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Author(s)	Kato, Kazuya; Usui, Sampei
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A LIST OF ERRATA OF CLASSIFYING SPACES OF DEGENERATING POLARIZED HODGE STRUCTURES

Kazuya Kato, Sampei Usui

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Corrections

1. Corrections for §7.1

There is a problem: The second convergence of 7.1.2 (3) is not justified. In order to solve this, §7.1 should be corrected as follows.

Proposition 7.1.1 and Corollary 7.1.3 are correct.

We replace 7.1.2 in the book by the following 7.1.2, Proposition 7.1.2.1 and 7.1.2.2.

7.1.2. We will prove Theorem A (i) in 7.1.2, 7.1.2.1 and 7.1.2.2.

Assume x_λ converges in $\tilde{E}_{\sigma,\mathrm{val}}$ to $x \in E_{\sigma,\mathrm{val}}$.

Let $\overline{x} = (A, V, Z) \in D^\sharp_{\sigma,\mathrm{val}}$ be the image of x and take an excellent basis $(N_s)_{s\in S}$ for $\overline{x}$ such that $N_s \in \sigma(q)$ for any s (6.3.9). Let S_j $(1 \le j \le n)$ be as in 6.3.3. Take an $\mathbf{R}$-subspace B of $\sigma_{\mathbf{R}}$ such that $\sigma_{\mathbf{R}} = A_{\mathbf{R}} \oplus B$.

We have a unique injective open continuous map

$$(\mathbf{R}^S_{\ge 0})_{\mathrm{val}} \times B \to |\mathrm{toric}|_{\sigma,\mathrm{val}}$$

which sends $((e^{-2\pi y_s})_{s\in S}, b)$ $(y_s \in \mathbf{R}, b \in B)$ to $\mathbf{e}((\sum_{s\in S} iy_sN_s) + ib)$ (cf. 3.3.5). Let U be the image of this map. Define the maps $t_s : U \to \mathbf{R}_{\ge 0}$ $(s \in S)$ and $b : U \to B$ by

$$(t, b) = ((t_s)_{s\in S}, b) : U \simeq (\mathbf{R}^S_{\ge 0})_{\mathrm{val}} \times B \to \mathbf{R}^S_{\ge 0} \times B.$$

Let $|\ \ | : \mathrm{toric}_{\sigma,\mathrm{val}} \to |\mathrm{toric}|_{\sigma,\mathrm{val}}$ be the canonical projection induced by $\mathbf{C} \to \mathbf{R}$, $z \mapsto |z|$. Then, $|q| \in U$ and $t(|q|) := (t_s(|q|))_{a\in S} = 0$. Since $|q_\lambda| \to |q|$, we may assume $|q_\lambda| \in U$.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Proposition 7.1.2.1. *Assume $x_\lambda = (q_\lambda, F_\lambda)$ converges in $\tilde{E}_{\sigma,\mathrm{val}}$ to $x = (x, F) \in E_{\sigma,\mathrm{val}}$. We use the notation in* 7.1.2. *Take $c_j \in S_j$ for each j. Fix $1 \le j \le n+1$. Assume $t_s(q_\lambda) \neq 0$ for any λ and any $s \in S_{\ge j}$* (6.3.11). *($S_{\ge n+1}$ is defined to be the empty set.) Define $y_{\lambda,s} \in \mathbf{R}$ by $t_s(q_\lambda) = e^{2\pi y_{\lambda,s}}$. For each $k \in \mathbf{Z}$ such that $j \le k \le n$, let $N_k = \sum_{s\in S_k} a_s N_s$ where $a_s \in \mathbf{R}$ is the limit of $y_{\lambda,s}/y_{\lambda,c_k}$. Let $\tilde{\rho} : \mathbf{G}_{m,\mathbf{R}}^n \to G_{\mathbf{R}}$ be the homomorphism of the* SL(2)*-orbit* (5.2.2) *associated to (q, F). Let*

$$e_{\lambda,\ge j} = \exp(\textstyle\sum_{s\in S_{\ge j}} iy_{\lambda,s}N_s) \in G_{\mathbf{C}},$$

$$\tau_{\lambda,k} = \tilde{\rho}_k\left(\sqrt{y_{\lambda,c_{k+1}}/y_{\lambda,c_k}}\right) \in G_{\mathbf{R}} \ \ (1 \le k \le n), \quad \tau_{\lambda,\ge j} = \textstyle\prod_{k=j}^n \tau_{\lambda,k} \in G_{\mathbf{R}}.$$

(i) *In $\check{D}$, $\tau_{\lambda,\ge j}^{-1} e_{\lambda,\ge j} F_\lambda$ converges to $\exp(iN_j)\hat{F}_{(j)}$. Here in the case $j = n+1$, N_{n+1} denotes 0, and $\hat{F}_{(n+1)}$ denotes F.*

(ii) *Assume $((N_s)_{s\in S_{\le j-1}}, F_\lambda)$ satisfies Griffiths transversality for any λ. Then, for any sufficiently large λ, $((N_s)_{s\in S_{\le j-1}}, e_{\lambda,\ge j}F_\lambda)$ generates a nilpotent orbit.*

Proof. We prove this proposition by downward induction on j. We write the above (i) and (ii) for j as (i)$_j$ and (ii)$_j$, respectively.

First, (i)$_{n+1}$ means that F_λ converges to F, which is evident.

For any $1 \le j \le n+1$, we deduce (ii)$_j$ from (i)$_j$. Since $((N_s)_{s\in S_{\le j-1}}, F_\lambda)$ satisfies Griffiths transversality by the assumption, $((N_s)_{s\in S_{\le j-1}}, \tau_{\lambda,\ge j}^{-1} e_{\lambda,\ge j}F_\lambda)$ satisfies Griffiths transversaslity. Since $\tau_{\lambda,\ge j}^{-1} e_{\lambda,\ge j}F_\lambda$ converges to $\exp(iN_j)\hat{F}_{(j)}$ by (i)$_j$ and since $((N_s)_{s\in S_{\le j-1}}, \exp(iN_j)\hat{F}_{(j)})$ generates a nilpotent orbit (6.1.3), Proposition 7.1.1 shows that for any sufficiently large λ, $((N_s)_{s\in S_{\le j-1}}, \tau_{\lambda,\ge j}^{-1} e_{\lambda,\ge j}F_\lambda)$ generates a nilpotent orbit. Hence for a sufficiently large λ, $((N_s)_{s\in S_{\le j-1}}, e_{\lambda,\ge j}F_\lambda)$ generates a nilpotent orbit.

