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Author(s)	Hamamoto, Hideki
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HIDEKI HAMAMOTO

A FUZZY-THEORETIC APPROACH TO ADJECTIVAL COMPARATIVES*

This paper is a study of the meaning of adjectival comparatives from the perspective of fuzzy set theory. The concept of fuzzy subsets is especially useful for the representation of imprecise knowledge of the type which is prevalent in human concept formation. In many respects, natural languages are flexible, and this flexibility is sometimes considered to be beyond the reach of ordinary logic, which can analyze crisp aspects of language but not its many fuzzy areas. In particular, adjectives have in general resisted our scientific approach to the meaning component, basically because many words of this syntactic category represent relations which speakers perceive as holding to varying degrees, according to various circumstances. For instance, *cold* may generally be used to describe ice water, but solid ice has even more of the salient quality that makes this adjective applicable. Fuzzy set theory provides a means of formalizing the notion that an adjective like *cold* may hold of one object to a greater degree than it holds of another. This observation has actually become a commonplace of semantic studies of adjectives. Less well known, however, are other benefits afforded by the adoption of a fuzzy-theoretic approach, particularly in the area of adjectival comparison. The present paper investigates such an application of fuzzy theory.

We begin by presenting some basic definitions of fuzzy-theoretic notions to be employed in the present discussion. The subsequent section introduces a fuzzy-subset-based definition of the meaning of adjectives, which will be taken for granted in all that follows. After an examination and rejection of some classical approaches to the semantics of adjectival comparison, a fuzzy-theoretic analysis will be presented. The subsequent discussion will include an exploration of the problem of relativity of meaning in the light of a fuzzy-theoretic notion known as the extension principle, which can capture the context dependency of adjectives in a fairly rational manner. It will also be shown that the fuzzy-theoretic view opens up possibilities for insightfully analyzing licensing conditions for comparatives.

1 Preliminary Definitions

In this section, we will briefly introduce some basic concepts assumed in fuzzy set theory, which are minimally necessary for the understanding of the discussion in the following sections. We will generally follow the development of fuzzy theory given by Yager (1986).

A FUZZY SET is a generalization of the notion of an ordinary or crisp set. A FUZZY SUBSET can be regarded as a predicate whose truth values are drawn from the unit interval $[0, 1]$ rather than the set $\{0, 1\}$, as in the case of an ordinary set.

(1) Definition of Fuzzy Subsets

Assume X is a set corresponding to the universe. A fuzzy subset A of X is a subset in which the membership grade of any element $x \in X$ is drawn from the unit interval $[0, 1]$.

(2) Example

Assume $X = \{x_1, x_2, x_3, x_4\}$ and $A = \{0.8/x_1, 0.5/x_2, 0.1/x_3, 0.3/x_4\}$. Then A is an example of a fuzzy subset of X.

In the framework of the theory of fuzzy subsets, expressions of the form α/x are understood to indicate that the element x has membership grade α in the relevant fuzzy subset. The larger the membership grade of an element the more strongly it is a member of the fuzzy subset. For any

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fuzzy subset A, μ_A denotes a function from members of A to their membership grades. Therefore, the membership grade of a given x in A may be denoted $\mu_A(x)$.

At this point it is already easy to see how fuzzy theory might apply to the meaning of adjectives. Fuzzy subsets allow for the description of concepts in which the boundary between having a property and not having a property is not sharp. Rather than providing a standard set-theoretic interpretation of such gradient predicates as *cold*, we could apply a fuzzy-theoretic evaluation, so that, for instance $\mu_{cold}(ice\ water)$ might be 0.75, while $\mu_{cold}(solid\ ice)$ might be 0.9, or some such similar values. Of course, in the application of fuzzy theory that we propose here, the values of membership grades for a fuzzy subset will be subjective and context dependent. Furthermore, in many cases it is the shape of the membership function that is of significance rather than the actual values. This will be of critical importance in the discussion of various forms of adjectival comparison.

One more definition that will be useful for describing the context-dependent nature of adjectival interpretation is the EXTENSION PRINCIPLE. This will provide a means of implementing the effect of comparison classes on adjectives in the present system (e.g. in the phrase *tall for a basketball player*, the set of all basketball players serves a comparison class).

(3) EXTENSION PRINCIPLE

Assume X and Y are two sets. Let f be a mapping from X into Y, i.e. $f : X \mapsto Y$ such that for each $x \in X$, $f(x) = y \in Y$. Suppose that A is a fuzzy subset of X. We can define $f(A)$ as follows: $f(A) = \bigcup_{x \in X} \mu_A(x)/f(x)$.

With these definitions in hand, we are ready to proceed to empirical applications of fuzzy theory to natural language data.

2 THE MEANING OF ADJECTIVES AND FUZZY THEORY

This section introduces the basic background framework within which various analyses in the following sections are carried out. As a starting point, we focus on the application of fuzzy subsets to the interpretation of the meaning of words.

2.1 AN INFORMAL OVERVIEW

In order to facilitate a deeper intuitive understanding of the subsequent formal discussion, let us begin with some commonsense observations about plausible disagreements on interpretation that might arise in discourse. Consider the following samples of dialogs between two persons, A and B.

(4) A: It's *cold* this morning, isn't it?
B: You call this *cold*?!

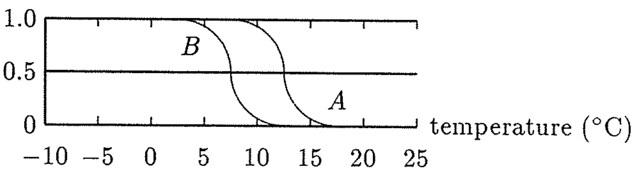
(5) A: This VCR is *cheap*—20% off the list price.
B: *Cheap* my foot! I bought the same one at a 40% discount.

(6) A: I need to talk to that *middle-aged* gentleman over there.
B: You mean that yuppy kid?
B': He's slightly passed that stage, wouldn't you say?

In all these cases there is a disagreement between the interlocutors caused by a discrepancy in the interpretation of a word on the part of the two participants. In other words, the interlocutors are at odds over the interpretation of the words *cold*, *cheap*, and *middle-aged*. Although all of the concepts seem rather basic, one can easily imagine personal differences in interpretation. Fuzzy theory allows us plausibly to model this sort of subjectivity, while giving an account of aspects of interpretation that remain constant across speakers. The commonality will be inherent in membership curves.

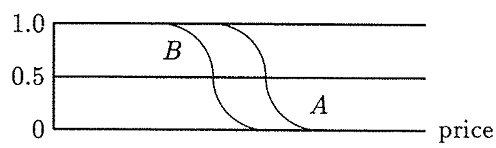
Consider the fuzzy graph below, which denotes two possible membership functions for *cold*; these functions could plausibly represent the respective interpretations of our interlocutors, A and B, in (4). The horizontal axis designates the scale of temperature. The vertical axis shows the degree of compatibility. Therefore each point on the curve depicts the degree of the membership of individuals bearing the temperature associated with the corresponding point on the horizontal scale.

(7) FUZZY GRAPH OF A AND B'S NOTIONS OF 'COLD'



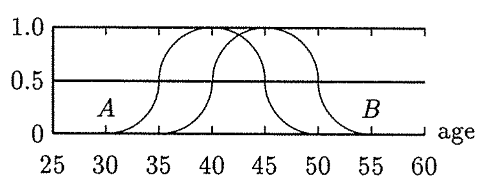
Observing the graph above, we can easily see that the shape of membership functions assumed by A and B are similar but that there is an offset of about five degrees separating points of equivalent non-extreme membership grades between the two curves. Take one point on the temperature scale for example: at 10°C, A feels cold to an elevated degree (approximately 0.9), while B's perception of coldness is much less acute (only about 0.1). Therefore, under the same physical conditions, A feels cold, while B does not. In this manner, we can attribute the discontinuity in conversation (4) to the difference in membership functions between the two participants. In the same way, the discrepancy in (5) is also explained by a difference in membership functions.

