



Title	Initial Fatigue Crack Growth Behavior in a Notched Component (Report IV) : Effect of Tensile Residual Stress(Mechanics, Strength & Structural Design)
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Citation	Transactions of JWRI. 1987, 16(2), p. 375-382
Version Type	VoR
URL	https://doi.org/10.18910/7380
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Initial Fatigue Crack Growth Behavior in a Notched Component (Report IV)[†]

— Effect of Tensile Residual Stress —

Kohsuke HORIKAWA*, Sang-moung CHO**

Abstract

The present report was intended to evaluate the effect of tensile residual stress on initial fatigue crack propagation in a notch field.

Initial fatigue crack propagation rate in tensile residual stress field on center notched strip (SS41) is affected by both residual stress and elasto-plastic behavior. In low cycle (high stress) region, the effect of elasto-plastic behavior was remarkable. But in high cycle (low stress) region, the effect of residual stress was conspicuous.

The relaxation of residual stress by yielding of notch field was modeled and confirmed by experiments. ΔJ to initial fatigue crack in notch field was calculated by considering both of elasto-plastic effect and residual stress.

As the results, initial fatigue crack propagation behavior in notch field with tensile residual stress could be interpreted by using the ΔJ .

KEY WORD: (Fatigue Crack Propagation) (Notch) (Notch Field) (Initial Fatigue Crack) (Elasto-Plastic Effect) (Residual Stress) (ΔJ)

1. Introduction

In the case of considering fatigue crack initiation life in structural discontinuity (Notch) which is free from defect, many studies have been performed on fatigue damage parameters to include residual stress effect^{1, 2)}. It can be explained that the effect of residual stress is conspicuous in the region of high cycle (low stress) fatigue, but elasto-plastic effect is remarkable in the region of low cycle (high stress) fatigue³⁻⁶⁾. However, there is not the study to interpret the effect of residual stress on initial fatigue crack propagation in notch field, and the mechanical modelization on that is needed.

When the effect of residual stress on long crack propagation is assessed, it is assumed that the relaxation of residual stress by plastic deformation is not devolped. And it is generalized method that stress intensity factor K_{res} calculated by initial residual stress is considered to determine stress ratio R ⁶⁻¹⁰⁾. This method is regarded as to consider initial residual stress as mean stress. However in notch field, plastic strain is developed due to stress concentration, and initial residual stress is considerably relaxed. As the consequence it follows that residual stress

as mean stress decreased.

In the present report, center notched strip, in which notch was located in tensile residual stress field, was used. And the present report is intended to investigate and to evaluate the effect of residual stress on initial fatigue crack propagation in notch field. The modelization for relaxation of residual stress due to plastic deformation was tried. Crack opening ratio U was determined in consideration of residual stress, and it was used to calculate ΔJ . Moreover, initial crack propagation in notch field with tensile residual stress was characterized by using ΔJ .

2. Specimens and Experimental Procedure

The material used in this report was 5.5 mm thickness SS41 mild steel plate, which had been tested in the previous report¹¹⁾. Melt-run by automatic TIG welder was made in longitudinal direction on the strip of 250 mm length and 55 mm breadth. A weld line was located at the center of the strip. The welding condition is given in Table 1. The bead of 5.5 mm width was concave. These welded strips were machined to 240 mm length, 50 mm breadth (=2B) and 5 mm thickness. A circular hole of 7 mm

[†] Received on Nov. 4, 1987

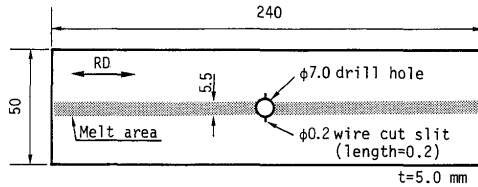
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Transactions of JWRI is published by Welding Research Institute of Osaka University, Ibaraki, Osaka 567, Japan

Table 1 TIG welding condition

Position	Electrode	Arc Length	Current	Voltage	Speed
Flat	$\phi 2.4$ mm	1.3 mm	200 A	17 V	50 cm/min

