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Title	L1(0,∞) ノ函數ノ Fourier transform Ⅱ.
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311. $L_1(0,\infty)$ ノ函數ノ Fourier Transform II.

高橋龍夫 (東北大)

$L_1(0,\infty)$ ノ函数ノ conjugate function ノ研究カラ始メル．ソノ方法ハ Titchmarsh (Reciprocal formulae involving series and integrals, Math. Zeits, Bd. 25) ニ做フ、他ノ方法ヲ用ヘバ或ハモット簡單ニユクカモ知レナイ。

$\phi(x)=\dfrac{x}{|\log x|^{1+\varepsilon}+1}$ トオク．之ハ0ク丈ク C_0 デ monotone increasing 且ツ convex デアル、斯様ナ $C_0(<1)$ ノ存在ハ 2nd deriative ヲトッテ容易ニ verify スル事ガ出

キル。$\sum_{m=1}^{\infty} a_m$ ヲ absolutely convergent series トスルト次ノ定理ガ得ラレル。

Theorem 3.

$$\sum_{n=1}^{\infty} \frac{\left| \sum_{m=1}^{\infty} \frac{a_m}{m-n+\frac{1}{2}} \right|}{\left| \log \left| \sum_{m=1}^{\infty} \frac{a_m}{m-n+\frac{1}{2}} \right| \right|^{1+\varepsilon}+1} \leqq A \sum_{n=1}^{\infty} |a_n|.$$

proof I ト同ジ notation ヲ用ヒテ

$$\sum_{n=1}^{\infty} \frac{\left| \sum_{m=1}^{\infty} \frac{a_m}{m-n+\frac{1}{2}} \right|}{\left| \log \left| \sum_{m=1}^{\infty} \frac{a_m}{m-n+\frac{1}{2}} \right| \right|^{1+\varepsilon}+1} = \sum_{n=1}^{\infty} \phi\left(\left| \sum_{m=1}^{\infty} \frac{a_m}{m-n+\frac{1}{2}} \right| \right)$$

$$\leqq A \sum_{n=1}^{\infty} \psi\left(\left| \sum_{m=1}^{\infty} \frac{a_m}{m-n+\frac{1}{2}} \right| \right) \leqq A \sum_{n=1}^{\infty} \psi\left(\left| \sum_{m=1}^{\left[\frac{n}{2}\right]} \frac{a_m}{m-n+\frac{1}{2}} \right| \right.$$

$$\left. + \left| \sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]} \frac{a_m}{m-n+\frac{1}{2}} \right| + \left| \sum_{m=n+\left[\frac{n}{2}\right]+1}^{\infty} \frac{a_m}{n-m+\frac{1}{2}} \right| \right)$$

$$\leqq A \sum_{n=1}^{\infty} \frac{\sum_{m=1}^{\left[\frac{n}{2}\right]} \frac{|a_m|}{\left|m-n+\frac{1}{2}\right|}}{\left| \log \sum_{m=1}^{\left[\frac{n}{2}\right]} \frac{|a_m|}{\left|m-n+\frac{1}{2}\right|} \right|^{1+\varepsilon}+1}$$

$$+ A \sum_{n=1}^{\infty} \frac{\sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]} \frac{|a_m|}{\left|m-n+\frac{1}{2}\right|}}{\left| \log \sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]} \frac{|a_m|}{\left|m-n+\frac{1}{2}\right|} \right|^{1+\varepsilon}+1}$$

$$+A\sum_{n=1}^{\infty}\frac{\sum_{m=n+\left[\frac{n}{2}\right]+1}^{\infty}\frac{|a_m|}{\left|n-m+\frac{1}{2}\right|}}{\left|\log\sum_{m=n+\left[\frac{n}{2}\right]+1}^{\infty}\frac{|a_m|}{\left|n-m+\frac{1}{2}\right|}\right|^{1+\varepsilon}+1}$$