(8) FUZZY GRAPH OF A AND B'S NOTIONS OF 'CHEAP'



Now we come to a slightly different problem, having to do with (6). The two rejoinders exhibited above show that the membership curve for the adjective *middle-aged* must be more complex than those illustrated in (7) and (8). There are two sources of possible disagreement when this form is used: the individual of whom it is asserted may be perceived as either too young (B) or too old (B'). For the sake of visual clarity, the following graph shows only the curves relevant for utterances A and B. The curve for B' would be similarly bell-shaped and would be situated a little further to the left than either of the curves shown.

(9) FUZZY GRAPH OF A AND B'S NOTIONS OF 'MIDDLE-AGED'



By looking at the membership curves for the various adjectives above, we can see what is common and different among the individual interpretations of the words in question: the correspondence between membership grades and individual points on the horizontal scale can vary significantly from speaker to speaker, but the overall shape of the membership curve will remain constant across speakers. Therefore, we submit that this membership curve is the proper focus of semantic investigation.

(10) HYPOTHETICAL DEFINITION OF THE MEANING OF ADJECTIVES

> The meaning of an adjective P is the membership function which characterizes the fuzzy subset of P.

Of course the above definition should ultimately be extended to other syntactic categories, especially nouns, verbs, and adverbs, but we shall content ourselves with the above form for convenience.

2.2 A Formal Approach to the Notion of Meaning

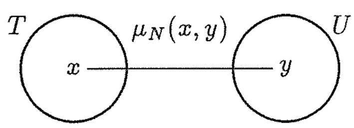
In this section we shall examine the problem of meaning from a more formal perspective, following roughly the argument developed by Zadeh (1987). Let's start with the definition of the fuzzy language L. The fuzzy language L is a system which has 3 components.[1]

(11) $L = (U, T, N)$

> U is a set of objects called the universe of discourse;
>
> T is a set of terms;
>
> N is a naming relation.

Let U be a universe of discourse, i.e., a collection of objects generically denoted by y, e.g., a set of integral numbers, a set of things in a room, a set of relations between these things, etc. Furthermore, let T be a set of terms generically denoted by x. We will regard a language L as a correspondence between a set of terms T and the universe of discourse U. This correspondence will be defined by a naming relation N, which associates with each term x in T and each object y in U the degree $\mu_N(x, y)$. This degree is assumed to be a number in the interval $[0, 1]$, so that N is a fuzzy relation from T to U. A fuzzy relation R from a set X to a set Y is a fuzzy subset of the Cartesian product $X \times Y$. The relation between these components will be shown in the following diagram.

(12) The Naming Relation



When x is chosen to be a particular term in T, say $x = young$, the function $\mu_N(young, y)$ defines a fuzzy subset of U whose membership function $\mu_{young}(y)$ is given by $\mu_{young}(y) = \mu_N(young, y)$.

(13) Meaning of a Term

> The meaning of a term x in T is a fuzzy subset $M(x)$ of U characterized by the membership function $\mu_{M(x)}(y) = \mu_N(x, y)$, where $x \in T$ and $y \in U$. (Zadeh 1987:469)

Following the definition above, the meaning of a word x is a fuzzy subset $M(x)$. When no confusion will result, we shall simplify explicit references to fuzzy subsets of terms, such as $M(young)$, by replacing them with the term itself. Thus, $\mu_{M(young)}$ and μ_{young} may be taken as being the same. Now, since a fuzzy set is completely characterized by its membership function, Zadeh's definition accords well with the intuitive observation set forth in the preceding subsection on the meaning of adjectives.

Now we have briefly examined intuitions about the lexical meanings of gradable adjectives and also a fuzzy-theoretic formalization of their interpretation. We may now proceed to the principal claims of this paper which concern various complexities in the semantics of adjectival comparison. However, before setting out our own proposals, we shall review some of the existing analyses of this phenomenon in the literature.

3 Previous Analyses of Adjectival Comparatives

Existing theories of the semantics of comparatives can be roughly classified into two different types according to the way in which they treat degree. The first type of theory treats degrees as quantifiable individuals: we may therefore call it the individual theory of degree. The second type adopts an approach which uses an abstracted form of degree modifier: we will consequently call it the modifier theory of degree.

[1]A more detailed discussion may include a fourth component E as an embedding set. A set of terms T may be considered to be a fuzzy subset of E. But for our purposes, it will be convenient to assume that T is a non-fuzzy set, regarding L merely as a correspondence between T and U.

3.1 The Individual Theory of Degree

There are at least two major approaches to the problem of the semantics of scalar adjectives that involve positing degrees as quantifiable individuals. The first technique, due to McCawley (1973) and others treats degrees as equivalence classes within a partition that a given scalar predicate imposes on a set of individuals. This approach also involves an ordering on degrees, $<$ (or its converse $>$) which allows one to express the truth condition of a simple comparative like (14a) in roughly the manner illustrated in (14b).

(14) a. John is taller than Tom.

b. $\exists x \exists y$[John is tall to $x \wedge$ Tom is tall to $y \wedge x > y$]

This approach, however, inevitably leads to circular reasoning, since in order to set up the partition on which the definition of degree itself is based, one would have to appeal to some notion of comparison.[2] In other words, analyzing a comparative in terms of degree cannot help utilizing the notion of comparison; this thought process is, of course, circular and therefore provides no explanation of the concept.

Seuren (1973) provided an alternative view of degree. His approach would replace (14b) with (15).

(15) $\exists x$[John is tall to $x \wedge \neg$[Tom is tall to x]]

Though Seuren is vague about the exact nature of degrees, it may be inferred from (15) that his notion of degree does not involve equivalence classes. The reason for this is that, for (15) to represent the meaning of (14a), x would have to be the degree for all individuals as tall as or taller than John. If x were comparable to the equivalence class containing John and all individuals of identical height, (15) would wind up being paraphrasable by 'John and Tom are not of equal height.' However, even though Seuren is not employing equivalence classes, there nonetheless arises the same sort of problematic circular reasoning as was encountered above. His concept of degree requires some form of implicational scale which could only be set in place through some prior notion of comparison, whence the circularity.

For reasons like those sketched here, the individual theory of degree appears to provide little useful insight into the notion of comparison.

3.2 The Modifier Theory of Degree

In contrast to theories that depict degree as an individual, Klein (1980, 1982) proposes to represent this concept with a logical element comparable to a certain kind of natural language adverbial, exemplified by *very* and *quite*. The leading idea underlying this approach is that the features of the logical language should resemble as much as possible the natural language forms whose truth conditions they are employed to describe. This leads Klein to posit degree modifiers that apply directly to adjective meanings, in a manner reminiscent of the adjunction of adverbs in syntax. Such degree modifiers map their arguments into logical expressions with modified truth conditions but the same logical type. For instance, *tall*(*John*) and *tall*(*Bill*) may both be true, and yet, when we consider *very*(*tall*), which is of the same logical type as the simple form *tall*, it may be the case that *very*(*tall*)(*John*) is true, but *very*(*tall*)(*Bill*) is not.[3]

Slightly more formally, we may say, omitting any consideration of intensionality, that a property like *tall* is a member of the set $\{0,1\}^U$, where $\{0,1\}^U$ is the set of all functions from U to $\{0,1\}$, and U is the universe of individuals. Degree modifiers are functions from properties to properties, so they will be members of the set $(\{0,1\}^U)^{\{0,1\}^U}$: *very* and *quite* will be members of this set, but so will a variety of other functions not necessarily corresponding to lexicalized degree modifiers. In what follow, we shall use the variable d to range over members of $(\{0,1\}^U)^{\{0,1\}^U}$

Entailments like the following lead to the basic inspiration for Klein's treatment of comparatives.