**Fig. 1** Center notched specimen with residual stress by melt-run.

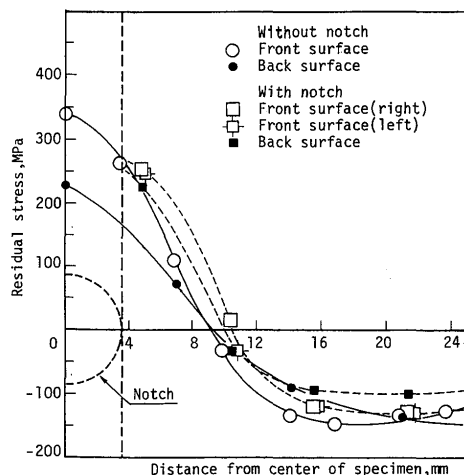
diameter($=2a_0$) was drilled at the center of the bead. The ratio a_0/B of this specimen was the same value with it in the previous report¹¹⁾, and elastic stress concentration factor K_t of the circular hole was 2.7. And as can be seen in **Fig. 1**, slit of 0.2 mm width and 0.2 mm length as initial defect was processed by wire cutter.

Fatigue crack propagation tests were carried out under constant amplitude of fully reversed axial load control ($R=-1$). Frequency of sinusoidal load was 3 ~ 10 Hz. Crack length was measured on both surface of front and back by travelling microscope (X50). All tests were performed at ambient room temperature.

3. Distribution of Welding Residual Stress in Notched Specimen

3.1 Distribution of initial residual stress

Fig. 2 shows the trend of residual stress measured before (\circ and \bullet marks) and after (\square and \boxplus marks) processing the notch. And \circ marks are distribution of residual stress on the front surface where the melt-run was done, and \bullet marks are that on the back surface. After processing the notch, the residual stress measured on the front surface is indicated by \square and \boxplus marks, which were

**Fig. 2** Distribution of residual stress before fatigue test in specimens without and with notch.

obtained from strain gages bonded with symmetry to the center of the notch. And \blacksquare marks are the trend measured on the back surface for the same specimen. Residual stress in notched specimens were measured on the plane of the maximum stress by using strain gages with gage length 0.3 mm. The peak value of tensile residual stress on the front surface was decreased slightly due to the process of notch. But the difference of the residual stress on the front and back surface became smaller, and the symmetric property of the stress to the center of notch was confirmed.

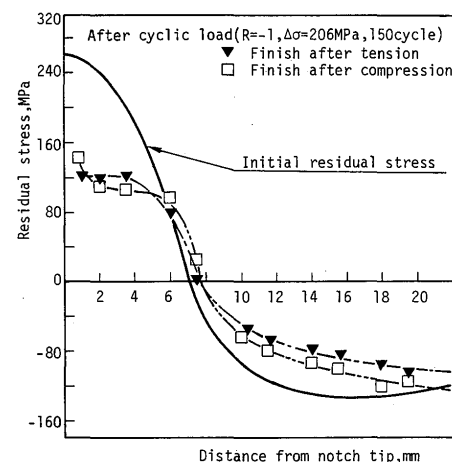
3.2 Change of residual stress by cyclic load.

In notch field, initial residual stress is high, and may be considerably relaxed due to plastic deformation when external force is loaded.

Fig. 3 shows the experimental results to examine the relaxation of initial residual stress by cyclic load in notched specimen. Solid line means initial residual stress. After nominal stress range $\Delta\sigma = 206$ MPa was loaded by 150 cycle, residual stress was measured. In **Fig. 3**, the residual stresses shown by \blacktriangledown and \square marks were measured when cyclic load test was finished after tensile and compressive load had been given. In any case, the residual stress after relaxation was clearly small in comparison with the initial value. Especially, tensile residual stress in notch field was very relaxed. From the trend of the residual stress shown by \blacktriangledown and \square marks, it appears that the residual stress after relaxation does not very change as tensile or compressive load is given before finish.

Next, the relaxation model of residual stress is treated for notch field only.

