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SPHERES NOT ADMITTING SMOOTH ODD-FIXED-POINT ACTIONS OF S_5 AND $SL(2, 5)$

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Abstract

Let G be a finite group and Σ a homology sphere with smooth G -action. If the G -fixed-point set of Σ consists of odd-number points then the dimension of Σ could be restrictive. In this article we confirm the claim in the cases where $G = S_5$ or $SL(2, 5)$.

1. Introduction

Let G be a finite group. For a smooth manifold M , we refer to a smooth G -action on M simply as a G -action on M . A G -action on M is called an *odd-fixed-point action*, or *odd-f.p. action* for short, (resp. *one-fixed-point action*, or *one-f.p. action* for short) if the G -fixed-point set of M consists of odd-number points (resp. exactly one point). It is easy to see that for a G -action on the sphere of dimension 0, 1, or 2, if the G -fixed-point set contains at least one isolated point, there are exactly two G -fixed points. Contrary to an expectation by D. Montgomery–H. Samelson [14, Remarks 7], E. Stein [26] found one-f.p. actions on S^7 of $SL(2, 5)$, and next T. Petrie [22] constructed one-f.p. actions on spheres of odd-order abelian groups having at least three non-cyclic Sylow subgroups.

Let A_5 and S_5 denote the alternating group and the symmetric group on five letters, respectively. The fact that the n -dimensional sphere S^n , $n \geq 6$, has one-f.p. actions of A_5 was shown by the first author in [15, 17] for $n = 6$; in [17, 18] for $n \geq 9$; and by A. Bak and the first author in [1, 2] for $n = 7, 8$. On the other hand, M. Furuta [10] proved the non-existence of one-f.p. actions on S^4 for any finite group, cf. [16]. M. Furuta, S. Demichelis [8] and N.P. Buchdahl–S. Kwasik–R. Schultz [6] showed the non-existence of one-f.p. actions on S^n , where $n = 3, 4, 5$, for any finite group.

A closed smooth manifold is called a *homotopy sphere* if it is homotopy equivalent to the sphere of same dimension. E. Laitinen–P. Traczyk [13] showed that for $n = 5$ or ≥ 7 , if $G \not\cong A_5$ then there never exist one-f.p. actions of G on n -dimensional homotopy spheres Σ such that $\dim \Sigma^H \leq 2$ for all subgroups $H \neq \{e\}$ of G . For a commutative ring R , a closed smooth manifold M is called an *R -homology sphere* if the homology groups $H_i(M; R)$ are isomorphic to the homology groups $H_i(S^n; R)$ of the sphere S^n of same dimension, for all $i \geq 0$. A $\mathbb{Z}$ -homology sphere is usually called a *homology sphere*. A. Borowiecka [3, Theorem 1.1] showed that any $\mathbb{Z}$ -homology sphere of dimension 8 does not admit one-f.p. actions of $SL(2, 5)$.

In this paper we will prove the following theorem.

Theorem 1.1. *Let G be the group S_5 (resp. $SL(2, 5)$) and Σ a $\mathbb{Z}$ -homology sphere. If $\dim \Sigma \in \{0, 1, 2, 3, 4, 5, 7, 8, 9, 13\}$ (resp. $\{0, 1, 2, 3, 4, 5, 6, 8, 9\}$) then Σ does not admit effective odd-f.p. actions of G .*

Recall that E. Stein [26] proved the existence of an effective one-f.p. $SL(2, 5)$ -action on S^7 . On the other hand, it has not yet proved in a literature that there is a one-f.p. S_5 -action on S^6 .

Theorem 1.1 follows from Theorems 4.2 and 5.2 in the present article.

REMARK. After the submission of the first draft of this paper, we were informed that a recent work [4] of A. Borowiecka–P. Mizerka shows that for some finite Oliver groups G of order up to 216, and for $G = A_5 \times C_k$ with $k = 3, 5$, or 7 , there is no one-f.p. G -action on a sphere S^n , where n is chosen within a subset of $\{6, 7, 8, 9, 10\}$, and the choice depends on G .

