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Simplified Measuring Methods of Three Dimensional Welding Residual Stresses†

— Proposal of nL_y Method and Simple L_y Method —

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Abstract

The authors developed the measuring theory of three dimensional residual stresses induced in a long welded joint based on the theory of inherent strain, and two methods, L_y method and L_z method, were presented. Although these methods enable us to obtain very accurate residual stress distributions, they require so many measurements and machinery cutting. In many practical cases, the distribution at a specific point is sufficient for special purposes.

To this end, in this paper, the authors developed an approximate measuring method of three dimensional residual stresses along the thickness direction at a specific point, based on the L_y method.

According to this method which is nemed nL_y method, the object is cut into a specimen with the specified aspect ratio. As the result, the cross section of the new specimen is in the state of plane deformation and effective longitudinal inherent strains can be measured in a simple way. Then, three dimensional residual stresses can be estimated by two dimensional stress analysis in stead of three dimensional one, using the finite element method.

Furthermore, a simple estimation method which is named Simple L_y method is proposed, admitting inclusion of estimation errors by simplification. By this method, three dimensional stresses can be estimated only by performing simple manual calculation.

KEY WORDS: (Measurement of Residual Stresses) (Residual Stresses) (Inherent Strains) (L_y Method) (nL_y Method) (Simple L_y Method)

1. Introduction

It is very important to measure three dimensional residual stresses produced in the interior of a welded joint in order to investigate the safety of heavy structures. However, there being no direct method of measurement, the residual stresses should be indirectly estimated by any means.

In connection with this, the authors have carried out a series of researches in order to develop theoretically accurate measuring methods of three dimensional residual stresses. In the first place, based on the theory of inherent strain, a new general principle in measurement of residual stresses, in which inherent strains are dealt as parameters of measurement, has been proposed besides the already introduced principle of measurement in which released section-forces of the object are dealt as its parameters. Nextly, based on these respective principles, the general theories have been formulated with the aid of the finite element method and further developed introducing a statistic approach. Thus, the measuring method of residual stresses in a body of an arbitrary shape has become basically possible and the reliability of the estimated stress values has been investigated.

In the next place, the general measuring theory of three dimensional residual stresses with the aid of inherent strains has been examined of its practical use for actual welded joints. The general theory has been simplified by taking advantage of characteristics of the distribution of inherent strains produced in a long welded joint, which are uniform in the longitudinal direction, and it has been shown that the three dimensional inherent strain distribution can be divided into the inherent strains in the transverse cross section and the inherent strains in the longitudinal (weld line) direction. As the result, the three dimensional stresses $\{\sigma^A\}$ produced by the former can be directly observed from the stresses remained in a sliced thin plate parallel to the transverse cross section of the weld line (T-specimen) and the three dimensional stresses $\{\sigma^B\}$ produced by the latter can be estimated by measuring the inherent strains in sliced plates one of which sides is parallel to the weld line (L_i-specimen), (**Figure.1**). Furthermore, as two practical methods of cutting out L_i-specimen, L_z method by which the cutting surfaces become perpendicular to the thickness (z) direction and L_y method by

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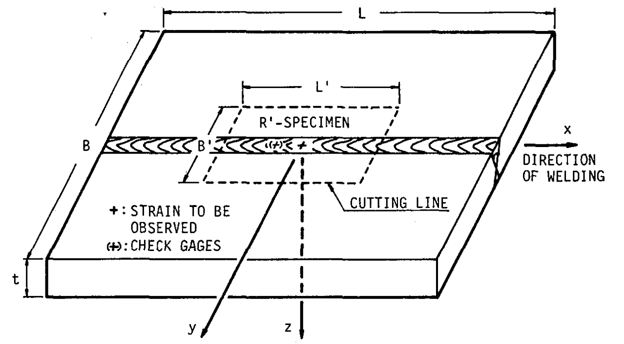
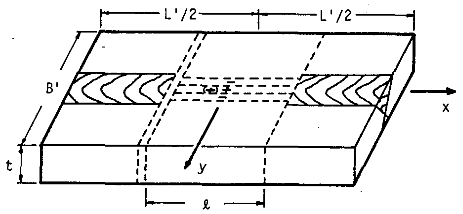
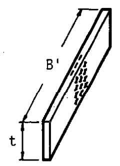


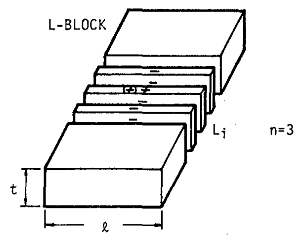
(a) Test model (R-specimen)



(b) R'-specimen produced by cutting
($L'>2B'>4t$)



(c) T-specimen



(b) L_i-specimen
($l>2t$)

Fig. 1 Procedure of approximate measuring method (nL_y method) for three dimensional welding residual stresses and locations of observed strains

which the cutting surfaces become perpendicular to the transverse (y) direction have been presented. By actual application of these methods to measurement of three dimensional residual stresses in actual welded joints, their usefulness and the reliability of the estimated values have been validated.

The above-mentioned practical measuring methods of three dimensional residual stresses needs a complicated theoretical calculation and costs a considerable amount of money and labor, because it has been developed with the purpose of measuring three dimensional residual stresses at every point of a transverse cross section in the weld line. However, in general, the demand for simple measurement of local distribution of three dimensional residual stresses (for example the distribution in the vicinity of the weld line in the thickness direction) exceeds that for measurement of stresses at every point of a cross section of a weld line. Such being the case, it is considered to be wasteful to apply the above-mentioned measuring methods as they are.