Fig. 4 shows the relation of local stress and strain in notch field having residual stress under fully reversed cyclic load condition. $\bar{\sigma}_{\text{mean}}$ to act as mean stress after

**Fig. 3** Distribution of residual stress after cyclic load test in notched specimen.

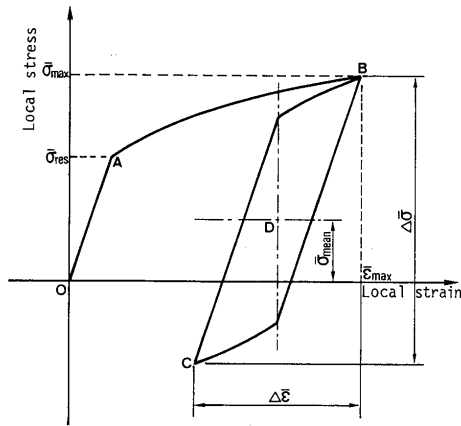


Fig. 4 Relation of local stress and strain in notch field with residual stress by cyclic load.

local stress and strain were stabilized by cyclic load was calculated here by the following modelization. The material constants obtained in the previous report¹¹⁾ were used to relate cyclic stress and strain. The approach can be written as follows.

- (1) (Initial residual stress) + (Maximum tensile load) → Maximum tensile stress $\bar{\sigma}_{\max}$ (Point B in Fig. 4)
- (2) (Maximum tensile stress $\bar{\sigma}_{\max}$) – (Stress amplitude $\bar{\sigma}_{\text{amp}}$ by load amplitude) → Mean stress $\bar{\sigma}_{\text{mean}}$ (Point D in Fig. 4)

The descriptions on two procedures are as follows.

- (1) Calculation of the maximum tensile stress $\bar{\sigma}_{\max}(x)$ (Point B in Fig. 4) in notch field.

In order to calculate the maximum tensile stress when maximum tensile load is given to notched strip having residual stress, it is needed to superpose residual stress and external force in any form. But as can be seen in Fig. 4, it is difficult to superpose those in form of local stress. Consequently, the maximum tensile load was superposed to residual stress taken in form of external force. It was tried to apply strain energy density as intermediary quantity used in the first report¹²⁾. And maximum tensile stress $\bar{\sigma}_{\max}(x)$ in notch field was obtained from the strain energy density W .

When $W(0)$ was given by the strain energy density at $x=0$ of notch tip, the distribution $W(x)$, in which residual stress was considered, could be written as follows.

$$W(x) = W(0)/(1 + g_{wp} \cdot x/\rho), x/\sqrt{a_0\rho} \leq 1.0 \quad (1.1)$$

$$\text{where, } W(0) = K_w \cdot W_n \quad (1.2)$$

$$K_w = K_t^2 + \left(\frac{1-n}{1+n}\right) \left(K_t^2 - \frac{1}{\phi^2}\right) \cdot C_w \quad (1.3)$$

$$C_w = 1 + \frac{1}{2} \sin \left\{ \frac{\pi}{2} K_t \left(\frac{\phi-1}{K_t-1} \right) \right\} \quad (1.4)$$

$$\phi = (S_a + \bar{\sigma}_{\text{rp}/K_t})/\sigma_{YC} \quad (1.5)$$

$$W_n = (S_a + \bar{\sigma}_{\text{rp}/K_t})^2/2E \quad (1.6)$$

And S_a is the stress amplitude in net section by external force, $\bar{\sigma}_{\text{rp}/K_t}$ is the initial residual stress at the tip of notch. Moreover g_{wp} in Eq. (1.1) is the gradient of strain energy density in notch field at elasto-plastic state, and is calculated by using notch shape and $W(0)$ ¹²⁾. If equivalent stress is calculated from the strain energy density estimated by the above method, and then the maximum tensile stress $\bar{\sigma}_{\max}(x)$ of point B in Fig. 4 is obtained.

Also in the study by Lawrence et al⁵⁾, it had been tried to replace residual stress with external force, and to predict fatigue crack initiation life by estimating the fatigue damage parameter at the spot of crack initiation. But in the present study, it was tried to extend the approach on the superposition of residual stress up to distribution as well as a point.

