}{\rho''}) = -\frac{P_x'}{E} \times [(\frac{1}{A'} + \frac{1}{A''}) + \frac{(h'+h'')^2}{I'+I''}]$$

Consequently, we obtain the following equation:

$$\frac{d^2P_x'}{dx^2} = \frac{D}{E}[(\frac{1}{A'} + \frac{1}{A''}) + \frac{(h'+h'')^2}{I'+I''}]\cdot P_x' \quad \text{--------------} \quad (11)$$

The general solution of the equation (11) is as follows;

$$P_x' = c_1 e^{x/x_0} + c_2 e^{-x/x_0} \quad \text{-----------------------------} \quad (12)$$

in which

$$1/x_0 = \sqrt{\frac{D}{E}[(\frac{1}{A'} + \frac{1}{A''}) + \frac{(h'+h'')^2}{I'+I''}]} \quad \text{----} \quad (13)$$

Substituting the boundary conditions

$x = 0, \quad P_x' = P$
$x = \infty, \quad P_x' = 0$

to the equation (12), the following equation is given:

$$P_x' = Pe^{-x/x_0} \quad \text{--------------------------------------} \quad (14)$$

from the equation (3),

$$T_x = \frac{P}{x_0}\cdot e^{-x/x_0} \quad \text{-------------------------------------} \quad (15)$$

When $\frac{x}{x_0} \geqq 4$, $\frac{P_x'}{P} \leqq 0.018 \ll 1$, therefore P_x' can be neglected. Thus, when the length of the fillet weld joint is more than $4x_0$, it can be deemed as an infinite length of fillet welds. The shearing displacement at the cross section **S—S** becomes as follows from the equations (2) and (15):

$$\delta_x = \frac{P}{Dx_0}\cdot e^{-x/x_0} \quad \text{-------------------------------------} \quad (16)$$

The shearing displacement λ_b' between plates I and II at the end by the restraint force P is given by the following equation:

$$\lambda_b' = \delta_0 = \frac{P}{Dx_0} \quad \text{--------------------------------} \quad (17)$$

where

δ_0 : shearing displacement at the end section (mm)

Next we consider the weld deferment Δ and the root gap G of the butt weld joint shown in **Fig. 1 (b).** Supposing that a uniform force P acts on plates I and II in the same way as the first case, the displacement λ_b'' between plates I and II can be given by the following equation:

$$\lambda_b'' = \frac{P}{E}(\frac{\Delta - G}{A'} + \frac{\Delta}{A''}) \quad \text{---------------------} \quad (18)$$

Accordingly the contraction of the weld joint by the restraint force P can be given by the following equation:

$$\lambda_b = 2(\lambda_b' + \lambda_b'') \quad \text{-------------------------------} \quad (19)$$

the restraint intensity K of the butt weld joint in shear restraint condition shown in **Fig. 1 (b)** and **(c)** can be given by the following equation:

$$K = P/(\lambda_b \cdot B_w) = 1/[2B_w\cdot\{\frac{1}{E}(\frac{\Delta - G}{A'} + \frac{\Delta}{A''}) + \frac{1}{Dx_0}\}] \quad \text{---------------------} \quad (20)$$

where

B_w : length of butt weld joint (mm)

Besides, when the bending moment by the restraint force P can be neglected, the equation (13) becomes

$$1/x_0 = \sqrt{\frac{D}{E}\left(\frac{1}{A'} + \frac{1}{A''}\right)} \quad \text{------------------------} (13')$$

Fig. 3 shows an example of numerical calculation of the relation between the restraint intensity K and the weld deferment $\varDelta$, where $\frac{D}{E} = 0.5$. The restraint intensity K increases remarkably as compared with the decrease of the weld deferment $\varDelta$. It is clear that the restraint intensity K is less when the bending moment is taken into consideration.

To confirm the accuracy of the theoretical solution of the restraint intensity of shear restrained condition shown in **Fig. 1 (b)** and **(c),** the following experiments were carried out. As shown in **Fig. 4,** two types of specimens were prepared; one was for the case in which the bending moment by the restraint force could be neglected (H-type specimen) and another for the case in which the bending moment could not be neglected (T-type specimen) with **Y**-shaped groove and 2 mm root gap and the size of the weld deferment was varied. The leg length of fillet welds was 3 mm.

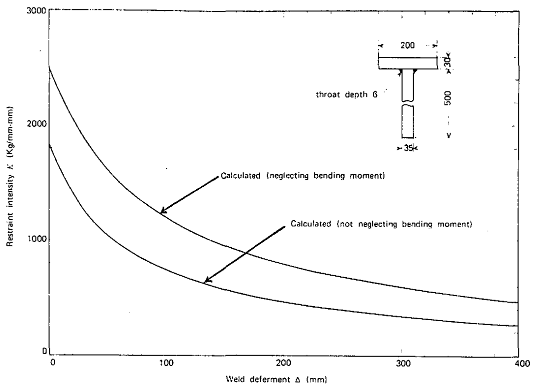


Fig. 3. Effect of weld deferment on restraint intensity.

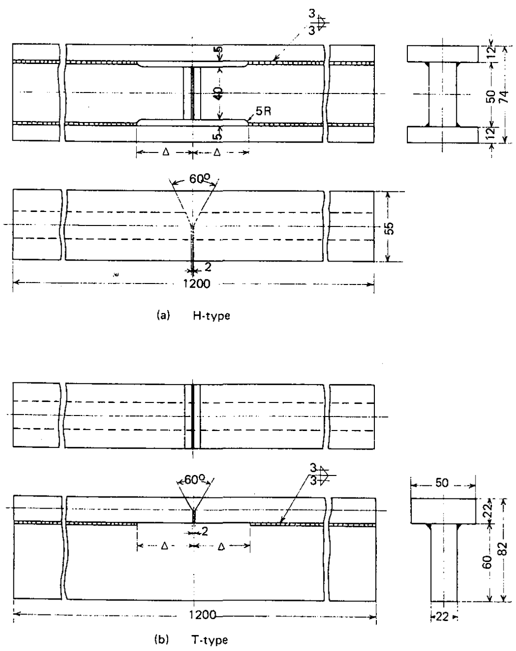


Fig. 4. Specimen design.

