1120. 河口空間ノ共形幾何學

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§1. 次數Mノ河口空間即チ曲線 $x^i(t)$ $(i=1,2,\cdots\cdots,n)$ ノ $t_0 \leqq t \leqq t_1$ ノ間ノ弧ノ長サガ積分

$$(1.1)\qquad S=\int_{t_0}^{t} F(x, x^{(1)}, \cdots\cdots, x^{(M)})\,dt$$

デ定義サレル空間ヲ考ヘル．コヽデ(1.1)ガ幾何學的意味ヲモツタメニハ，(1.1)ノ形ガ媒介變數 t ノスベテノ變換

$$(1.2)\qquad \bar{t}=\bar{t}(t)$$

ニ對シテ不變デナケレバナラナイ．(1.1)ニ對應シテ次數Mノ線素 $(x^{(1)}, x^{(2)}, \cdots\cdots, x^{(M)})$ ノウケル變換

$$x^{i(\bar{\mu})}=\sum_{\nu=1}^{\mu} a_{\nu}^{\mu}\, x^{i(\nu)}$$

ハM－經數變換群ヲナシ，無限小演子 $\Delta_1, \cdots\cdots, \Delta_M$ ハ

$$(1.3)\qquad \Delta_K=\sum_{\lambda \geqq K}\binom{\lambda}{K} x^{(\lambda-K+1)i}\frac{\partial}{\partial x^{(\lambda)i}}$$

デ與ヘラレル．Δ_K, Δ_H ノ Kommutator $[\Delta_K \Delta_H]$ ハ

$$(1.4)\quad [\Delta_K \Delta_H] = \frac{(K-H)(K+H-1)!}{K!\,H!}\Delta_{K+H-1}$$

トナル．之ヲ用ヒレバ，(1.1)ガ媒介変数ノ変換(1.2)デ不変デアルタメノ條件，即チ與ヘラレタスカラーFガ(1.2)デ

$$\overline{F}(x, x^{(\bar{1})}, \cdots\cdots, x^{(\bar{M})}) = \frac{dt}{d\bar{t}} F(x, x^{(1)}, \cdots\cdots, x^{(M)})$$

ノ如ク変換スルタメノ條件ハ次ノ形ニカクコトガ出来ル。

$$(1.5)\quad \Delta_1 F = F,\quad \Delta_K F = 0,\quad (2 \leqq K \leqq M)$$

$\rho(x, x^{(1)}, \cdots\cdots, x^{(\alpha)})$ハ座標$(x)$及ビ$\alpha$次マデノ線素ニdependスル任意ノ恒等的ニ零デナイスカラートスル。即チ

$$\Delta_K \rho = 0,\quad (1 \leqq K \leqq \alpha)$$

ヲ満足スル量トスルトキ，Fノ変化

$$(1.6)\quad \overline{F} = \rho F$$

ヲ河口空間Fノ次数αノ共形変換トイフ．コヽデ$0 \leqq \alpha \leqq M-1$トスル。

任意ノ点デ互ニα次ノ接触ヲナス微小ナ曲線ノ弧ハ，α次ノ共形変換デstreckentreuニ寫像サレル。

(1.6)デ不変ナ量ヲ求メルタメニスカラーFニ関シテ次ノ假定ヲオク。

$$\Delta_M F = 0 \text{ カラ } F_{(M)i}\, x^{(1)i} = 0,$$

従ッテ $F_{(M)i(M)j}\, x^{(1)i} = 0$　従ッテ行列$(F_{(M)i(M)j})$

階数ハ高々 $n-1$ デアル。我々ハソレガ丁度 $n-1$ ニ等シイト假定スル。然ルトキハ $F_{(M)i(M)j}$ ニ關スル行列 $(F_{(M)i(M)j})$ ノ代數余因子ヲ H_1^{ij} トスレバ

$$H_1^{ij}=F_1\, x^{(1)i} x^{(1)j}$$

トナルヤウナ函數 F_1 ヲ求メルコトガ出來ル。

F_1 ハ再ビ $F_{1(M)i(M)j}\, x^{(1)i}=0$ ナル關係ヲ滿足スル.