Next, for $1 \le j \le n$, we deduce (i)$_j$ from (i)$_{j+1}$ and (ii)$_{j+1}$. By a proposition on the strong topology in §3, for any fixed $e \ge 1$, there exist $F_\lambda^* \in \check{D}$ (for any λ) such that $((N_s)_{s\in S_{\le j}}, F_\lambda^*)$ satisfies Griffiths transversality and $y_{\lambda,c_j}^e d(F_\lambda, F_\lambda^*) \to 0$. Hence for the proof of (i)$_j$, we may assume that $((N_s)_{s\in S_{\le j}}, F_\lambda)$ satisfies Griffiths transversality. By (ii)$_{j+1}$, for a sufficiently large λ, $((N_s)_{s\in S_{\le j}}, e_{\lambda,\ge j+1}F_\lambda)$ generates a nilpotent orbit and hence $((N_s)_{s\in S_{\le j}}, \tau_{\lambda,\ge j+1}^{-1} e_{\lambda,\ge j+1}F_\lambda)$ generates a nilpotent orbit. By (i)$_{j+1}$, $\tau_{\lambda,\ge j+1}^{-1} e_{\lambda,\ge j+1}F_\lambda$ converges to $\exp(iN_{j+1})\hat{F}_{(j+1)}$. By this, $\tau_{\lambda,j}^{-1}\tau_{\lambda,\ge j+1}^{-1} e_{\lambda,\ge j+1}F_\lambda$ converges to $\hat{F}_{(j)}$ (6.1.11 (ii)). Hence $\tau_{\lambda,\ge j}^{-1} e_{\lambda,\ge j}F_\lambda = \exp(\sum_{s\in S_j} i(y_s/y_{c_j})N_s)\tau_{\lambda,j}^{-1}\tau_{\lambda,\ge j+1}^{-1}$ $e_{\lambda,\ge j+1}F_\lambda$ converges to $\exp(iN_j)\hat{F}_{(j)}$. □

7.1.2.2. We prove now that E_σ is open in $\tilde{E}_\sigma$ for the strong topology.

Since $\tilde{E}_{\sigma,\mathrm{val}} \to \tilde{E}_\sigma$ is proper surjective and $E_{\sigma,\mathrm{val}} \subset \tilde{E}_{\sigma,\mathrm{val}}$ is the inverse image of $E_\sigma \subset \tilde{E}_\sigma$, it is sufficient to prove that $E_{\sigma,\mathrm{val}}$ is open in $\tilde{E}_{\sigma,\mathrm{val}}$. Assume $(q_\lambda, F_\lambda) \in \tilde{E}_{\sigma,\mathrm{val}}$ converges in $\tilde{E}_{\sigma,\mathrm{val}}$ to $(q, F) \in E_{\sigma,\mathrm{val}}$. We prove that $(q_\lambda, F_\lambda) \in E_{\sigma,\mathrm{val}}$ for any sufficiently large λ. We may assume that, for some j $(1 \le j \le n+1)$, $t_s(q_\lambda) = 0$ for any λ and $s \in S_{\le j-1}$ and $t_s(q_\lambda) \neq 0$ for any λ and $s \in S_{\ge j}$. Then, since $(q_\lambda, F_\lambda) \in \tilde{E}_{\sigma,\mathrm{val}}$,

$((N_s)_{s\in S_{\le j-1}}, F_\lambda)$ satisfies Griffiths transversality for any λ. Hence by Proposition 7.1.2.1 (ii), for a sufficiently large λ, $((N_s)_{s\in S_{\le j-1}}, e_{\lambda,\ge j}F_\lambda)$ generates a nilpotent orbit, that is, $(q_\lambda, F_\lambda) \in E_{\sigma,\mathrm{val}}$.

2. Change of the organizations, etc.

There is a problem in the third limit of Proposition 6.4.1 (5). In order to solve this, we reorganize the materials as follows.

We should put the above corrected §7.1 just before §6.4.

3. Change in §7.2

Professor J.-P. Serre kindly pointed out that our book should not use [BS, §10], for errors are there (cf. 10.10. Remark in the version of [BS] contained in "Armand Borel oevres Collected papers, Vol. III, Springer-Verlag, 1983"). We used a result [BS, 10.4] in the proof of Lemma 7.2.12 of our book. In order to correct our argument, we change as follows.

We put the following assumption in Theorem 7.2.2 (i):
"Assume that σ is a nilpotent cone associated to a nilpotent orbit."

We replace Lemma 7.2.12 and its proof in our book by the following proposition and its proof which does not use [BS, §10].

Proposition 7.2.12. *Let σ be a nilpotent cone associated to a nilpotent orbit and let $W(\sigma)$ be the associated weight filtration. Then, by assigning the Borel-Serre splitting, we have a continuous map $E^\sharp_{\sigma,\mathrm{val}} \to \mathrm{spl}(W(\sigma))$.*

Proof. The composite map $E^\sharp_{\sigma,\mathrm{val}} \to D^\sharp_{\sigma,\mathrm{val}} \xrightarrow{\psi} D_{\mathrm{SL}(2)}$ is continuous by the definition of the first map and by 6.4.1 for the CKS map ψ. Let $N_1, \dots, N_n$ be a generator of the cone σ. Let s be a bijection $\{1,\dots,n\} \to \{1,\dots,n\}$. Then the image of the map $E^\sharp_{\sigma,\mathrm{val}} \to D_{\mathrm{SL}(2)}$ is contained in the union U of $D_{\mathrm{SL}(2)}(\{W(N_{s(1)} + \cdots + N_{s(j)}) \mid j = 1,\dots,n\})$ where s runs over all bijections $\{1,\dots,n\} \to \{1,\dots,n\}$. Since $N_{s(1)} + \cdots + N_{s(n)} = N_1 + \cdots + N_n$, the filtration $W(N_1 + \cdots + N_n) = W(\sigma)$ appears for any s. By [part 2, Proposition 3.2.12], the Borel-Serre splitting gives a continuous map $U \to \mathrm{spl}(W(\sigma))$. Thus we our assertion. □

We replace the third paragraph in 7.2.13 by the following:

"Since the action of $\sigma_{\mathbf{R}}$ on $\mathrm{spl}(W(\sigma))$ is proper and $E^\sharp_{\sigma,\mathrm{val}}$ is Housdorff, the action of $\sigma_{\mathbf{R}}$ on $E^\sharp_{\sigma,\mathrm{val}}$ is proper by applying Lemma 7.2.6 (ii) to the continuous map $E^\sharp_{\sigma,\mathrm{val}} \to \mathrm{spl}(W(\sigma))$ in Proposition 7.2.12. Hence $\mathrm{Re}(h_\lambda)$ converges in $\sigma_{\mathbf{R}}$ by Lemma 7.2.7."

Add the following sentence at the top of the fourth paragraph in 7.2.13:

"Let $|\ \ | : E_{\sigma,\mathrm{val}} \to E^\sharp_{\sigma,\mathrm{val}}$ be the continuous map $(q, F) \to (|q|, F)$ in 7.1.3."