$$=S_1+S_2+S_3$$

トオク。

$$S_1\leqq A\sum_{n=1}^{\infty}\frac{\sum_{m=1}^{\infty}|a_m|}{n\left(\left|\log\frac{A}{n}\right|^{1+\varepsilon}+1\right)}\leqq A\sum_{m=1}^{\infty}|a_m|\sum_{n=1}^{\infty}\frac{1}{n(\log n)^{1+\varepsilon}}$$

$$\leqq A\sum_{m=1}^{\infty}|a_m|.$$

$$S_3\leqq A\sum_{n=1}^{\infty}\frac{\sum_{m=n+\left[\frac{n}{2}\right]+1}^{\infty}|a_m|}{n(\log n)^{1+\varepsilon}}\leqq A\sum_{m=1}^{\infty}|a_m|\sum_{n=1}^{\infty}\frac{1}{n(\log n)^{1+\varepsilon}}$$

$$\leqq A\sum_{m=1}^{\infty}|a_m|.$$

次= S_2 ヲ計算スル、今 $n=1,2,3,\cdots\cdots$ ，中 $|na_n|<\frac{1}{2}c_0$ トナル n ノ set ヲ E_1 トシ，$|na_n|\geqq\frac{c_0}{2}$ トナル n ノ set ヲ E_2 トスル。

$$S_2=A\sum_{n=1}^{\infty}\psi\left(\sum_{\substack{m\in E_1\\ m\in\left(\left[\frac{n}{2}\right]+1,\ n+\left[\frac{n}{2}\right]\right)}}+\sum_{\substack{m\in E_2\\ m\in\left(\left[\frac{n}{2}\right]+1,\ n+\left[\frac{n}{2}\right]\right)}}\right)$$

$$\leqq A\sum_{n=1}^{\infty}\phi\left(\sum_{\substack{m\in E_1\\ m\in\left(\left[\frac{n}{2}\right]+1,\ n+\left[\frac{n}{2}\right]\right)}}\right)+A\sum_{n=1}^{\infty}\phi\left(\sum_{\substack{m\in E_2\\ m\in\left(\left[\frac{n}{2}\right]+1,\ n+\left[\frac{n}{2}\right]\right)}}\right)$$

$$=K_1+K_2$$

トオク．

今 $m\in E_1$ ナルトキ $a_m^*=a_m$，他デハ $a_m^*=0$ トスル．$\phi(x)$ ハ $0<x<C_0$ デ convex．且ツ $|ma_m^*|<\frac{C_0}{2}$ ナル故ニ $|na_m^*|<C_0$，コゝニ $m\in\left(\left[\frac{n}{2}\right]+1,\ n+\left[\frac{n}{2}\right]\right)$．

$$K_1=A\sum_{n=1}^{\infty}\frac{\sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]}{}'\dfrac{|a_m^*|}{\left|m-n+\frac{1}{2}\right|}}{\left|\log\sum\dfrac{|a_m^*|}{\left|m-n+\frac{1}{2}\right|}\right|^{1+\varepsilon}+1},$$

之ニ Jensen ノ不等式ヲ apply シテ

$$K_1\leqq A\sum_{n=1}^{\infty}\frac{1}{n}\sum_{\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]}{}'\frac{\dfrac{n|a_m^*|}{\left|m-n+\frac{1}{2}\right|}}{\left|\log n\dfrac{|a_m^*|}{\left|m-n+\frac{1}{2}\right|}\right|^{1+\varepsilon}+1}$$

$$\leqq A\sum_{n=1}^{\infty}\sum_{m=\left[\frac{n}{2}\right]}^{\infty}{}'\frac{|a_m|}{\left|m-n+\frac{1}{2}\right|\log^{1+\varepsilon}n}$$

$$\leqq A\sum_{m=1}^{\infty}|a_m|\sum_{n=\left[\frac{m}{2}\right]}^{\infty}{}'\frac{1}{\left|m-n+\frac{1}{2}\right|\log^{1+\varepsilon}n}\leqq A\sum_{m=1}^{\infty}|a_m|.$$