Then, the butt weld joint of the thus prepared specimens was welded with 4 mmϕ illuminite type electrodes; the welding conditions: 160 A welding current, 22~28 V arc voltage and 150 mm/min. welding speed.

After complete cooling, the butt welded joint was cut loose with a machine in the vicinity of the weld and the shrinkage of the weld at that moment was measured by means of a contact gauge to calculate the restraint stress σ_w acting on the butt weld.

Fig. 5 shows the numerical values of the weld deferment $\varDelta$ and the measured restraint stress σ_w. The values in the figure were calculated by the equations (13), (13′) and (20) and the relation of $\sigma_w = m \cdot K$[2] by taking $\frac{D}{E} = 0.5$, $m = 4.5 \times 10^{-2}$. The measured values and the calculated values show considerably good agreement, which suggests that the theoretical solution is appropriate. In **H**-type specimens with $\varDelta = 15$ mm, the measured restraint stress is low because of weld cracking. It was considered that the restraint intensity of these specimens would be higher than the critical restraint intensity value of mild steel (Kcr= 1560 kg/mm·mm[3]).

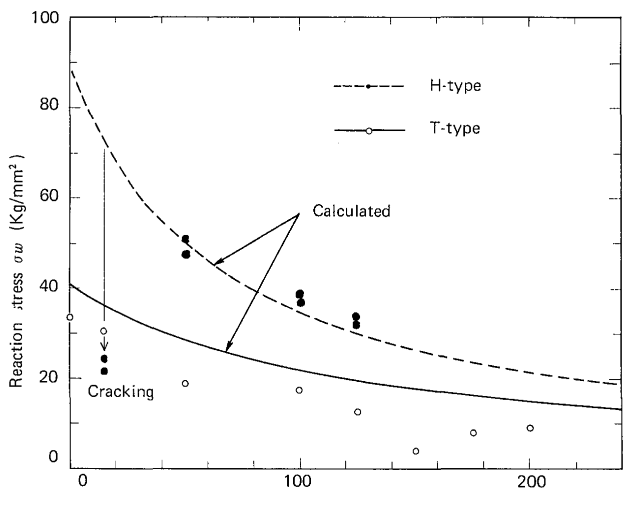


Fig. 5. Relation of reaction stress vs. weld deferment.

3. Effective width

In the theoretical solution as derived in section 2, the restraint intensity K increases in proportion to the increase of the width $2B$ of plate II in **Fig. 1 (c).** Consequently, when B becomes infinitely great, K becomes infinitely great. In reality, as the shrinkage of plate II is caused by the force transmitted through the fillet welds, it will occur only localy near the intersection of the fillet welds because the force acting at plate II is not even but local along the fillet welds. This means that when the width of plate II is relatively wide the actual restraint intensity at the intersection of butt weld joint and fillet weld joint will be much lower than that calculated with the actual width.

In this section a consideration is made on the limit of the width which affect the restraint intensity(named "effective width") when the width of plate II is relatively great, as shown in **Fig. 1 (c).**

First, let us consider an infinite plate acted by a shearing force Fdx' along the x-axis at the origin, as shown in **Fig. 6 (a).** The stresses of x-direction and y-direction are given by the following equations:[5)]

$$\sigma x = \frac{Fdx' \; Cos\theta}{4\pi r}\left[-(3+\nu)+2(1+\nu)\;Sin^2\theta\right]$$

$$\sigma y = \frac{Fdx' \; Cos\theta}{4\pi r}\left[(1-\nu)-2(1+\nu)\;Sin^2\theta\right]$$

where

Fdx' : shearing force per unit length (kg/mm)

On the A—A line,

$$r=\sqrt{x'^2+y^2}$$

$$Cos\theta = \frac{x'}{r} \quad (x'>0\;)$$

$$Sin\theta = \frac{y}{r}$$

so, the stress of x-direction on **A—A** line is as follows:

$$\sigma x = \frac{Fx'dx'}{4\pi(x'^2+y^2)}\left[-(3+\nu)+2(1+\nu)\frac{y^2}{x'^2+y^2}\right] \qquad \text{------------- (21)}$$

In the case of **Fig. 6 (b),** the stress of the x-direction on the y-axis is given by the equation (22) by taking $x' \rightarrow -x'$ in the equation (21):

$$\sigma x = \frac{Fx'dx'}{4\pi(x'^2+y^2)}\left[(3+\nu)-2(1+\nu)\frac{y^2}{x'^2+y^2}\right] \qquad \text{----------- (22)}$$

Also, for **Fig. 6 (c),** the same result as the equation (22) is derived.

Therefore, the stress of the x-direction on the y-axis in the case of **Fig. 6 (d)** is twice that of the equation (22):

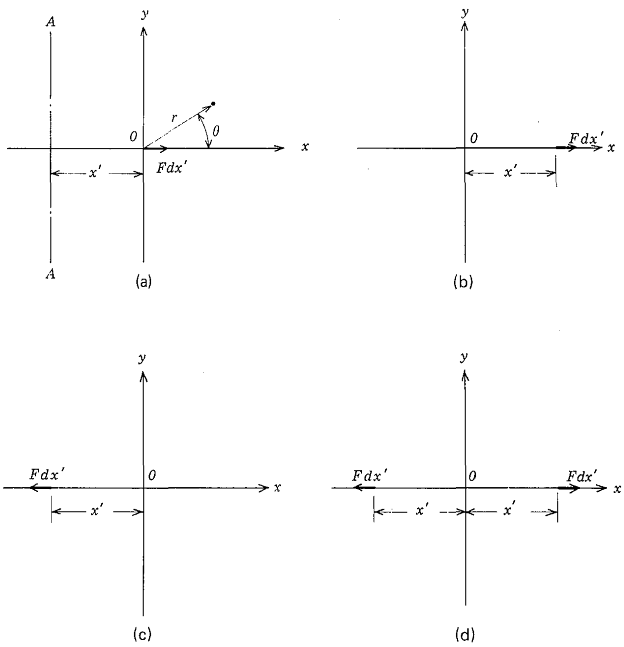


Fig. 6.