2. Necessary conditions for odd-f.p. actions on spheres

Let G be a finite group. The following three well-known facts are useful for our study.

Proposition 2.1 (cf. [5, Chapter III, Theorem 4.3]). *Let p be a prime, G a finite group of p -power order, and X a finite G -CW complex. Then the Euler characteristic $\chi(X^G)$ is congruent modulo p to the Euler characteristic $\chi(X)$.*

Lemma 2.2 (Smith's Theorem [25], cf. [5, Chapter III, Theorem 5.1]). *Let p be a prime, G a finite group of p -power order, and X a $\mathbb{Z}_p$ -homology sphere with G -action. Then X^G is a $\mathbb{Z}_p$ -homology sphere.*

Lemma 2.3 (Lefschetz' Formula, cf. [9, p.225, Exercises (6.17), 3]). *Let X be a finite G -CW complex and $g \in G$. Then the Lefschetz number $L(g : X \rightarrow X)$ coincides with the Euler characteristic $\chi(X^g)$. Therefore if X is $\mathbb{Q}$ -acyclic then $\chi(X^g) = 1$.*

Let $S(G)$ denote the set of all subgroups of G . For a prime p , let $\mathcal{P}_p(G)$ denote the family of all subgroups of G with p -power order and set $\mathcal{P}(G) = \bigcup_p \mathcal{P}_p(G)$ where p ranges over all primes. After [20], for p and q each of which is a prime or 1, we denote by $\mathcal{G}_p^q$ the family of finite groups G having a sequence $P \trianglelefteq H \trianglelefteq G$ such that $|P|$ is a power of p , H/P is cyclic, and $|G/H|$ is a power of q . Set $\mathcal{G}_p = \bigcup_q \mathcal{G}_p^q$, $\mathcal{G}^q = \bigcup_p \mathcal{G}_p^q$, $\mathcal{G} = \bigcup_{p,q} \mathcal{G}_p^q$, $\mathcal{G}_p^q(G) = \mathcal{G}_p^q \cap S(G)$, $\mathcal{G}_p(G) = \mathcal{G}_p \cap S(G)$, $\mathcal{G}^q(G) = \mathcal{G}^q \cap S(G)$, and $\mathcal{G}(G) = \mathcal{G} \cap S(G)$.

Let Σ be a $\mathbb{Z}$ -homology sphere with G -action. For $x \in \Sigma^G$, let $T_x(\Sigma)$ denote the tangent space at x in Σ with induced linear G -action.

Proposition 2.4. *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action and $x_0 \in \Sigma^G$. Then the following properties hold.*

- (1) *For every $H \in \mathcal{G}^1(G)$, $\chi(\Sigma^H)$ is equal to $1 + (-1)^k$, where $k = \dim T_{x_0}(\Sigma)^H$.*
- (2) *For every $K \in \mathcal{G}^q(G)$, $\chi(\Sigma^K)$ is congruent modulo q to $1 + (-1)^k$, where $k = \dim T_{x_0}(\Sigma)^K$.*

Proof. Let U be a closed disk G -neighborhood of x_0 in Σ and take the pinching map

$$f : \Sigma \rightarrow \Sigma / (\Sigma \setminus \text{int}(U)) \cong S(\mathbb{R} \oplus T_{x_0}(\Sigma)).$$

Clearly, f is a $\mathbb{Z}$ -homology equivalence. Let C_f denote the mapping cone of f . It follows that C_f is $\mathbb{Z}$ -acyclic. Thus by Lemmas 2.2 and 2.3, we obtain the property (1). Applying Proposition 2.1 to K/H -spaces Σ^H and $S(\mathbb{R} \oplus T_{x_0}(\Sigma))^H$ for $H \trianglelefteq K$ such that $|K/H|$ is a power of q and $H \in \mathcal{G}^1$, we obtain the property (2) from the property (1). $\square$