行列 $(F_{1(M)i(M)j})$ ノ階數ガ $n-1$ ニ等シイト假定シテ

$$\mathrm{Adj}\; F_{1(M)i(M)j}=H_2^{ij}$$

トオケバ $H_2^{ij}=F_2 x^{(1)i}x^{(1)j}$ トナル樣ナ F_2 ガ求マル。

F_1, F_2 ハ共形變換，媒介變數ノ變換及ビ座標變換デ次ノ如ク變換スル。

$$\overline{F}_1=\rho^{n-1}\left(\frac{d\bar{t}}{dt}\right)^{(2M-1)(n-1)+2}\left|\frac{\partial x}{\partial \bar{x}}\right|^2 F_1$$

$$\overline{F}_2=\rho^{(n-1)^2}\left(\frac{d\bar{t}}{dt}\right)^{((2M-1)n+3)(n-1)+2}\left|\frac{\partial x}{\partial \bar{x}}\right|^{2n} F_2$$

從ツテ $\overline{F_2^{-1}F_1^{n}}=\rho^{n-1}\left(\dfrac{dt}{d\bar{t}}\right)^{n-1}F_2^{-1}F_1^{n}$

故ニ $\theta=\mathrm{sgn}(F).\ F^{-1}\left|F_2^{-1}F_1^{n}\right|^{\frac{1}{n-1}}$

デ定義サレルθハ intrinsic ナスカラーデ且ツ共形不變量デアル。

$$\theta^{(1)}=\psi,\quad \kappa=F/\psi$$

トオケバ ψ ハ共形不変量デ媒介変数ノ変換デFト同ジ変換ヲウケ，κ ハ intrinsic デ共形変換デ

$$\bar{\kappa}=\rho\kappa$$

ノ如ク変換スル。

定理2.(1.9) $$t_{i\mu}=\sum_{\lambda=\mu}^{M}\binom{\lambda}{\mu}\left(\frac{1}{F}\right)^{(\lambda-\mu)}E_{i\lambda},$$

$$(\mu=\alpha+1,\cdots\cdots,M)$$

ナルM−αコノ共変ベクトルハ，次数αノ共形変換ニ對シテ不変デアル。*

証明. $\bar{F}=\rho F$ カラ

$$\log\bar{F}=\log\rho+\log F$$

従ッテ $\mu\geqq\alpha+1$ ナラバ

$$\sum_{\lambda=\mu}^{M}(-1)^{\lambda}\binom{\lambda}{\mu}(\log\bar{F})_{(\lambda)i}^{\cdots(\lambda-\mu)}=\sum_{\lambda=\mu}^{M}(-1)^{\lambda}\binom{\lambda}{\mu}(\log F)_{(\lambda)i}$$

シカルニ

$$\sum_{\lambda=\mu}^{M}(-1)^{\lambda}\binom{\lambda}{\mu}(\log F)_{(\lambda)i}^{\cdots(\lambda-\mu)}=\sum_{\lambda=\mu}^{M}(-1)^{\lambda}\binom{\lambda}{\mu}\left(\frac{F_{(\lambda)i}}{F}\right)^{(\lambda-\mu)}$$

$$=\sum_{\lambda=\mu}^{M}\sum_{\nu=0}^{\lambda-\mu}(-1)^{\lambda}\binom{\lambda}{\mu}\binom{\lambda-\mu}{\nu}\left(\frac{1}{F}\right)^{(\nu)}F_{(\lambda)i}^{\cdots(\lambda-\mu-\nu)}$$

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* $E_{i\lambda}=\sum_{\mu=\lambda}^{M}(-1)^{\mu}\binom{\mu}{\lambda}\left(F_{(\mu)i}\right)^{(\mu-\lambda)}$ ハ $0\leqq\lambda\leqq M$ ニ對シ共変ベクトルノ成分ヲナシ Synge ノベクトルトヨバレル。

$$=\sum_{\lambda=\mu}^{M}\sum_{\nu=0}^{\lambda-\mu}-\binom{\mu+\nu}{\mu}\left(\frac{1}{F}\right)^{(\nu)}(-1)^{\lambda}\binom{\lambda}{\mu+\nu}F_{(\lambda)i}^{\cdots(\lambda-\mu-\nu)}$$

$$=\sum_{\alpha=\mu}^{M}\binom{\alpha}{\mu}\left(\frac{1}{F}\right)^{(\alpha-\mu)}\sum_{\lambda=\alpha}^{M}(-1)^{\lambda}\binom{\lambda}{\alpha}F_{(\lambda)i}^{\cdots(\lambda-\alpha)}$$

$$=\sum_{\lambda=\mu}^{M}\binom{\lambda}{\mu}\left(\frac{1}{F}\right)^{(\lambda-\mu)}E_{i\lambda}$$