$$\sigma x = \frac{Fx'dx'}{2\pi(x'^2+y^2)}\left[(3+\nu)-2(1+\nu)\frac{y^2}{x'^2+y^2}\right]$$

Here, we take the following relation derived from the equation (14),

$$F = P\cdot e^{-\beta(x'-\Delta)}$$

where

$$\beta = 1/x_0$$

the stress of the x-direction on the y-axis due to the total shearing force of fillet welds can be expressed by the following equation:

$$\sigma x = \int_\Delta^\infty \frac{Pe^{-\beta(x'-\Delta)}x'dx'}{2\pi(x'^2+y^2)}\left[(3+\nu)-2(1+\nu)\frac{y^2}{x'^2+y^2}\right]$$

$$= \frac{Pe^{\beta\Delta}}{2\pi}\left[(3+\nu)\int_\Delta^\infty \frac{e^{-\beta x'}x'dx}{x'^2+y^2} - 2(1+\nu)\,y^2 \times \int_\Delta^\infty \frac{e^{-\beta x'}x'dx'}{x'^2+y^2}\right]$$

The x-direction stress at the origin is follows by taking y=0:

$$(\sigma_x)_{y=0} = \frac{Pe^{\beta\Delta}}{2\pi}(3+\nu)\int_\Delta^\infty \frac{e^{-\beta x'}}{x'}dx' \qquad \text{----------- (23)}$$

Letting

$$\beta x' \equiv u$$

we have

$$\int_{\Delta}^{\infty} \frac{e^{-\beta x}}{x'} dx' = \int_{\beta\Delta}^{\infty} \frac{e^{-u}}{u} du = -Ei(-u)$$

where

Ei: integral expornential function

Then, the equation (23) is expressed as follows:

$$(\sigma_x)_{y=0} = \frac{Pe^{\beta\Delta}}{2\pi}(3+\nu)\{-Ei(-\beta\Delta)\} \quad \text{--------- (24)}$$

In the same manner, the y-direction stress at the origin is expressed by the following equation:

$$(\sigma_y)_{x=0} = \frac{Pe^{\beta\Delta}}{2\pi}(\nu-1)\{-Ei(-\beta\Delta)\} \quad \text{-------- (25)}$$

Therefore, the x-direction strain at the origin is expressed by the following equation:

$$(\varepsilon_x)_{y=0} = \frac{1}{E}\{(\sigma_x)_{y=0} - \nu\cdot(\sigma_y)_{x=0}\}$$

$$= \frac{Pe^{\beta\Delta}}{2\pi E}(3+2\nu-\nu^2)\{-Ei(-\beta\Delta)\} \quad \text{---- (26)}$$

Now, the total shearing force acting on the portion of fillet welds can be expressed by the following equation:

$$\int_{\Delta}^{\infty} Fdx' = Pe^{\beta\Delta}\int_{\Delta}^{\infty} e^{-\beta x'} = Px_0 \quad \text{---------------- (27)}$$

Here, the effective width Beq is defined as half the width of a rectangular plate which causes the even strain $(\varepsilon_x)_{y=0}$ when the tensile force Px_0 acts on it, as shown in **Fig. 7 (a)** and **(b).** In this manner, the elongation of plate II along the weld deferment Δ is estimated on a lower side and this means that the restraint intensity is estimated on a higher side. This, however, can be taken to be on the safe side in plactical cases. Namely,

$$(\varepsilon_x)_{y=0} \cdot E = \frac{Px_0}{2\,Beq} \quad \text{------------------------------ (28)}$$

Substituting the equations (26) and (27) in (28), the effective width Beq can be expressed by

$$Beq/x_0 = \pi/[(3+2\nu-\nu^2)\{-Ei(-\Delta/x_0)\}\, e^{\Delta/x_0}] \quad \text{------------------------ (29)}$$

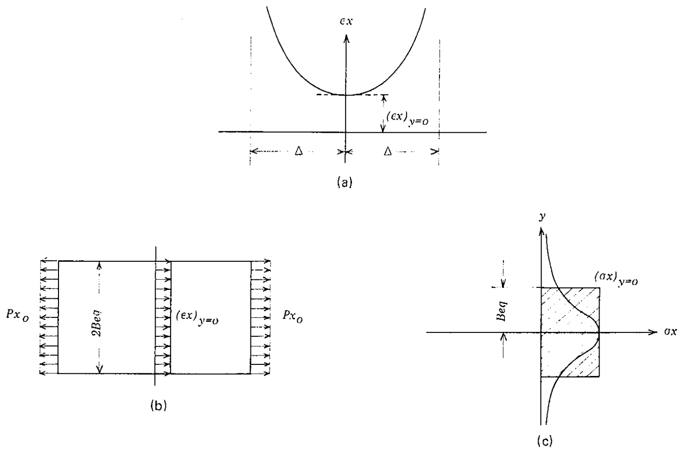


Fig. 7.

In the equation (29), the effective width Beq is derived from the strain theory. However, it can be got from the stress theory, that is, by dividing the total x-direction stress acting on the y-axis by $(\sigma_x)_{y=0}$ as shown in **Fig. 7 (c).** In the stress theory, the effective width Beq is given by the equation (31), using the equations (24) (27) and (30)·

$$(\sigma_x)_{y=0} \cdot 2\,Beq = Px_0 \quad \text{-------------------------- (30)}$$

$$Beq/x_0 = \pi/[(3+\nu)\{-Ei(-\Delta/x_0)\}\, e^{\Delta/x_0} \quad \text{----- (31)}$$