Corollary 2.5 (cf. [20, 11]). *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action. If Σ^G consists of exactly one point then G does not belong to $\mathcal{G}$.*

Lemma 2.6 (P. Conner–E. Floyd [7, Theorem 25.1]). *Let C be a group of order 2 and M a connected closed smooth manifold of positive dimension with C -action. If M^C is a non-empty finite set then $|M^C|$ is an even integer.*

Proposition 2.7. *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action and L a subgroup of G with $|G : L| = 2$. If $|\Sigma^G| \equiv 1 \pmod{2}$ then there exists a G -fixed point x_0 such that $\dim T_{x_0}(\Sigma)^L = 0$, hence the point x_0 is an isolated L -fixed point of Σ .*

Proof. There is a connected component M of Σ^L such that $|M \cap \Sigma^G| \equiv 1 \pmod{2}$. Note that G/L acts on M and $M^{G/L} = M \cap \Sigma^G (= M^G)$. By Lemma 2.6, M does not have positive dimension. Thus $\dim M = 0$ and M consists of one point, say x_0 . Then x_0 is a G -fixed point and $T_{x_0}(\Sigma)$ satisfies the desired property. $\square$

Corollary 2.8 (cf. [19, Lemma 2.1]). *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action and K the intersection of all subgroups H of G with $|G : H| \leq 2$. If $\Sigma^K = \{x_0\}$ then $\dim T_{x_0}(\Sigma)^K = 0$, hence the point x_0 is an isolated K -fixed point of Σ .*

Proof. Without any loss of generality, we may assume $G \neq K$. If $\dim T_{x_0}(\Sigma)^K > 0$ then there exists a subgroup L of G with $|G : L| = 2$ such that $\dim T_{x_0}(\Sigma)^L > 0$. But this contradicts Proposition 2.7. $\square$

Proposition 2.9. *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action. Suppose G satisfies the following conditions.*

- (1) *Each element of G is of prime-power order.*
- (2) *For every element $g \in G$ of 2-power order, the order of g is less than or equal to 4, cf. the 8-condition in [21].*

Then the tangential G -representations at G -fixed points of Σ are mutually isomorphic as real G -modules.

Proof. If $\Sigma^G = \{x, y\}$ then by a slight generalization of Sanchez' theorem on the Smith equivalence, see [24, Corollary 1.11], [23, p.194, Theorem 0.4], [12], [21, 8-condition Lemma], $T_x(\Sigma)$ is isomorphic to $T_y(\Sigma)$. Now we suppose $\Sigma^G \supsetneq \{x, y\}$ ($x \neq y$). For an arbitrary element $g \in G$ of prime-power order, say $\text{ord}(g) = p^k$ (p a prime), the g -fixed-point set Σ^g is a $\mathbb{Z}_p$ -homology sphere. Since $|\Sigma^g| \geq 3$, we see that Σ^g is connected and that $\text{res}_{\langle g \rangle}^G T_x(\Sigma)$ is isomorphic to $\text{res}_{\langle g \rangle}^G T_y(\Sigma)$. Therefore $T_x(\Sigma)$ is isomorphic to $T_y(\Sigma)$ as real G -modules. $\square$

Note that the conditions (1) and (2) in Proposition 2.9 are fulfilled for $G = A_5$. Also remark that $|S_5/A_5| = 2$.

Proposition 2.10. *Let G and L be the groups S_5 and A_5 , respectively, and Σ a $\mathbb{Z}$ -homology sphere with G -action. If $|\Sigma^G| \equiv 1 \pmod{2}$ then the following holds.*

- (1) $T_x(\Sigma) \cong T_y(\Sigma)$ as real L -modules for all $x, y \in \Sigma^L$.
- (2) $|\Sigma^L| \equiv 1 \pmod{2}$.

