



Title	函数方程式 $f(x+y)+g(x-y)=2h(x)+2k(y)$ 二就イ テ
Author(s)	春木, 博
Citation	全国紙上数学談話会. 1941, 224, p. 492-493
Version Type	VoR
URL	https://doi.org/10.18910/74900
rights	
Note	

The University of Osaka Institutional Knowledge Archive : OUKA

<https://ir.library.osaka-u.ac.jp/>

The University of Osaka

970 函數方程式

$$f(x+y) + g(x-y) = 2h(x) + 2k(y)$$

ニ就イテ

春 木 博 (神戸高等商船)

今、函數方程式

$$(1) f(x+y) + g(x-y) = 2h(x) + 2k(y)$$

ノ可測解 $f(x)$, $g(x)$, $h(x)$, $k(x)$ ヲ求メテ見ヨウ。

(1) = 3 11

$$k(y) = \frac{1}{2} \{ f(z+y) + g(z-y) - 2h(z) \}$$

$$\text{故ニ } h(x+y) + h(x-y)$$

$$= \frac{1}{2} \{ f(z+x+y) + g(z-x-y) - 2h(z) \\ + f(z+x-y) + g(z-x+y) - 2h(z) \}$$

シカルニ (1) = 3 11

$$f(z+x+y) + g(z-x+y) = 2h(z+y) + 2k(x)$$

$$f(z+x-y) + g(z-x-y) = 2h(z-y) + 2k(x)$$

ナル故

$$h(x+y) + h(x-y)$$

$$= 2h(x) + h(z+y) + h(z-y) - 2h(z)$$

$$h(z+y) + h(z-y) - 2h(z) = 2p(y) \text{ トオケバ}$$

$$(2) h(x+y) + h(x-y) = 2h(x) + 2p(y)$$

(2) ノ筆者ノ本誌 954 談話「ニ次式ノ函數方程式ニ関シ

テ」ニ於テ命ジタル函數方程式ナル故、 a, b, c ヲ任意ノ實
常數トシタルトキ

$$(3) \quad h(x) = ax^2 + bx + c$$

$$p(x) = ax^2$$

$$\text{故} = \text{又} \quad h(x+y) + h(x-y) = 2h(x) + 2ay^2$$

之ヨリ a, b, c 任意ノ實數トシテルトキ

$$(4) \quad h(x) = ax^2 + dx + e$$

(3), (4) $\Rightarrow$ (1) $\sim$ 代入スルバ

$$\begin{aligned} (5) \quad f(x+y) + g(x-y) \\ = 2(ax^2 + dx + e) + 2(ay^2 + by + c) \end{aligned}$$

$y = 0$ トオケバ

$$f(x) + g(x) = 2(ax^2 + dx + e) + 2c$$

$$\text{故} = \quad g(x) = 2(ax^2 + dx + e + c) - f(x)$$

上式ヲ (5) $\sim$ 代入スルバ

$$f(x+y) - f(x-y) = 4axy + 2(b+d)y + 2c$$

$y = 0$ トオケバ

$$y = x \text{ トオケバ} \quad f(2x) = 4ax^2 + 2(b+d)x + l$$

$$\therefore f(x) = ax^2 + (b+d)x \quad (l = f(0) \text{ トオケル})$$

$$\therefore f(x) = ax^2 + (b+d)x + l$$

結局 a, b, d, e, l $\Rightarrow$ 任意ノ實數トスルトキ

$$\begin{cases} f(x) = ax^2 + (b+d)x + l \\ g(x) = ax^2 - (b-d)x + 2e - l \\ h(x) = ax^2 + dx + e \\ k(x) = ax^2 + bx \end{cases}$$

(完)