The relation between Beq/x_0 and Δ/x_0 can be shown as a graph; see **Fig. 8**

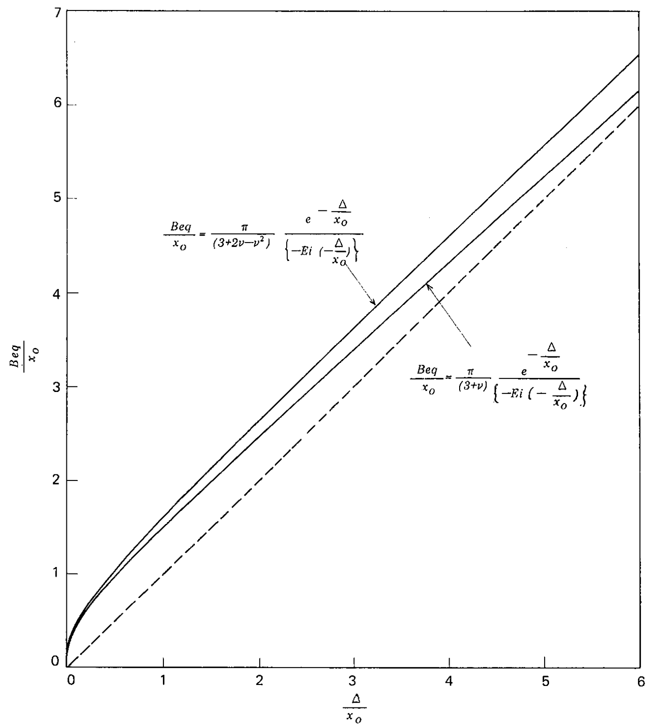


Fig. 8. Relation between Beq/x_0 and Δ/x_0.

In equation (13), letting

$$A' = 2h't$$
$$A'' = 2h'' \cdot 2Beq$$
$$I' = \frac{2th'^3}{3}$$
$$I'' = \frac{4Beq\,h''^3}{3}$$

we have

$$1/x_0 = \sqrt{\frac{D}{E}\{(\frac{1}{2h't} + \frac{1}{4h''Beq}) + \frac{3(h'+h'')^2}{2th'^3+4\,Beq\,h''^3}\}} \qquad \text{------------------ (32)}$$

This equation indicates that the effective width Beq is determined by the dimensions of weld joints.

The theoretical solution so far obtained gives the average restraint intensity of the butt weld joint in shear restrained condition but the serious subject is the local restraint intensity of the butt weld joint at the intersection of the fillet weld joint where weld cracking sometimes occurrs in practical welds.
So, for this problem we propose the following hypothesis;

"Even if the restraining force by welding acts on the butt joint, the restraining force which gives much influence to the local restraint intensity at the intersection will not be the remove force but the neighboring. Therefore, further expanding the concept of the effective width we assume an effective width Beq for plate I too which is the same as the value calculated for plate II. It is assumed that only the restraining force whithin the limit of the effective width of plate I affects the restraint intensity at the intersection."

Then in the equation (13), letting

$$A' = Beq \cdot t$$
$$A'' = 2h''Beq$$
$$I' = \frac{1}{12}\,Beq^3 \cdot t$$
$$I'' = \frac{4}{3}\,Beq\,h''^3$$

we have

$$1/x_0 = \sqrt{\frac{D}{E}\{(\frac{1}{Beq\,t} + \frac{1}{4h''\,Beq}) + \frac{(Beq/2+h'')^2}{\frac{1}{12}Beq^3\,t + \frac{4}{3}\,Beq\,h''^3}\}} \qquad \text{---------- (33)}$$

4. Experiments with actual structure

To evaluate the theoretical solution, measurements of actual local restraint intensities at the intersection of butt weld joint and fillet weld joint were taken in an actual structure. It was a deck longitudinal stiffener with a butt weld joint, located beneath the upper deck of a 230,000 DW-ton tanker under construction; the weld deferment length of the fillet welds connecting the longitudinal to the deck plate was varied (see **Fig. 9**). The consumable nozzle electroslag welding method (CES welding method) is used to this kind of joint; a 25 mmϕ hole is made in the deck plate right above each butt weld joint of the longitudinal for the nozzle. The butt weld joint of deck plates was welded with 3-layer shielding welds to prevent the displacement of the deck plates accompanying the shrinkage during the CES welding. High restraint intensity being presumed, a 35 mmR scallop is provided, as it is shipbuilding practice, at the upper end of the joint, as shown in **Fig. 9 (b)**. The neighboring longitudinals on both sides of the test longitudinal being welded together in advance, the measured butt weld joint was considered to be in a most severe restraint consition.

The tests were conducted by pulling the joint by means of oil jacks. The actual restraint intensity was calculated from the shrinkage of the gap and the applied force, both actually measured. Restraining fixtured were installed to pull the joint and crip gauges to measure the shrinkage of the gap (see **Figs. 10** and **11**). The fixtures were on 6 positions near the joint, 3 positions on either side of the stiffener and the oil jacks were arranged to get an equal pressure. The pressure acting on each jack was measured by a pressure gauge. **Fig. 12** shows the arrangement of the

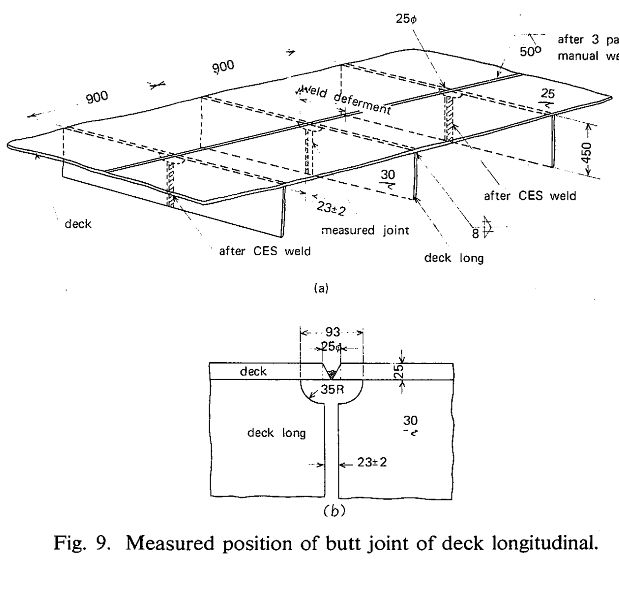


Fig. 9. Measured position of butt joint of deck longitudinal.