Proof. The property (1) follows from Proposition 2.9. By Proposition 2.7, there is a G -fixed point x_0 of Σ being isolated in Σ^L . By the property (1), all points in Σ^L are isolated, and hence $|\Sigma^L| < \infty$. By Proposition 2.1, we obtain $|\Sigma^L| \equiv 1 \pmod{2}$. $\square$

3. A_5 -actions on homology spheres

In this section we let $L = A_5$. There are precisely five irreducible real L -modules (up to isomorphisms), say $\mathbb{R}$, $U_{3,1}$, $U_{3,2}$, U_4 and U_5 , which are mutually non-isomorphic and $\dim \mathbb{R} = 1$, $\dim U_{3,1} = \dim U_{3,2} = 3$, $\dim U_4 = 4$, and $\dim U_5 = 5$.

Theorem 3.1. *Let Σ be a $\mathbb{Z}$ -homology sphere with L -action. If $|\Sigma^L| \equiv 1 \pmod{2}$ then for any $x \in \Sigma^L$, $T_x(\Sigma)$ contains an irreducible real L -submodule of dimension 3.*

Proof. The data of $\dim U^H$ for irreducible real L -modules U and subgroups H of L are as in the next table. In the table A_4 stands for the alternating group on four letters.

Table 3.1.

	E	C_2	C_3	C_5	D_4	D_6	D_{10}	A_4	A_5
$U_{3,i}$	3	1	1	1	0	0	0	0	0
U_4	4	2	2	0	1	1	0	1	0
U_5	5	3	1	1	2	1	1	0	0

Assume that $T_x(\Sigma)$ is isomorphic to $U_4^{\oplus a} \oplus U_5^{\oplus b}$ for non-negative integers a and b . Then Σ^{C_2} is a $\mathbb{Z}_2$ -homology sphere, and the equalities $\dim \Sigma^{C_2} = 2a + 3b$, $\dim \Sigma^{D_4} = a + 2b$, $\dim \Sigma_0^{D_6} = a + b$ hold, where $\Sigma_0^{D_6}$ is the union of all connected components of Σ^{D_6} intersecting with Σ^{D_4} . The intersection form

$$H_{a+2b}(\Sigma^{C_2}; \mathbb{Z}_2) \times H_{a+b}(\Sigma^{C_2}; \mathbb{Z}_2) \longrightarrow \mathbb{Z}_2$$

is trivial. On the other hand, $|\Sigma^{D_4} \cap \Sigma^{D_6}| \equiv 1 \pmod{2}$, which shows a contradiction. Thus $T_x(\Sigma)$ contains an L -submodule isomorphic to $U_{3,1}$ or $U_{3,2}$. $\square$

According to [2, 16, 17], the spheres S^n , $n \geq 6$, have one-f.p. actions of L .

Proposition 3.2. *Let Σ be an n -dimensional homology sphere with L -action such that $|\Sigma^L| < \infty$. Then the following holds.*

- (1) If $n = 3$ or 6 then $|\Sigma^L| \leq 2$.
- (2) If $n = 9$ and $|\Sigma^L| \equiv 1 \pmod{2}$ then $|\Sigma^L| = 1$.

Proof. Let Σ be a $\mathbb{Z}$ -homology sphere with an L -action of dimension 3, 6 or 9 such that $0 < |\Sigma^L| < \infty$ and $x_0 \in \Sigma^L$. Then $T_{x_0}(\Sigma)$ is isomorphic to a direct sum of

- (Case 1) 3-dimensional irreducible real L -modules, or
 (Case 2) 4- and 5-dimensional irreducible real L -modules.

In Case 1, we have $\dim T_{x_0}(\Sigma)^{D_4} = 0$ and hence, by Smith's theorem, $|\Sigma^{D_4}| = 2$, which shows $|\Sigma^L| \leq 2$. For Σ satisfying $\dim \Sigma = 9$ and $|\Sigma^L| \equiv 1 \pmod{2}$, Theorem 3.1 implies that Case 2 does not happen. Therefore we conclude $|\Sigma^L| = 1$. $\square$

4. S_5 -actions on homology spheres

In this section we let $G = S_5$ and $L = A_5$. There are precisely seven irreducible real G -modules (up to isomorphisms), say $\mathbb{R}, \mathbb{R}_\pm, V_{4,1}, V_{4,2}, V_{5,1}, V_{5,2}, V_6$, where $\dim \mathbb{R} = \dim \mathbb{R}_\pm = 1$, $\dim V_{k,i} = k$, and $\dim V_6 = 6$.