- (2) Calculation of mean stress $\bar{\sigma}_{\text{mean}}(x)$ (Point D in Fig. 4) in notch field.

For calculation of point C in Fig. 4, kinematic hardening rule was used as a hardening rule of material. And it was assumed that the size of yield surface is constant, and diameter of the yield surface is $2\sigma_{YC}$. Therefore, reversed stress to be based on the point B in Fig. 4 decreases with linear relation to strain as far as $2\sigma_{YC}$. Below $2\sigma_{YC}$, the reversed stress decreases with hardening rule depended upon cyclic hardening exponent n' , and a point C can be obtained.

Accordingly, ϕ of Eq. (1.5) and W_n of Eq. (1.6) were given by

$$\phi = (\Delta S)/2\sigma_{YC} \quad (2.1)$$

$$W_n = (\Delta S)^2/2E \quad (2.2)$$

By using these parameter, strain energy density $W(x)$ was calculated. Equivalent stress to be determined from this strain energy density $W(x)$, becomes $\Delta\bar{\sigma}(x)$, which is the range between the stress of point B and point C. Therefore, position of point C can be determined by cyclic stress-strain relation. Furthermore, when $\bar{\sigma}_{\text{amp}}(x) = \Delta\bar{\sigma}(x)/2$ is given, $\bar{\sigma}_{\text{mean}}(x)$ is obtained as follows.

$$\bar{\sigma}_{\text{mean}}(x) = \bar{\sigma}_{\max}(x) - \bar{\sigma}_{\text{amp}}(x) \quad (3)$$

By calculation using the above approach, maximum tensile stress $\bar{\sigma}_{\max}(x)$, stress amplitude $\bar{\sigma}_{\text{amp}}(x)$ and mean stress $\bar{\sigma}_{\text{mean}}(x)$ to be residual stress after relaxation are indicated in Fig. 5. In the case of Fig. 5, the nominal stress range $\Delta\sigma = 206 \text{ MPa}$ was given. And ▼ and □

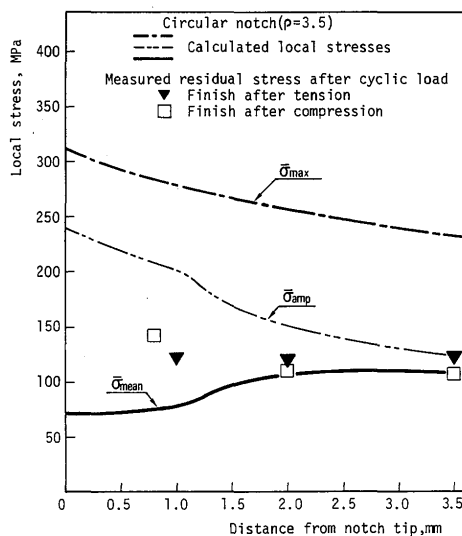


Fig. 5 Various calculated local stresses and measured residual stress after cyclic load in notch field.

marks are the experimental data referred from Fig. 3. It may be difficult to compare directly $\bar{\sigma}_{mean}(x)$ of solid line and measured residual stress in Fig. 5. Because $\bar{\sigma}_{mean}(x)$ was equivalent stress, but measured residual stress was obtained by multiplying maximum principal strain by elastic modulus. However, it may be mentioned that estimated $\bar{\sigma}_{mean}(x)$ corresponds well with measured trend.

In consequence, it was tried to employ the trend of solid line in Fig. 5 as mean stress acting during fatigue crack propagation.

4. Fatigue Crack Propagation in Notch Field

4.1 Effect of residual stress on crack opening ratio U

In this section, by evaluating effect of residual stress on crack opening ratio U, it was intended to consider the effect of residual stress on ΔJ .

If nominal stress is low, and a crack is long, then small scale yield condition is satisfied. In this case, initial residual stress may be considered as mean stress by assuming that residual stress is not relaxed due to plastic deformation. That approach is very generalized⁶⁻¹⁰. The feature of that approach can be said that stress ratio R is not determined by only external forces, but is computed by stress intensity factors considering residual stress. That is to say, stress ratio R is computed as follows.

$$R = \frac{K_{min} + K_{res}}{K_{max} + K_{res}} \quad (4)$$

where, K_{res} is the stress intensity factor calculated by initial residual stress.

