



Title	二次式ニ関スル函数方程式ニ就イテ
Author(s)	春木, 博
Citation	全国紙上数学談話会. 1941, 221, p. 404-405
Version Type	VoR
URL	https://doi.org/10.18910/74884
rights	
Note	

The University of Osaka Institutional Knowledge Archive : OUKA

<https://ir.library.osaka-u.ac.jp/>

The University of Osaka

954. 二次式=開スル函数方程式=就イテ

森 本 博(神戸商船)

$f(x) = ax^2 + bx + c$, $g(x) = ax^2$ ハ、次ノ函数方程式

$$(F) \quad f(x+y) + f(x-y) = 2f(x) + 2g(y)$$

ヲ満足セシメル。

今逆ニ (F) ヲ充ス可測函数 $f(x)$, $g(x)$ ヲ求メテ見ヨシ。

$$(F) \text{ヨリ} \quad g(y) = \frac{1}{2} \{ f(x+y) + f(x-y) - 2f(x) \}$$

$$\begin{aligned}
\text{故} = & g(x+y) + g(x-y) - 2g(x) - 2g(y) \\
= & \frac{1}{2} \{ f(2x+y) + f(-y) - 2f(x) \\
& + f(2x-y) + f(y) - 2f(x) \\
& - 2f(2x) - 2f(0) + 4f(x) \\
& - 2f(x+y) - 2f(x-y) + 4f(x) \}
\end{aligned}$$

シカル =

$$\begin{aligned}
f(2x+y) + f(2x-y) &= 2f(2x) + 2g(y) \\
f(x+y) + f(x-y) &= 2f(x) + 2g(y)
\end{aligned}$$

ナル故

$$\begin{aligned}
& g(x+y) + g(x-y) - 2g(x) - 2g(y) \\
= & \frac{1}{2} \{ f(y) + f(-y) - 2f(0) - 2g(y) \}
\end{aligned}$$

(F) = 於て $x=0$ トオケル

$$f(y) + f(-y) - 2f(0) - 2g(y) = 0$$

$$\text{故} = g(x+y) + g(x-y) = 2g(x) + 2g(y)$$

$g(x)$ ハ可測ナル故、 a 任意ノ實數トスル、

$$g(x) = ax^2$$

$$(F) = \text{ヨリ } f(x+y) + f(x-y) = 2f(x) + 2ay^2$$

之ヨリ可測解 $f(x)$ ヲ求ムル、

$$f(x) = ax^2 + bx + c \quad (a, b, c \text{ ハ任意ノ實數})$$