(16) John is very tall, and Bill is not very tall $\rightarrow$ John is taller than Bill.

[2] See Klein (1980:3) for more details.

[3] These expressions should be read in a left-associative manner: first apply *very* to *tall*, and then apply the result to *John* or *Bill*. This sort of interpretation may be assumed whenever the above bracketing syntax is encountered in this paper.

The idea is that among the set of functions $(\{0,1\}^U)^{\{0,1\}^U}$ from which degree modifiers are drawn, there is a series of functions corresponding to all possible gradations along a dimension. Therefore, the comparative in (17) could be rendered as in (18).

(17) John is taller than Bill.

(18) $\exists d[d(\mathit{tall})(\mathit{John}) \wedge \neg d(\mathit{tall})(\mathit{Bill})]$

The intended interpretation is that somewhere along the dimension of gradations of tallness there is a point corresponding to a particular degree modifier which describes John's level of tallness, but which surpasses Bill's.

Moving back to the formal perspective, however, we can easily see that the total membership of $(\{0,1\}^U)^{\{0,1\}^U}$ contains certain groups of functions that would be mutually incompatible. For instance, one could easily construct three different degree modifier functions that would even make the following statement true.

(19) #John is taller than Tom, Tom is taller than Bill, and Bill is taller than John.

Thus, it is necessary to impose a certain order on the set of degree modifiers, and to do this Klein employs the following meaning postulate, where the variables x and y range over individuals and Q ranges over properties.

(20) CONSISTENCY POSTULATE

$\forall x, y, Q\,[\exists d[d(Q)(x) \wedge \neg d(Q)(y)] \rightarrow \forall d[d(Q)(y) \rightarrow d(Q)(x)]]$ (Klein 1982:126)

In this manner the correct interpretation for the comparative construction is ensured, although a definite problem arises on this approach, when one examines the semantics of lexicalized degree modifiers a little more closely.

Since Klein's meaning postulate in (20) affects all expressions of a particular logical type, it will apply across the board to all degree modifiers, including those that have actually been lexicalized in English. Therefore, his approach makes the explicit claim that all such modifiers should obey (20). This unfortunately leads the analysis into error. Among degree modifying adverbials there is one group that appears to obey Klein's postulate, (21a), and another which clearly goes against it, (21b).

(21) TWO DIFFERENT SUBSETS OF DEGREE MODIFIERS

a. *very*, *considerably*, *quite*, ...

b. *moderately*, *averagely*, ...

The behavior of two types of modifiers is exemplified in (22).

(22) a. [John is *very* tall $\wedge$ Bill is not *very* tall] $\rightarrow$ John is taller than Bill.

b. [John is *moderately* tall $\wedge$ Bill is not *moderately* tall] $\not\rightarrow$ John is taller than Bill.

To see that the antecedent of (22b) really does not imply the consequent, note that the antecedent would be true if John and Bill's respective heights were six and seven feet, but in such a case the consequent would be false. Thus, Klein's analysis proves untenable due to its dependency on a meaning postulate that cannot be maintained in the face of the data provided by the lexicalized degree modifiers in (21b).

Since approaches based on standard logic appear not to provide an adequate handle on the problem of adjectival comparison we propose to shift our focus to fuzzy set theory. In the sections that follow we shall lay out the basics of the fuzzy-set-theoretic analysis along with some further refinements.

4 THE FUZZY-SET-THEORETIC APPROACH TO ADJECTIVAL COMPARISON

This section will introduce a new perspective on the analysis of adjectival comparison, i.e., the fuzzy-set-theoretic approach. In a foregoing section we already mentioned that the basic notion of fuzzy subsets affords a simple and satisfying rendering of our basic intuitions about the meaning of scalar predicates. Let us consider this issue through a comparison. In classical set theory, the denotation of a property like that of *being rich*, would be a set: all and only individuals who possessed the property in question would be in that set. This view provides no means of representing relative properties. For instance, if John is somewhat rich, and Mary is extremely wealthy, then putting them into the same set ignores an obvious difference in status. Even to approach the problem of scalar properties, semantic analyses based on classical logic have to resort to some representation of degree. Consequently the attribution of a scalar property is portrayed as being logically complex, even though all languages with which we are familiar realize such scalar notions with simple lexical items. Fuzzy set theory offers a notable improvement on this state of affairs. If the denotation of *rich* is a fuzzy set, then the membership function will provide a representation of relative wealth. As for the John and Mary of our example, John's membership grade will be inferior to Mary's, because the former is not so rich as the latter. However, beyond the elegance of the fuzzy-theoretic treatment of the positive use of adjectives, it provides a straightforward implementation of adjectival comparison. We now turn to this topic, beginning with the exposition of some fundamental aspects of the approach.

4.1 The Basic Analysis of Comparatives

Under the fuzzy-theoretic approach to scalar relations, the implementation of adjectival comparison could hardly be simpler: it amounts to numeric comparison of membership degrees. For instance, the sentence in (23a) would have roughly the same truth conditions as the formal expression in (23b).

(23) a. John is kinder than Tom.

b. $\mu_{kind}(John) > \mu_{kind}(Tom)$

Recall that the symbol μ above signifies a membership function. If A is a fuzzy subset, we use μ_A to indicate the membership function of A. Furthermore, the meaning of a term like *kind*, denoted $M(kind)$, is a fuzzy set. Therefore, $\mu_{M(kind)}(John)$ or more simply $\mu_{kind}(John)$ indicates the membership grade of John in terms of kindness. The reader will also recall that the greater the membership grade of an element is, the more strongly it is a member of the fuzzy subset. Therefore, (22) indicates that the statement *John is kinder than Tom* is true if and only if the membership grade of John with respect to $M(kind)$ is greater than that of Tom.

A variety of comparative constructions receive straightforward renderings on the fuzzy-theoretic approach. Representations for degree expressions based on *more*, *less*, and *as* are provided in (24).

(24) a. x is more Q than y : $\mu_Q(x) > \mu_Q(y)$

b. x is less Q than y : $\mu_Q(x) < \mu_Q(y)$

c. x is as Q as y : $\mu_Q(x) \geq \mu_Q(y) \wedge \mu_Q(x) \approx \mu_Q(y)$

The rendering of the *as... as* construction presented in (24c) is perhaps somewhat controversial. It says that x's membership degree must not be inferior to y's—$\mu_Q(x) \geq \mu_Q(y)$—and moreover that the two membership degrees must be approximately the same—$\mu_Q(x) \approx \mu_Q(y)$. An alternative view would be to suppose that only the former of the above conditions need be satisfied. This a point worthy of some discussion, but this shall be postponed until after a consideration of subjectivity.

4.2 Subjectivity and Degrees of Comparison

In the present subsection, we shall consider the matter of semantic differences among variations in comparative expressions which are sensitive to the magnitude of differences in degree. An array of examples in decreasing order of assumed magnitude of degree difference is presented in (25).

