Recovery of Vanishing Cycles by Log Geometry: Case of Several Variables

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For the memory of Dr. Wolfgang Vogel

Abstract

This article is a generalization of the author's work [U] to the case of several variables. We first construct compatible actions of monoid S on a "several-variables-version of semi-stable degeneration of pairs"and on the associated log topological spaces introduced by K. Kato and C. Nakayama in [KN]. Here S is the product of the unit interval and the unit circle. Then we show that the associated log topological family is locally piecewise C^∞ trivial over the base, i.e., the associated log topological family recovers the vanishing cycles, in the most naive sense, of the original degeneration. Using this result together with the theory of canonical extensions by Deligne [D], we introduce two types of integral structure of the variation of mixed Hodge structure associated to "several-variables-version of semi-stable degeneration of pairs". We only sketch the proof here. The complete proof will appear soon somewhere.

1 Log Structures

In this section, we prepare some notations concerning about log structures for our later use. For general theory of log structures, see, for example, [K].

Let $X \supset D$ be a d-dimensional complex manifold and a divisor with normal crossings. The associated *fine saturated log structure* (cf. [K]) is defined by

$$\mathcal{M}_X := \{f \in \mathcal{O}_X \mid f \text{ is invertible outside } D\} \overset{\alpha}{\hookrightarrow} \mathcal{O}_X.$$

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Let T be a point Spec $\mathbf{C}$ with a log structure

$$\mathbf{R}_{\geq 0} \times \mathbf{C}_1 \to \mathbf{C}, \quad (r, u) \mapsto ru,$$

where $\mathbf{C}_1 \subset \mathbf{C}$ is the unit circle. Notice that this log structure is not fine saturated. K. Kato and C. Nakayama introduced in [KN] a *log topological space* $X^{\log}$ as the set of T-valued points in the category of log schemes:

$$X^{\log} := \mathrm{Hom}(T, X) \xrightarrow{\tau_X} X, \quad \text{forgetting morphism.}$$

Let $\tilde{x} \in X^{\log}$ and $x := \tau_X(\tilde{x})$. Choose a local coordinates $z_1, \ldots, z_d$ at $x \in X$ such that D has a local equation $\prod_{1 \leq i \leq s(x)} z_i^{m(i)}$, $m(i) \geq 1$. Then we see that

$$\mathcal{M}_{X,x} = \coprod \Big\{ \mathcal{O}_{X,x}^{\times} \prod_{1 \leq i \leq s(x)} z_i^{b(i)} \Bigm| b \in \mathbf{N}^{s(x)} \Big\} \simeq \mathcal{O}_{X,x}^{\times} \oplus \mathbf{N}^{s(x)}, \quad \text{where } \mathbf{N} := \mathbf{Z}_{\geq 0}.$$

$$X^{\log} \overset{\text{locally}}{\simeq} (\mathbf{R}_{\geq 0})^{s(x)} \times (\mathbf{C}_1)^{s(x)} \times \mathbf{C}^{d-s(x)} \xrightarrow{\tau_X} X \overset{\text{locally}}{\simeq} \mathbf{C}^d,$$
$$\tau_X\big((r_i, u_i)_{1 \leq i \leq s(x)}, (z_j)_{s(x)+1 \leq j \leq d}\big) = \big((r_i u_i)_{1 \leq i \leq s(x)}, (z_j)_{s(x)+1 \leq j \leq d}\big),$$

where $r_i := |z_i|$ and $r_i u_i := z_i$. This induces a topology on the set $X^{\log}$, and $\tau_X : X^{\log} \to X$ can be regarded as a real blowing-up (cf. [M]) and $X^{\log}$ as a manifold with corners (cf. [AMRT]).