即チ $t_{i\mu}=E_{i\mu}(\log F)$ $\quad \bar{t}_{i\mu}=t_{i\mu}$ トナル。

$t_{i\alpha+1},\cdots\cdots,t_{iM}$ ハ媒介変数ノ変換デ不変デハナク

$$(1.10)\qquad \begin{aligned}&\Delta_K t_{i\mu}=-\binom{\mu+K}{K}t_{i\mu+K-1} \qquad 2\leqq K\leqq M-\mu+1\\ &\Delta_1 t_{i\mu}=-\mu t_{i\mu}.\end{aligned}$$

ナル関係ガ成立スル．即チ

$$\Delta_K E_{i\mu}=-\binom{\mu+K-1}{K}E_{i\mu+K-1}-\delta_K^1 E_{i\mu}$$

及ビ

$$\Delta_K\left(\frac{1}{F}\right)^{(H)}=\frac{H+1-2K}{H+1}\binom{H+1}{K}\left(\frac{1}{F}\right)^{(H-K+1)}$$

ナル関係ヲ用フレバ

$$\Delta_K t_{i\mu}=\Delta_K\sum_{\nu=\mu}^{M}\binom{\nu}{\mu}\left(\frac{1}{F}\right)^{(\nu-\mu)}E_{i\nu}$$

$$=\sum_{\nu\geqq\mu}\left[\binom{\nu}{\mu}\frac{\nu-\mu-2K+1}{\nu\ \mu+1}\binom{\nu-\mu+1}{K}\left(\frac{1}{F}\right)^{(\nu-\mu-K+1)}E_{i\nu}\right.$$

$$\left.-\binom{\nu}{\mu}\left(\frac{1}{F}\right)^{(\nu-\mu)}\binom{\nu+K-1}{K}E_{i\nu+K-1}\right]$$

$$=\sum_{\nu\geq\mu+K-1}\left[\binom{\nu}{\mu}\frac{\nu-\mu-2K+1}{\nu-\mu+1}\binom{\nu-\mu+1}{K}-\binom{\nu-K+1}{\mu}\binom{\nu}{K}\right]\left(\frac{1}{H}\right)^{(\nu-\mu-K+1)}E_{i\nu}$$

$$=-\binom{\mu+K}{K}t_{i\mu+K-1}$$

$$\Delta_1 t_{i\mu}=-(\mu+1)\,t_{i\mu}+t_{i\mu}=-\mu\,t_{i\mu}$$

§2. 次ニρヲ係函數ト制限シタ場合ヲ論ジヨウ。コノ場合ニハMコノSyngeノベクトルノミノ線狀結合トシテ，共形不變デ且intrinsicナMコノ共變ベクトルヲ導クコトガ出來ル。

$E_{i\mu}$ハ共形變換デ次ノ如ク變換スル。

$$(2.1)\quad \overline{E}_{i\mu}=c^{\lambda}_{\mu}E_{i\lambda},\quad c^{\lambda}_{\mu}=\binom{\lambda}{\mu}\rho^{(\lambda-\mu)}$$

之ハ(1.9)ヲ出シタト全ク同ジ計算デ証明スルコトガ出來ル。

$\overline{\kappa}=\rho\kappa$ カラ

$$\overline{\kappa}^{(\nu)}=\sum_{\mu=0}^{\nu}\binom{\nu}{\mu}\rho^{(\mu)}\kappa^{(\nu-\mu)}$$

從ッテ

$$(2.2)\quad \frac{\rho^{(\nu)}}{\rho}=\frac{\overline{\kappa}^{(\nu)}}{\overline{\kappa}}-\frac{\kappa^{(\nu)}}{\kappa}-\sum_{\mu=1}^{\nu-1}\binom{\nu}{\mu}\frac{\rho^{(\mu)}}{\rho}\frac{F\kappa^{(\nu-\mu)}}{\kappa}$$

之ト $$\overline{E}_{i1}=\sum_{0\leq\nu\leq M-1}(\nu+1)E_{i\nu+1}\rho^{(\nu)}$$

或ハ

$$\frac{\bar{E}_{i1}}{\bar{\varkappa}} = \sum_{\nu=0}^{M-1} (\nu+1)\, \frac{E_{i\nu+1}}{\varkappa}\, \frac{\rho^{(\nu)}}{\rho}$$