(25) a. John is far richer than Tom.

b. John is richer than Tom.

c. John is as rich as Tom.

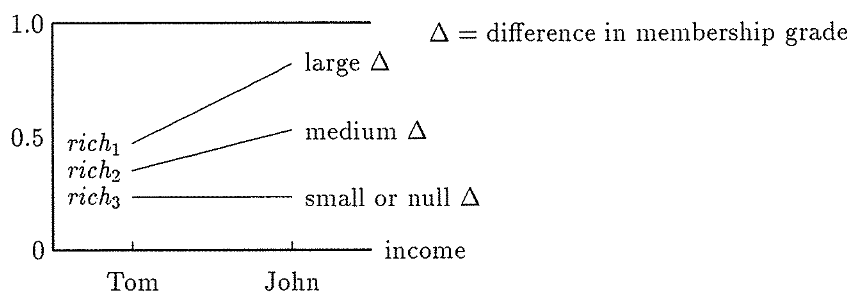
d. John is roughly as rich as Tom.

e. John is exactly as rich as Tom.

An examination of the conditions that govern the usage of such expressions as these provides further motivation for our fuzzy-theoretic analysis of scalar relations. One may demonstrate that the differences in degree to which expressions such as those above implicitly allude are not directly based on conditions in the real world, but rather are determined by subjective evaluations which correspond directly to the membership function μ in our fuzzy-theoretic model. The ease with which this facet of the semantics of scalar predicates can be rendered in fuzzy theory is an indication of the utility of the approach.

Next let us turn to a more concrete example. Consider two objects, Tom and John, and the degree to which they possess the property of richness. For the sake of perspicuity, we shall assume that richness is judged simply according to a person's income. In the following graph whose horizontal axis represents income and whose vertical axis indicates degree of membership, three distinct possible versions of the membership function of *rich* are drawn as curves increasing from left to right. Let us assume that John's income is greater than Tom's.

(26) SUBJECTIVE VARIATION IN MEMBERSHIP FUNCTIONS



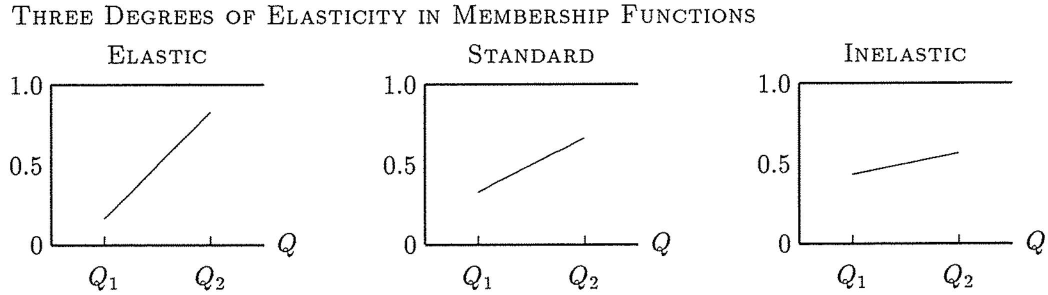
Given the single real-world situation described by the horizontal axis of (26) regarding John and Tom's respective incomes, a variety of different ways of describing the situation are conceivable according to the observer's (=speaker) subjective judgement.

(27) a. John is far richer than Tom.
b. John is richer than Tom.
c. John is (roughly) as rich as Tom.

To a billionaire the difference in John and Tom's incomes may be insignificant, even though it might be great enough for an ordinary person to utter *John is richer than Tom.* The former person might then say *John is (roughly) as rich as Tom.* Meanwhile, to a person of modest means, the difference in question might merit the expression *John is far richer than Tom.* The different choices of utterances are attributable to variations in the membership functions that distinct individuals associate with the predicate *rich.* This observation indicates that comparative expressions do not reflect the real world directly but rather represent each person's subjective judgement about the world. In summary, we may say that the comparative should be construed as a representation of differences in membership grades subjectively assigned to objects, not as representations of mere differences among the objects concerned.

Now we would like to introduce the notion of ELASTICITY OF MEMBERSHIP FUNCTIONS. This is a concept devised to indicate the degree of responsiveness of membership functions to changes in quantity on the horizontal scale (e.g., changes in income, to take the specific example of (27)). The continuum of elasticity is divided qualitatively into three categories, depicted in (28).

(28) THREE DEGREES OF ELASTICITY IN MEMBERSHIP FUNCTIONS



In increasing Q from Q_1 to Q_2, the difference Δ, $\mu_A(Q_2)-\mu_A(Q_1)$, may be large, medium, or small, in which case we say that the membership function is ELASTIC, STANDARD, or INELASTIC. These categories are useful when we describe for a given adjective the varieties of membership functions for the same adjective that derive from personal judgments about the property in question.

In terms of linguistic descriptions, these categories of curves provide us with a vocabulary for expressing cognitive and pragmatic constraints on the appropriate use of adjectival comparative constructions. For instance, if one person has an inelastic membership function for a certain property (e.g. *rich*), some difference existing in Q (e.g. income) will not be reflected in his choice of linguistic expression. He may use an equative construction. On the other hand, if a person has an elastic membership function for the property he may respond to the difference in Q so much that he will use a comparative expression, as exemplified as in (27a) above. These observations are intended to show the potential utility of regarding the membership curves of scalar adjectives as objects of linguistic description. However, a fully articulated analysis based on this perspective will have to await another study.

It is also plausible that such adjectives as *tall*, *long*, and *big*, which describe objectively observable properties like height, length, and physical magnitude will have a standard elastic membership function. In contrast to adjectives which describe subjectively judged properties like 'being rich,' the membership functions of the more objective adjectives are to be delimited by human perception, and therefore will tend toward standardization.

4.3 On the Analysis of Equatives

At this point, let us return to the matter of the analysis of equatives. The gist of our approach is that the truth conditions for a form like *John is as rich as Tom* are roughly those conveyed by $\mu_{rich}(John) \geq \mu_{rich}(Tom) \wedge \mu_{rich}(John) \approx \mu_{rich}(Tom)$. In effect, we are proposing that the *as... as* construction imposes both a lower bound and an upper bound on the value of $\mu_{rich}(John)$: the former conjunct provides the lower bound, and the latter provides the upper one. However, some researchers would claim that the second conjunct should be eliminated from the above rendering. Let us call this the lower bound analysis. The debate centers around the proper treatment of the anomaly in the following example.

(29) #John is as rich as Tom. In fact, John is far and away the richest man in the world.

Our claim is that the infelicity in (29) is semantic, since the context described by the discourse would not satisfy the second conjunct of the truth conditions of the *as... as* construction—$\mu_{rich}(John) \approx \mu_{rich}(Tom)$. The opposing view, advanced by such researchers as Klein (1980:38) and Horn (Horn 1989:387ff), is that (29) violates the Gricean maxim of quantity, in that a more informative utterance like *John is richer than Tom* could have been used. Thus, adherents of the lower bound theory assume that the empirical perception of upperboundedness derives from pragmatic factors.

One way of deciding the issue would be to consider the interaction of the *as... as* construction with negation.

(30) a. John is not as rich as Tom. Tom is in fact far richer.

b. John is not as rich as Tom. John is in fact far richer.