Example (1.1) Let Δ be the open unit disc in the complex plane, and H the upper half plane. Let $\exp 2\pi\sqrt{-1}(\ ) : H \to \Delta^*$ be the universal cover of the punctured disc. Then the pair $(\Delta, \{0\})$ induces the following diagram:

$$\begin{array}{ccccl} H & \subset & \hat{H} & := & \mathbf{R} + \sqrt{-1}(\mathbf{R}_{>0} \coprod \{\infty\}) \\ & & \downarrow & & \\ \Big\downarrow & & \Delta^{\log} & \simeq & \hat{H}/\mathbf{Z} \\ & & \downarrow & & \\ \Delta^* & \subset & \Delta. & & \end{array}$$

2 Recovery of vanishing cycles

Let $n \geq 1$ and $a(k)$ $(-1 \leq k \leq n)$ be integers such that

(2.1) $$0 = a(-1) \leq a(0) < a(1) < \cdots < a(n).$$

Set

(2.2) $$A := \{1, 2, \ldots a(n)\}, \quad A(k) := \{a(k-1)+1, \ldots, a(k)\} \quad (0 \leq k \leq n).$$

Let

(2.3) $$f : X \to P$$

be a proper, flat morphism of a d-dimensional complex manifold X to a polydisc $P := \Delta^n$ with coordinates $t_1, \ldots, t_n$. Let B_k be the divisor on P defined by $t_k = 0$, and set $B := \sum_{1 \leq k \leq n} B_k$. Set $D := f^*B$ and let

(2.4) $$f^*B_k =: \sum_{i \in A(k)} m(i) D_i \qquad (1 \leq k \leq n)$$

be the irreducible decomposition. Let $Y = \sum_{i \in A(0)} D_i$ be a divisor on X, flat with respect to f. We assume that f is smooth over $P^* := P - B$ and that

$$Y + D = \sum_{i \in A(0)} D_i + \sum_{1 \le k \le n} \sum_{i \in A(k)} m(i) D_i \tag{2.5}$$

is a divisor with simple normal crossings whose distinct prime divisors are D_i $(i \in A)$. The fine saturated log structures associated to the pairs $X \supset D$, $Y \supset D \cap Y$ and $P \supset B$ induce a commutative diagram:

$$\begin{array}{ccc} (X \supset Y) & \xleftarrow{\tau_X} & (X^{\log} \supset Y^{\log}) \\ {\scriptstyle f}\big\downarrow & & \big\downarrow{\scriptstyle f^{\log}} \\ P & \xleftarrow{\tau_P} & P^{\log}. \end{array} \tag{2.6}$$

Let $[0,1] \subset \mathbf{R}$ be the unit interval regarded as a monoid by multiplication. The monoid

$$S := ([0,1] \times \mathbf{C}_1)^n \tag{2.7}$$

has natural actions on the polydisc P and on $P^{\log}$. These actions can be lifted to the diagram (2.6), and we have

Theorem 1 *We use the above notation. Assume that the divisor D in* (2.5) *is reduced. Then, the family of open spaces*

$$\overset{\circ}{f}{}^{\log} \colon (X^{\log} - Y^{\log}) \to P^{\log}$$

is locally piecewise C^∞ trivial over the base $P^{\log}$. This means that $\overset{\circ}{f}{}^{\log}$ recovers the vanishing cycles, in the most naive sense, of the degenerating family

$$\overset{\circ}{f} \colon (X - Y) \to P.$$

We will sketch the construction of the liftings of S-actions to the diagram (2.6) and the proof of Theorem 1 in Section 4 below.

3 Integral structure of degenerate VMHS

In the famous pioneer work [St1], Steenbrink constructed the limiting mixed Hodge structure associated to a semi-stable degeneration of algebraic varieties. In that paper, people found some parts which are not clear. One of them is the part of integral structures and later the original author himself rewrote that part correctly in [St2] by using a fine saturated log structure. But he used fractions there and hence he obtained rational structures rather than integral ones. In this section, we introduce integral structures on degenerating variations of mixed Hodge structures as an application of canonical extensions of Deligne in [D] and our Theorem 1.

We use the notation in Section 2. Then, it can be verified that

$$\mathcal{V} := R^q f_* \Omega^\bullet_{X/P}(\log(Y + D)) \tag{3.1}$$

is the canonical extension of Deligne [D, (II.5.2)] of $\mathcal{V}|P^*$, whose Gauss-Manin connection ∇ is obtained as the differential $d_1 : E_1^{0,q} = \mathcal{V} \to E_1^{1,q} = \Omega_P^1(\log B) \otimes_{\mathcal{O}_P} \mathcal{V}$ of the spectral sequence of hypercohomology of the complex $\Omega_X^\bullet(\log(Y + D))$ with respect to a filtration $G^k := f^*\Omega_P^k(\log B) \wedge \Omega_X^\bullet(\log(Y + D))[-k]$.

