



Title	二三ノ函数方程式ノ可測解ニ就イテ
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955. 二三ノ函数方程式ノ可測解ニ就イテ

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§ 1. 明ニ $(x+y)^4 + (x-y)^4 = 2x^4 + 2y^4 + 12x^2y^2$ が成立スル。今逆ニ次ノ函数方程式

$$(F) \quad f(x+y) + f(x-y) = 2f(x) + 2f(y) + \varphi(x)y^2$$

ヲ満足セシメル可測函数 $f(x)$, $\varphi(x)$ ヲ求メヲ見ヨウ。

$$\varphi(x) = \frac{1}{y^2} \{f(x+y) + f(x-y) - 2f(x) - 2f(y)\}$$

$$\begin{aligned} \text{故ニ} \quad & \varphi(x+y) + \varphi(x-y) - 2\varphi(x) - 2\varphi(y) \\ &= \frac{1}{y^2} \{f(x+2y) + f(x) - 2f(x+y) - 2f(y) \\ &\quad + f(x) + f(x-2y) - 2f(x-y) - 2f(y) \\ &\quad - 2f(x+y) - 2f(x-y) + 4f(x) + 4f(y) \\ &\quad - 2f(2y) - 2f(0) + 4f(y) + 4f(y)\} \end{aligned}$$

$$\text{シカルニ} (F) = 0$$

$$f(x+2y) + f(x-2y) = 2f(x) + 2f(2y) + 4\varphi(x)y^2$$

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) + \varphi(x)y^2$$

$$f(0) = 0$$

ナル故

$$\varphi(x+y) + \varphi(x-y) - 2\varphi(x) - 2\varphi(y) = 0$$

$$\therefore \varphi(x+y) + \varphi(x-y) = 2\varphi(x) + 2\varphi(y)$$

$\varphi(x)$ ハ可測ナル故 $\varphi(x) = ax^2$ (a ハ任意ノ実数)

$\varphi(x)$ が判レバ $f(x)$ が可測ナル故、種々函数方程式26

号卜同様, 方針 = ヨリ結局

$$\begin{cases} f(x) = \alpha x^4 + \beta x^2 \\ g(x) = 12 \alpha x^2 \end{cases} \quad (\alpha, \beta \text{ は任意, 實数})$$

§ 2. 明 = $\cos(x+y) - \cos(x-y) = -2 \sin x \sin y$
が成立スル。今逆ニ次, 函数方程式

$$(G) \quad f(x+y) - f(x-y) = -2g(x)g(y)$$

ヲ満足セシメル可測函数 $f(x)$, $g(x)$ ヲ求メテ見ヨ。

$$g(x) = -\frac{1}{2g(y)} \{f(x+y) - f(x-y)\}$$

$$\text{故} = g(x+y) + g(x-y)$$

$$= -\frac{1}{2g(y)} \{f(x+2y) - f(x) + f(x) - f(x-2y)\}$$

$$= -\frac{1}{2g(y)} \{f(x+2y) - f(x-2y)\}$$

$$= -\frac{1}{2g(y)} \times (-2g(x)g(2y)) \quad ((G) = \text{ヨル})$$

$$= \frac{g(x)g(2y)}{g(y)}$$

$$\text{故} = g(y) = \frac{1}{2} \frac{g(2y)}{g(y)} \quad \text{トオケバ}$$

$$g(x+y) + g(x-y) = 2g(x)g(y)$$

$g(x)$, $g(y)$ は可測ナル故、コレヨリヨク知ラレタル如ク

$$g(x) = \alpha x + \beta \quad (\alpha, \beta \text{ は任意, 實数})$$

$$\text{又ハ} \quad g(x) = \alpha \sin \beta x + \gamma \cos \delta x \quad (\alpha, \beta, \gamma, \delta \text{ は任意})$$

實数)

$$\text{又ハ } g(x) = \alpha \sin h \beta x + \gamma \cos h \delta x \quad (\alpha, \beta, \gamma, \delta \text{ 任意, 實数})$$

$$g(0) = 0 + \gamma \text{ 故 } g(x) = \alpha x$$

$$\text{又ハ } g(x) = \alpha \sin \beta x$$

$$\text{又ハ } g(x) = \alpha \sin h \beta x$$

$$(G) = \text{於テ } y = x \text{ トオケバ}$$

$$f(2x) = -2g^2(x) + f(0)$$

$$\therefore f(x) = -2g^2\left(\frac{x}{2}\right) + f(0)$$

結局 α, β, γ 任意, 実数トスルトキ

$$\begin{cases} f(x) = -\frac{1}{2} \alpha^2 x^2 + \beta \\ g(x) = \alpha x \end{cases}$$

$$\text{又ハ } \begin{cases} f(x) = -2\alpha^2 \sin^2 \frac{1}{2} \beta x + \gamma \\ g(x) = \alpha \sin \beta x \end{cases}$$

$$\text{又ハ } \begin{cases} f(x) = -2\alpha^2 \sin^2 h \frac{1}{2} \beta x + \gamma \\ g(x) = \alpha \sin h \beta x \end{cases}$$