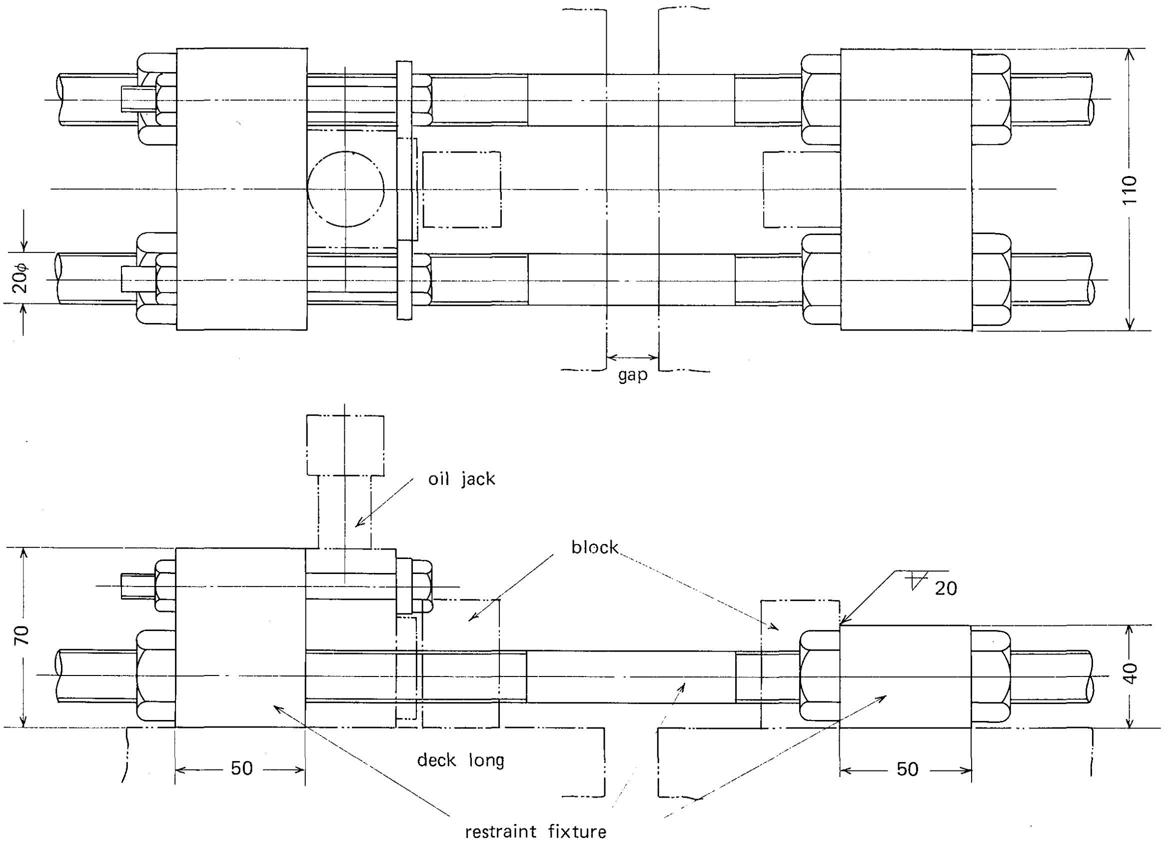


Fig. 10. Schematic view of restraining fixture.

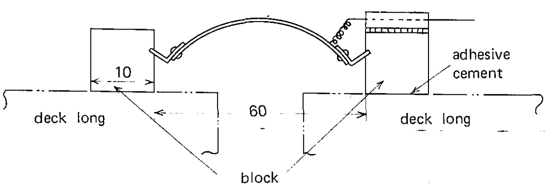


Fig. 11. Schematic view of clip gauge.

fixtures and the clip gauges.

As shown in **Table 1,** the measurement was carried out the same joint by varying the weld deferment Δ. The fillet welds were made with 6 mmϕ low hydrogen type electrodes (50 kg/mm^2), with a leg length of about 8 mm. At both ends of the fillet welds 3-axis strain gauges were affixed to check and keep the pressure loading within the elastic condition of the fillet welds.

Fig. 13 illustrates an example of measurement for the case of Δ=100 mm. **Fig. 13 (a)** shows the distribution of the gap shrinkage and the force value in the figure means the force per unit joint length which was calculated by dividing the total load of the oil jacks by the butt joint length (450 mm). **Fig. 13 (b)** shows the distribution of the local restraint intensity which was calculated depending upon the values given by **Fig. 13 (a).** At the top of the butt weld joint is a high restraint intensity value.

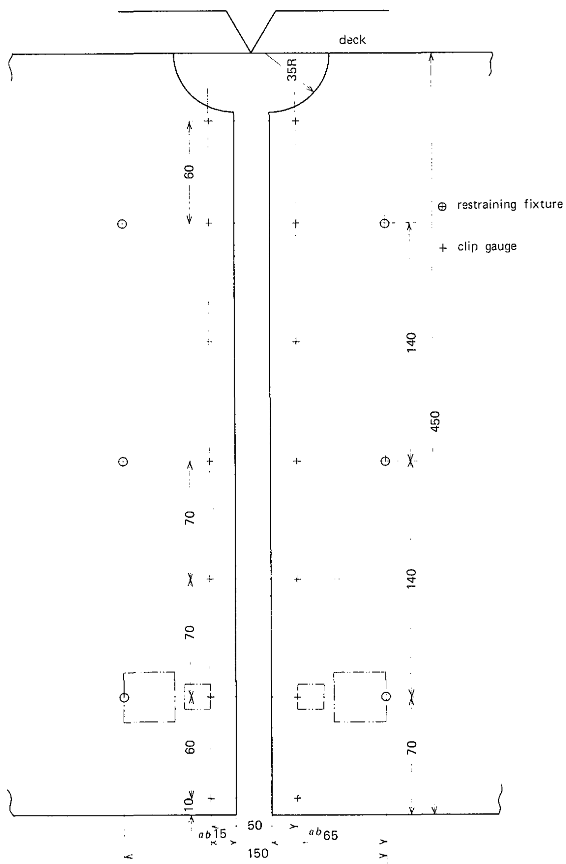
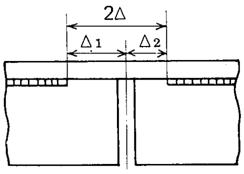


Fig. 12. Arrangement of restraining fixture and clip gauge.