4. Typographical Errors and Comments

The first paper after the cover paper, the french translation of the short poem, the second line from the bottom: l'in ni → l'infini.

Contents, Chapter 4, 4.1, the end of the title: $\Gamma\backslash D_{\Sigma^\sharp} \to \Gamma\backslash D^\sharp_\Sigma$.

Page viii, Chapter 6: $D_{\mathrm{SL}}(2) \to D_{\mathrm{SL}(2)}$.

0.2.15, at the end of the 4th line after the display (2): Add "(see 0.1.3)".

0.2.15, the 4th and the 5th lines after the display (3): $\theta \to \alpha$. (There are three such changes.)

0.2.15, the the 6th line after the display (3): $x \to \theta$. (There are two such changes.)

0.5.7, the 7th line from the bottom: $K_{\mathbf{R}} \to K_{\mathbf{r}}$.

0.5.21, the 2nd paragraph, the 3rd line: C_∞ where → C_∞, where.

0.5.21, the 3rd paragraph, the 7th and the 8th lines: $\cup_{s'\neq s} \to \bigcup_{s'\neq s}$.

0.5.21, the 3rd paragraph, the 9th line: Then for → Then, for.

0.5.21, the 3rd paragraph, the last line: $\mathbf{Z} \to \mathbf{Z}^2$.

0.5.21, the 4th paragraph, the 6th line from the bottom: $(0, z) \to (1, z)$. (There are two such typos.)

0.5.21, the 4th paragraph, the 2nd line from the bottom: if for → if, for.

0.7.5, the 2nd paragraph, the 2nd line from the bottom: Add "with $a \neq b$" after X.

Definition 1.3.7, (2): $z_j \to iy_j$. (There are three such typos. Especially, the final $\mathrm{Im}(z_j)$ should be simply y_j.)

2.2.3, at the end of the 1st paragraph: (2.2.2) → 2.2.2.

2.2.3, (2), 2nd line: Add "," after $h(f)$.

Proposition 2.2.3, Proof, the 11th line: $\left(R^m\tau_*\left(\mathcal{O}_X\otimes_{\mathbf{C}}R\right)\right)_x \to (R^m\tau_*(\mathcal{O}_X\otimes_{\mathbf{C}}R))_x$.

Proposition 2.3.2, the 1st display: Insert colon ":" between L_0 and =.

Proposition 2.3.2, the line after the 1st display: following → below.

Proposition 2.3.2, the line after the 1st display: Let → Let A' be the subring of $\mathbf{C}$ generated by A and $\mathbf{Q}$, let $L_{0,A'} = A' \otimes_A L_0$, and let

Proposition 2.3.2, the line after the 3rd display: $\xi = \exp(\sum_{j=1}^n (2\pi i)^{-1}\log(q_j)\otimes N_j)$ depends on the local choices of the branches of $\log(q_j) \to \xi$ depends on the choices of the branches of $\log(q_j)$ in $\mathcal{O}_X^{\log}$ locally on $X^{\log}$.

Proposition 2.3.2, the line after the display in (1): branches of → branches in $\mathcal{O}^{\log}_{X,y}$ of.

Proposition 2.3.2, the 2nd line after the display in (1): satisfying (1) as above which satisfies furthermore the following condition (2) → which satisfies above (1) and also the following (2).

Proposition 2.3.2, (2), the 1st line: branch of ξ_y defined by the fixed branches of $\log(q_j)_y$ satisfies → branch of the germ ξ_y, defined by the fixed branches of the germs of $\log(q_j)_y$, satisfies.

Proposition 2.3.2, (2), the last display: $v \in L_0 \to v \in L_y = L_0$.

Proposition 2.3.2, Proof, the 2nd line: branch of $\log(q_j)_y$ → branch of the germ $\log(q_j)_y$.

Proposition 2.3.2, Proof, the 3rd line: Add a comma "," before "where".

Proposition 2.3.2, Change the 1st display of Proof with its comments as follows:

$$\begin{aligned}\gamma_k(\nu(v)) &= \gamma_k(\xi_y^{-1}(1\otimes v)) = \gamma_k(\xi_y)^{-1}\cdot(1\otimes v)\\ &= \exp(-(\textstyle\sum_{j=1}^{n}((2\pi i)^{-1}\log(q_j)_y - \delta_{jk})\otimes N_j)\cdot(1\otimes v)\\ &= \xi_y^{-1}\exp(1\otimes N_k)(1\otimes v) = \xi_y^{-1}(1\otimes\gamma_k(v)) = \nu(\gamma_k(v))\end{aligned}$$

(δ_{jk} is the Kronecker symbol, and for the signature before δ_{jk}, see Appendix A1). Here the second equality follows from the monodromy action of γ_k on the locally constant sheaf L', the fifth equality follows from the endmorphism N_k of the constant sheaf L_0, and $\gamma_k(v)$ in the second last and in the last is the image of the element $v \in L_0 = L_y$ by the monodromy action of γ_k on the locally constant sheaf L at y.

Example after Proposition 2.3.2, the 6th line: Add "on $\Delta^{\log}$" after "locally".

Example after Proposition 2.3.2, the line before the 1st display: Add "Let $y \in x^{\log}$ and let $L_0 := L_y$" at the top of this line.

Example after Proposition 2.3.2, the line before the 1st display: branch of $\log(q) \to$ branch in $\mathcal{O}^{\log}_{X,y}$ of the germ of $\log(q)$.

Example after Proposition 2.3.2: $e_{1,y} \to e_1$. (There are eight such changes.)

Example after Proposition 2.3.2, the line after the 1st display: base $\to$ basis.

Example after Proposition 2.3.2, the 2nd display: $e_{2,y} \to 1\otimes e_{2,y}$.

Example after Proposition 2.3.2, the 2nd line from the bottom: Insert "$= (2\pi i)^{-1}\log(q)_y \otimes e_1 + 1\otimes e_{2,y}$ (cf. 0.2.15 (2))" after "$(2\pi i)^{-1}\log(q)e_1 + e_{2,y}$".

Example after Proposition 2.3.2, the last line: $-(2\pi i)^{-1}\log(q)e_{1,y} + e_{2,y} \to -(2\pi i)^{-1}\log(q)_y \otimes e_1 + 1\otimes e_{2,y}$.

Proposition 2.3.3, Proof, the line after the 1st display and also the line after the 2nd display: proposition $\to$ Proposition.

Proposition 2.3.3, Proof, the 2nd line after the 2nd display: Add $\{x\}$ after "fs logarithmic point".

Proposition 2.3.3, Proof, the 3rd line after the 2nd display: $\mathcal{O}^{\log}_X \to \mathcal{O}^{\log}_x$.

Proposition 2.3.3, Proof, the last line: $M^{\mathrm{gp}}_{X,x} \to M^{\mathrm{gp}}_x$.