Therefore from Eq. (4), it can be mentioned that initial residual stress is employed as mean stress. The stress ratio R affects crack opening ratio U, and is considered in determining ΔK_{eff} as follows.

$$\Delta K_{eff} = \Delta K \cdot U(R, \dots) \quad (5)$$

It was tried to extend the above approach based on LEFM to the case which LEFM was not defined.

There are several reports to emphasize that crack opening ratio U is affected stress level as well as stress ratio R¹³⁻¹⁵. In the present report, that approach was extended and applied to notch field having residual stress. For that approach, following hypotheses were given.

- (1) Residual stress influences crack opening ratio U only.
- (2) Crack opening ratio U depends upon two parameters of stress ratio R and strain level $\bar{\epsilon}_{max}(a)/\epsilon_{YC}$.
- (3) Initial residual stress affects the $\bar{\epsilon}_{max}(a)/\epsilon_{YC}$, and $\bar{\sigma}_{mean}(a)$ after relaxation.
- (4) Stress ratio R depends upon both of external force and residual stress $\bar{\sigma}_{mean}(a)$ after relaxation.

In order to obtain crack opening ratio U, evaluation on the above (3) and (4) is needed.

At first, $\bar{\epsilon}_{max}(a)/\epsilon_{YC}$ could be calculated by using initial residual stress, external force, notch shape and material constants as shown in Fig. 4 (Point B) and Fig. 5 (dot-and-dash line). And stress ratio R was calculated by Eq. (4). In Eq. (4), K_{max} and K_{min} were computed by elasto-plastic stress in notch field when only external force was considered. Moreover, K_{res} was obtained by using residual stress $\bar{\sigma}_{mean}(a)$ after relaxation (solid line in Fig. 5). It was supposed that only crack opening ratio U_e was determined by stress ratio R from Eq. (4). Practical experimental results may be employed for the determination of the U_e . But, in the present report the empirical formulas by Katoh et al¹⁶) were used.

$$U_e = 1.0/(1.5 - R), \quad R \leq 0.5 \quad (6.1)$$

$$U_e = 1.0, \quad R > 0.5 \quad (6.2)$$

Considering both of $\bar{\epsilon}_{max}(a)/\epsilon_{YC}$ of stress level and U_e by stress ratio R, crack opening ratio U, as the previous report¹¹), could be given as follows.

$$U = U_e + (1 - U_e) \tanh [0.7 \log \{ \bar{\epsilon}_{max}(a)/\epsilon_{YC} + 0.6 \}] \quad (7)$$

Fig. 6 indicates the trend of U obtained by experiment and by estimation from Eq. (7). Measured data have some scattering, but as a whole there is good correspondence between the two trends of U. When external force is high,

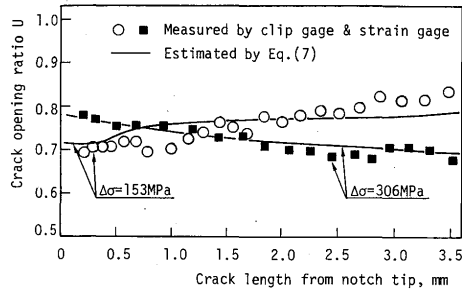


Fig. 6 Crack opening ratio U measured and estimated in notch field with residual stress ($\rho = 3.5$).

the U becomes low as crack grows because of large dependence on $\bar{\epsilon}_{\max}(a)/\epsilon_{YC}$ rather than on R or U_e . While in the case of low load, because the U depends mainly upon R or U_e , the U becomes high as crack grows.