Table 1. Weld deferment.

test No.	Δ1 (afterward)	Δ2 (forward)	2Δ
1	300 (mm)	46.5 (mm)	346.5 (mm)
2	150	150	300
3	100	100	200
4	46.5	46.5	93



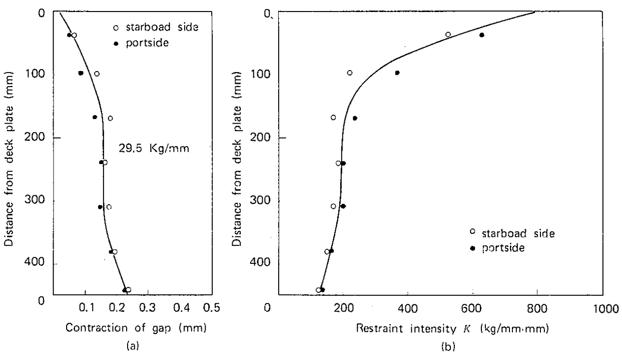


Fig. 13. Contraction of gap and restraint intensity ($\varDelta$=100).

Fig. 14 show the relation between the weld deferment $\varDelta$ and the effective width *Beq* calculated by applying the effective width to the deck plate and the logitudinal plate. For the sake of convenience, *Beq* was derived from the strain theory and $\frac{D}{E}=0.2$ was employed.

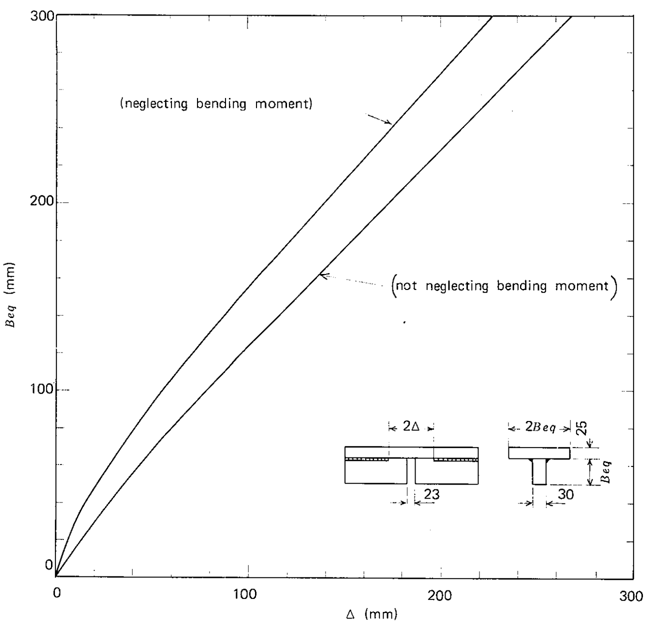


Fig. 14. $\varDelta$ *vs*. *Beq*.

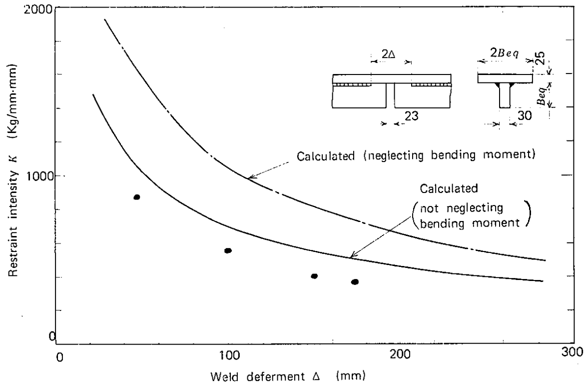


Fig. 15. Effect of weld deferment on restraint intensity.

Fig. 15 shows the relation between the local restraint intensity at the intersection and the weld deferment. The values in the figure were calculated by the equation (33), employing the *Beq* in **Fig. 14.** It shows that the local restraint intensity decreases as the weld deferment increases.

Because of the fact that the effective width was taken on the safe side and the assumption that uniform force acts in plates I and II, the calculated values of the local restraint intensity are considerably higher than the measured values. However, the proposed method will serve satisfactorily in estimating the local restraint intensity of weld joints in connection with the susceptibility of weld cracking.

5. Estimation of restraint intensity in structural members

In section 4, it was learned that the theoretical solution of the local restraint intensity, which introduced the concept of the effective width, would give slightly higher values than the actually measured ones. Therefore, in the evaluation of the local restraint intensity of the weld joint in actual structural members with regard to weld crack susceptibility, the theoretical solution can be applied on the safe side.

In this section, we try to apply this consept of the effective width to a general shear restraint butt weld joint and to derive a general equation for estimating the local restraint intensity at the intersection of butt weld joint and fillet weld joint. In introducing the general equation, the bending moment by restraint force and the root gap of the weld joint are neglected.

Let us consider the following geometry for the joint:

$$\frac{D}{E}=\mu$$

$$A'=a_1\,Beq\,h_1$$
$$A''=a_2\,Beq\,h_2$$

$$\frac{h_2}{h_1}=\beta$$

where

a_1, a_2: coefficients of effective width with respect to plates I and II (=1 or 2)

h_1, h_2: thicknesses of plates I and II (mm)

then, the equation (13′) becomes as follows:

$$1/x_0=\sqrt{\frac{J}{Beq\ h_1}} \quad \text{-------------------------------- (34)}$$

where

$$J=\mu J_1$$

$$J_1=(1+\frac{a_1}{a_2\beta})/a_1$$

The theoretical equation of the local restraint intensity is derived as follows:

$$K_0=E\mu/(2a_1\cdot\frac{Beq}{x_0}) \quad \text{---------------------------- (35)}$$

$$K=K_0/(1+\frac{\Delta}{x_0})=E\mu/\{2a_1\cdot\frac{Beq}{x_0}\cdot(1+\frac{\Delta}{x_0})\} \quad \text{-- (36)}$$

where

K_0: the local restraint intensity when weld deferment $\Delta=0$ (kg/mm·mm)