Consider (30a), where Tom is richer than John. This is unproblematic on either analysis of the *as... as* construction. For any Q, x, and y, $\mu_Q(x) < \mu_Q(y)$ implies $\neg\mu_Q(x) \geq \mu_Q(y)$, which in turn would imply $\neg[\mu_Q(x) \geq \mu_Q(y) \wedge \mu_Q(x) \approx \mu_Q(y)]$. Consequently Tom's being richer than John would make *John is not as rich as Tom* true on either analysis. If we move on to (30b), where John is far richer than Tom, we then find a difference in predictions. On our view *John is not as rich as Tom* and *John is far richer (than Tom)* may be simultaneously true, since the latter implies $\neg\mu_{rich}(John) \approx \mu_{rich}(Tom)$, which would in turn imply the truth of *John is not as rich as Tom.* The competing, lower bound hypothesis, would find itself in difficulty, however, because it supposes that the truth conditions of *John is not as rich as Tom* would be $\neg\mu_{rich}(John) \geq \mu rich(Tom)$, which is equivalent to $\mu_{rich}(John) < \mu rich(Tom)$, which in turn contradicts *John is far richer (than Tom)*. Consequently, the non-contradictoriness of (30b) is predicted on the approach adopted here, but not on the lower bound analysis; this appears to be an advantage for our view. However, data like (30b) have not gone unnoticed by such researchers as Horn (1989): still we believe that the alternative approach he offers does not measure up to the actual data.

Horn suggests that (30b) is a case of what he calls METALINGUISTIC NEGATION, which, according to his definition, refers to the extended use of negation as a way for speakers to announce their unwillingness to assert something in a given way, or to accept another's assertion of it.

(31) a. Jude is as tall as Mona.
b. No, he's not as tall as Mona, he's taller. (Horn 1989:387)

Contra Horn, Kempson (1986) suggests that not all cases of what Horn calls metalinguistic negation should be treated as exceptional uses of negation, but rather that they should be regarded as cases of usual, descriptive negation. Although she does not specifically mention the case of equatives, Kempson takes issue with various cases where Horn claims to discern metalinguistic negation. Indeed, several of her arguments which undermine various of Horn's claims about supposed instances of metalinguistic negation may be successfully reapplied to the case of negated equatives to show that they are in fact instances of normal, descriptive negation.

Of course there are cases of negation that are well and truly metalinguistic. These include such examples as (32) and (33).

(32) (So, You [mĩʸənɨǰd] to solve the problem.) No, I didn't [mĩʸənɨǰ] to solve the problem—I [mænɨǰd] to solve the problem. (Horn 1985:132)

(33) I'm not his daughter—he's my father. (ibid.:133)

In (32) the speaker uses negation not to assert the contrary of a given proposition, but rather to reject a low prestige pronunciation. In (33), the speaker employs negation to deemphasize the relevance of the former proposition and emphasize that of the latter. Kempson provides diagnostic tests to distinguish such true case of metalinguistic negation from those which she claims have been erroneously included in this category by Horn. For instance, sentences with typical metalinguistic negation cannot be turned into *it is not true that* constructions, as we see in (34).

(34) a. I'm not his daughter: he is my father. (Kempson 1986:84)
b. ?It's not true that I'm his daughter: he is my father. (ibid.:87)

Notice that, unlike the metatlinguistic form in (34), negated *as... as* constructions do not exhibit this behavior: the *it is not true that* version in (35b) is just as acceptable as the form in (35a).

(35) a. Jude is not as tall as Mona: he is taller.
b. It is not true that Jude is as tall as Mona: he is taller.

Kempson also asserts that with descriptive negation, but not the metatlinguistic variety, it is possible to employ sentence continuations expressing evidence for holding the belief expressed by the negated clause.

(36) John didn't hit the target: he hit the wall. (Kempson 1986:85)

This pattern is compatible with negated *as... as* constructions, a fact which suggests that this is descriptive negation.

(37) Jude is not as tall as Mona: he is taller.

The trailing clause clearly expresses the speaker's reason for negating the preceding proposition.

Kempson also notes that metalinguistic negation admits contradiction, as in the following example, where *it's not that* manifests a definite metalinguistic quality.

(38) It's not that Mark ate three biscuits—though he did—it's that I'm too tired to cook breakfast. (Kempson 1986:86)

When the metalinguistic negation marker *it's not that* is removed, yielding a case of descriptive negation, the contradictory *though* clause is perceived as illogical.

(39) #Mark didn't eat three biscuits, though he did. (Kempson 1986:86)

Turning to negated *as... as* constructions, we find the same pattern.

(40) It's not that Jude is as tall as Mona—though he is—it's that he is so much taller that he dwarfs her.

(41) #Jude is not as tall as Mona, though he is.

According to Kempson's line of argument, it follows that the negation in (41) is not metalinguistic.

Finally, Kempson adopts two diagnostics borrowed from Horn (1985). First only descriptive negation admits the use of negative polarity items such as *any*. The metalinguistic negation in (42) is therefore unacceptable.

(42) He didn't [mĩʸənɨǰ] to open $\left\{\begin{matrix}\text{??any}\\ \text{some}\end{matrix}\right\}$ doors, he [mænɨǰd] to open some doors.
(Kempson 1986:87, Phonetic script has been altered for consistency.)

Contrast this with the following negated *as... as* construction which contains the negative polarity item *any more*.

(43) Jude is not as tall as Mona any more; he has become much taller.

The acceptability of (43) suggests that this is an example of descriptive and not metalinguistic negation. Furthermore, Horn asserts that lexically incorporated negation cannot take on a metalinguistic reading. Hence, *impossible* is infelicitous in (44).

(44) It's $\left\{\begin{matrix}\text{??impossible}\\ \text{not possible}\end{matrix}\right\}$ to see him—it's essential. (Kempson 1986:87)

Therefore, since *impossible* yields an acceptable result in (45), it follows that we have a case of descriptive negation.

(45) It is impossible to be as tall as Mona, since she is the shortest person in the world.

We have seen five different arguments that all suggest that negation of the *as... as* construction is descriptive and not metalinguistic. This means in particular that those who would claim that (30b) is an instance of metalinguistic negation are mistaken. This implies in turn that the felicity of (30b) can be explained neither in the pragmatics nor in the semantics, if one adopts the lower bound analysis. In contrast, our approach predicts the acceptability of (30b) as a consequence of the semantic analysis and is therefore to be preferred over the lower bound theory.

At this point we have now covered most of the salient points that needed to be considered with respect to the fundamental fuzzy-theoretic analysis of adjectives and adjectival comparison. We shall therefore move on to a discussion of some elaborations to the theory.

5 Contextual Problems with Adjectives

The interpretation of adjectives is dependent on contexts. Even a simple statement like *John is tall* cannot be given a truth value unless the class with respect to which tallness is asserted is specified. Also, the comparative sentence *Tarō is cleverer than Hanako* cannot be judged as true or false, until it has been established what aspects of cleverness are at issue. The former type of contextual problem concerns comparison classes, which will be discussed in the following subsection. The latter problem concerns contextual meaning specification, which will be treated in its turn.

5.1 Comparison Classes and the Extension Principle

A person who would be judged as being *old* according to our ordinary criterion of age, could be called an *up-and-coming, young politician.* And a person who is taller than the average may be called a *short* basketball player. How can we explain these cases? So far we have assumed that the meaning of adjectives can be represented in the form of a fuzzy membership function. Then in order to handle the problems above, we are forced to assume that a fuzzy membership function must be context-dependent. Furthermore, this context-dependence may be modeled by means of the extension principle for fuzzy subsets, which is repeated here for convenience.