トカラ

$$\frac{\bar{E}_{i1}}{\bar{\varkappa}} = \sum_{\nu=0}^{M-2}{}' (\nu+1) \frac{E_{i\nu+1}}{\varkappa}\, \frac{\rho^{(\nu)}}{\rho} + \Bigg[\frac{\bar{\varkappa}^{(M-1)}}{\bar{\varkappa}} - \frac{\varkappa^{(M-1)}}{\varkappa}$$

$$- \sum_{\nu=1}^{M-2} \binom{M-1}{\nu} \frac{\rho^{(\nu)}}{\rho}\, \frac{\varkappa^{(M-\nu-1)}}{\varkappa} \Bigg] M \frac{E_{iM}}{\varkappa}$$

$$= \frac{E_{i1}}{\varkappa} + \sum_{\nu=1}^{M-2} \frac{\rho^{(\nu)}}{\rho} \Bigg[(\nu+1) \frac{E_{i\nu+1}}{F\varkappa} - \binom{M-1}{\nu} \frac{\varkappa^{(M-\nu-1)}}{\varkappa} M \frac{E_{iM}}{\varkappa} \Bigg]$$

$$+ \Bigg[\frac{\bar{\varkappa}^{(M-1)}}{\bar{\varkappa}} - \frac{\varkappa^{(M-1)}}{\varkappa} \Bigg] M \frac{E_{iM}}{\varkappa}$$

從ツテ

$$(23) \quad \frac{\bar{E}_{i1}}{\bar{\varkappa}} - \frac{\bar{\varkappa}^{(M-1)}}{\bar{\varkappa}} M\, \bar{\mathfrak{C}}_{iM} = \frac{E_{i1}}{\varkappa} - \frac{\varkappa^{(M-1)}}{\varkappa} M\, \mathfrak{C}_{iM}$$

$$+ \sum_{\nu=1}^{M-3} \frac{\rho^{(\nu)}}{\rho} \Bigg[(\nu+1)\, \frac{E_{i\nu+1}}{\varkappa} - \binom{M-1}{\nu} \frac{\varkappa^{(M-\nu-1)}}{\varkappa} M \mathfrak{C}_{iM} \Bigg]$$

$$+ (M-1)\, \mathfrak{C}_{iM-1}\, \frac{\rho^{(M-2)}}{\rho}$$

$$\mathfrak{C}_{iM} = \frac{E_{iM}}{\varkappa}, \quad \mathfrak{C}_{iM-1} = \frac{E_{iM-1}}{\varkappa} - \frac{\varkappa^{(1)}}{\varkappa} M\, \mathfrak{C}_{iM}$$

$E_{iM} = (-1)^M F_{(M)i}$, ヨリ直ニ $\bar{\mathfrak{C}}_{iM} = \mathfrak{C}_{iM}$

又 $\bar{\mathfrak{C}}_{iM-1} = \mathfrak{C}_{iM-1}$ デアル. 之ヲ見ルニハ $F = \left(\frac{1}{\rho}\right)\bar{F}$, $\varkappa = \left(\frac{1}{\rho}\right)\bar{\varkappa}$

トシテ上ト同ジ計算ヲ繰返セバ

$$\frac{E_{i1}}{\varkappa}-\frac{\varkappa^{(M-1)}}{\varkappa}M\mathfrak{C}_{iM}=\frac{\overline{E}_{i1}}{\overline{\varkappa}}-\frac{\overline{\varkappa}^{(M-1)}}{\varkappa}M\overline{\mathfrak{C}}_{iM}$$

$$+\Phi\left(E_{iM},E_{iM-1},\cdots\cdots,\varkappa,\varkappa^{(1)},\cdots\cdots,\rho,\rho^{(1)},\cdots\cdots\rho^{(M-3)}\right)$$

$$-(M-1)\overline{\mathfrak{C}}_{iM-1}\frac{\rho^{(M-2)}}{\rho}$$

コノ式ト(2.3)トノ $\rho^{(M-2)}$ ヲ含ム項及ビ $\rho,\rho^{(1)}\cdots\cdots$ ヲ含マナイ項ヲ比較スレバ直ニ $\overline{\mathfrak{C}}_{iM-1}=\mathfrak{C}_{iM-1}$ ヲ得ル。之ヲ繰返シテ行ケバ

$$\frac{\overline{E}_{i1}}{\overline{\varkappa}}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\overline{\varkappa}^{(M-\nu)}}{\overline{\varkappa}}\overline{\mathfrak{C}}_{iM-\nu+1}\tag{2.4}$$

$$=\frac{E_{i1}}{\varkappa}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\varkappa^{(M-\nu)}}{\varkappa}\mathfrak{C}_{iM-\nu+1}$$

$$-\sum_{\nu=0}^{M-\mu-1}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{\varkappa}-\sum_{\alpha=0}^{\mu-1}\binom{M-\alpha-1}{\nu}\frac{\varkappa^{(M-\alpha-\nu-1)}}{\varkappa}(M-\alpha)\mathfrak{C}_{iM-\alpha}\right]$$