The locally constant sheaf of $\mathbf{C}$-modules $\mathrm{Ker}(\nabla|P^*)$ lifts to $\tau_P^{-1}(P^*)$ and extends one on $P^{\log}$. We denote the latter by $L'_{\mathbf{C}}$. On the other hand, by Theorem 1, we have locally constant sheaf of $\mathbf{Z}$-modules on $P^{\log}$:

$$L_{\mathbf{Z}} := R^q(\overset{\circ}{f}{}^{\log})_*\mathbf{Z}. \tag{3.2}$$

By construction, $L'_{\mathbf{C}}$ and $\mathbf{C} \otimes_{\mathbf{Z}} L_{\mathbf{Z}}$ coincide on $\tau_P^{-1}(P^*)$, hence they coincide on whole $P^{\log}$ because they are locally constant.

Let $N_i := \log \gamma_i$ $(1 \le i \le n)$ be the monodromy logarithms of $L_{\mathbf{Z}}$ induced by the action of the group $(\mathbf{C}_1)^n$ on $P^{\log}$. Let $\varpi : \hat{H}^n \to P^{\log}$ be the universal covering (cf. Example (1.1)) and let $l_1, \ldots, l_n$ be coordinates on $\hat{H}^n$ with $\exp(2\pi\sqrt{-1}l_i) = t_i$. Choose a flat frame $e_1, \ldots, e_r$ of $\varpi^{-1}L_{\mathbf{Z}}$ and modify

$$\tilde{e}_j := \exp\Big(-\sum_{1\le i\le n} l_iN_i\Big) \cdot e_j \quad (1 \le j \le r). \tag{3.3}$$

Then, this drops to a single-valued frame of $\mathcal{O}_P^{\log} \otimes_{\mathbf{Z}} L_{\mathbf{Z}}$ on $P^{\log}$, where $(\mathcal{O}_P^{\log})_{\tilde{t}} := \mathcal{O}_{P,t}[l_1, \ldots, l_n]$ for $\tilde{t} \in P^{\log}$ and $t = \tau_P(\tilde{t}) \in P$. Hence this still drops to a frame of $\mathcal{V}$ on P. We also denote this frame of $\mathcal{V}$ by the same symbol $\tilde{e}_1, \ldots, \tilde{e}_r$.

It is easy to see, by the definition (3.2), that under the identification

$$\mathbf{C} \otimes_{\mathbf{Z}} (\varpi^{-1}L_{\mathbf{Z}})(h) \tilde{\to} \mathcal{V}(O), \quad \tilde{e}_j(h) \mapsto \tilde{e}_j(O), \tag{3.4}$$

where $h \in \hat{H}^n$ and $O \in P$ the origin, we have

$$N_i = -2\pi\sqrt{-1}\mathrm{Res}(t_i = 0)(\nabla) \quad \text{(cf. [D, (II.1.17), (II.5.2)])}. \tag{3.5}$$

Thus we have

Theorem 2 *We use the notation of Sections* 2 *and* 3*. Assume that the divisor D in* (2.4) *is reduced. Then, $\mathcal{V}$ has two types of integral structure:*

(i) $$\mathcal{O}_P^{\log} \otimes_{\mathbf{Z}} L_{\mathbf{Z}} \simeq (\tau_P)^*\mathcal{V} \quad \textit{on} \quad P^{\log}.$$

The local monodromies are induced by $(\mathbf{C}_1)^n$-action on $P^{\log}$.

(ii) $$\mathcal{O}_P \otimes_{\mathbf{Z}} (\tau_P)_*R^q(\overset{\circ}{f}{}^{\log})_*(\overset{\circ}{f}{}^{\log})^{-1}\mathbf{Z}[l_1, \ldots, l_n] \simeq \mathcal{V} \quad \textit{on} \quad P.$$

The monodromy logarithms are given by $-2\pi\sqrt{-1}Res(t_i = 0)(\nabla)$ $(1 \le i \le n)$.