Consequently, the local restraint intensity at the intersection of butt weld joint and fillet weld joint is calculated by the values of μ, $\frac{\Delta}{x_0}$ and $\frac{Beq}{x_0}$, which are decided by the geometry of the joint. For example, when these values are calculated by the equation (29), or (31):

$$\frac{\Delta}{x_0}=a$$

$$\frac{Beq}{x_0}=b$$

substituting these values in the equation (34), the following relation is derived:

$$\frac{\Delta}{h_1}\cdot J=a\cdot b$$

That is, among these values, $\frac{\Delta}{x_0}$, $\frac{Beq}{x_0}$, and $\frac{\Delta}{h_1}\cdot J$, there is a correlation, and if one of these values is decided, the others are settled naturally. **Figs. 16** and **17** show the relations of $\frac{\Delta}{x_0}$, $\frac{Beq}{x_0}$, $\frac{Beq}{\Delta}$, $\frac{Ka_1}{E\mu}$, to $\frac{\Delta}{h_1}J$, i. e. the local restraint intensity K is obtained readily from the values of $\frac{\Delta}{h_1}J$ using **Figs. 16** and **17.** There is almost no difference in the local restraint intensity whether the effective width Beq is obtained from the strain theory or the stress theory.

Fig. 18 explains how to choose the coefficients of the effective width a_1 and a_2 in actual application. The suffixes 1 and 2 are fixed in the following manner: 1 for plate I on which tensile stress acts owing to the shrinkage of welds and 2 for plate II on which com-

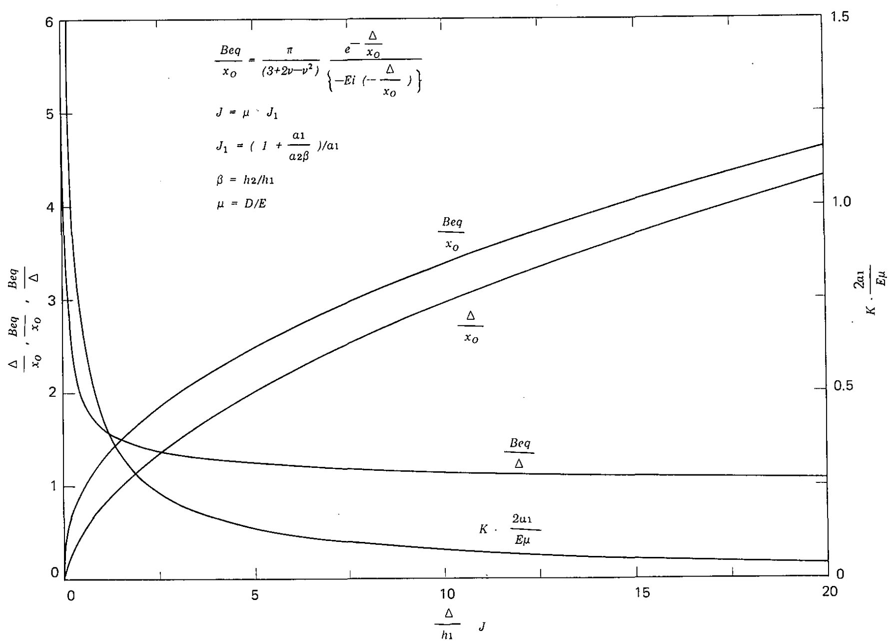


Fig. 16. $\frac{\Delta}{x_0}$, $\frac{Beq}{x_0}$, $\frac{Beq}{\Delta}$, vs. $\frac{\Delta}{h_1}\cdot J$ (strain)

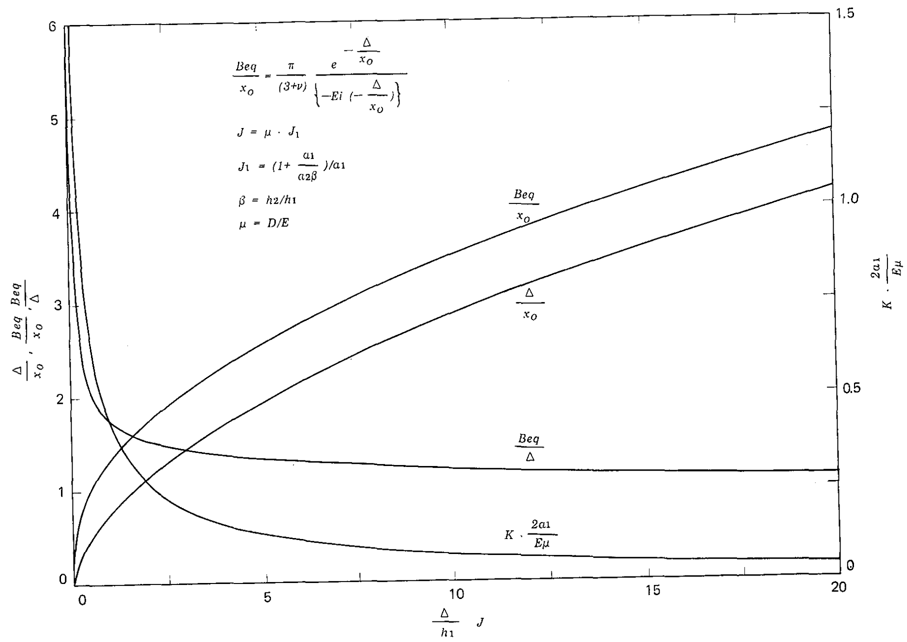


Fig. 17. $\frac{\Delta}{x_0}, \frac{Beq}{x_0}, \frac{Beq}{\Delta}$ vs. $\frac{\Delta}{h} J$ (stress).

pressive stress acts owing to the same. Concerning the values of the coefficients of the effective width a_1 and a_2, 1 is applied when the fillet welds are located at the edge of plate and 2 is applied when they are located in the middle of plate.

Fig. 19 illustrates the relation between Ka_1 and $\frac{h_1}{\Delta J_1}$ based on the effective width from the stress theory in the cases of $\mu=\frac{1}{6}$ and $\mu=\frac{1}{3}$. Naturally, the local restraint intensity becomes lower as the μ value becomes lower.

