

Title	Analysis of Temperature Distribution with Radial Symmetrical Cooling Terminal around Moving Heat Source
Author(s)	Matsunawa, Akira; Takemata, Hiroyuki; Okamoto, Ikuo
Citation	Transactions of JWRI. 7(2) P.275-P.277
Issue Date	1978-12
Text Version	publisher
URL	http://hdl.handle.net/11094/7261
DOI	
rights	本文データはCiNiiから複製したものである
Note	

Osaka University Knowledge Archive : OUKA

<https://ir.library.osaka-u.ac.jp/>

Osaka University

Analysis of Temperature Distribution with Radial Symmetrical Cooling Terminal around Moving Heat Source†

Akira MATSUNAWA*, Hiroyuki TAKEMATA** and Ikuo OKAMOTO***

Nomenclature: The following nomenclature will be used throughout this paper;

- q : heat input, [cal/s]
- q' : heat input per unit length (= q/h), [cal/s·cm]
- h : thickness of plate, [cm]
- v : speed of source, [cm/s]
- t : time, [s]
- T : temperature, [°C]
- T_0 : temperature at cooling terminal, [°C]
- T_f : reference temperature, [°C]
- K : heat conductivity, [cal/s·cm·°C]
- $1/2\lambda$: thermal diffusivity, [cm²/s]
- R : radius of cooling terminal, [cm]
- n : non-dimensional heat input (= $\lambda v q / 2\pi K (T_f - T_0)$)
- θ : normalized temperature (= $(T - T_0) / (T_f - T_0)$)
- Λ : non-dimensional radius of cooling terminal (= $\lambda v R$)
- ρ : normalized radius (= r/R) ($0 \leq \rho \leq 1$)

Assumptions: Mathematical analyses were conducted under the following assumptions;

- 1) The physical properties of the plate, i.e., K and λ are independent of the temperature and the position.
- 2) The moving speed v and the rate of heat input q are constant.
- 3) Heat losses from the surface by convection and radiation are neglected.
- 4) Cooling terminal of constant temperature T_0 is placed over the equidistant plane from the moving source.

Point Heat Source: The basic differential equation of heat conduction is expressed as the following form in the fixed co-ordinate (x, y, z).

$$2\lambda \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \quad (1)$$

Supposing that a point heat source q is supplied on the surface of semi-infinite plate and moves along the x -axis

with the constant speed of v , one can rewrite Equation (1) into the moving co-ordinate system (ξ, y, z) whose origin is taken at the heat source.

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 2\lambda \left(-v \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t} \right) \quad (2)$$

where, $\xi = x - vt$.

Taking

$$T - T_0 = \exp. (-\lambda v \xi) \cdot \Phi(\xi, y, z) \quad (3),$$

Equation (2) under the quasi-stationary state becomes

$$\frac{\partial^2 \Phi}{\partial \xi^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} - (\lambda v)^2 \Phi = 0 \quad (4),$$

or in polar co-ordinate (r, α, β),

$$\frac{d^2 (r\Phi)}{dr^2} - (\lambda v)^2 (r\Phi) = 0 \quad (5).$$

Therefore, the temperature is solved from the general solution of Equation (4) and Equation (3).

$$T - T_0 = \frac{1}{r} \exp. (-\lambda v r \cos \alpha \cos \beta) \times [C_1 \exp. (\lambda v r) + C_2 \exp. (-\lambda v r)] \quad (6)$$

The boundary conditions are

$$\begin{cases} T = T_0 \text{ at } r = R \\ -2\pi r K (\partial T / \partial r) \rightarrow q \text{ as } r \rightarrow 0 \end{cases} \quad (7).$$

Therefore, the solution under question becomes

$$T - T_0 = \frac{q}{2\pi K} \frac{1}{r} \exp. [-\lambda v r (1 + \cos \alpha \cos \beta)] \times \frac{1 - \exp. [-2\lambda v (R - r)]}{1 - \exp. [-2\lambda v R]} \quad (8),$$

or in non-dimensional form¹⁾

† Received on October 23rd, 1978
* Lecturer
** Graduate Student
*** Professor

$$\frac{\theta}{n} = \frac{1}{\rho\Lambda} \exp. [-\rho\Lambda (1 + \cos \alpha \cos \beta)] \times \frac{1 - \exp. [-2\Lambda(1 - \rho)]}{1 - \exp. [-2\Lambda]} \quad (9).$$

Here, the term, i.e.,

$$\gamma \equiv \frac{1 - \exp. [-2\Lambda(1 - \rho)]}{1 - \exp. [-2\Lambda]} \quad (10)$$

represents the cooling factor that depends on the radius Λ . It is evident that

$$\lim_{\Lambda \rightarrow \infty} \lambda = 1 \text{ for } 0 \leq \rho < 1,$$

and Equation (9) coincides completely with Rosenthal's solution²⁾ of

$$\frac{\theta}{n} = \frac{1}{\lambda\nu r} \exp. [-\lambda\nu r (1 + \cos \alpha \cos \beta)] \quad (11),$$

since $\rho\Lambda = (r/R)(\lambda\nu R) = \lambda\nu r$.

In Figures 1 and 2 are shown the behaviors of γ against Λ and the temperature distribution behind a source along the moving axis (ξ -axis) for several Λ -values. In these calculations, T_0 was taken to zero.