$$\mathfrak{C}_{iM}=\frac{E_{iM}}{E\varkappa}\tag{2.5}$$

$$\mathfrak{C}_{iM-1}=\frac{E_{iM-1}}{F\varkappa}-\frac{\varkappa^{(1)}}{\varkappa}M\mathfrak{C}_{iM}$$

$$\mathfrak{C}_{iM-2}=\frac{E_{iM-2}}{\varkappa}-\frac{\varkappa^{(2)}}{\varkappa}\binom{M}{2}\mathfrak{C}_{iM}-\frac{\varkappa^{(1)}}{\varkappa}\binom{M-1}{1}\mathfrak{C}_{iM-1}$$

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$$\mathfrak{C}_{iM-\mu-1}=\frac{E_{iM-\mu-1}}{\varkappa}-\frac{\varkappa^{(\mu-1)}}{\varkappa}\binom{M}{\mu-1}\mathfrak{C}_{iM}$$

$$-\frac{\kappa^{(\mu-2)}}{\kappa}\binom{M-1}{\mu-2}\Gamma_{iM-1}-\cdots\cdots-\frac{\kappa^{(1)}}{\kappa}(M-\mu+2)\Gamma_{iM-\mu+2}$$

ヲ終ル．即チ（2.4）ヲ假定スレバ

$$\frac{\bar{E}_{i1}}{\bar{\kappa}}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\bar{\kappa}^{(M-\nu)}}{\bar{\kappa}}\bar{\Gamma}_{iM-\nu+1}=\frac{E_{i1}}{\kappa}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\kappa^{(M-\nu)}}{\kappa}\Gamma_{iM-\nu+1}$$

$$+\sum_{\nu=0}^{M-\mu-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{\kappa}-\sum_{\lambda=0}^{\mu-1}\binom{M-\lambda-1}{\nu}\frac{\kappa^{(M-\lambda-\nu-1)}}{\kappa}(M-\lambda)\Gamma_{iM-\lambda}\right]$$

$$+\frac{\rho^{(M-\mu-1)}}{\rho}(M-\mu)\left[\frac{E_{iM-\mu}}{\kappa}-\sum_{\lambda=0}^{\mu-1}\binom{M-\lambda}{M-\mu}\frac{\kappa^{(\mu-\lambda)}}{\kappa}\Gamma_{iM-\lambda}\right]$$

$$\Gamma_{iM-\mu}=\frac{E_{iM-\mu}}{\kappa}-\sum_{\lambda=0}^{\mu-1}\binom{M-\lambda}{M-\mu}\frac{\kappa^{(\mu-\lambda)}}{\kappa}\Gamma_{iM-\lambda}$$

トオケバ之ハ共形不変デ

$$\frac{\bar{E}_{i1}}{\bar{\kappa}}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\bar{\kappa}^{(M-\nu)}}{\bar{\kappa}}\bar{\Gamma}_{iM-\nu+1}=\frac{E_{i1}}{\kappa}-\sum_{\nu=1}^{\mu}\frac{(M-\nu+1)\kappa^{(M-\nu)}}{\kappa}\Gamma_{iM-\nu+1}$$

$$+\sum_{\nu=0}^{M-\mu-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{\lambda\nu+1}}{\kappa}-\sum_{\lambda=0}^{\mu}\frac{\kappa^{(M-\lambda-\nu-1)}}{\kappa}(M-\lambda)\Gamma_{iM-\lambda}\right]$$

$$+\left[\frac{\bar{\kappa}^{(M-\mu-1)}}{\bar{\kappa}}-\frac{\kappa^{(M-\mu-1)}}{\kappa}\right](M-\mu)\Gamma_{iM-\mu}$$

或ハ

$$\frac{\bar{E}_{i1}}{\bar{\kappa}}-\sum_{\nu=1}^{\mu+1}\frac{(M-\nu+1)\bar{\kappa}^{(M-\nu)}}{\bar{\kappa}}\Gamma_{iM-\nu+1}=\frac{E_{i1}}{\kappa}-\sum_{\nu=1}^{\mu+1}\frac{(M-\nu+1)\kappa^{(M-\nu)}}{\kappa}\Gamma_{iM-\nu+1}$$

$$+\sum_{\nu=0}^{M-\mu-2}\frac{\rho^{(\nu)}}{\rho}\left[(\nu+1)\frac{E_{i\nu+1}}{\kappa}-\sum_{\lambda=0}^{\mu+1}\binom{M-\lambda-1}{\nu}\frac{\kappa^{(M-\lambda-\nu-1}}{\kappa}(M-\lambda)\Gamma_{iM-\lambda}\right]$$