Theorem 4.1. *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action. If $|\Sigma^G| \equiv 1 \pmod{2}$ then for any $x \in \Sigma^G$, $T_x(\Sigma)$ contains an irreducible real G -submodule of dimension 6.*

Proof. We remark $\text{res}_L^G V_{4,i} \cong U_4$, $\text{res}_L^G V_{5,i} \cong U_5$, and $\text{res}_L^G V_6 \cong U_{3,1} \oplus U_{3,2}$, where $U_4, U_5, U_{3,1}$, and $U_{3,2}$ are irreducible real L -modules in the previous section. Since $|\Sigma^G| \equiv 1 \pmod{2}$, we obtain $|\Sigma^L| \equiv 1 \pmod{2}$ by Proposition 2.10. Theorem 3.1 implies that $\text{res}_L^G T_x(\Sigma)$ contains an L -submodule isomorphic to $U_{3,1}$ or $U_{3,2}$. Therefore $T_x(\Sigma)$ contains a G -submodule isomorphic to V_6 . $\square$

Theorem 4.2. *Let Σ be a $\mathbb{Z}$ -homology sphere with G -action. If $|\Sigma^G| \equiv 1 \pmod{2}$ then $\dim \Sigma$ does not lie in $T = \{0, 1, 2, 3, 4, 5, 7, 8, 9, 13\}$.*

Proof. Set $n = \dim \Sigma$ and let a, b, c be non-negative integers such that $n = 4a + 5b + 6c$. If $n \in T$ then we get $c = 0$. Thus Theorem 4.2 follows from Theorem 4.1. $\square$

5. $SL(2, 5)$ -actions on homology spheres

Let G be the group $SL(2, 5)$ and Z the center of G . There are precisely nine irreducible real G -modules (up to isomorphisms), say $\mathbb{R}, U_{3,1}, U_{3,2}, U_4, U_5, W_{4,1}, W_{4,2}, W_8$, and W_{12} , where $\dim \mathbb{R} = 1$,

$$\dim U_{3,i} = 3, U_{3,i}^Z = U_{3,i}, \dim W_{4,i} = 4 \text{ and } W_{4,i}^Z = 0 \text{ for } i = 1, 2,$$

$$\dim U_k = k \text{ and } U_k^Z = U_k \text{ for } k = 4, 5,$$

$$\dim W_k = k \text{ and } W_k^Z = 0 \text{ for } k = 8, 12.$$

For an integer $m \geq 2$, let C_m denote the cyclic group of order m and Q_m the quaternion group or generalized quaternion group of order m . We remark that for a real G -module W with $W^Z = 0$, the equality $W^H = 0$ holds for all subgroups H of G isomorphic to C_4, Q_8 , or Q_{12} .

The next result is a generalization of [3, Theorem 1.2 (ii)].

Theorem 5.1. *Let Σ be a $\mathbb{Z}_2$ -homology sphere with G -action. If $|\Sigma^G| \equiv 1 \pmod{2}$ then the set of all points $x \in \Sigma^G$ such that $T_x(\Sigma)$ contains an irreducible real G -submodule of dimension 3 consists of odd-number points.*

Proof. By the hypothesis $|\Sigma^G| \equiv 1 \pmod{2}$, it suffices to consider the case $\dim \Sigma > 0$. Let F denote the set consisting of points $x \in \Sigma^G$ such that each tangential representation $T_x(\Sigma)$ contains an irreducible real G -submodule of dimension 3, and set $F' = \Sigma^G \setminus F$. It suffices to prove $|F'| \equiv 0 \pmod{2}$.