In consequence, crack opening ratio U could be calculated by considering residual stress, and then the crack extension force ΔJ , as the previous report, could be obtained as follows.

$$\Delta J = \Delta J_e = \Delta K_{\text{eff}}^2 / E = (U \Delta K)^2 / E, \phi_a \leq 1.0 \quad \dots (8.1)$$

$$\frac{\Delta J}{\Delta J_e} = 1.0 + \frac{1}{C_N^2 \pi} \{ \phi_a (1+n')/n' - 1 \} \frac{h(n') C_p}{\phi_a^2},$$

$$\phi_a > 1.0 \quad \dots (8.2)$$

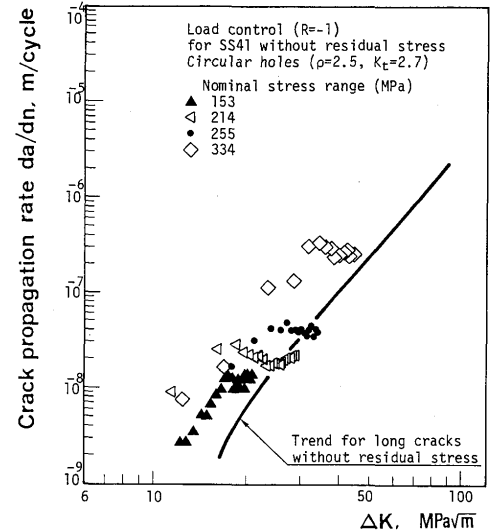
where, $\phi_a = \Delta \bar{\sigma}(a)/2\sigma_{YC}$

4.2 Characterization of initial crack propagation rate in notch field

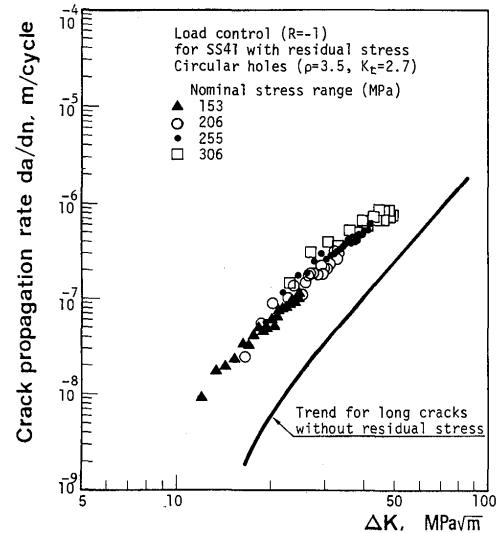
In this section, experimental results on initial crack propagation in notch field were characterized by crack extension force.

Fig. 7 (a) (b) show the initial crack propagation rates arranged by ΔK . In this case, both effects of elasto-plastic behavior and residual stress were disregarded. (a) shows the case in the circular notch field ($\rho = 2.5$ mm) that residual stress is not existent as referred in the third report¹¹⁾. On account of disregard for only elasto-plastic effect, initial crack propagation rate approaches to the master curve as crack becomes long and then ΔK becomes high in each nominal stress level. However, in (b) of the case that residual stress is existent, initial crack propagation rate does not approach to the master curve although crack becomes long and then ΔK becomes high in each nominal stress level. This tendency could result from the disregard of residual stress as well as elasto-plastic effect.

Fig. 8 (a) (b) are the initial crack propagation rates arranged by ΔJ . In Fig. 8, only elasto-plastic effect was considered, but residual stress was disregarded. In (a) of