定理3.
$$\mathfrak{T}_{iM-\nu}=\frac{E_{iM-\nu}}{\varkappa}-\sum_{\alpha=0}^{\nu-1}\binom{M-\alpha}{M-\nu}\frac{\varkappa^{(\nu-\alpha)}}{\varkappa}\mathfrak{T}_{iM-\alpha}$$

$$\mathfrak{T}_{iM}=\frac{E_{iM}}{\varkappa}$$

デ定義サレルMコノ共変ベクトル $\mathfrak{T}_{i1},\cdots\cdots,\mathfrak{T}_{iM}$ ハ共形不変量デアル。

(2.5) ヲ書キ直セバ

$$E_{iM}=\varkappa\,\mathfrak{T}_{iM}$$

$$E_{iM-1}=\varkappa^{(1)}M\,\mathfrak{T}_{iM}+\varkappa\,\mathfrak{T}_{iM-1}$$

$$E_{iM-2}=\varkappa^{(2)}\binom{M}{2}\mathfrak{T}_{iM}+\varkappa^{(1)}\binom{M-1}{1}\mathfrak{T}_{iM-1}+\varkappa\,\mathfrak{T}_{iM-2}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$E_{iM-\nu}=\varkappa^{(\nu)}\binom{M}{\nu}\mathfrak{T}_{iM}+\varkappa^{(\nu-1)}\binom{M-1}{\nu-1}\mathfrak{T}_{iM-1}+\varkappa^{(\nu-2)}\binom{M-2}{\nu-2}\mathfrak{T}_{iM-2}+\cdots\cdots+\varkappa\,\mathfrak{T}_{iM-\nu}$$

従ッテ

$$\mathfrak{T}_{iM-\nu}=\frac{1}{\varkappa^{\nu+1}}\begin{vmatrix} \varkappa & & & E_{iM} \\ M\varkappa^{(1)} & \varkappa & & E_{iM-1} \\ \binom{M}{2}\varkappa^{(2)} & \binom{M-1}{1}\varkappa^{(1)} & \varkappa & E_{iM-2} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \\ \binom{M}{\nu}\varkappa^{(\nu)} & \binom{M-1}{\nu-1}\varkappa^{(\nu-1)} & \binom{M-2}{\nu-2}\varkappa^{(\nu-2)} & E_{iM-\nu} \end{vmatrix}$$

$$\mathfrak{T}_{iM-\nu}=\sum_{\mu=0}^{\nu}C^{M-\mu}_{M-\nu}E_{iM-\mu}$$

$$C^{M-\mu}_{M-\nu}=\frac{1}{\kappa^{\nu+1}}\begin{vmatrix} \kappa & & & & 0 \\ M\kappa^{(1)} & & & & \vdots \\ \cdots\cdots & & & & \vdots \\ \binom{M}{\mu}\kappa^{(\mu)} & \binom{M-1}{\mu-1}\kappa^{(\mu-1)} & \cdots\kappa\cdots & & 1 \\ \cdots\cdots & & & & \vdots \\ \binom{M}{\nu}\kappa^{(\nu)} & \binom{M-1}{\nu-1}\kappa^{(\nu-1)} & \cdots\cdots & \kappa^{(1)} & 0 \end{vmatrix}$$

$$=\binom{M-\mu}{M-\nu}\left(\frac{1}{\kappa}\right)^{(\nu-\mu)}$$

即チ

定理4. $\quad \mathfrak{C}_{i\mu}=\sum_{\lambda=\mu}^{M} C^{\lambda}_{\mu} E_{i\lambda}, \quad C^{\lambda}_{\mu}=\binom{\lambda}{\mu}\left(\frac{1}{\kappa}\right)^{(\lambda-\mu)}$

$E_{i\mu}$ ニ関シテハ基本的ナ関係

$$E_{i\mu}x^{(1)i}=0 \quad (2\leqq\mu\leqq M),\quad E_{i1}x^{(1)i}=-F$$

ガ成立スル。之レカラ

$$\mathfrak{C}_{i\mu}x^{(1)i}=0,\quad (2\leqq\mu\leqq M),\quad \mathfrak{C}_{i1}x^{(1)i}=-\frac{F}{\kappa}$$

定理5. $\quad \mathcal{E}_{i\lambda}=\frac{\kappa}{F}\sum_{\mu=\lambda}^{M} A^{\mu}_{\lambda}\mathfrak{C}_{i\mu}$