又 $m\in E_2$ ナルトキ $a_m^{**}=a_m$，他デハ $a_m^{**}=0$ トスル．ソウスルト

$$K_2=A\sum_{n=1}^{\infty}\frac{\left|\sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]}{}'\dfrac{a_m^{**}}{m-n+\frac{1}{2}}\right|}{\left|\log\left|\sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]}{}'\dfrac{a_m^{**}}{m-n+\frac{1}{2}}\right|\right|^{1+\varepsilon}+1}$$

$$\leqq A\sum_{n=1}^{\infty}\sum_{m=\left[\frac{n}{2}\right]+1}^{n+\left[\frac{n}{2}\right]}\frac{|a_m^{**}|}{\left|m-n+\frac{1}{2}\right|}$$

$$\leqq A\sum_{m=1}^{\infty}|a_m^{**}|\sum_{\nu=1}^{m}\frac{1}{\nu}\leqq A\sum_{\substack{m=1\\ m\in E_2}}^{\infty}\frac{A_m^{**}}{m}\leqq A\sum_{m=1}^{\infty}A_m^{**}|a_m|$$

$$\leqq A\sum_{m=1}^{\infty}|a_m|,$$

コヽニ $A_m^{**}=\sum_{n=1}^{m}|a_m^{**}|$. 之デ theorem ガ証明サレタ。

Lemma 2. $0\leqq X_1\leqq X_2<X_3\leqq X_4$ トシ

$|f(t)|\leqq M$ for $X_1\leqq t\leqq X_4$,

$f(t)=a$ for $X_2\leqq t\leqq X_3$,

$f(t)=0$ for $t<X_1$, $t>X_4$.

且ツ $X_2+\delta<x<X_3-\delta$, $\nu x<k\leqq \nu x+1$, $\nu>\frac{3}{\delta}$ トスルヽ。ν, k ハ正ノ整数デアル。

今 $m_i=\nu\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}}f(t)\,dt$ トオケバ

$$\left|\int_0^{\infty}\frac{f(t)}{t-x}\,dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|<\frac{6M}{\nu\delta-3}$$

proof. $\left|x-\frac{k\pm N}{\nu}\right|<\delta$ トスルヽ。 $N+1\leqq\nu\delta$ ナラバ之ハ満足サレル。サテ $\frac{k-N}{\nu}<t<\frac{k+N}{\nu}$ デハ $f(t)=a$ デアルカラ

$$\left|\sum_{i=k-N}^{k+N}\frac{m_i}{i-k-\frac{1}{2}}\right|=a\left|\sum_{i=k-N}^{k+N}\frac{1}{i-k-\frac{1}{2}}\right|$$

$$= \frac{a}{N+\frac{1}{2}} < \frac{a}{\nu\delta-3} \leqq \frac{M}{\nu\delta-3},$$

又 $$\int_{\frac{k-N}{\nu}}^{\frac{k+N}{\nu}} \frac{f(t)}{t-x}dt = a\int_{x-\frac{k-N}{\nu}}^{\frac{k+N}{\nu}-x} \frac{dy}{y} \leqq \frac{2a\left(\frac{k}{\nu}-x\right)}{x-\frac{k-N}{\nu}}$$

$$\leqq \frac{2a}{N-1} \leq \frac{2M}{\nu\delta-3}.$$

故= (1) $$\left|\int_{\frac{k-N}{\nu}}^{\frac{k+N}{\nu}} \frac{f(t)}{t-x}dt - \sum_{i=k-N}^{k+N} \frac{m_i}{i-k-\frac{1}{2}}\right| < \frac{3M}{\nu\delta-3}.$$