Proposition 2.3.3, Proof, the last line: in $\to$ on.

Proposition 2.3.4, (i), the display: $\otimes L \to \otimes_A L$.

Proposition 2.3.4, (ii), the line after the display: **C**-linear $\to$ A-linear.

Proposition 2.3.4, (ii), the line after the display: $\otimes L \to \otimes_A L$. (There are two such changes.)

Proposition 2.3.4, (ii): $1\otimes\mathcal{N}' \to 1_{\mathcal{O}^{\log}_x}\otimes\mathcal{N}'$. (There are two such changes.)

Proposition 2.3.4, (ii), the 2nd line from the bottom: $\mathcal{O}^{\log}_X \to \mathcal{O}^{\log}_x$

Proposition 2.3.4, (ii), the last line: $\omega^{1,\log} \to \omega^{1,\log}_x$.

Proposition 2.3.4, Proof: Do not change the line between "Proof." and "(i)".

Proposition 2.3.4, Proof, the 2nd line after the display: being $\to$ be.

Proposition 2.3.4, Proof, the last sentence of the proof of (i): Insert "(Proposition 2.3.2)," before "and".

Proposition 2.3.4, Proof, at the end of the proof of (i), add the following:

The last equation follows from Lie derivative. In fact, putting $\ell_j = (2\pi i)^{-1}\log(q_j)_{y'}$, $\xi_{y'}^{-1} = \exp(\sum_{j=1}^{n}(-\ell_j)\otimes N_j)$ by the definition in 2.3.2, and we have

$$\log(\gamma_k)(-\ell_j)^m = \lim_{a\to 0}\frac{\exp(a\log(\gamma_k))(-\ell_j)^m - (-\ell_j)^m}{a} = \delta_{kj}m(-\ell_j)^{m-1}$$

(δ_{kj} is the Kronecker symbol).

Proposition 2.3.4, Proof of (ii), the 4th line: Add the following after "with $-d$.":

In fact, in the above notation, we have the following by the formula of differential of composite map, since the N_j are commutative.

$$-d(\xi^{-1}(1\otimes v)) = -d(\xi^{-1})(1\otimes v) = \sum_j d\ell_j \otimes \xi^{-1}(1\otimes N_j)(1\otimes v) = \mathcal{N}'(\xi^{-1}(1\otimes v)).$$

2.3.6, the 4th line: X - U $\to$ $X - U$.

2.3.7, the 3rd paragraph, the 2nd line: $L_0 \to L_1$ (denoted by L_0 in 2.3.2).

2.3.7, the 3rd paragraph, the display: $L_0 \to L_1$.

2.3.7, the 3rd paragraph, the 2nd line from the bottom: Change "$N_j : L_{0,\mathbf{Q}} \to L_{0,\mathbf{Q}}$ be the logarithm of γ_j" to "$N_j : L_{1,A'} \to L_{1,A'}$ be the logarithm of γ_j, where A' is the subring of $\mathbf{C}$ generated by A and $\mathbf{Q}$".

2.3.7, (1): $L_0 \to L_1$.

2.3.7, (1): $\nu = \to \xi =$.

2.3.7, (2), the 1st line: branch of $\to$ branch in $\mathcal{O}_{X,1}^{\log}$ of.

2.3.7, (2), the last line: Change "$\xi_{1,0}\circ\nu_y : 1\otimes L_y \to 1\otimes L_0$" to "$\xi_{1,0}\circ\nu_1 : 1\otimes L_1 \to 1\otimes L_1$".

2.3.7, the last paragraph, the 2nd line: $L' = \to L' :=$.

2.3.7, the last paragraph, the 2nd line: $L_0 \to L_1$. (There are two such changes in this line.)

2.3.7, the last paragraph, the 2nd line: $\xi_{0,1}^{-1} \to \xi_{1,0}^{-1}$.

2.3.7, the 2nd line from the bottom: $\mathcal{O}^{\log} \to \mathcal{O}_X^{\log}$.

2.3.7, the 2nd line from the bottom: $L_0 \to L_1$.

2.3.8, the 6th line: L_0 be the stalk $L_p \to L_p$ be the stalk of L at p.

2.3.8, the 1st display, the right hand side: $\otimes L_0 \to \otimes_A L_p$.

2.3.8, the 1st line in (1): $\otimes L_0 \to \otimes L_p$.

2.3.8, the 2nd line from the bottom in (2): branch of $\to$ branch in $\mathcal{O}_{X,\tilde{p}}^{\log}$ of.

2.3.8, the last line in (2): at p is $\tilde{p}$ $\to$ at $\tilde{p}$ is 0.

Definition 2.4.7, (2), the 1st line: $M_x \to M_x^{\mathrm{gp}}$.

Proposition 2.5.1, Proof, the 3rd line: $M_x \to M_x^{\mathrm{gp}}$.

Proposition 2.5.1, Proof, display (2): $\xi_{0,y} \to \xi_{y,0}$.

2.5.3, the 5th line after the 2nd display: Add "(2.3.3)" after "$\langle\ ,\ \rangle_0$".

2.5.3, the 6th line after the 2nd display: $\{\tilde{\mu}_y(F(s)) \mid s \in \mathrm{sp}(y)\} \to Z := \{\tilde{\mu}_y(F(s)) \mid s \in \mathrm{sp}(y)\}$.

Proposition 2.5.5, Proof, the 5th line from the end: Add "(2.2.9 (2))" after "h_j".

Proposition 2.5.5, Proof, the 4th line from the end: $q_j \to q_k$. (There are three such changes.)

Proposition 2.5.5, Proof, the 3rd line from the end: $1 \le j \le n \to 1 \le k \le m$.

Proposition 2.5.5, Proof, the 2nd line from the end: $\mathbf{R} \to \mathbf{R}^{\mathrm{add}}$.

Lemma 2.5.6, Proof, the 2nd line: exist → exists.

2.5.13, the last line: proposition → Proposition.

2.5.11, The 3rd line: $\mathfrak{g}_{\mathbf{R}} \to \mathfrak{g}_{\mathbf{Q}}$.

3.3.2, at the end of the 6th line: Change a comma to a period.

3.3.3, the line after the 2nd display: Change the line at "On the other hand ...".

3.3.3, display (1): $\xi \to \xi^{-1}$.

3.3.4, the 4th line: $\check{E} \to \check{E}_\sigma$.

Proposition 3.3.6, Proof, the 2nd line: $\mu_\sigma(v) \to 1 \otimes \mu_\sigma(v)$.

Proposition 3.3.6, Proof, the 2nd line: $\xi \to \xi^{-1}$. (There are two such changes.)

Proposition 3.3.6, Proof, the 2nd line: Add "for $s \in \mathrm{sp}(y)$" at the end of this sentence.