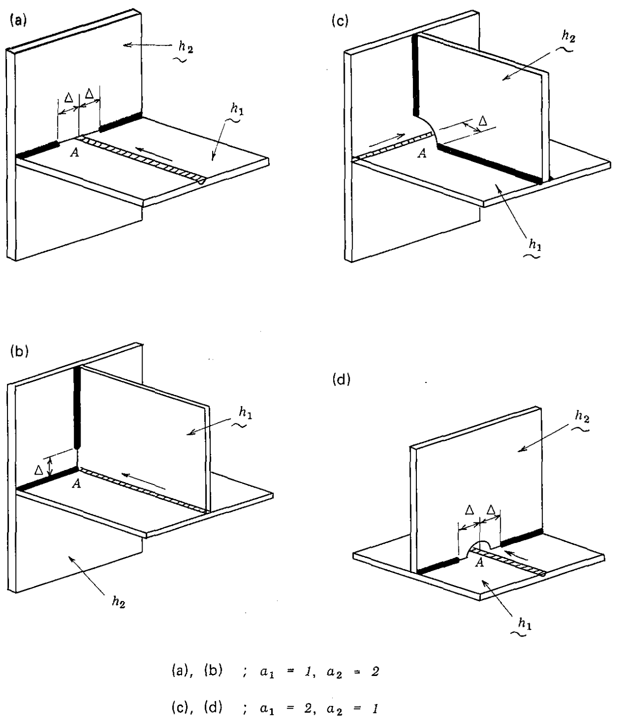


Fig. 18. Example of weld joint restrained by shearing force through fillet welds.

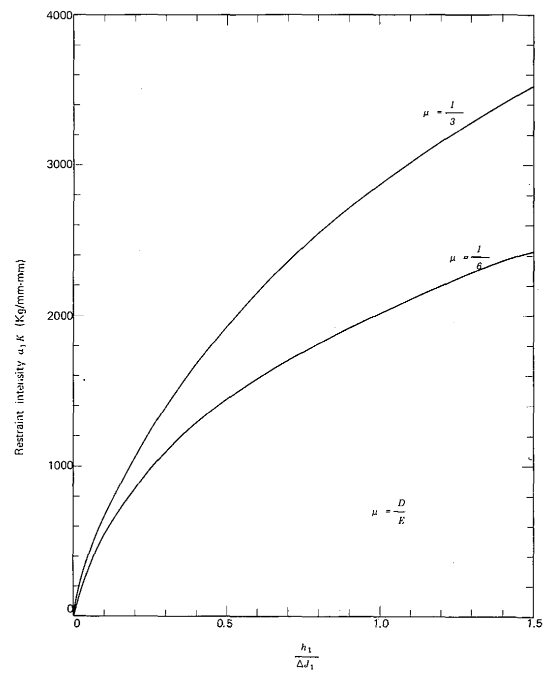


Fig. 19. Restraint intensity at the intersection of butt and fillet Joints.

Table 2. Restraint intensity of the weld joint in steel construction ($a_1=1$)

type of steel construction	location of weld joint	plate thickness h_1 (mm)	coefficient of restraint K_o (Kg/mm²·mm)	restraint intensity * K (Kg/mm·mm)
ships[6]	side shell	20	44.3	890
	bottom shell	28	24.7	690
	bottom shell	28	26.1	730
	bottom shell	28	28.0	780
	upper deck	32	39.9	1280
	upper deck	32	38.2	1220
building[7] frames	pillar & beam	12	40.8	490
	pillar & beam	28	39.0	1092
	pillar & beam	34	18.5	629
bridges[4]	flange & diaphragm	40	—	600

note * $K = K_o h_1$

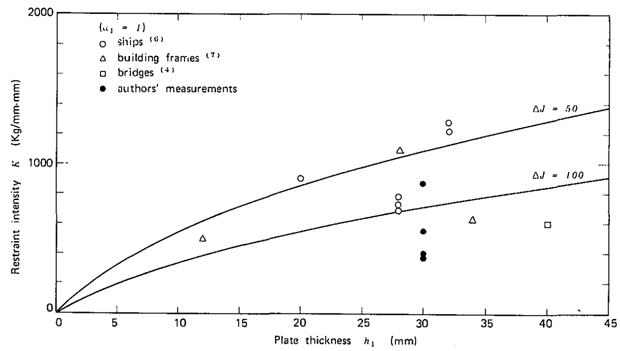


Fig. 20. Restraint intensity of the weld joint in steel constructions.

Table 2 gives a summary of publicized past data of actual measurement of the local restraint intensity in the intersection of butt weld joint and fillet weld joint in structural members. **Fig. 20** shows a graph of the past data and our measurements. The calculated values in the figure were computed on the condition of $\mu=\frac{1}{6}$. The plotted points in **Fig. 20** shows that the proposed method can give satisfactory results.

Thus, by applying the effective width to plates I and II, the estimation of the local restraint intensity in the intersection of butt weld joint and fillet weld joint which has been thought hitherto almost impossible, will become feasible.

6. Conclusion

For dealing with weld cracking in structural members, the general equation of estimating the local restraint intensity at the intersection of butt weld joint and fillet weld joint has been derived in the following manner.

(1) The theoretical solution of the restraint intensity in the butt weld joint which is restrained by shearing force in the fillet welds having a weld deferment Δ is derived. (equations (13), (13′) and (20))

(2) For the case in which the width of restraining plate is wide and great, the concept of the effective width is introduced. The effective width *Beq* is much influenced by the length of the weld deferment Δ. (equation (29) and (31))

(3) The concept of the effective width is further expanded to the welding plate to derive an equation for the local restraint intensity. Thus, the general equation of rough estimation for the local restraint intensity at the intersection of butt weld joint and fillet weld joint was derived. (equation (36))

(4) To confirm the accuracy of the derived general equation, actual measurements of the restraint intensity were made of a butt weld joint of a deck longitudinal stiffener of an actual ship. The results are sufficiently satisfactory. **(Fig. 15)**

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