又 $i \geqq k+N+1$ トシテ

$$\left|\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}} \frac{f(t)}{t-x}dt - \frac{m_i}{i-\nu x-\frac{1}{2}}\right| = \left|\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}} \frac{f(t)}{t-x}dt - \frac{\nu}{i-\nu x-\frac{1}{2}}\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}} f(t)dt\right|$$

$$= \left|\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}} \left(\frac{1}{t-x} - \frac{1}{\frac{i-\frac{1}{2}}{\nu}-x}\right) f(t)dt\right| \leqq \left|\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}} \frac{\frac{i-\frac{1}{2}}{\nu}-t}{(t-x)\left(\frac{i-\frac{1}{2}}{\nu}-x\right)} f(t)dt\right|$$

$$\leqq \frac{M}{2\nu}\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}} \frac{dt}{\left(t-x-\frac{1}{2\nu}\right)^2}.$$

故= (2) $$\left|\int_{\frac{k+N}{\nu}}^{\infty} \frac{f(t)}{t-x}dt - \sum_{i=k+N+1}^{\infty} \frac{m_i}{i-\nu x-\frac{1}{2}}\right| \leqq \frac{M}{2\nu}\int_{\frac{k+N}{\nu}}^{\infty} \frac{dt}{\left(t-x-\frac{1}{2\nu}\right)^2}$$

$$= \frac{M}{2(k+N-\nu x-\frac{1}{2})} < \frac{M}{2(N-1)} < \frac{M}{2(\nu\delta-3)}.$$

又 (3) $\left|\sum_{i=K+N+1}^{\infty}\left(\frac{m_i}{i-\nu x-\frac{1}{2}}-\frac{m_i}{i-K-\frac{1}{2}}\right)\right|<\frac{M}{\nu\delta-3}$.

故ニ (2)(3) カラ

$$\left|\int_{\frac{K+N}{\nu}}^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=K+N+1}^{\infty}\frac{m_i}{i-K-\frac{1}{2}}\right|<\frac{3M}{2(\nu\delta-3)}.$$

又 $\left|\int_{0}^{\frac{K-N}{\nu}}\frac{f(t)}{t-x}dt-\sum_{i=1}^{K-N}{}'\frac{m_i}{i-\nu x-\frac{1}{2}}\right|$

$$=\left|\int_{0}^{\frac{K-N}{\nu}}\frac{f(t)}{t-x}dt-\sum_{i=1}^{K-N}\frac{1}{(i-\frac{1}{2})\frac{1}{\nu}-x}\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}}f(t)\,dt\right|,$$

且ツ

$$\left|\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}}\frac{f(t)}{t-x}dt-\frac{m_i}{i-\nu x-\frac{1}{2}}\right|<\frac{M}{2\nu}\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}}\frac{dt}{(t-x-\frac{1}{2\nu})^2},$$

故ニ $\left|\int_{0}^{\frac{K-N}{\nu}}\frac{f(t)}{t-x}dt-\sum_{i=1}^{K-N}\frac{m_i}{i-\nu x-\frac{1}{2}}\right|<\frac{M}{2\nu}\int_{0}^{\frac{K-N}{\nu}}\frac{dt}{(t-x-\frac{1}{2\nu})^2}<\frac{M}{\nu\delta}$.

故ニ以上ノ計算ヲ綜合シテ

$$\left|\int_{0}^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-K-\frac{1}{2}}\right|<\frac{6M}{\nu\delta-3}.\quad q.e.d.$$

次ニ $f(t)=a_\mu$ for $0\leqq x_\mu\leqq t<x_{\mu+1}$ $(\mu=1,2,\cdots\cdots m-1)$.

$f(t)=0$ for $0\leqq t<x_1$, $t>x_m$.