Note that the integral structure (i) is a generalization of Schmid's type, and (ii) is a generalization of Steenbrink's type.

Remark (3.6) (i) $\mathbf{C} \otimes_{\mathbf{Z}} L_{\mathbf{Z}}$ and $(\mathcal{V}, \nabla)$ correspond under the log Riemann-Hilbert correspondence in [KN], by using the monodromy weight filtration in [CK] in case $Y = \emptyset$ and in general case the convolution of the relative monodromy weight filtrations in [SZ] or the weight filtration constructed in [F].
(ii) The author was communicated by Morihiko Saito, on May 24, 1996, that there is a correction of [St, (5.9)] in [Sa, 4.2].
(iii) Fujisawa, in [F], has generalized the result in [St2] into the case of several variables in a similar method as in [St2], and he has introduced a 'rational structure' on $\mathcal{V}$.

4 Outline of Proof of Theorem 1

The proof is analogous to the argument of Clemens [C], but there are some points in the proof of [C, Theorem 5.7] which are not clear. The readers can find a complete proof in the case of $\dim P = 1$ in [U].

We use the notation in Section 2. For $I \subset A$, we denote

$$D_I := \bigcap_{i \in I} D_i, \qquad I(k) := I \cap A(k) \qquad (0 \le k \le n).$$

The following proposition plays a key role.

Propositon 3 *In the above notation, shrinking the polydisc P, we have the following:*
(a) *There exist a family $\{U_I\}_{I \subset A}$ of open neighborhoods U_I of D_I and a family $\{\pi_I : U_I \to D_I\}_{I \subset A}$ of C^∞ projections which satisfy*

(i) $U_I \cap U_J = U_{I \cup J}$,

(ii) $\pi_I \circ \pi_J | U_I = \pi_I$ *for* $I \supset J$.

(b) *There exists a family $\{z_i\}_{i \in A}$ of multi-valued C^∞ global equations z_i of D_i in X which has the following properties:*

(iii) *Let $J \subset A - A(0)$, $x \in D_J$ and $F := \pi_J^{-1}(x)$. Then, choosing branches of the multi-valued functions, $\{z_j | F\}_{j \in J}$ form a system of holomorphic coordinates on F and*

$$\prod_{j \in J(k)} z_j^{m(j)} = (\text{constant})\, t_k \circ f \quad \text{on} \quad F \quad (1 \le k \le n),$$

where the (constant) *depends on F and on the choices of the z_j, of their branches and of the t_k.*

(iv) *For $i, j \in A$ with $i \ne j$, any branch of z_i is constant on each fiber of $\pi_j : U_j \to D_j$.*

We omit here the proof of this proposition, because it is rather complicated though elementary and also the argument is essentially the same as in the case of $\dim P = 1$ (see [U, §3, §4], in this case). In order to lift the action of monoid $S = ([0,1] \times \mathbf{C}_1)^n$ to the whole diagram (2.6), we should prepare two more things.

For each integer $1 \le k \le n$ and a number $0 \le \delta < 1$, let

$$\begin{aligned} C(k) &:= [0,1]^{a(k)-a(k-1)} \quad \text{unit cube in } \mathbf{R}^{a(k)-a(k-1)}, \\ C(k)_\delta &:= \Big\{ (r_i)_{i \in A(k)} \in C(k) \;\Big|\; \prod_{i \in A(k)} r_i^{m(i)} = \delta \Big\}, \\ E(k)_\delta &:= \bigcup_{\delta' \in [0,\delta]} C(k)_{\delta'}. \end{aligned} \tag{4.1}$$

For each $i \in A - A(0)$, we choose a number

$$0 < \varepsilon_i < 1. \tag{4.2}$$

In the following, we assume that all the cuboids contained in $C(k)$ are parallel to the cube $C(k)$. Let $D(k)$ be the cuboid in $C(k)$ with the two points $B(k) := (\varepsilon_i)_{i \in A(k)}$ and