トオケバ $\mathcal{E}_{i1}$ ハ *intrinsic* ナ共変ベクトルデ共形変換ニヨリ

$$\bar{\mathcal{E}}_{i\lambda}=\rho^{\,}\mathcal{E}_{i\lambda}$$

ノ如ク変換サレル。コヽニ $x^{(\mu)i}$, $\bar{t}=\bar{t}(t)$ ニ對スル変換ヲ

$$x^{(\bar{\mu})i}=\sum_{\lambda=1}^{\mu}a^{\mu}_{\lambda}\left(\frac{dt}{d\bar{t}},\frac{d^2t}{d\bar{t}^2},\cdots\cdots,\frac{d^{\mu-\lambda+1}t}{d\bar{t}^{\mu-\lambda+1}}\right)x^{(\lambda)i}$$

トシ A^{μ}_{λ} ハ a^{μ}_{λ} ノ $\frac{dt}{d\bar{t}}, \frac{d^2t}{d\bar{t}^2},\cdots\cdots$ ノ所ヘ $\psi, \psi^{(1)}\cdots\cdots$ ヲ

代入シタモノ

即チ $\quad A^{\mu}_{\lambda}=a^{\mu}_{\lambda}(\psi, \psi^{(1)},\cdots\cdots\psi^{(\mu-\lambda)})$

トスル。

証明. $E_{i\mu}$, C^{λ}_{μ}, A^{μ}_{λ} ハ $\bar{t}=\bar{t}(t)$ デ夫々次ノ如キ変換ヲウケルコトカラ明カデアル。*

$$\bar{E}_{i\mu}=\frac{dt}{d\bar{t}}\bar{a}^{\lambda}_{\mu}E_{i\lambda}$$

$$\bar{C}^{\lambda}_{\mu}=a^{\lambda}_{\alpha}\bar{a}^{\beta}_{\mu}C^{\alpha}_{\beta}$$

$$\bar{A}^{\mu}_{\lambda}=a^{\mu}_{\nu}A^{\nu}_{\lambda}$$

$M-\alpha$ コノ intrinsic ナ共変ベクトル

$$\varepsilon_{i\alpha+1},\cdots\cdots,\varepsilon_{iM}$$

ハ次数 α ノ共形変換ニ對シ $\bar{\varepsilon}_{i\mu}=\rho\,\varepsilon_{i\mu}$ ノ如ク変換サレル。

$\varepsilon_{i\lambda}$ ハ又 $\xi_{i\lambda}$ ノ線状結合トシテ次ノ形ニカクコト

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* A. Kawaguchi, Differentialgeometrie hoherer Ordnung. III

Jour Fac. Sci. Hokkaido Imp Univ (1941)

ガ出来ル。

$$\varepsilon_{i\lambda}=\sum_{\mu=\lambda}^{M}K_{\lambda}^{\mu}t_{i\mu}\qquad K_{\lambda}^{M}=\frac{\kappa}{\psi}\sum_{\alpha=\lambda}^{\mu}\binom{\mu}{\alpha}\psi^{(\mu-\alpha)}A_{\lambda}^{\alpha}(\psi,\psi^{(1)},\cdots,\psi^{(\alpha-\lambda)})$$

即チ $$\frac{F}{\kappa}\varepsilon_{i\lambda}=\sum_{\lambda\leq\nu\leq M}\sum_{\nu\geq\mu}A_{\lambda}^{\mu}\binom{\nu}{\mu}\left(\frac{1}{n}\right)^{(\nu-\mu)}E_{i\nu}$$

$$=\sum_{\lambda\leq\mu\leq M}\sum_{\mu\leq\nu\leq M}A_{\lambda}^{\mu}\binom{\nu}{\mu}\left(\frac{\psi}{F}\right)^{(\nu-\mu)}E_{i\nu}$$

$$=\sum_{\mu=\lambda}^{M}\sum_{\nu\geq\mu}^{M}\sum_{\alpha=0}^{\nu-\mu}\left(A_{\lambda}^{\mu}\binom{\nu}{\mu}\binom{\nu-\mu}{\alpha}\psi^{(\alpha)}\left(\frac{1}{F}\right)^{(\nu-\mu-\alpha)}E_{i\nu}\right.$$

$$=\sum_{\mu=\lambda}^{M}\sum_{\nu\geq\mu}^{M}\sum_{\alpha=0}^{\nu-\mu}A_{\lambda}^{\mu}\binom{\mu+\alpha}{\mu}\psi^{(\alpha)}\binom{\nu}{\mu+\alpha}\left(\frac{1}{F}\right)^{(\nu-\mu-\alpha)}E_{i\nu}$$