Proposition 3.3.6, Proof, the 4th line from the bottom: Erase "(we take $\Gamma(\sigma)^\vee$ as $\mathcal{S}$ in 2.3.7)".

3.4.2, As a newline add the following at the end:

Note that the structure sheaf $\mathcal{O}_{E_\sigma}$ is injectively embedded in the sheaf of $\mathbf{C}$-valued functions on E_σ. In fact, let V be the subset of E_σ consisting of all points with trivial logarithmic structure, i.e., $V = E_\sigma \cap (\mathrm{torus}_\sigma \times \check{D})$. If local sections f and g of $\mathcal{O}_{E_\sigma}$ coincide as local functions on E_σ, then their restrictions to V coincide as holomorphic functions on V and hence all their corresponding higher partial derivatives coincide on V. Since V is dense in E_σ (in the strong topology), the values of their corresponding higher partial derivatives coincide on E_σ, hence f and g coincide as local sections of $\mathcal{O}_{E_\sigma}$.

After Proposition 3.6.1, the 2nd line before 3.6.2: proposition → Proposition.

3.6.4, at the end of the 2nd paragraph: 3.6.1 → 3.6.1 (i).

3.6.5, the last line: Add a comma "," after (xy, y).

Proposition 3.6.6, Proof, the 1st paragraph, the 2nd line from the bottom: $\cup \to \bigcup$.

3.6.19, the 1st line: a finite → a nonempty finite.

Lemma 3.6.21, Proof: Do not change a line between "Proof" and "(i)".

Lemma 3.6.21, Proof, the 2nd line of the 1st paragraph: 3.6.7 → 3.6.8.

3.6.22, the 2nd line: of local → of logarithmic local.

3.6.22, the 3rd line: exist → exists.

3.6.22, display: Add a comma "," at the end.

3.6.26, the 2nd line: $M^{\mathrm{gp}}_{X,x} \to M^{\mathrm{gp}}_{X_{\mathrm{val}},x}$.

4.1, the title, the last: $\Gamma\backslash D_{\Sigma^\sharp} \to \Gamma\backslash D^\sharp_\Sigma$.

4.1.1, Theorem A, (iii): $(\mathbf{e}(a)q, \exp(-a)F) \to (\mathbf{e}(-a)q, \exp(a)F)$. (Cf. 2.2.9, the action of γ on $\log(q)$.)

4.1.1, Theorem A, (iv): Delete "open and".

4.3.4, (i), the last line: $\mathbf{Z} \to \mathbf{R}$.

Proposition 4.3.6, Proof, the first line: condition → conditions.

Proposition 4.3.6, Proof, the 3rd and the 5th lines: pullback of $H' \to$ pullback H' of H.

Proposition 4.3.8, (i), the 2nd line from the bottom: $X' \to W'$.

Proposition 4.3.8, Proof, the 2nd paragraph, the 2nd line and the 2nd line from the bottom: $\cup \to \bigcup$.

4.4.2, the line after the 2nd display: $\oplus \to \bigoplus$.

Proposition 4.4.3, Proof, the 5th line from the bottom: $\tau^{-1}(\mathcal{M}^p_y) \to \tau^{-1}(\mathcal{M}^p)_y$.

4.4.5, the 2nd line: X $\to$ X (italic).

Theorem 4.4.8, the bottom of the display, exponent: $m \to w$.

5.1.11, the last line: $x \in V$ to $a(x)h(x) \to \chi \in V$ to $a(\chi)h(\chi)$. (Change x to χ. There are three such changes.)

5.2.16, at the end add the following:

In their old proofs depend on [KU2, 4.12] where there is a mistake. For a correction, see 3.3.6 in the following paper: K. Kato, C. Nakayama and S. Usui, Classifying spaces of degenerating mixed Hodge structures, II: Spaces of SL(2)-orbits, Kyoto J. Math. 51 (1): Nagata Memorial Issue, 2011, 149–261.

5.3.2, the line after (2): Insert the following at the end of the first sentence.

, where h is considered as an $\mathbf{R}$-linear map $A_{\mathbf{R}} \to \mathbf{R}$.

5.3.2, the 2nd line from the bottom: then V $\to$ we have V.

Lemma 5.3.4, Proof, the 2nd line: $\cup \to \bigcup$.

5.3.6, the 3rd paragraph, the last line: Add one more ")" after $\log(\Gamma(\sigma)^{\mathrm{gp}})$.

5.3.6, the 4th paragraph, the 2nd line: Add a comma "," before "where".

5.4.1, the 3rd line: 1.3.6 $\to$ 1.3.7.

Proposition 6.4.1, (5), the 3rd line: (for λ, for sufficiently large) $\to$ (for sufficiently large λ).

Lemma 6.4.2, Proof the 2nd line after the display (1): in $y_{\lambda,s}$ $\to$ in the $y_{\lambda,s}$.

6.4.4, (C_j), (2): Add the following sentence after the end: Here, for elements $h = (h_k)_{1\le k\le j}$ and $h' = (h'_k)_{1\le k\le j}$ of $\mathbf{N}^j$, $h \le h'$ for the product oder in $\mathbf{N}^j$ means $h_k \le h'_k$ for all k.

6.4.5, the 3rd line: basis $\to$ base.

Lemma 6.4.7, the 2nd line: a map $\to$ a continuous map.

6.4.8, the 2nd line: $D_{\sigma,\mathrm{val}} \to D^{\sharp}_{\sigma,\mathrm{val}}$.

6.4.8, the 4th line: 6.4.10 $\to$ 6.4.10 below.

6.4.8, (1), the last line: $\psi \to \tilde{\psi}$. (There are two such.)

Lemma 6.4.9, Proof, the first display: Delete "." in the first line, add "(resp." at the top of the 2nd line, and add ")" before the period of the second line.

Lemma 6.4.9, Proof, the 5th line from the bottom and the 2nd line from the bottom: $\sum_{s\in S_{\le b'(j)}} \to \sum_{s\in S'_{\le b'(j)}}$. ($S$ is changed into S'.)

Lemma 6.4.11, the 2nd line: $(N_j)_{j\in S} \to (N_s)_{s\in S}$.

Lemma 6.4.11, Proof, the 3rd paragraph, the 2nd line: $\Gamma(\sigma)^{\mathrm{gp}} \otimes \mathbf{Q} \to \mathbf{Q} \otimes \Gamma(\sigma)^{\mathrm{gp}}$.

Lemma 6.4.11, Proof, the last line: $e(iy) \to \mathbf{e}(iy)$.

7.2.1, Shift "and $\mathbf{e}$ in 3.3.5" on the 8th line, to the end of the second sentence on the 2nd line.

The 2nd line before Lemma 7.2.4: $,; (h,x) \mapsto (x,hx) \to , (h,x) \mapsto (x,hx),$.