If F' is empty then it is obvious that $|F| = |\Sigma^G| \equiv 1 \pmod{2}$. Therefore we may assume that F' is not empty. Let x and y be points in F and F' , respectively. Then we have

$$T_x(\Sigma^Z) \cong U \oplus U_4^{\oplus a} \oplus U_5^{\oplus b}$$

for a direct sum $U \neq 0$ of 3-dimensional irreducible real G -modules, say $\dim U = 3k > 0$, and non-negative integers a and b , and

$$T_y(\Sigma^Z) \cong U_4^{\oplus s} \oplus U_5^{\oplus t}$$

for some non-negative integers s and t with $4s + 5t > 0$. Note that for $H \cong Z, C_4, Q_8$, the H -fixed point sets Σ^H are $\mathbb{Z}_2$ -homology spheres and $G/Z \cong A_5$ acts on Σ^Z . By Table 3.1 we get the equalities

$$\begin{aligned} \dim \Sigma &= 3k + 4a + 5b, \quad \dim \Sigma = 4s + 5t, \\ \dim \Sigma^{C_4} &= k + 2a + 3b, \quad \dim \Sigma^{C_4} = 2s + 3t, \\ \dim \Sigma^{Q_8} &= a + 2b, \quad \dim \Sigma^{Q_8} = s + 2t, \end{aligned}$$

as well as

$$(5.1) \quad \begin{aligned} \dim T_x(\Sigma^{Q_8}) + \dim T_x(\Sigma^{Q_{12}}) &= \dim \Sigma^{C_4} - k, \\ \dim T_y(\Sigma^{Q_8}) + \dim T_y(\Sigma^{Q_{12}}) &= \dim \Sigma^{C_4}. \end{aligned}$$

It is remarkable that the integers s and t are uniquely determined by $\dim \Sigma^{C_4}$ and $\dim \Sigma^{Q_8}$. In addition, we note

$$\Sigma^{Q_8} \cap \Sigma^{Q_{12}} = \Sigma^G.$$

Let $\Sigma_0^{Q_{12}}$ denote the union of all connected components C of $\Sigma^{Q_{12}}$ such that $C \cap \Sigma^G \neq \emptyset$ and $\dim \Sigma^{Q_8} + \dim C = \dim \Sigma^{C_4}$. The equality (5.1) implies that $F \cap \Sigma_0^{Q_{12}} = \emptyset$ and $F' = \Sigma_0^{Q_{12}} \cap \Sigma^G$, and furthermore that the mod-2 intersection number of Σ^{Q_8} and $\Sigma_0^{Q_{12}}$ in Σ^{C_4} is equal to $|F'|$ in $\mathbb{Z}_2$. Let

$$\varphi : H_{s+2t}(\Sigma^{C_4}; \mathbb{Z}_2) \times H_{s+t}(\Sigma^{C_4}; \mathbb{Z}_2) \longrightarrow \mathbb{Z}_2$$

be the canonical mod-2 intersection form on Σ^{C_4} . Since Σ^{C_4} is a $\mathbb{Z}_2$ -homology sphere, the intersection form φ is trivial, hence the mod-2 intersection number of Σ^{Q_8} and $\Sigma_0^{Q_{12}}$ in Σ^{C_4} is zero. Therefore we have proved $|F'| \equiv 0 \pmod{2}$. $\square$

Theorem 5.2. *Let G be the group $SL(2, 5)$ and Σ a $\mathbb{Z}_2$ -homology sphere with effective G -action. If $|\Sigma^G| \equiv 1 \pmod{2}$ then $\dim \Sigma$ does not lie in $\{0, 1, 2, 3, 4, 5, 6, 8, 9\}$.*

Proof. By Theorem 5.1 there exists a G -fixed point x of Σ such that $T_x(\Sigma)$ contains a 3-dimensional irreducible real G -submodule. Since the action is effective, $T_x(\Sigma)$ contains a G -submodule isomorphic to $W_{4,1}$, $W_{4,2}$, W_8 , or W_{12} . Now we can readily see $\dim \Sigma \geq 7$ and $\dim \Sigma \neq 8, 9$. $\square$

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