今 $0\leqq x_0<x_1$, $x_{m+1}>x_m$ トシ E ヲ $(x_\mu+\delta, x_{\mu+1}-\delta)$ ヨリ成ル set トシ E′ ヲ (x_0, x_{m+1}) ニ関スル E ノ comple-

mentary set トスル。

$$I=\int_0^{\infty}\frac{\left|\int_0^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|}{\left|\log\left|\int_0^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right|^{1+\varepsilon}+1}dx,$$

$$m_i=\nu\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}}f(t)\,dt$$

トスル。 $I=\int_E+\int_{E'}+\int_0^{x_0}+\int_{x_{m+1}}^{\infty}=I_1+I_2+I_3+I_4$

トオク。 Lemma 2 デ $X_1=x_0$, $X_2=x_\mu$, $X_3=x_{\mu+1}$, $X_4=x_{m+1}$ トオクト

$$\int_{x_\mu+\delta}^{x_{\mu+1}-\delta}\frac{\left|\int_0^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|}{\left|\log\left|\int_0^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right|^{1+\varepsilon}+1}dx$$

$$\leqq A\int_{x_\mu+\delta}^{x_{\mu+1}-\delta}\psi\left(\left|\int_0^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right)dx$$

$$\leqq A\int_{x_\mu+\delta}^{x_{\mu+1}-\delta}\psi\left(\frac{6M}{\nu\delta-3}\right)dx$$

$$\leqq A\phi\left(\frac{6M}{\nu\delta-3}\right)(x_{\mu+1}-x_\mu-2\delta)$$

$$=A\frac{\frac{M}{\nu\delta-3}}{\log^{1+\varepsilon}\frac{6M}{\nu\delta-3}+1}(x_{\mu+1}-x_\mu-2\delta).$$

故=

$$|I_1| \leqq \frac{\frac{AM}{\nu\delta-3}}{\left(\log\frac{6M}{\nu\delta-3}\right)^{1+\varepsilon}+1}(x_{m+1}-x_0) \leqq A(x_{m+1}-x_0)\frac{1}{\nu\delta(\log^{1+\varepsilon}\nu\delta+1)}$$

$$|I_2| \leqq A\int_{E'}\phi\left(\left|\int_0^\infty \frac{f(t)}{t-x}dt - \sum_{i=1}^{\infty}\frac{m_i}{i-kx-\frac{1}{2}}\right|\right)dx$$

$$\leqq A\int_{E'}\phi\left(\left|\int_0^\infty \frac{f(t)}{t-x}dt\right|\right)dx + A\int_{E'}\phi\left(\left|\sum_{i=1}^{\infty}\frac{m_i}{i-kx-\frac{1}{2}}\right|\right)dx$$

$\int_0^\infty \frac{f(t)}{t-x}dt$ ハ finite interval デ ϕ ニ関シテ integrable デアル（何者，logarithmic infinity ノ有限個，ミヲ持ツカラ）。故ニ $\delta\to 0$ ノトキ第一項ハ 0 ニ tend スル。又

$$\left|\sum_{i=1}^{\infty}\frac{m_i}{i-kx-\frac{1}{2}}\right| < 4M\log\{2\nu(x_m-x_1)+4\}$$

（何者，$m_i=0$ for $i<\nu x_1$, $i>1+\nu x_m$）. 故=

$$|I_2| \leqq O(1) + \frac{AM\log\{2\nu(x_m-x_1)+4\}m\delta}{|\log|\log\{2\nu(x_m-x_1)+4\}|\,|^{1+\varepsilon}+1}$$

$$\leqq Am\delta\log 2\nu + O(1), \quad (\delta\to 0).$$

$$|I_3| < A\int_0^{x_0}\phi\left(\left|\int_0^\infty \frac{f(t)}{t-x}dt\right|\right)dx + A\int_0^{x_0}\phi\left(\left|\sum_{i=1}^{\infty}\frac{m_i}{i-kx-\frac{1}{2}}\right|\right)dx.$$

然ルニ=

$$\int_0^\infty \frac{f(t)}{t-x}dt \leqq M\int_{x_1}^{x_m}\frac{dt}{t-x} \leqq M\frac{x_m-x_1}{x_1-X}.$$

$$\left|\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right| < M\,\frac{\sqrt{x_m}-\sqrt{x_1}+1}{\sqrt{x_1}-\sqrt{x}-2}\,.$$