Proposition 7.2.10. Add the following sentence after the display: Here the suffix "triv" means the subspace consisting of the points with trivial logarithmic structure.

Proposition 7.2.10, Proof, the 3rd line after Claim A: the limit → and the limit.

Proposition 7.2.10, Proof, the 2nd paragraph after Claim C_l, the 2nd line: N'_s $(s \in S')$ → elements of σ'.

Proposition 7.2.10, Proof, the 2nd line above (1) and the line above (1): $B_{\mathbf{R}} \to B$.

Proposition 7.2.10, Proof, (1): $(y'_\lambda, -y'^*_\lambda) \to (y'_\lambda - y'^*_\lambda)$. (Delete a comma before −.)

Proposition 7.2.10, Proof, (1): $d \to d'$. (There are twice such changes.)

7.2.13, the 3rd paragraph, the 4th line: $\cup \to \bigcup$.

7.3.1, Theorem A (v) for $\Gamma(\sigma)^{-1}\backslash D_\sigma$ → Theorem A (v) for $\Gamma(\sigma)^{-1}\backslash D_\sigma$. (add a period).

7.3.1, Theorem A (vi) for $\Gamma(\sigma)^{-1}\backslash D_\sigma$ → Theorem A (vi) for $\Gamma(\sigma)^{-1}\backslash D_\sigma$. (add a period).

Lemma 7.3.3, Proof, Claim: Add a comma "," at the end of the display

7.3.7, the 3rd line from the bottom: Change the first "F" in this line to "$\exp(-z)F$".

7.3.8, At the end of the proof, add the following sentence as a newline: The proof for $D^\sharp_{\sigma,\mathrm{val}} \hookrightarrow D^\sharp_{\Sigma,\mathrm{val}}$ is similar and we omit it.

7.4.1, the end of the 2nd paragraph: The rest follow. → The remaining are the proofs of the following four assertions.

7.4.1, after Theorem A (v), add the following sentence: The Hausdorffness of $\Gamma\backslash D_\Sigma$ can be proved similarly and we omit it.

7.4.5, the line after the formula (2): $\mathrm{gr}(\gamma) \to \mathrm{gr}(\log(\gamma))$.

7.4.10: Add a period at the end.

7.4.11, the 3rd and the 4th lines: Delete "it follows ... that".

8.2.3, just before Claim 1: Delete $\square$.

8.2.3, at the end: This proves Claim 2. → This proves Claim 2, and completes the proof of Step 1.

10.1.2, the last line: satisfying → with.

Theorem 10.1.6: Shift "(2 bis)" and "(3 bis)" to the right in order to line up vertically with the other numbers "(1), (2), (3), (4)".

10.2.1, the 2nd display from the bottom: $\cup \to \bigcup$.

10.2.11, at the end add the following:

There is amistake in [KU2, 4.12]. For a correction, see §3.3 especially 3.3.6 in the following paper: K. Kato, C. Nakayama and S. Usui, Classifying spaces of degenerating mixed Hodge structures, II: Spaces of SL(2)-orbits, Kyoto J. Math. 51 (1): Nagata Memorial Issue, 2011, 149–261.

Lemma 10.2.12, Proof, the 2nd line from the bottom: Lemma 10.2.8 → Proposition 10.2.8.

Lemma 10.2.14, (ii), the 1st line: Add "the" before "restriction".

Lemma 10.2.14, Proof, formula (4): $\mathrm{Int}(\tilde\rho(t)) \to \mathrm{Int}(\tilde\rho(t))^{-1}$.

Lemma 10.2.14, Proof, There are no (1), (2). Run up the numbers of reference by 2 so that (3), ... , (7) are shifted to (1), ... , (5).

Lemma 10.2.15: The style of condition (1) is as same as that of condition (2).

Lemma 10.2.15: Insert "medskip" between the end of condition (1) (i.e., the 5th line of (1)) and the next sentence "Let $\check{D}^W$ ... ".

10.3.1, the 2nd line before the display (5): $\rho \to \tilde\rho$.

Lemma 10.3.4, Proof, the 3rd line: L $\to$ L (italic).
10.3.5, the 2nd paragraph, the 4th line: $\cup \to \bigcup$.
Lemma 10.3.6, Proof, the 2nd line from the bottom: $\chi^{-1}(t_\lambda) \to \chi(t_\lambda)^{-1}$.
12.2.2, the 8th line: W $\to$ W (italic).
Proposition 12.2.15, Proof, (1): $\cup \to \bigcup$.
Lemma 12.3.5, Proof, the 6th paragraph, the 1st line: $\sigma_a \to \sigma_\alpha$.
Lemma 12.3.5, Proof, the 6th paragraph, the 1st line: $\sigma_b \to \sigma_\beta$.
12.3.8, the 3rd display: Add a period "." at the end.
12.3.9, (1): Change

$$\gamma F = F, \quad \text{where} \quad F := F\left(\begin{pmatrix} 0 & 0 \\ 0 & -it' \end{pmatrix}, 0\right)(\mathrm{gr}_0^W).$$

to

$$\gamma F = F', \quad \text{where} \quad F \text{ (resp. } F') := F\left(\begin{pmatrix} 0 & 0 \\ 0 & -it \text{ (resp. } -it') \end{pmatrix}, 0\right)(\mathrm{gr}_0^W).$$

12.3.9, at the end of the 3rd display: Change a period "." to a comma ",".

Added in the proof after 12.7.7: The conjecture here is known to be false. For this and the related discussions, see 7.3 of the following paper. Kato, K., Nakayama, C. and Usui, S., Classifying spaces of degenerating mixed Hodge structures, III: Spaces of nilpotent orbits, to appear in J. Alg. Geom., 2012.

A1.5, the 3rd line: action of γ $\to$ action of γ on L_a.
A1.6, before Example: $M_{X,x} \to M_x$.
A1.6, Example, the 2nd line: After M_Δ, add "associated with the divisor $\{0\}$ of Δ".
A1.7, the last line: Change "$[0,1] \mapsto \Delta^*$" to "$[0,1] \to \Delta^*$".
A1.10, the 2nd line after the 2nd display: $\log(f) \to \log(q)$.
A2.5, the 2nd line: Add a period "." after "Proof of (iii)".
A2.5, Claim 1, the 2nd line: $\cup \to \bigcup$.
A2.5, Claim 2, Proof, the 4th line and the the 7th line: $\cup \to \bigcup$. (There are two such changes.)
A2.5, Claim 2, Proof, the 2nd paragraph, the 1st line: proposition $\to$ Proposition.
REFERENCES, [Ak] $\to$ [AK].
REFERENCES, [BJ]: spces $\to$ spaces.
REFERENCES, [D4]: 1972 $\to$ 1971.
REFERENCES, [G1]: modular varieties $\to$ the modular varieties.
REFERENCES, [G1], [G2], [G3], [G4]: manifolds. $\to$ manifolds, (comma).
REFERENCES, [I2], Critstante $\to$ Cristante.
REFERENCES, [Kk1]: Delete "Perspectives in Mathematics,".
REFERENCES, [Og]: correspondences $\to$ correspondence.
LIST OF SYMBOLS, page 324, after the 7th line: Insert "$\check{\varphi} : \check{E}_\sigma \to \Gamma(\sigma)^{\mathrm{gp}}\backslash\check{D}_{\mathrm{orb}}$ 3.3.6."