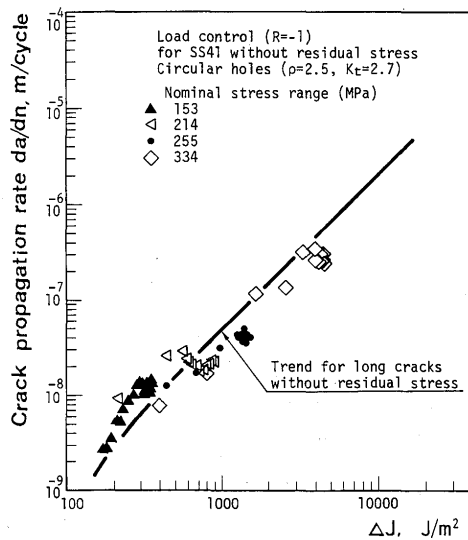
(a) Without residual stress



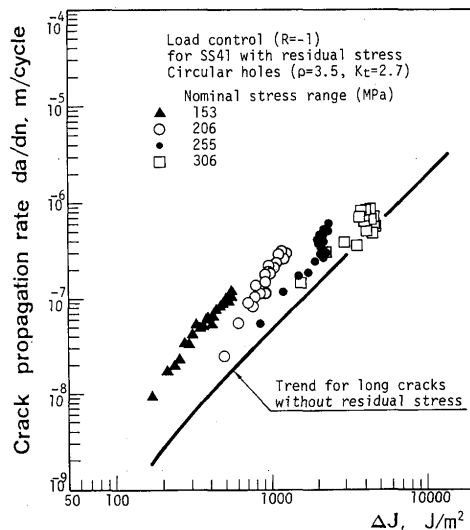
(b) With residual stress

Fig. 7 da/dn vs. ΔK calculated in disregard of residual stress in notch field.

the case that residual stress is not existent, the initial crack propagation rates correspond well with the master curve. While, in (b) of the case that residual stress is existent, the lower the load is and the longer the crack from notch tip is, the more the initial crack propagation rates are deviated from the master curve. This tendency may be caused by the relaxation of initial residual stress. That is to say, the lower the load is and the longer the crack is, the smaller the initial residual stress is relaxed. Because of just this reason, the effect of residual stress on initial crack propagation varies with load level and crack length from notch tip. On the other hand, when load level is high in Fig. 8 (b), initial crack propagation rates become close to the master curve. This tendency can be caused by not only the large relaxation of residual stress,



(a) Without residual stress



(b) With residual stress

Fig. 8 da/dn vs. ΔJ calculated in disregard of residual stress in notch field.

but also that crack propagation rate in high propagation rate region is not much susceptible to stress ratio $R^{17)}$.

In Fig. 7 (a) and Fig. 8 (b), the deviation of initial crack propagation rate from the master curve was examined for crack length.

In Fig. 7 (a) of the case that residual stress is not existent, because elasto-plastic effect is disregarded, the shorter the crack is, the more the deviation is. While in Fig. 8 (b) of the case that the residual stress is existent, because elasto-plastic effect is considered, but residual stress is not done, the longer the crack is, the more the deviation is. This is contrary to Fig. 7 (a).

From the above examination, it can be mentioned that if notch field with residual stress takes elasto-plastic behavior, initial crack propagation rate is affected by both of elasto-plastic behavior and residual stress.

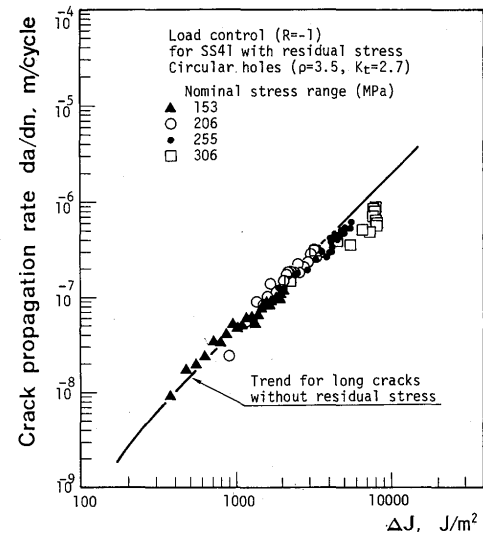


Fig. 9 da/dn vs. ΔJ calculated in consideration of residual stress in notch field.

Fig. 9 shows the relation of ΔJ - da/dn in notch field with residual stress. This ΔJ was computed by considering residual stress in notch field as shown in Eq. (4) ~ (7) and Eq. (8). As a whole, initial crack propagation rates coincide well with the master curve obtained from long crack propagation property of the material in elastic state. It can be referred that initial crack propagation rate can be characterized by ΔJ regardless of nominal stress level, crack length and residual stress.

In consequence, initial crack propagation rate in notch field with residual stress may be predicted by the relation of ΔJ - da/dn for long crack in the state which residual stress is not existent.