故=

$$|I_3| \leqq \frac{A\left|\frac{x_m-x_1}{x_1-x_0}\right|}{\left|\log\frac{x_m-x_1}{x_1-x}\right|^{1+\varepsilon}+1} + \frac{A\,\frac{\sqrt{x_m}-\sqrt{x_1}+1}{\sqrt{x_1}-\sqrt{x}-2}}{\left|\log\frac{\sqrt{x_m}-\sqrt{x_1}+1}{\sqrt{x_1}-\sqrt{x}-2}\right|^{1+\varepsilon}+1}$$

$$\leqq A\frac{1}{x_1-x_0}\;\frac{1}{\left|\log\frac{x_m-x_1}{x_1-x}\right|^{1+\varepsilon}+1} \leqq \frac{A}{|\log x_0|^{1+\varepsilon}}$$

同様= $|I_4| \leqq \dfrac{A}{|\log x_m|^{1+\varepsilon}}$.

上ノ計算ヲ綜合シテ

$$|I| \leqq A\frac{x_m-x_0}{\nu\delta(\log^{1+\varepsilon}\nu\delta+1)}+O(1)+A\delta\log\nu+\frac{A}{|\log x_0|^{1+\varepsilon}} + \frac{A}{|\log x_m|^{1+\varepsilon}}\,.$$

今 $\delta=\dfrac{1}{\sqrt{\nu}}$, $x_0=\dfrac{1}{\sqrt{\nu}}$, $x_{m+1}=\sqrt{\nu}$ トスルト $x_{m+1}-x_0=O(\sqrt{\nu})$,

故=

$$|I| \leqq \frac{A}{\log^{1+\varepsilon}\nu}+O(1)+\frac{A\log\nu}{\sqrt{\nu}}+\frac{A}{\log^{1+\varepsilon}\nu}+\frac{A}{\log^{1+\varepsilon}\nu} \to 0.$$

故=

$$(4)\quad \lim_{\nu\to\infty}\int_0^{\infty}\phi\left(\left|\int_0^{\infty}\frac{f(t)}{t-x}dt-\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right)dx=0.$$

Lemma 3. $\chi(x)$ ヲ continuous monotone

increasing function トシ $\chi(0)=0$, $\chi(2x)\leqq A\chi(x)$ ナラバ

$$\lim_{m\to\infty}\int_0^\infty \chi(f_m(x)-f(x))\,dx$$

ナラバ

$$\lim_{m\to\infty}\int_0^\infty \chi(f_m(x))\,dx=\int_0^\infty \chi(f(x))\,dx.$$

コレハ良ク知ラレテ居ル。(拙著，数物會誌，綜合報告)．

コノ Lemma ト (4) カラ

$$\int_0^\infty \phi\left(\left|\int_0^\infty \frac{f(t)}{t-x}\,dt\right|\right)dx=\lim_{\nu\to\infty}\int_0^\infty \phi\left(\left|\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right)dx$$

Theorem 3 ニヨリ

$$\int_0^\infty \phi\left(\left|\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right)dx=\frac{1}{\nu}\sum_{k=1}^{\infty}\phi\left(\left|\sum_{i=1}^{\infty}\frac{m_i}{i-k-\frac{1}{2}}\right|\right)$$

$$\leqq\frac{1}{\nu}A\sum_{i=1}^{\infty}|m_i|$$

故ニ

$$\int_0^\infty \phi\left(\left|\int_0^\infty \frac{f(t)}{t-x}\,dt\right|\right)dx\leqq A\sum_{i=1}^{\infty}\int_{\frac{i-1}{\nu}}^{\frac{i}{\nu}}|f(t)|\,dt=A\int_0^\infty|f(t)|\,dt.$$

<u>Theorem 4.</u> $f(t)\in L_1(0,\infty)$ トシ ソノ conjugate function ヲ $g(t)$ トスルト

$$\int_0^\infty \frac{|g(t)|}{\left|\log|g(t)|\right|^{1+\varepsilon+1}}\,dt\leqq A\int_0^\infty|f(t)|\,dt.$$