$$=\sum_{\alpha=\lambda}^{M}\left(\sum_{\mu=\lambda}^{\alpha}A_{\lambda}^{\mu}\binom{\alpha}{\mu}\psi^{(\alpha-\mu)}\right)\left(\frac{\nu}{\alpha}\right)\sum_{\nu=\alpha}^{M}\binom{\nu}{\alpha}\left(\frac{1}{F}\right)^{(\nu-\alpha)}E_{i\nu}$$

$$=\sum_{\alpha=\lambda}^{M}K_{\lambda}^{\alpha}t_{i\alpha}$$

§3. $g_{ij}-\psi^{2(M-1)}FF_{(M)i(M)j}+\varepsilon_{i1}\varepsilon_{j1}$

トオケバ g_{ij} ハ二階対称テンソルデソノ行列式ハ一般ニ0デナク，

$$g_{ij}x^{(1)i}=-F\varepsilon_{j1}\,,\quad g_{ij}x^{(1)i}x^{(1)j}=F^{2}$$

ナル関係ガ成立シ，且ツ共形変換デ

$$\bar{g}_{ij}=\rho^{2}g_{ij}$$

ノ如ク変換スル。従ッテ g_{ij} ヲ我々ノ空間ノ基本テンソルトシテ採用スルコトが出来ル。

X^j ヲ任意ノ *intrinsic* デ共形不変ナベクトルトスレバ

$$\psi^{2(M-1)}F.D_{jM-1}\left(F_{(M)i}\right)X^j=\psi^{2(M-1)}F.F_{(M)i(M)j}\frac{dX^j}{dt}$$

$$+\frac{1}{M}\psi^{2(M-1)}F\left[F_{(M)i(M-1)j}+\psi^{-1}\cdot\psi^{(1)}F_{(M)i(M)j}\right]X^j$$

$$\kappa\varepsilon_{i1}\left\{\left(\frac{\varepsilon_{j1}}{\kappa}X^j\right)^{(1)}\right\}=\varepsilon_{i1}\varepsilon_{j1}\frac{dX^j}{dt}+\kappa\varepsilon_{i1}\left[\left(\frac{\varepsilon_{j1}}{\kappa}\right)^{(1)}\right]X^j$$

ハ何レモ t ノ変換デ F ト同ジ変換ヲスル反変ベクトルデアル。之等カラ X^j ノ曲線ニ共形不変ナ共変微分

$$\frac{\delta X^j}{dt}=\frac{dX^j}{dt}+\Gamma^j_iX^i$$

$$\Gamma^j_i=\frac{1}{M}\psi^{-(M-1)}F\cdot g^{ja}\left[F_{(M)a(M-1)i}+\psi^{-1}\psi^{(1)}F_{(M)a(M)i}\right]-\frac{\kappa}{F}x^{(1)j}\left(\frac{\varepsilon_{i1}}{\kappa}\right)^{(1)}$$

ヲ定義スルコトが出来ル。

Γ^i_j ハ $2M+1$ 次ノ線素ニマデ *depend* スル函数デアル。

$$D_p\Gamma^i_j=\sum_{\lambda\geqq p}\binom{\lambda}{p}\Gamma^i_{j(\lambda)k}dx^{(\lambda-p)k}\qquad(p=1,2,\cdots)$$

カラ intrinsic デ共形不変ナ線型接續ノ係數

$$\sum_{0\leqq\alpha\leqq 2M}\Gamma^{i(\alpha)}_{jk}\,dx^{(\alpha)k}$$

ヲ導クコトガ出来ル. 之ヲ用ヒテ X^i ノ共変微分

$$\delta X^i = dX^i + \sum_{a=0}^{2M+1}\Gamma^{i(a)}_{jk}X^j dx^{(a)k}$$

ガ定義サレル。

共形不変ナ基本接續ノ決定, ρ ガ一般ノ函數ノ場合ノ理論, 或ハ共変微分ノ次數ヲモ少シ低クスルコトハ出来ナイカ, 一点 (x) ニ於テ相隣ルニツノ線素 $(x^{(1)}, -----, x^{(\nu)})$, $(x^{(1)}+dx^{(1)}, -----, x^{(\nu)}+dx^{(\nu)})$ ノ間ノ角ヲ定義スルニハドウスレバヨイカ等ノ問題ガ残ッテヰルガ今回ハ之レデ筆ヲオク。