Title	円系表面ニツイテ
Author(s)	松村, 宗治
Citation	全国紙上数学談話会. 1937, 121, p. 59-62
Version Type	VoR
URL	https://doi.org/10.18910/74471
rights	
Note	

546. 円系表面ニツイテ

松村　宗治（台北大）

（I）円系表面上ノ極小曲線ハ

(1)　$(\theta_t\theta_t)\,dt^2+2(\theta_t\theta_\tau)\,dt\,d\tau+(\theta_\tau\theta_\tau)\,d\tau^2=0$

デ表ハサレルコトヲ自分ハ前ニ述ベタ。

サテ、ソレヲ

(2)　$d\varphi^2+d\psi^2=0$

ニテ表ハシ得ベシ。ソレニハ φ ハ

(3)　$$\frac{\partial}{\partial t}\left(\frac{(\theta_\tau\theta_\tau)\frac{\partial\varphi}{\partial t}-(\theta_t\theta_\tau)\frac{\partial\varphi}{\partial\tau}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}\right)+\frac{\partial}{\partial\tau}\left(\frac{(\theta_t\theta_t)\frac{\partial\varphi}{\partial\tau}-(\theta_t\theta_\tau)\frac{\partial\varphi}{\partial t}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}\right)=0$$

ノ解デアル。コノトキ

$$-\frac{(\theta_t\theta_t)\dfrac{\partial\varphi}{\partial\tau}-(\theta_t\theta_\tau)\dfrac{\partial\varphi}{\partial t}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}dt+\frac{(\theta_\tau\theta_\tau)\dfrac{\partial\varphi}{\partial t}-(\theta_t\theta_\tau)\dfrac{\partial\varphi}{\partial\tau}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}d\tau$$

ハ完全微分トナリ、ソレヲ $d\psi$ トスル。此ノ φ, ψ ヲトレバ

(2) ノ如ク極小曲線ヲ表シ得ベシ。

ツマリ、ソコニ

$$\begin{cases}\dfrac{\partial\psi}{\partial t}=-\dfrac{(\theta_t\theta_t)\dfrac{\partial\varphi}{\partial\tau}-(\theta_t\theta_\tau)\dfrac{\partial\varphi}{\partial t}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}\\[2ex]\dfrac{\partial\psi}{\partial\tau}=\dfrac{(\theta_\tau\theta_\tau)\dfrac{\partial\varphi}{\partial t}-(\theta_t\theta_\tau)\dfrac{\partial\varphi}{\partial\tau}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}\end{cases}$$

$$\begin{cases}\dfrac{\partial\varphi}{\partial t}=\dfrac{(\theta_t\theta_t)\dfrac{\partial\psi}{\partial\tau}-(\theta_t\theta_\tau)\dfrac{\partial\psi}{\partial t}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}\\[2ex]\dfrac{\partial\varphi}{\partial\tau}=\dfrac{-(\theta_\tau\theta_\tau)\dfrac{\partial\psi}{\partial t}-(\theta_t\theta_\tau)\dfrac{\partial\psi}{\partial\tau}}{\sqrt{(\theta_t\theta_t)(\theta_\tau\theta_\tau)-(\theta_t\theta_\tau)^2}}\end{cases}$$

デアル。

(II) 円系表面ノ法曲率ヲ $\dfrac{1}{R}$ トセバ

$$(1)\quad \frac{1}{R}=\frac{\lambda\left[L\,dt^2+2M\,dt\,d\tau+N\,d\tau^2\right]}{(\theta_t\theta_t)dt^2+2(\theta_t\theta_\tau)dt\,d\tau+(\theta_\tau\theta_\tau)dt^2}$$

デアル。記法ハ前々ノ通リデアル。λノ決定方ニツイテハ以前ニノベタカラ其様ニ定メルモノトスル。(1) カラ

$$(2)\quad -\frac{1}{R}\left\{(\theta_t\theta_t)dt^2+2(\theta_t\theta_\tau)dt\,d\tau+(\theta_\tau\theta_\tau)d\tau^2\right\}$$
$$+L\,dt^2+2M\,dt\,d\tau+N\,d\tau^2=0$$

(2)ヲ dt ＝ツイテ、マタ $d\tau$ ＝ツイテ微分シ、$\frac{1}{R}$ ノ極大極小ニ對スル條件ヲ求メ R ヲ消去セバ

$$(3)\quad \begin{vmatrix}(\theta_t\theta_t)dt+(\theta_t\theta_\tau)d\tau & L\,dt+M\,d\tau\\ (\theta_t\theta_\tau)dt+(\theta_\tau\theta_\tau)d\tau & M\,dt+N\,d\tau\end{vmatrix}=0$$

ヲ得。但シ $\lambda\neq0$ トス。

(3)ハ $\frac{1}{R}$ ノ極大及ビ極小ニ對スル $dt:d\tau$ ノ方向ヲ與フル二次方程式トナリ、ツマリ曲率曲線ノ方程式デアル。

(III) M. Chini ノ論文 (*Sulla determinazione delle geodetische di talune superficie, Rendiconti Instituto Lombardo* (2) 59, 298-302) ヨリ吾々ノ円系表面ノ基本量 $(\theta_t\theta_t)$, $(\theta_\tau\theta_\tau)$, $(\theta_t\theta_\tau)$ ノ間ニ

$$(\theta_\tau\theta_\tau)=1,\quad (\theta_t\theta_\tau)=0,$$

$$(\theta_t\theta_t)=\frac{1}{\{t+f(\tau)\}^2}\text{ ---------(1)}$$

ガ成立ツ場合ニ其上ノ測地線ノ一般方程式ハ

$$\frac{(\theta_t\theta_t)\sqrt{\sin\alpha}}{\sin\tau}-\int\frac{f(\tau)\sqrt{\sin\alpha}}{\sin\tau}d\tau=\text{const.}\quad(2)$$

デ與ヘラルルコトガワカル。

但シ α ハ

$$\sqrt{\sin\alpha}\,\cot\tau+\frac{1}{2}\int\sqrt{\sin\alpha}\;d\alpha=C\text{ ----------(3)}$$

ヲ満足スル。コヽニCハ常數デアル。