(46) EXTENSION PRINCIPLE

Assume X and Y are two sets. Let f be a mapping from X into Y, i.e. $f : X \mapsto Y$ such that for each $x \in X$, $f(x) = y \in Y$. Suppose that A is a fuzzy subset of X. We can define $f(A)$ as follows: $f(A) = \bigcup_{x \in X} \mu_A(x)/f(x)$. [=(3)]

Consider the concrete example of *tall* and *short* to illustrate how the principle works. Assume that we have the following membership functions for *tall* and *short* with respect to people in general. Let us call the set consisting of the general population *GP*, and the relevant versions of the membership functions $tall_{GP}$ and $short_{GP}$.

(47) $tall_{GP} = \{0/140\text{ cm}, 0.3/160\text{ cm}, 0.6/165\text{ cm}, 0.7/175\text{ cm}, 0.9/180\text{ cm}, 1.0/185\text{ cm}\}$

$short_{GP} = \{1.0/140\text{ cm}, 0.9/145\text{ cm}, 0.7/150\text{ cm}, 0.6/160\text{ cm}, 0.3/165\text{ cm}, 0/185\text{ cm}\}$

Furthermore, we shall call the set of basketball players *BP* and assume the function f as a mapping from *GP* to *BP*, which takes the form of $f(x) = x + 15$. Assume that for all x, $x \in GP$ implies $f(x) \in BP$, and that $tall_{GP}$ is a fuzzy subset of *GP*. We can define $f(tall_{GP})$ as a fuzzy subset of *BP* such that $f(tall_{GP}) = tall_{BP} = \bigcup_{x \in GP} \mu_{GP}(x)/f(x)$. We get the membership functions of $tall_{BP}$ and $short_{BP}$ for basketball players by means of the extension principle.

(48) $tall_{BP} = \{0/155\text{ cm}, 0.3/175\text{ cm}, 0.6/180\text{ cm}, 0.7/190\text{ cm}, 0.9/195\text{ cm}, 1.0/200\text{ cm}\}$

$short_{BP} = \{1.0/155\text{ cm}, 0.9/160\text{ cm}, 0.7/165\text{ cm}, 0.6/175\text{ cm}, 0.3/180\text{ cm}, 0/200\text{ cm}\}$

These functions can predict the possibility that a basketball player of height 175 cm, who is tall enough to be called *a tall man* among ordinary people, will be described as *a short basketball player*, because his membership grade amounts only to 0.3 on $tall_{BP}$ while it is 0.6 on $short_{BP}$. Fuzzy-set-theoretically, the contextual shift which causes a change in comparison class concerned is accounted for in terms of mapping from one set to another. We should note that an adjective is not contextually bound until it is related to a comparison class. Therefore, whenever an adjective is used in a context, it has an index to show this class. In order to capture this observation in our theory, we should revise the formulations of the adjectival comparison constructions in such a way that an adjective is followed by an index which shows its comparison class.

(49) a. x is more Q than y : $\mu_{Q_A}(x) > \mu_{Q_B}(y)$

b. x is less Q than y : $\mu_{Q_A}(x) < \mu_{Q_B}(y)$

c. x is as Q as y : $\mu_{Q_A}(x) \geq \mu_{Q_B}(y) \wedge \mu_{Q_A}(x) \approx \mu_{Q_B}(y)$

The new feature of the above translations is to be found in the indices A and B added to Q. Furthermore, a pragmatic constraint must be added, to the effect that in the absence of any contradictory specification A and B are to be taken as equal. Let us consider some examples.

(50) a. John is 170 cm tall.

b. Tom is 175 cm tall.

c. John is taller than Tom.

Now let's see how our revised formulation works. First let f be a mapping from the set *GP* to the set of jockeys (J), $f : GP \mapsto J$. Assume $x \in GP$, $y \in J$, and $f(x) = y = x - 10$. By applying the extension principle we get $tall_J$ as follows.

(51) $tall_J = \{0/130\text{ cm}, 0.3/150\text{ cm}, 0.6/155\text{ cm}, 0.7/165\text{ cm}, 0.9/170\text{ cm}, 1.0/175\text{ cm}\}$

The premises (50a,b) allow us to conclude that the membership grade of John on $tall_J$, 0.9, is greater than that of Tom on $tall_{BP}$, 0.3. Upon hearing (50c), the hearer is faced with the task of interpreting what comparison classes are assumed with respect to the adjective *tall*. For instance, one possible, if unlikely, way to fix the relevant comparison classes might be as in (52), where $tall_J$ is assumed for John, and $tall_{BP}$ for Tom.

(52) $\mu_{tall_J}(John) > \mu_{tall_{BP}}(Tom)$

However, recall the comment above, in which we called for a pragmatic constraint that would set comparison classes equal, unless the linguistic form or else the surrounding context imposed a different specification. Consequently, on a context-free reading, an extra condition would be imposed through the pragmatics, in addition to (52).

(53) $J = BP$

Since, (53) does not hold, (52) cannot be maintained as a viable interpretation on a context-free reading. In place of (52), the reader would have to substitute an interpretation where the comparison class indices matched, e.g. any of the readings in (54).

(54) a. $\mu_{tall_{GP}}(John) > \mu_{tall_{GP}}(Tom)$

b. $\mu_{tall_J}(John) > \mu_{tall_J}(Tom)$

c. $\mu_{tall_{BP}}(John) > \mu_{tall_{BP}}(Tom)$

Probably, (54a) is the most likely default interpretation for a sentence like (50c), since it seems natural to suppose the existence of pragmatic norms governing this choice. However, we shall not attempt to go into the matter any further.

Nonetheless, (52) could be a valid reading of (50c), provided either that the surrounding context were rich enough to specify the non-matching comparison classes, or else the form of the sentence contained extra information along the following lines.

(55) John is taller for a jockey than Bill is for a basketball player.

The sentence (55) explicitly mentions comparison classes, and this cancels the pragmatic constraint that would otherwise force the indices to be the same.

Thus, we have seen in this subsection that the effects of comparison classes on adjective meanings can be modeled very naturally within the fuzzy-theoretic approach by means of a standard mechanism provided by the framework, i.e., the extension principle. Next we shall move on to an examination of some further contextual effects.

5.2 Meaning Specification of Evaluative Adjectives

As is well known, relative adjectives belong to two different classes, descriptive adjectives, such as *tall*, *long*, *heavy*, etc., and evaluative adjectives (henceforth E adjectives), such as *clever*, *nice*, *pretty*, etc. Comparison of E adjectives is the central problem in this section. Since the issues surrounding E adjectives are complex, we shall attempt to keep the discussion simple by restricting our attention largely to the case of *clever*. According to Kamp (1975), *clever* has at least two distinctive meaning elements, which we shall label $clever_q$ and $clever_s$. Following Kamp, let us assume the distinction in (56).

(56) a. $clever_q$: quick-wittedness

b. $clever_s$: ability to solve problems

We propose that the membership function of *clever* be based on the Cartesian product $clever_q \times clever_s$ and that the membership grade of the general predicate *clever* be the sort of object described in (57).

(57) $\mu_{clever} : x \times y \mapsto [0,1]$, where $x \in clever_q$, $y \in clever_s$

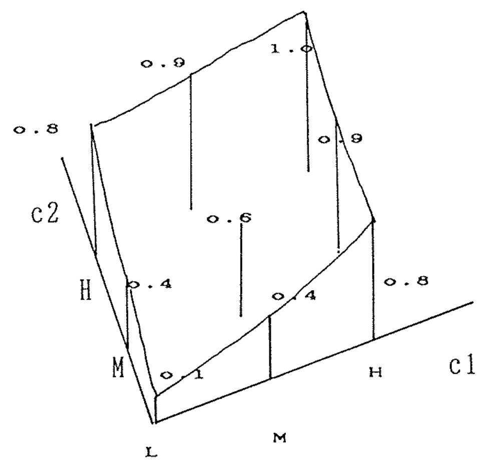
As an illustration, consider the following simplified model. Assume quick wittedness and ability to solve problems are quantitatively judged in terms of three distinctive levels, L, M, and H, $L < M < H$.