$(1,\ldots,1)$ as the extreme vertices. We construct a family of projections from each face of $D(k)$ passing through the vertex $B(k)$ to the union of the faces of $C(k)$ passing through the origin O in the following way:
For $I \subset A(k)$, we denote by $B(I)$ the vertex of the cuboid $D(k)$ whose i-th coordinate is 1 $(i \in I)$ and the other coordinate is ε_j $(j \in A(k) - I)$. Let $D(I)$ be the face of $D(k)$ with the two points $B(k)$ and $B(I)$ as the extreme vertices, and let $C(I)$ be the face of $C(k)$ passing through O, parallel to $D(I)$ and with the same dimension as $D(I)$. For each point $Q \in D(I)$, let $D(I)^{\perp}+Q$ be the affine subspace which is the orthoginal complement of $D(I)$ passing through Q, and let p_Q be the projection in $D(I)^{\perp}+Q$ from the point Q whose rays are in the cuboid in $D(I)^{\perp}+Q$ with the two points Q and $(D(I)^{\perp}+Q)\cap C(I)$ as the extreme vertices. We denote by p_I the collection of the projections p_Q $(Q \in D(I))$. We thus have a family $\{p_I\}_{I\subset A(k)}$ of projections.
Then, for a fixed non-negative number $\delta \le \delta_0$ and any fixed point $(r_i)_{i\in A(k)} \in C(k)_{\delta_0}$, the hypersurface $C(k)_\delta$ and the unique ray of the family of projections $\{p_I\}_{I\subset A(k)}$ passing through the point $(r_i)_{i\in A(k)}$ intersect at one point and, moreover, they are transversal except at the points of the singular locus of $C(k)_0$. Denote this intersection point by

$$\langle r, (r_i)_{i\in A(k)}\rangle, \quad \text{where} \quad r := \delta/\delta_0, \tag{4.3}$$

and call this the *hyperbolic polar coordinates* of the point in $E(k)_{\delta_0}$. Define

$$R(k) : [0,1]\times E(k)_{\delta_0} \to E(k)_{\delta_0} \quad \text{by} \quad R(k)(s, \langle r, (r_i)_{i\in A(k)}\rangle) := \langle sr, (r_i)_{i\in A(k)}\rangle. \tag{4.4}$$

Here we may assume that the above number δ_0 is chosen so small that, for every $1 \le k \le n$,

$$(r_i)_{i\in A(k)} \in E(k)_{\delta_0} \text{ implies } r_i < \varepsilon_i/2 \text{ for some } i \in A(k). \tag{4.5}$$

Then, for each $1 \le k \le n$,

$$\{(r_i)_{i\in A(k)} \in C(k)_{\delta_0} \,|\, r_j < \varepsilon_j/2\}_{j\in A(k)} \tag{4.6}$$

forms an open covering of $C(k)_{\delta_0}$. Take a C^∞ partition of unity

$$\{\lambda_j\}_{j\in A(k)} \tag{4.7}$$

on $C(k)_{\delta_0}$ which is subordinate to the covering (4.6), and extend this over $E(k)_{\delta_0}$ by

$$\lambda_j(\langle r, (r_i)_{i\in A(k)}\rangle) := \lambda_j((r_i)_{i\in A(k)}) \quad \text{for all } r \in [0,1].$$

Let r_i $(i \in A - A(0))$ be as in Proposition 3 (b). We choose the positive numbers ε_i in (4.2) so small that $\{y \in X \,|\, r_i(y) \le \varepsilon_i\}$ is contained in the neighborhood U_i in Proposition 3 $(i \in A - A(0))$, and we shrink the polydisc $P = \Delta^n$ so that $X \subset \bigcup_{i\in A-A(0)} U_i$, $r_i(y) \le 1$ $(y \in X, i \in A - A(0))$ and the radius of each factor Δ is less than or equal to δ_0.

