

Title	On delta-unknottting operation
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Δ -unknotting operations is infinite.

Proof. We consider the Δ -unknotting operations $\Delta_n (n \geq 0)$ as illustrated in Figure 2.

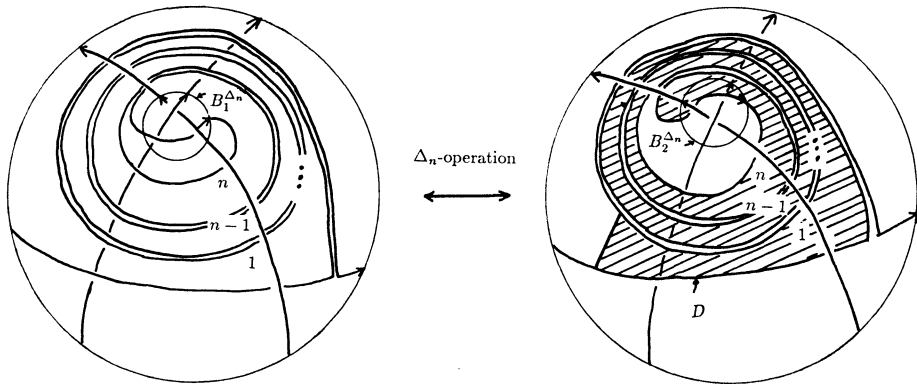


Figure 2

Considering the disk D in Figure 2, it is easy to show that k_{Δ_n} is ambient isotopic to k_{Δ_0} . Now we will prove that if $n \neq m$ then Δ_n is not homeomorphic to Δ_m .

We consider the following graph. (See Figure 3(a).) It is an embedding of the graph indicated in Figure 3(b). If Δ_1 is homeomorphic to Δ_2 , then there is a homeomorphism of S^3 such that $h(k) = k$, $h(G_{\Delta_1}) = G_{\Delta_2}$. To prove that G_{Δ_n} is not equivalent to G_{Δ_m} , it is sufficient to consider the three constituent knots, which spun all vertices, illustrated in Figure 3(c).

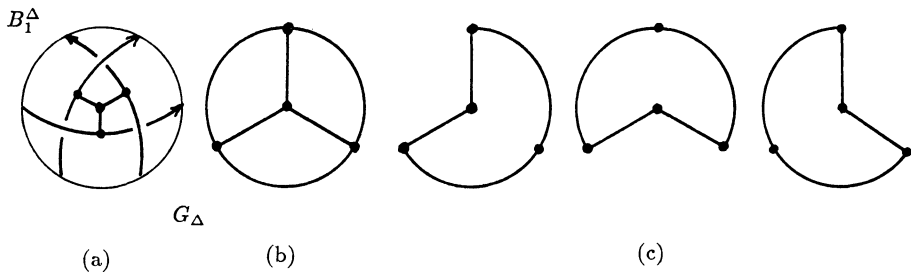


Figure 3

Since k is a knot, it is sufficient to consider two cases as indicated in Figure 4.

In the case (i), after moving by an ambient isotopy, G_{Δ_n} and its three constituents knots are illustrated in Figure 5. It is easy to show that $k_{n,1} \cong k_{m,1}$ and $k_{n,2} \cong k_{m,2}$. Now we will prove that $k_{n,3} \not\cong k_{m,3}$, if $n \neq m$. Let a_n be the second