5. Corrected versions of §2.3

There are only typographical errors and no other problems in Proposition 2.3.2 of [KU09]. But since they make this fundamental part uncomfortable to read, we add the corrected version here for readers' convenience. The notation and the indications in this appendix are all those loc.cit.

Proposition 2.3.2. *Let X be an object of $\overline{\mathcal{A}}_1(\log)$ (which contains $\mathcal{B}(\log)$), let A be a subring of $\mathbf{C}$, and let L be a locally constant sheaf on $X^{\log}$ of free A-modules of finite rank. Let $x \in X$, let y be a point of $X^{\log}$ lying over x, and assume that the local monodromy of L at y is unipotent. Let $(q_j)_{1\leq j\leq n}$ be a finite family of elements of $M^{\mathrm{gp}}_{X,x}$ whose image in $(M^{\mathrm{gp}}_X/\mathcal{O}^{\times}_X)_x$ is a $\mathbf{Z}$-basis, and let $(\gamma_j)_{1\leq j\leq n}$ be the dual $\mathbf{Z}$-basis of $\pi_1(x^{\log})$ in the duality in 2.2.9. Then if we replace X by some open neighborhood of x, we have an isomorphism of $\mathcal{O}^{\log}_X$-modules*

$$\nu : \mathcal{O}^{\log}_X \otimes_A L \xrightarrow{\sim} \mathcal{O}^{\log}_X \otimes_A L_0, \quad L_0 := \textit{the stalk } L_y$$

where L_0 is regarded as a constant sheaf, satisfying the condition (1) below. Let A' be the subring of $\mathbf{C}$ generated by A and $\mathbf{Q}$, let $L_{0,A'} = A' \otimes_A L_0$, and let

$$N_j : L_{0,A'} \to L_{0,A'}$$

be the endmorphism of constant sheaf which is induced by the logarithm of the monodromy action of γ_j on the stalk L_y of the locally constant sheaf L. Lift q_j in $\Gamma(X, M^{\mathrm{gp}}_X)$ (by replacing X by an open neighborhood of x), and let

$$\xi = \exp(\textstyle\sum_{j=1}^{n}(2\pi i)^{-1}\log(q_j)\otimes N_j) : \mathcal{O}^{\log}_X \otimes_{A'} L_{0,A'} \xrightarrow{\sim} \mathcal{O}^{\log}_X \otimes_{A'} L_{0,A'}.$$

Note that the operator ξ depends on the choices of the branches of $\log(q_j)$ in $\mathcal{O}^{\log}_X$ locally on $X^{\log}$, but that the subsheaf $\xi^{-1}(1\otimes L_0)$ of $\mathcal{O}^{\log}_X \otimes_A L_0$ is independent of the choices and hence is defined globally on $X^{\log}$.

(1) *The restriction of ν to $L = 1\otimes L$ induces an isomorphism of locally constant sheaves*

$$\nu : L \xrightarrow{\sim} \xi^{-1}(1\otimes L_0).$$

If we fix branches $\log(q_j)_{y,0}$ in $\mathcal{O}^{\log}_{X,y}$ of the germs $\log(q_j)_y$ at y $(1\leq j\leq n)$, we can take an isomorphism ν which satisfies above (1) and also the following (2).

(2) *The branch $\xi_{y,0}$ of the germ ξ_y, defined by the fixed branches $\log(q_j)_{y,0}$ of the germs $\log(q_j)_y$, satisfies*

$$\nu(1\otimes v) = \xi^{-1}_{y,0}(1\otimes v) \quad \textit{for any } v \in L_y = L_0.$$

Proof. Let L' be the locally constant subsheaf $\xi^{-1}(1\otimes L_0)$ of $\mathcal{O}^{\log}_X \otimes L_0$. Fix a branch $\log(q_j)_{y,0}$ of the germ $\log(q_j)_y$ at y for $1\leq j\leq n$, and let $\nu : L_y \to (L')_y$ be the isomorphism of A-modules $v \mapsto \xi_{y,0}^{-1}(1 \otimes v)$, where $\xi_{y,0}$ is defined by the fixed branches $\log(q_j)_{y,0}$ of $\log(q_j)_y$. Then ν preserves the local monodromy actions of $\pi_1(x^{\log})$ on these stalks of the locally constant sheaves L and L'. In fact, for $v \in L_0$ and for $1 \le k \le n$,

$$\begin{aligned}\gamma_k(\nu(v)) &= \gamma_k(\xi_{y,0}^{-1}(1 \otimes v)) = \gamma_k(\xi_{y,0})^{-1} \cdot (1 \otimes v)\\ &= \exp(-(\textstyle\sum_{j=1}^n ((2\pi i)^{-1} \log(q_j)_{y,0} - \delta_{jk}) \otimes N_j) \cdot (1 \otimes v)\\ &= \xi_{y,0}^{-1} \exp(1 \otimes N_k)(1 \otimes v) = \xi_{y,0}^{-1}(1 \otimes \gamma_k(v)) = \nu(\gamma_k(v))\end{aligned}$$

(δ_{jk} is the Kronecker symbol, and for the signature before δ_{jk}, see Appendix A1). Here the second equality follows from the monodromy action of γ_k on the locally constant sheaf L', the fifth equality follows from the endmorphism N_k of the constant sheaf L_0, and $\gamma_k(v)$ in the second last and in the last is the image of the element $v \in L_0 = L_y$ by the monodromy action of γ_k on the locally constant sheaf L at y. Hence there is a unique isomorphism $\nu : L|_{x^{\log}} \to L'|_{x^{\log}}$ between the pullbacks of L and L' to $x^{\log}$ which induces the above isomorphism ν on the stalks at y.