Now an action of the monoid S on $X^{\log}$ is defined in the following way. For $y \in X$, let

$$\begin{aligned} &I := \{i \in A - A(0) \,|\, U_i \ni y\}, \quad x := \pi_I(y), \quad F := \pi_I^{-1}(x), \\ &F^{\log} : \text{the closure of } \tau_X^{-1}(F - F\cap D) \text{ in } X^{\log} \end{aligned} \tag{4.8}$$

Let z_i $(i \in I)$ be as in Proposition 3 (b) and let

$$z_i(y) =: r_i(y)u_i(y), \quad y \in X, \tag{4.9}$$

be the decompositions into the absolute values and the arguments. Notice that the $r_i(y)$ are single-valued, whereas the $u_i(y)$ aremulti-valued. For each u_i, we choose abranch and regard u_i as a single-valued function on $F^{\log}$. We thus have coordinates $((r_i(\ \ ), u_i(\ \ ))_{i \in I}$ on $F^{\log}$. We define an action $S \times F^{\log} \to F^{\log}$ by

$$\begin{aligned} \Big(r_i((s,v)\cdot\tilde{y})\Big)_{i\in A(k)} &:= R(k)\Big(s(k), (r_j(\tilde{y}))_{j\in A(k)}\Big), \\ u_i((s,v)\cdot\tilde{y}) &:= v(k)^{\lambda_i(\tilde{y})/m(i)}u_i(\tilde{y}) \qquad (i \in A(k)) \end{aligned} \tag{4.10}$$

for $1 \le k \le n$, where

$$(s,v) = (s(k), v(k))_{1\le k\le n} \in S = ([0,1]\times \mathbf{C}_1)^n, \quad \lambda_i(\tilde{y}) := \lambda_i((r_j(\tilde{y}))_{j\in A(k)}) \quad (i \in A(k)).$$

Here in the left side of the second equation in (4.10), the complex power is understood as one determined by a choice of a branch of $\log v(k)$.

Then we can verify the following claim:

Claim (4.11) The monoid action (4.10) is compatible with the restricted morphism $f^{\log} : F^{\log} \to P^{\log}$, and these actions on the fibers $F^{\log}$ fit together to give a piecewise C^∞ action on $X^{\log}$.

The S-action on $X^{\log}$ preserves the subspace $Y^{\log}$ by Proposition 3 (iv), and they drop down to induce S-actions on X and on Y. We see that these S-actions are compatible with the natural ones on P and on $P^{\log}$. Let $O \in P$ be the origin. We denote

$$O^{\log} := \tau_P^{-1}(O) \simeq (\mathbf{C}_1)^n, \qquad X^{\log}_{O^{\log}} := (f^{\log})^{-1}(O^{\log}). \tag{4.12}$$

For $(\mathbf{0}, \mathbf{1}) = ((0,\dots,0),(1,\dots,1)) \in S$, we define a piecewise C^∞ map

$$\tilde{\pi} : X^{\log} \to X^{\log}_{O^{\log}} \quad \text{by} \quad \tilde{\pi}(\tilde{y}) := (\mathbf{0},\mathbf{1})\cdot\tilde{y}. \tag{4.13}$$

By Proposition 3 (iv), $\tilde{\pi}$ is compatible with the inclusion $Y^{\log} \subset X^{\log}$. Let $\tilde{t} \in P^{\log}$ and $\tilde{t}_0 := (\mathbf{0},\mathbf{1})\cdot\tilde{t} \in O^{\log}$, and let $X^{\log}_{\tilde{t}}$ and $X^{\log}_{\tilde{t}_0}$ be the fibers of $f^{\log}$ over $\tilde{t}$ and $\tilde{t}_0$, respectively. Then we can verify the following claim:

Claim (4.14) Assume that the divisor D in (2.5) is reduced. Then, the restricted map $\tilde{\pi} : X^{\log}_{\tilde{t}} \to X^{\log}_{\tilde{t}_0}$ is piecewise C^∞ isomorphic.

From this, we see that the map $\tilde{\pi}$ in (4.13) yields a horizontal projection of the family $f^{\log} : X^{\log} \to P^{\log}$, compatible with the inclusion $Y^{\log} \subset X^{\log}$. Thus we get Theorem 1.

The above argument is essentially the same as in the case of the $\dim P = 1$ and the details in this case can be found in [U, §5].

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