Proof. 上述ノコトカラ argument ガ充分大ナルトキ0デアルヤウナ step function ニ付テコノ Theorem ガ証明サレタ。次ニ一般ノ場合ヲ証明スル。$f(t)\in L_1(0,\infty)$. ソウスルト step functions ノ sequence $f_n(x)$ ガアツテ

$$\lim_{n\to\infty}\int_0^\infty |f(x)-f_n(x)|\,dx=0.$$

サテIノ Theorem 1 カラ，Nヲ fixed constant トスルト（Theorem ハ $(0,2\pi)$ トシテヤツテキルガ finite interval デヨイ）

$$\int_0^N \phi\left(\left|\int_0^N \frac{f_n(t)}{t-x}\,dt-\int_0^N \frac{f_{rm}(t)}{t-x}\,dt\right|\right)dx$$
$$\leqq A\int_0^N |f_n(t)-f(t)\,dt\longrightarrow 0 \quad (n\to\infty)$$

故ニヨク知ラレタ mean convergence ノ Theorem カラ (n_i) ナル (n) ノ subsequence ガ exist シテ $(0,N)$ ノ殆ンドスベテノ x ニ付テ

$$\lim_{i\to\infty}\int_0^N \frac{f_{n_i}(t)}{t-x}\,dt=\int_0^N \frac{f(t)}{t-x}\,dt$$

$x<N-\delta$ $(\delta>0)$, $\displaystyle\int_N^\infty \frac{f_{n_i}(t)-f(t)}{t-x}\,dt\leqq\frac{1}{\delta}\int_N^\infty |f_{n_i}(t)-f(t)|\,dt\to 0.$

故ニ $(0,N)$ ノ殆ンドスベテノ x ニ付テ

(5) $$\lim_{i\to\infty}\int_0^\infty \frac{f_{n_i}(t)}{t-x}\,dt=\int_0^\infty \frac{f(t)}{t-x}\,dt.$$

ソウスルト diagonal method ニヨリ (n_i') ナル seq ガアッテ，$(0,\infty)$ ノ殆ドスベテノ x ニ對シテ

$$(6)\quad \lim_{i\to\infty}\int_0^\infty \frac{f_{n_i'}(t)}{t-x}\,dt=\int_0^\infty \frac{f(t)}{t-x}\,dt$$

サテ $f_{n_k'}(t)-f_{n_i'}(t)$ ハ step function デアルカラ

$$\int_0^\infty \phi\left(\left|\int_0^\infty \frac{f_{n_k'}(t)}{t-x}\,dt-\int_0^\infty \frac{f_{n_i'}(t)}{t-x}\,dt\right|\right)dt$$

$$\leqq A\int_0^\infty \left|f_{n_k'}(t)-f_{n_i'}(t)\right|dt\to 0\quad (i,k\to\infty).$$

故ニ

$$\int_0^\infty \phi\left(\left|F(x)-\int_0^\infty \frac{f_{n_i'}(t)}{t-x}\,dt\right|\right)dt\to 0$$

ナル ϕ ニ関シテ integrable ナ $F(x)$ ガアル。(6) カラ

$$F(x)=\int_0^\infty \frac{f(t)}{t-x}\,dt\ (\text{almost everywhere}).$$

故ニ

$$\lim_{i\to\infty}\int_0^\infty \phi\left(\left|\int_0^\infty \frac{f_{n_i'}(t)}{t-x}\,dt\right|\right)dx=\int_0^\infty \phi\left(\left|\int_0^\infty \frac{f(t)}{t-x}\,dt\right|\right)dx.$$

且ツ $$\int_0^\infty \phi\left(\left|\int_0^\infty \frac{f_{n_i'}(t)}{t-x}\,dt\right|\right)dx\leqq A\int_0^\infty \left|f(t)\right|dt$$

之デ Theorem ガ完全ニ証明サレタ。