(58)

		$clever_s$		
		L	M	H
$clever_q$	L	0.1	0.4	0.8
	M	0.4	0.6	0.9
	H	0.8	0.9	1.0

The relevant membership graph then becomes three-dimensional, as shown in (59). Assume that the axis marked 'c1' corresponds to $clever_q$ and that the one marked 'c2' corresponds to $clever_s$.

(59) THE MEMBERSHIP FUNCTION FOR 'CLEVER'



Now assume the situation postulated as shown in the table (60).

(60)

	Tom	Jane
$clever_q$	H	L
$clever_s$	L	H
μ_{clever}	0.8	0.8

Tom is not so clever in the sense of $clever_s$ as in the sense of $clever_q$, while Jane is just the opposite. In this situation, both *Tom is cleverer than Jane* and *Jane is cleverer than Tom* may be feasible, depending on the context. When we select a context which places weight on $clever_q$ (exemplified as context δ_1 in (61) below) (61B) is possible. Then if we choose a context which puts more emphasis on $clever_s$ (e.g. δ_2 in (62) below), (62B) is more appropriate.

(61) CONTEXT δ_1

A: Who understands things more quickly, Tom or Jane?

B: Tom is cleverer than Jane.

(62) CONTEXT δ_2

A: Who can solve this difficult problem, Tom or Jane?

B: Jane is cleverer than Tom.

Now it has become clear that comparatives with E adjectives cannot be given a truth value until the context which decides what aspects of meaning are at issue has been specified. This observation makes us add another condition to the formulation of comparatives. The new condition requires the context which constrains the meaning to be specified.

(63) MEANING CONDITION ON E ADJECTIVES IN COMPARATIVES

a. x is more Q than y : $\mu_{Q_A^\delta}(x) > \mu_{Q_B^\delta}(y)$

b. x is less Q than y : $\mu_{Q_A^\delta}(x) < \mu_{Q_B^\delta}(y)$

c. x is as Q as y : $\mu_{Q_A^\delta}(x) \geq \mu_{Q_B^\delta}(y) \wedge \mu_{Q_A^\delta}(x) \approx \mu_{Q_B^\delta}(y)$

where Q is an E adjective, and δ is an index of a context for meaning specification.

The same definitions could be used in the case of non-E adjectives. However, the effect of the context δ would be vacuous, exerting no influence on the choice of the fuzzy set to model the meaning of the adjective.

We should note that nothing excludes the possibility of a case where both meaning elements, $clever_q$ and $clever_s$, are concerned. Assume the same situation described in table (60) above, and consider (64) with respect to the context δ_3 in (65), where both $clever_q$ and $clever_s$ are involved.

(64) Tom is as clever as Jane.

(65) CONTEXT δ_3

A: Who in our class deserves most to be called clever, Tom or Jane?

B: Tom is as clever as Jane.

In this case $\mu_{clever^{\delta_3}}$ is the fuzzy membership function for the general *clever* predicate, i.e., the function whose values are set down in (60). Consequently $\mu_{clever^{\delta_3}}(Tom)$ and $\mu_{clever^{\delta_3}}(Jane)$ are given exactly the same value, 0.8, and the equative construction, whose truth condition is $\mu_{clever^{\delta_3}}(Tom) \geq \mu_{clever^{\delta_3}}(Jane) \wedge \mu_{clever^{\delta_3}}(Tom) \approx \mu_{clever^{\delta_3}}(Jane)$, will obtain.

6 LICENSING CONDITIONS FOR COMPARATIVES

A variety of researchers have reported that certain adjectives resist comparison. There follows a list of four categories of adjectives that have been recognized as being incompatible with the comparative construction, along with exemplars of each type.

(66) DENOMINAL ADJECTIVES (Quirk et al. 1985)

a. *This machine is more *atomic* than that.

b. *Bruno is more *Italian* than Giovanni.

(67) POLARIZED ADJECTIVES (Gnutzmann 1975)

a. *He is more *dead* than that man.

b. *She is more *married* than Jane.

c. *John is more *mortal* than Jack.

(68) SCALE-EXTREMITY ADJECTIVES (Bolinger 1967)

a. *This point is more *central* than that.

b. *It is more *unique.*

c. *It is more *perfect.*

d. *The two men more *identical.*

e. *My coffee is more *sugarless* than yours.

(69) CONVEX ADJECTIVES (ibid.)

a. *This water is *lukewarm*, but that water is more *lukewarm.*

b. *His report card was *middling*, but mine was more *middling.*

c. *John is more *middle-aged* than Jack.

d. *Your composition was *fair*, but mine was *fairer.*[4]

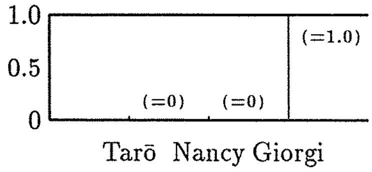
In this section we shall show that the incompatibility with comparison of each of these categories is predictable from an examination of the adjective's fuzzy membership function. Let us begin the discussion by considering how the semantic characteristics of each adjective type are reflected in its corresponding membership function.

Denominal adjectives such as *Italian* and *atomic* are derived from nouns, often by the addition of a suffix. These denominals maintain some of the semantic properties of the nouns from which they are derived. Of principal interest to us is the fact that nouns tend to represent crisp properties more than do adjectives.[5] For instance, the statement that *Giorgi is an Italian* is true if and only if the person in question bears a certain citizenship. The adjectival counterpart of the previous sentence, i.e., *Giorgi is Italian*, has the same truth conditions. If we consider the fuzzy membership graph for an adjective like *Italian*, points along the horizontal axis will correspond to individuals, and the curve will either remain at 0 along the baseline or spike all the way to 1, depending on the individual in question. Let us call such a curve DISCRETE.

[4] *Fair* in this example means 'reasonably good.'

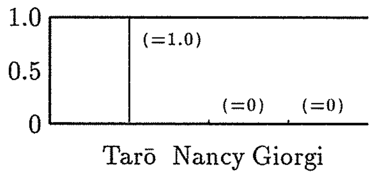
[5] Crisp properties can be represented by fuzzy sets as effectively as scalar ones: they are simply fuzzy sets where the membership function happens to relate all members to the membership values 0 and 1.

(70) DENOMIMAL: *Italian* $\Longrightarrow$ DISCRETE



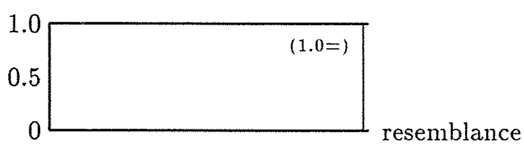
Now let us turn to polarized adjectives such as *married*, *dead*, and *mortal.* What we observed in the denominal type is also true for these polarized adjectives. Indeed, denominal adjectives should probably be considered a subtype of the polarized variety. The fuzzy membership function for a polarized adjective looks just like that pictured above for denominals..