By the proper base change theorem (Appendix A2) applied to the proper map $\tau : X^{\log} \to X$ and to the sheaf $\mathcal{F}$ of isomorphisms from L to L' on $X^{\log}$, the isomorphism ν extends to an isomorphism $\nu : L \xrightarrow{\sim} L'$ if we replace X by some open neighborhood of x in X. This isomorphism ν induces an isomorphism of $\mathcal{O}_X^{\log}$-modules

$$\nu : \mathcal{O}_X^{\log} \otimes_A L \xrightarrow{\sim} \mathcal{O}_X^{\log} \otimes_A L' = \mathcal{O}_X^{\log} \otimes_A L_0. \quad \square$$

Example. Let $f : E \to \Delta$ be the degenerating family of elliptic curves in 0.2.10 and consider the locally constant sheaf $L = R^1 f_*^{\log}(\mathbf{Z})$ on $\Delta^{\log}$. In 2.3.2, take $X = \Delta$, $x = 0 \in \Delta$, $A = \mathbf{Z}$, and take the coordinate function q of Δ as q_1 ($n = 1$ in this situation). Then the element γ_1, which we denote here by γ, is the positive generator of $\pi_1(\Delta^{\log})$ (represented by a circle in Δ^* in the counterclockwise direction, cf. Appendix A1). As is explained in 0.2, L has a $\mathbf{Z}$-basis (e_1, e_2) locally on $\Delta^{\log}$ (e_1 is defined globally but e_2 is determined by a local choice of the branch of $\log(q)$). Let $y \in x^{\log}$ and let $L_0 := L_y$. Fix a branch in $\mathcal{O}_{X,y}^{\log}$ of the germ of $\log(q)$ at y and take the corresponding $e_{2,y}$. We have

$$\begin{aligned}&\gamma(e_1) = e_1,\ \gamma(e_{2,y}) = e_1 + e_{2,y},\\ &N(e_1) = 0,\ N(e_{2,y}) = e_1, \ \text{ where } N = \log(\gamma).\end{aligned}$$

The $\mathcal{O}_X^{\log}$-module $\mathcal{O}_X^{\log} \otimes_{\mathbf{Z}} L$ has a global basis $(1 \otimes e_1, \omega)$ as in 0.2.15. We have an isomorphism of $\mathcal{O}_X^{\log}$-modules

$$\nu : \mathcal{O}_X^{\log} \otimes L \xrightarrow{\sim} \mathcal{O}_X^{\log} \otimes L_0, \quad 1 \otimes e_1 \to 1 \otimes e_1, \ \ \omega \mapsto 1 \otimes e_{2,y}.$$

This ν has the property stated in Proposition 2.3.2 globally on Δ. In fact, since $\omega = (2\pi i)^{-1} \log(q) e_1 + e_2 = (2\pi i)^{-1} \log(q)_y \otimes e_1 + 1 \otimes e_{2,y}$ (cf. 0.2.15 (2)), ν sends e_1 to $1 \otimes e_1 = \xi^{-1}(1 \otimes e_1)$ and e_2 to $-(2\pi i)^{-1} \log(q)_y \otimes e_1 + 1 \otimes e_{2,y} = \xi^{-1}(1 \otimes e_{2,y})$.

2.3.7. The isomorphism ν in 2.3.2 appears locally on X depending on a local choice of $(q_j)_j$. Here we see that in the case $X = \mathrm{Spec}(\mathbf{C}[\mathcal{S}])_{\mathrm{an}}$, a canonical ν exists globally on X.

Let $\mathcal{S}$ be an fs monoid, $X = \mathrm{Spec}(\mathbf{C}[\mathcal{S}])_{\mathrm{an}}$ with the canonical logarithmic structure, and let $U = X_{\mathrm{triv}} = \mathrm{Spec}(\mathbf{C}[\mathcal{S}^{\mathrm{gp}}])_{\mathrm{an}}$. Then $U = \mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{C}^\times)$, and via the exact sequence

$$0 \to \mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{Z}) \to \mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{C}) \to \mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{C}^\times) \to 0$$

(the third arrow is induced from $\mathbf{C} \to \mathbf{C}^\times$, $z \mapsto \exp(2\pi i z)$), $\mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{C})$ is regarded as a universal covering of U and the fundamental group of U is identified with $\mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{Z})$.

Let A be a subring of $\mathbf{C}$, let L be a locally constant sheaf on $X^{\log}$ of free A-modules of finite rank with unipotent local monodromy, and let L_1 (denoted by L_0 in 2.3.2) be the stalk of L at the unit point $1 \in U = \mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{C}^\times)$ regarded as a constant sheaf on $X^{\log}$. Then there is a unique isomorphism of $\mathcal{O}_X^{\log}$-modules

$$\nu : \mathcal{O}_X^{\log} \otimes_A L \xrightarrow{\sim} \mathcal{O}_X^{\log} \otimes_A L_1$$

satisfying the following (1) and (2) for *any* finite family $(q_j)_{1\le j\le n}$ of elements of $\mathcal{S}^{\mathrm{gp}}$ which is a $\mathbf{Z}$-basis of $\mathcal{S}^{\mathrm{gp}}/(\text{torsion})$. Let $(\gamma_j)_{1\le j\le n}$ be the $\mathbf{Z}$-basis of $\pi_1(U, 1) = \mathrm{Hom}(\mathcal{S}^{\mathrm{gp}}, \mathbf{Z})$ which is dual to $(q_j)_{1\le j\le n}$, and let $N_j : L_{1,A'} \to L_{1,A'}$ be the logarithm of γ_j, where A' is the subring of $\mathbf{C}$ generated by A and $\mathbf{Q}$.

(1) $\qquad \nu(1 \otimes L) = \xi^{-1}(1 \otimes L_1) \quad \text{with } \xi = \exp(\sum_{j=1}^n (2\pi i)^{-1} \log(q_j) \otimes N_j).$

(2) Let $\log(q_j)_{1,0}$ be the branch in $\mathcal{O}_{X,1}^{\log}$ of the germ of $\log(q_j)$ at $1 \in U$ which has the value 0 at 1, and let $\xi_{1,0} = \exp(\sum_{j=1}^n (2\pi i)^{-1} \log(q_j)_{1,0} \otimes N_j)$. Then the map $\xi_{1,0} \circ \nu_1 : 1 \otimes L_1 \to 1 \otimes L_1$ is the identity map.

The proof is similar to that of 2.3.2. First fix $(q_j)_{1\le j\le n}$. For the locally constant subsheaf $L' := \xi^{-1}(1 \otimes L_1)$ of $\mathcal{O}_X^{\log} \otimes L_1$, the isomorphism $\xi_{1,0}^{-1} : L_1 \to L'_1$ of stalks preserves the actions of $\pi_1(X^{\log}, 1) \simeq \pi_1(U, 1)$, and it is extended uniquely to an isomorphism $\nu : L \xrightarrow{\sim} L'$ on $X^{\log}$. This induces an isomorphism of $\mathcal{O}_X^{\log}$-modules $\nu : \mathcal{O}_X^{\log} \otimes_A L \xrightarrow{\sim} \mathcal{O}_X^{\log} \otimes_A L' = \mathcal{O}_X^{\log} \otimes_A L_1$. It is easy to check that ν is independent of the choice of $(q_j)_{1\le j\le n}$.