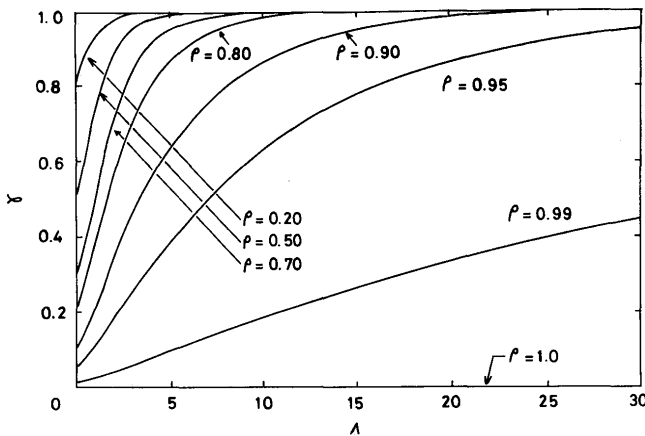


Fig. 1 Behavior of cooling factor against radius of cooling terminal (Three dimensional case)

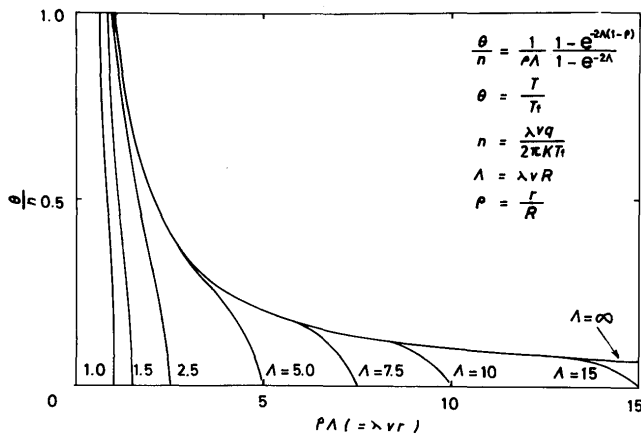


Fig. 2 Temperature distribution behind point heat source along moving axis when cooling terminal is placed around a source

Linear Heat Source: In two dimensional heat flow, there is no heat flux in the z -direction. Thus $\partial T/\partial z = 0$ and Equation (4) is expressed in cylindrical co-ordinate (r, α) as the form of

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r} \frac{d\Phi}{dr} - (\lambda\nu)^2 \Phi = 0 \quad (12).$$

The solution of temperature is, therefore, given by

$$T - T_0 = \frac{q}{2\pi h K} \exp. [-\lambda\nu r \cos \alpha] \cdot K_0(\lambda\nu r) \times \left[1 - \frac{K_0(\lambda\nu R) \cdot I_0(\lambda\nu r)}{I_0(\lambda\nu R) \cdot K_0(\lambda\nu r)} \right] \quad (13),$$

under the boundary conditions of

$$\begin{cases} T = T_0 \text{ at } r = R \\ -2\pi r K \left(\frac{\partial T}{\partial r} \right) \rightarrow q' = q/h \text{ as } r \rightarrow 0 \end{cases} \quad (14).$$

Equation (13) can be also written in non-dimensional form as

$$\frac{\theta}{n} = \frac{1}{\eta\Lambda} \exp. [-\rho\Lambda \cos \alpha] \cdot K_0(\rho\Lambda) \times \left[1 - \frac{K_0(\Lambda) \cdot I_0(\rho\Lambda)}{I_0(\Lambda) \cdot K_0(\rho\Lambda)} \right] \quad (15).$$

Here, the non-dimensional plate thickness η is defined by $\eta = (\lambda\nu h)/\Lambda = (\lambda\nu h)/(\lambda\nu R) = h/R$.

The cooling term of Equation (15) is

$$\gamma' = 1 - \frac{K_0(\Lambda) \cdot I_0(\rho\Lambda)}{I_0(\Lambda) \cdot K_0(\rho\Lambda)} \quad (16).$$

Figure 3 shows the change of γ' with Λ , in which the three dimensional cooling term, i.e., γ of Equation (10) is also represented by dotted lines. As seen in the figure, behaviors of γ and γ' are very similar except in smaller values of Λ , and they coincide at higher Λ -values. It is, therefore, possible to express Equation (15) as the following form in higher values of Λ .

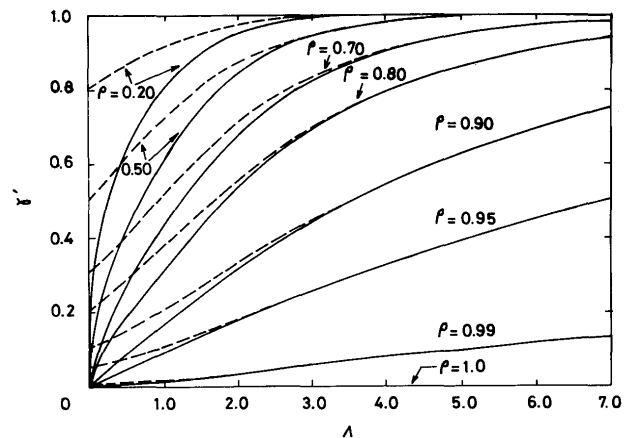


Fig. 3 Behavior of cooling factor against radius of cooling terminal (Solid line for linear source and dotted line for point source)

$$\frac{\theta}{n} = \frac{1}{\eta\Lambda} \exp. [-\rho\Lambda \cos \alpha] \cdot K_0(\rho\Lambda) \times$$

$$\left[\frac{1 - \exp. [-2\Lambda(1-\rho)]}{1 - \exp. (-2\Lambda)} \right] \text{ for } \Lambda > 4 \quad (17).$$

It is also evident that Equation (15) coincides with the solution by Rosenthal in two-dimensional heat flow at very high values of Λ .

References

- 1) N. Christensen et al.: "Distribution of Temperature in Arc Welding", British Welding J., No. 12, Vol. 54 (1965),
- 2) D. Rosenthal: "The Theory of Moving Sources of Heat and Its Application to Metal Treatment", Transactions of A.S.M.E., Nov., 1946, pp. 849-866.