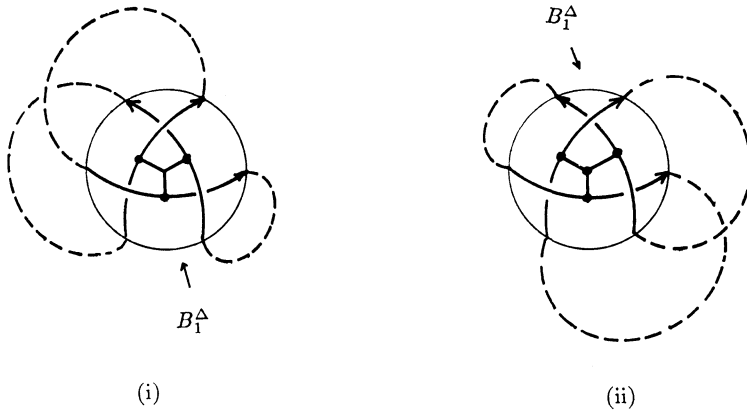


Figure 4

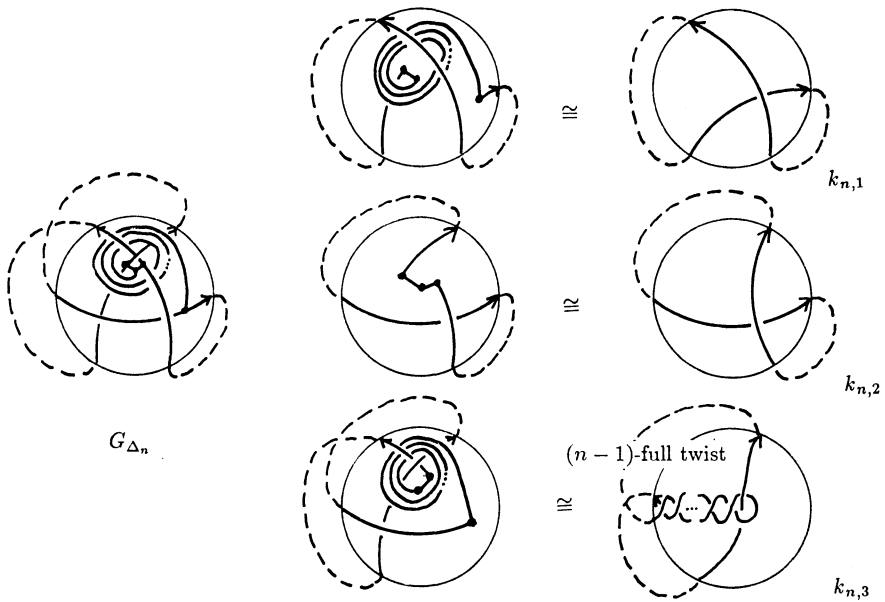


Figure 5

coefficient of the Conway polynomial of $k_{n,3}$. We have $a_n - a_{n-1} = 1$ i.e. $a_n = a_0 + (n-1)$. Then $k_{n,3} \not\cong k_{m,3}$ if $n \neq m$.

In the case (ii), we can prove that similarly. This completes the proof.

2. Note. In this section, we consider a Δ -unknotting operation as a local move on a knot diagram, ignoring the orientations [3]. Furthermore, we consider the mirror image of a Δ -unknotting operation as a Δ -unknotting operation, too. Suppose that Δ_l and Δ_r are like as illustrated in Figure 6, then

Δ_l and Δ_r are said to be twin-equivalent. The performances of Δ -unknotting operations on Δ_l and Δ_r are equivalent. Let k and k' be diagrams of a knot K , $\Delta(\Delta', \text{ resp.})$ Δ -unknotting operation for $k(k', \text{ resp.})$. Δ and Δ' are equivalent, write $\Delta \cong \Delta'$, if there exists a finite sequence $\{k_i, \Delta_i\}_{i=1,2,\dots,n}$ such that

- (1) Δ_i and Δ_{i+1} are Δ -unknotting operations of k_{i+1} ,
- (2) k_{i+1} is obtained from k_i by a combination of Reidemeister moves which fix Δ_i ,
- (3) Δ_i is twin-equivalent to Δ_{i+1} on k_{i+1} ,
- (4) $(k, \Delta) \cong (k_1, \Delta_1)$ and $(k', \Delta') \cong (k_n, \Delta_n)$,
- (5) (k_{i+1}, Δ_{i+1}) is obtained from (k_i, Δ_i) by the move illustrated as in Figure 7.

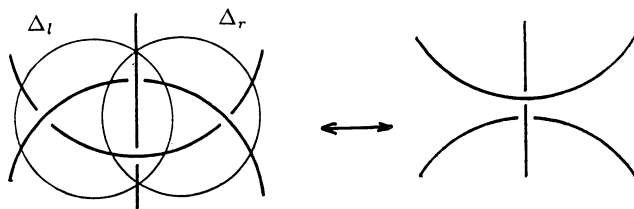


Figure 6

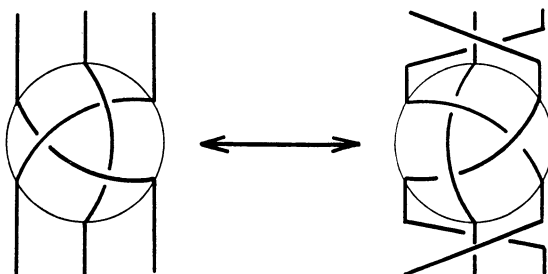


Figure 7

EXAMPLE 1. The knots as in Figure 8 have Δ -unknotting number one. The triangle regions marked by \blacktriangle are places to be performed by Δ -unknotting operations. For each knot, these Δ -unknotting operations are equivalent in the

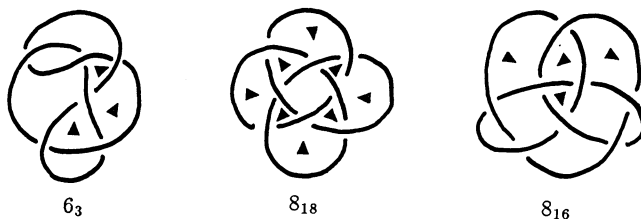


Figure 8

above sense.

EXAMPLE 2. Each Δ_n in the proof of Theorem is equivalent in the above sense.

Here, we raise the following problem.

Problem. *Let K be a knot with Δ -unknotting number one. Suppose that Δ and Δ' are Δ -unknotting operations which deform K into a trivial knot. Are Δ and Δ' equivalent in the above sense?*

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