§3. $\cos x, \sin x$ 間ニ

$$\cos(x+y) \cos(x-y) = \cos^2 y - \sin^2 x$$

が成立スル。

今逆ニ、次ノ函数方程式

$$(H) \quad f(x+y) f(x-y) = f^2(y) - g^2(x)$$

ヲ充テ可測函数 $f(x), g(x)$ ヲ求メテ見ヨウ。

$$f(0) = 0 + \text{ラベ } (F) = \text{於テ } y = 0 \text{ トオケユトニヨリ}$$

$$f^2(x) + g^2(x) = 0$$

$f(x), g(x)$ は一價有限實數値函数ナル故, $f(x) \equiv 0$,
 $g(x) \equiv 0$ ヲ得ル。

故ニ、以下ニ於テハ $f(0) \neq 0$ トシテヨイ。更ニ

$$f^*(x) = \frac{f(x)}{f(0)}, g^*(x) = \frac{g(x)}{f(0)} \text{ トオケバ, } f^*(0) = 1 =$$

シテ、シカモ $f^*(x), g^*(x)$ ハ (F) ヲ満足スル故、以下ニ
於テハ $f(0) = 1$ ナリトシテヨイ。

次ニ $f(x)$ ハ偶函数ナルコトヲ証明シヨウ。(F) = 於テ
 $y = 0$ トオケバ、 $f(0) = 1$ ナル故

$$(1) f^2(x) + g^2(x) = 1$$

(F) = 於テ x ト y トヲイレカヘレバ

$$(2) f(y+x)f(y-x) = f^2(x) - g^2(y)$$

シカルニ (1) = ヌリ

$$f^2(y) - g^2(x) = f^2(x) - g^2(y)$$

ナル故 (F) 及ビ (2) ヌリ

$$f(x+y)f(x-y) = f(y+x)f(y-x)$$

$y = -x$ トオケバ

$$f(0) = 1$$

ナル故

$$f(2x) = f(-2x)$$

故ニ $f(x)$ ハ偶函数ナル。

(F) = 於テ x, y ノ代リニ $\frac{x}{2}$ トオケバ

$$f(0) = 1$$

ナル故

$$f(x) = f^2\left(\frac{x}{2}\right) - g^2\left(\frac{x}{2}\right)$$

$$\text{故} = f(x+y) + f(x-y)$$

$$= \left\{ f^2\left(\frac{x+y}{2}\right) - g^2\left(\frac{x+y}{2}\right) \right\} + \left\{ f^2\left(\frac{x-y}{2}\right) - g^2\left(\frac{x-y}{2}\right) \right\}$$

$$= \left\{ f^2\left(\frac{x+y}{2}\right) - g^2\left(\frac{x-y}{2}\right) \right\} + \left\{ f^2\left(\frac{x-y}{2}\right) - g^2\left(\frac{x+y}{2}\right) \right\}$$

$$\text{シカル} = (F) = \exists \text{リ}$$

$$f^2\left(\frac{x+y}{2}\right) - g^2\left(\frac{x-y}{2}\right) = f(x)f(-y)$$

$$= f(x)f(y) \quad (f(x) \text{ハ偶函数})$$

$$\text{又} \quad f^2\left(\frac{x-y}{2}\right) - g^2\left(\frac{x+y}{2}\right) = f(x)f(y)$$

$$\text{故} = f(x+y) + f(x-y) = 2f(x)f(y)$$

$f(x)$ ハ可測ナル故、雜誌函数方程式第二十六号 = $\exists$ リ、 α ヲ任意ノ実数トシタルトキ

$$f(x) = \cos \alpha x \quad \text{或ハ} \quad f(x) = \cos h \alpha x$$

$$f(x) = \cos h \alpha x \text{ナラバ} f(x) \geq 1, \text{シカモ (1) = } \exists \text{リ}$$

$$f^2(x) + g^2(x) = 1 \text{ナル故, } f(x) \equiv 1, \text{即チ} f(x) =$$

$$\cos \alpha x = \text{於テ} \alpha = 0 \text{ノ場合ナアル。} f(x) \equiv 1 \text{ハ}$$

$$f(x) = \cos \alpha x = \text{於テ} \alpha = 0 \text{トシタ場合ナアルカラ}$$

結局求ムル解ハ α, β ヲ任意ノ実数トシタルトキ

$$f(x) = \alpha \cos \beta x, g(x) = \pm \alpha \sin \beta x$$

符号ノ取り方ハ任意ナアル。之レ等ハ明カニ (F)ヲ満足セシ

ナル。

—— (完) ——