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ADDENDUM TO "PERIOD MAP OF SURFACES WITH $p_g=1$, $c_1^2=2$ AND $\pi_1=Z/2Z$" IN THIS VOLUME

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In this addendum, we will give a table of the classification of the automorphisms on the surfaces with $p_g=1$, $c_1^2=2$, $\pi_1=\boldsymbol{Z}/2\boldsymbol{Z}$ and K ample.

Let X be a smooth surface with $p_g=1$, $c_1^2=2$, $\pi_1=\boldsymbol{Z}/2\boldsymbol{Z}$ and K ample. Denote by $\tilde{X}$ the universal cover of X and $\tilde{\tau}$ the covering transformation. Then, any $\sigma \in \mathrm{Aut}(X)$ has a lifting $\tilde{\sigma} \in \mathrm{Aut}(\tilde{X})$, which commutes with $\tilde{\tau}$ (another lifting is $\tilde{\sigma}\tilde{\tau}$), and $\tilde{\sigma} \in \mathrm{Aut}(\tilde{X})$ induces a projectivity of $\mathbf{P}=\mathbf{P}(1, 1, 1, 2, 2)$, since $\tilde{X}$ is a weighted complete intersection in $\mathbf{P}$ and $K_{\tilde{X}} \simeq \mathcal{O}_{\tilde{X}}(1)$. For $\sigma \in \mathrm{Aut}(X)$ and $\sigma' \in \mathrm{Aut}(X')$, we consider the following equivalence relation:

$$\text{(A.1)} \qquad \sigma \sim \sigma' \Longleftrightarrow {}^{\exists}\alpha\colon X \xrightarrow{\sim} X' \quad s.t. \quad \sigma' = \alpha\sigma\alpha^{-1}$$

Carrying out a similar program in [U.2], we get:

Theorem (A. 2) *Any automorphism $\sigma \neq id$ of a smooth surface X with $p_g=1$, $c_1^2=2$, $\pi_1=\boldsymbol{Z}/2\boldsymbol{Z}$ and K_X ample has a lifting $\tilde{\sigma}_i$ in the table below up to the equivalence relation* (A.1), *and such a $\tilde{\sigma}_i$ is uniquely determined by σ. The induced actions on $H^1(T_X)$, $H^0(\Omega_X^2)$ and $H^1_{prim}(\Omega_X^1)$ are as follows:*

a lifting of σ is $\tilde{\sigma}_i$ up to (A.1)	the induced actions of σ on $H^1(T_X)$, $H^0(\Omega_X^2)$ and $H^1_{prim}(\Omega_X^1)$ respectively
$\tilde{\sigma}_1=(1, 1, 1, 1, -1)$	$14(1)+2(-1)$, (-1), $13(-1)+4(1)$
$\tilde{\sigma}_2=(1, 1, 1, -1, -1,)$	$12(1)+4(-1)$, (1), $9(1)+8(-1)$
$\tilde{\sigma}_3=(1, i, i, 1, 1)$	$6(1)+6(-1)+4(i)$, (-1), $6(-1)+3(1)+4(-i)+4(i)$
$\tilde{\sigma}_4=(1, i, i, 1, -1)$	$6(1)+6(-1)+2(i)+2(-i)$, (1), $6(1)+3(-1)+4(i)+4(-i)$
$\tilde{\sigma}_5=(1, i, -i, 1, 1)$	$6(1)+6(-1)+2(i)+2(-i)$, (1), $5(1)+4(-1)+4(i)+4(-i)$

$\tilde{\sigma}_6=(1, i, -i, 1, -1)$	$6(1)+6(-1)+2(i)+2(-i),\quad (-1),$ $5(-1)+4(1)+4(-i)+4(i)$
$\tilde{\sigma}_7=(1, 1, -1, 1, 1)$	$10(1)+6(-1),\quad (-1),$ $10(-1)+7(1)$
$\tilde{\sigma}_8=(1, 1, i, 1, 1)$	$6(1)+4(-1)+5(i)+1(-i),\quad (i),$ $5(i)+5(-i)+6(-1)+1(1)$
$\tilde{\sigma}_9=(1, 1, i, 1, -1)$	$5(1)+5(-1)+4(i)+2(-i),\quad (-i),$ $5(-i)+5(i)+4(1)+3(-1)$
$\tilde{\sigma}_{10}=(1, 1, i, -1, -1)$	$4(1)+6(-1)+3(i)+3(-i),\quad (i),$ $5(i)+5(-i)+2(-1)+5(1)$
$\tilde{\sigma}_{11}=(1, 1, -1, 1, -1)$	$10(1)+6(-1),\quad (1),$ $10(1)+7(-1)$
$\tilde{\sigma}_{0'}=(1, 1, -1, (1, 1))$	$8(1)+8(-1),\quad (-1),$ $8(-1)+9(1)$
$\tilde{\sigma}_{2'}=(1, 1, -1, (1, -1))$	$6(1)+6(-1)+2(i)+2(-i),\quad (1),$ $4(1)+5(-1)+4(i)+4(-i)$
$\tilde{\sigma}_{3'}=(1, \varepsilon, \varepsilon^5, (1, 1))$	$3(1)+3(-1)+3(i)+3(-i)+2(\varepsilon)$ $+2(-\varepsilon),\quad (-i),$ $3(-i)+3(i)+1(1)+2(-1)+2(\varepsilon^7)$ $+2(\varepsilon^3)+2(\varepsilon)+2(\varepsilon^5)$
$\tilde{\sigma}_{5'}=(1, \varepsilon, \varepsilon^3, (1, 1))$	$2(1)+4(-1)+3(i)+3(-i)+1(\varepsilon)$ $+1(\varepsilon^3)+1(\varepsilon^5)+1(\varepsilon^7),\quad (-1),$ $2(-1)+3(1)+2(-i)+2(i)+2(\varepsilon^5)$ $+2(\varepsilon^7)+2(\varepsilon)+2(\varepsilon^3)$
$\tilde{\sigma}_{5''}=(1, \varepsilon, \varepsilon^7, (1, 1))$	$2(1)+4(-1)+3(i)+3(-i)+1(\varepsilon)$ $+1(\varepsilon^3)+1(\varepsilon^5)+1(\varepsilon^7),\quad (1),$ $1(1)+4(-1)+2(i)+2(-i)+2(\varepsilon)$ $+2(\varepsilon^3)+2(\varepsilon^5)+2(\varepsilon^7)$

where we use the notation:

$$i=\sqrt{-1} \quad \textit{and} \quad \varepsilon=\exp(2\pi i/8).$$

$(1, c_1, c_2, c_3, c_4)$ (*resp.* $(1, c_1, c_2, (c_3, c_4))$) $\in \mathrm{Aut}(\mathbf{P}(1, 1, 1, 2, 2))$ *sends* (w, x_1, x_2, z_3, z_4) *to* $(w, c_1x_1, c_2x_2, c_3z_3, c_4z_4)$ (*resp.* $(w, c_1x_1, c_2x_2, c_3z_4, c_4z_3)$).

$m_1(\lambda_1)+m_2(\lambda_2)+\cdots+m_r(\lambda_r)$ *indicates that the* λ_j*-eigen subspace has dimension* m_j $(j=1, 2,\dots, r)$.

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