4.3 Discussion

When the small scale yield condition is satisfied and tensile residual stress is considered for crack extension force, effective stress intensity factor range ΔK_{eff} may be used. It was discussed to apply ΔK_{eff} for notch field taking elasto-plastic behavior. In this case, the ΔK_{eff} may be obtained by two methods as follows.

$$\Delta K_{eff} = U \Delta K \dots \dots \dots (9.1)$$

$$\begin{aligned} \Delta K_{eff} &= \Delta \bar{\sigma}_{eff}(a) \sqrt{\pi a} \cdot C_N \\ &= \{U \Delta \bar{\sigma}(a)\} \sqrt{\pi a} \cdot C_N \dots \dots \dots (9.2) \end{aligned}$$

In Eq. (9.1), ΔK is computed in disregard of yielding in notch field (Yield strength of material is not used), namely ΔK is obtained by assuming only elastic behavior. But, in Eq. (9.1), crack opening ratio U can be the consequent measure upon elasto-plastic effect and residual

stress. That is to say, in order to obtain one parameter of ΔK_{eff} , the two behaviors (elastic and elasto-plastic) are considered.

Employing Eq. (9.2) in which elasto-plastic stress $\Delta \bar{\sigma}(a)$ in notch field is used, right term of Eq. (8.2) to calculate ΔJ becomes 1.0 regardless of elasto-plastic effect. Therefore, in cyclic plastic zone of notch field, small crack extension force is given.

Fig. 10 indicates that the effect of residual stress on crack extension force in notch field varies with stress level. The crack extension force computed in disregard of residual stress was given by ΔJ_0 , and that considering residual stress was given by ΔJ_{res} . If the ratio of ΔJ_{res} to ΔJ_0 becomes to 1.0, the effect of residual stress becomes to vanish. The ratios for two different nominal stress range ($\Delta \sigma$) were compared in notch field. The ratio is higher at $\Delta \sigma = 153$ MPa than at $\Delta \sigma = 306$ MPa. Namely, when nominal stress range is low, the effect of residual stress may become significant. And the ratio as a function of crack length (or distance from notch tip) becomes higher as crack propagates. Namely, when a crack is short, and propagates in high stress field of notch, the effect of residual stress may become insignificant. In the case of $\Delta \sigma = 306$ MPa, when crack is short, the ratio becomes close to 1.0. This endorses to some extent that Lawrence et al.⁵⁾ disregard the effect of residual stress on initial crack propagation in notch field. However, the error resulted from that residual stress is ignored, may become greater as local stress becomes lower.

5. Conclusions

In order to evaluate the effect of residual stress on initial crack propagation in notch field, center notched strips having tensile residual stress in the center were used for fatigue tests ($R=-1$). The crack extension force ΔJ was calculated in consideration of tensile residual stress in notch field. And it was tried to characterize initial crack

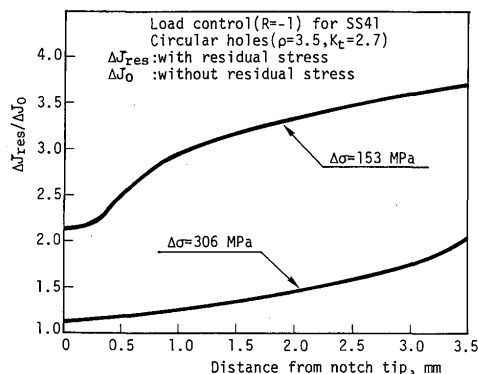


Fig. 10 Effect of residual stress on fatigue crack extension force ΔJ .

propagation rates by using the ΔJ .

The main results are as follows.

- (1) It was confirmed with experiment that tensile residual stress to exist in notch field is relaxed by plastic deformation, and only part of the stress persist. The present model for this relaxation corresponded comparably with experimental results (Fig. 5).
- (2) In initial crack propagation in notch field, elasto-plastic effect was distinguished when local stress range was high, but the effect of residual stress was significant when local stress range was low (Fig. 7 (a), Fig. 8 (b) and Fig. 10).
- (3) Crack opening ratio U was estimated in consideration of residual stress. Crack extension force ΔJ was calculated by using the U in notch field. And, by applying the ΔJ , initial crack propagation rate in notch field corresponded well with the master curve to be long crack propagation property in the material without residual stress.

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