(71) POLARIZED: *married* $\Longrightarrow$ DISCRETE



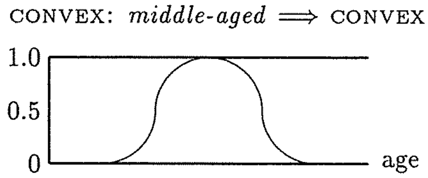
Next we shall analyze scale-extremity type adjectives, such as *central*, *perfect*, and *identical.* Take the case of the last form, *identical.* The property described by this adjective has to do with resemblance, but it is true in all and only cases where the resemblance is total, i.e., where no greater resemblance is conceptually possible. This 'all or nothing' truth condition may be found mutatis mutandis in all of the scale-extremity predicates. If we consider the fuzzy membership function for scale-extremity adjectives, we discover that the curve hugs the baseline, except for one point, where the curve spikes to 1. Let us call this pattern a VERTICAL curve.

(72) SCALE-EXTREMITY: *identical* $\Longrightarrow$ VERTICAL



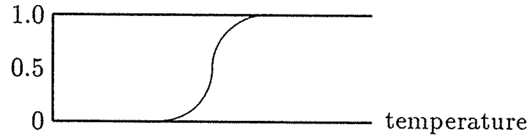
The three foregoing adjective categories have all been basically 'crisp,' in the sense that their fuzzy membership functions happen to map only onto the extreme values of 0 and 1. The final problematic category, however, is scalar, since fuzzy membership functions for this class indeed map onto non-extreme membership degrees. However, adjectives such as *middle-aged*, *tepid*, and *fair* present a special problem, which we have already briefly examined in a foregoing section. Recall that *middle-aged* does not hold at all of children and young adults at one end of the age scale or of senior citizens at the other. It is only between these age extremes that *middle-aged* holds at all. The effect is that the graph for the fuzzy membership function will feature a bell-shaped curve, which we shall describe as CONVEX.

(73) CONVEX: *middle-aged* $\Longrightarrow$ CONVEX

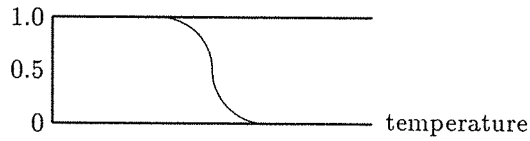


Having examined the fuzzy membership curves for the various problematic adjectives that resist comparison, let us reconsider some predicates that freely admit comparative forms. In our view, two good exemplars are *hot* and *cold*, which illustrate the two kinds of membership functions that we believe capable of supporting adjectival comparison. The graphs in (74) show the types of curves in question: they are characterized by a constant, gradual trend either upwards, as with *hot* in (74a), or downwards, as with *cold* in (74b). Let us call these patterns MONOTONIC-INCREASING and MONOTONIC-DECREASING, respectively.

(74) a. *hot* $\Longrightarrow$ MONOTONIC-INCREASING



b. *cold* $\Longrightarrow$ MONOTONIC-DECREASING



Let us say that a membership curve is MONOTONIC, if and only if it is monotonic-increasing or monotonic-decreasing.

Now, the generalization that we propose to account for the infelicity of comparative constructions in (66)–(69) is that the adjectives displayed there have inappropriate fuzzy membership curves: only when the membership curve is monotonic as described in (74) can a comparative form be employed. Let us next set about providing a formal implementation of this generalization.

We require a formalized notion of monotonicity which will basically describe curves with a constant inclination upward or downward. We may approach the problem as follows. The horizontal axes of the fuzzy graphs we are considering represent a variety of variables: we have seen temperature, price, age, income etc. In some cases, though not all, the points along the horizontal scale represent things that fall naturally into total orderings. This is certainly the case with temperature, price, age, and income, for instance, but not with 'the quality of being Italian.' Deciding when there is a (non-arbitrary) total ordering is a subtle matter into which we can offer no insights; we therefore choose to leave this matter vague for now and appeal to intuition to discover what orderings are available. Let us use the symbol $\prec$ in a general way to represent the relevant total ordering in any given case ($<$ and all related symbols will designate the usual numeric orderings). In cases where there is a total ordering $\prec$ available, we can define monotonic-increasing and monotonic-decreasing fuzzy membership functions as follows.

(75) a. MONOTONIC-INCREASING MEMBERSHIP FUNCTION
μ_Q is monotonic-increasing iff $\forall x, y[x \prec y \rightarrow \mu_Q(x) \leq \mu_Q(y)]$

b. MONOTONIC-DECREASING MEMBERSHIP FUNCTION
μ_Q is monotonic-decreasing iff $\forall x, y[x \prec y \rightarrow \mu_Q(x) \geq \mu_Q(y)]$

As already indicated, monotonic curves are either monotonic-increasing or monotonic-decreasing.

Now consider how the property of monotonicity applies to the various classes of adjectives considered in this section. *Hot* and *cold* are of course monotonic, since there membership curves show a constant inclination upwards or downwards in a manner that is in accord with the formal definition in (75). Our generalization therefore correctly predicts that they should be susceptible to comparison. In contrast, convex adjectives are clearly not monotonic, since their membership curves are bell-shaped, changing inclination in a manner incompatible with the formal definition of monotonicity. We therefore, accurately predict the infelicity of (69). The discrete membership functions for denominal and polarized adjectives lack any discernible total ordering for elements on the horizontal axis, so it would be rather unconvincing to say that the above definition rules out comparison in these cases. Furthermore, the final problematic category, the scale-extremity adjectives, are monotonic-increasing according to the foregoing definition, since the vertical curves for the relevant membership functions have only an upward inclination. Hence, we have not yet provided any means of predicting the ill-formedness of (68). For the three recalcitrant cases, we require another, very simple constraint.

A little reflection will reveal that the three cases that so far resist definitive analysis all involve crisp predicates, i.e., ones whose fuzzy membership functions happen to map strictly onto 0 and 1. If we propose that adjectival comparison be banned from applying to such crisp predicates, we will have succeeded in ruling out all of the remaining problematic cases in (66)–(68). We therefore offer the following definition of non-vacuous gradience, which is a property that holds of a membership function if at least one member of its range falls between 0 and 1 non-inclusively.

(76) NON-VACUOUS GRADIENCE

μ_Q is non-vacuously gradient iff $\exists x[0 < \mu_Q(x) < 1]$

All adjectives used in comparative constructions must be non-vacuously gradient. With this condition and the one above we prohibit all of the illicit comparatives in (66)–(69). In this manner we hope to have shown convincingly that the fuzzy-theoretic approach to adjectival semantics, by emphasizing the importance of the membership function as an essential element of the semantic analysis, provides the appropriate tool with which to analyze conditions on the choice of adjectives in comparative constructions.

7 CONCLUDING REMARKS

In this paper we have tried to formalize adjectival comparatives in terms of fuzzy semantics. This new approach, which is free from certain defects which the traditional approaches have, seems to be reliable and useful in the following points. The fuzzy-theoretic approach assumes that adjectival expressions do not directly reflect differences existent in the real world but reflect the utterer's subjective judgments on them. This enables us to give a feasible account of what is essential to the semantic content of adjectives, i.e., the shapes of their fuzzy membership curves, as opposed to things which are merely subjective. Also the framework of fuzzy sets provides the extension principle, which allows for a straightforward implementation of the notion of comparison classes. Another boon provided by the fuzzy-theoretic approach is the ease and simplicity with which it handles evaluative adjectives, whose meanings may be composed of many subordinate, gradient properties. Finally the focus placed in the fuzzy-theoretic framework on membership functions for fuzzy predicates emphasizes what we believe to be just the right formal mechanism for perspicuously stating the constraints on adjectival comparison. The various observations offered in the foregoing discussion of adjectival semantics underscores the utility of adopting fuzzy set theory as the basis for semantic analysis.