In this paper, a simplified measuring method of L_y method—nL_y method—for three dimensional residual stresses distributed in the thickness direction of a point, is proposed keeping the same degree of accuracy as the original L_y method. Furthermore, Simple L_y method is proposed, which enables the estimation of residual stresses by simple hand calculaion more or less at the sacrifice of accuracy of measurement. Finally, a numerical experiment using the results of actual measurement in a multipass welded jojnt is conducted in order to show applicability of the two simplified measuring methods.

2. Main Points in Simplification of Measurement of Three Dimensional Welding Residual Stresses

In this paper, simplification of measurement of stress distribution in z direction (the thickness direction) is studied on the basis of L_y method which best fulfills the purpose. Here, L_y method is called the basic measuring method. Here, L_y method is briefly summarized (Fig.1).

Assumptions adopted for development of the basic theory of L_y method are as follows.

1) Strains in the specimen change elastically due to cutting.
2) The remaining stresses in sliced thin plates are in the state of plane stress (to be sliced thin enough).
3) As each component of effective inherent strains is assumed constant along the weld line (x-axis), γ^*_{zx} and γ^*_{xy} vanish and the components ε^*_x, ε^*_y, ε^*_z, and γ^*_{yz} are functions of y and z coordinates only.

Here, the effective inherent strains mean those which produce residual stresses according to their magnitudes and not those which result in free expansion-contraction without producing residual stresses.

When L_y method is applied, a sliced thin plate, T-specimen (Fig. 1 (c)), and many other thin plates, L_i-specimens (Fig. 1 (d)), are cut out from the middle portion of a model joint, R-specimen, shown in Fig. 1 (a).

The remaining inherent strains in T-specimen are ε^*_y, ε^*_z and γ^*_{yz} and that in L_i-specimens is ε^*_x. Each group of the inherent strains induces three dimensional stresses, $\{\sigma^A\}$ and $\{\sigma^B\}$ respectively, in the original R-specimen. Then, the total three dimensional stresses $\{\sigma\}$ is expressed as the summation of these stresses,

$$\{\sigma\} = \{\sigma^A\} + \{\sigma^B\} \tag{1}$$

The remaining stresses in T-specimen, $\{\sigma^{AO}\}$, are at the state of plane stress. These can be transferred to those at the state of plane strain, $\{\sigma^A\}$, without knowing the inherent strains, ε^*_y, ε^*_z and γ^*_{yz}. This indicates that any simplification in measurement of $\{\sigma^A\}$ can not be expected.

On the other hand, in order to measure the three dimensional stresses $\{\sigma^B\}$ produced by the inherent strains along the weld line, the following procedure must be taken:

(i) L_l-specimens have to be prepared and released elastic strains in the specimens should be observed.

(ii) The estimation of their effective inherent strains $\{\varepsilon^*_x\}$ is necessary at every position of the cross section in the middle of a weld line constructing the observation equations.

(iii) Applying these estimated results of $\{\varepsilon^*_x\}$ to the original R-specimen, three dimensional elastic stresses are analyzed.

These processes require a great deal of work for the measurement.

Here, the following three points are taken into consideration in order to simplify the basic measuring method.

(i) Any method by which effective inherent strains $\{\varepsilon^*_x\}$ are directly estimated from observed elastic strains should be found out without using the observation equations, that is, for each L_l-specimen an independent direct relation between the linear component of effective inherent strains, ε^*_{xl}, and observed strains ε_{xl} should be made up.

(ii) The number of L_l-specimens to be furnished should be decreased.

(iii) Three dimensional elastic stress analysis should be replaced by two dimensional elastic stress analysis.

In order to satisfy these requirements, it is necessary to intentionally change the distribution of residual stresses to the specific one in such a way as to cut the original R-specimen ($L\times B\times t$, $L>B>t$) into R'-specimen ($L'\times B'\times t$). In this case, as the residual stresses are relaxed partially by cutting, changes of the stresses should be easily observed at least at the concerning measuring position including the interior of the plate.

The purpose to cut original R-specimen into R'-specimen is to satisfy the condition being $L'>B'$, so that the middle of the weld line of R'-specimen constitutes plane deformation. Plane deformation cited here implies that a plane section remains plane after deformation by imposition of the three dimensional inherent strains, ε^*_x, ε^*_y, ε^*_z and γ^*_{yz}, on R'-specimen.

In the next chapter, it will be shown that the new R'-specimen satisfies the requisite conditions for simplification of the basic method.

3. Development of Simplified Measuring Methods of Three Dimensional Welding Residual Stresses

3.1 Requisite dimensional ratio for reduced R'-specimen and the relaxed stresses $\{\sigma^C\}$ produced by cutting

A measuring position of three dimensional stresses distributed along z direction is assumed in the middle cross section of the weld line, that is $x=0$, $y=y_i$ in Fig.1 (a). For the convenience of explanation, the position is set at $x=0$, $y=0$. For other measurement, the origin of the coordinate system may be set to each measuring position and apply the same theory as will be described hereafter.

3.1.1 Requisite dimensional ratio for the state of plane deformation

As formerly mentioned in Chapter 2, R'-specimen is cut out from the original R-specimen by mechanical processing. One of the purposes of cutting out R'-specimen is to produce the state of plane deformation in the middle section of the weld line, so that this state facilitates the measurement of effective inherent strain component ε^*_x. Here, such dimensional ratio will be studied.

As indicated in Ref. 2), when the length L' of R'-specimen is longer than the double of its breadth B', the stresses produced by a set of inherent strains $\{\varepsilon^*_y, \varepsilon^*_z, \gamma^*_{yz}\}$ in the middle cross section are self-equilibrating and do not produce any deformation (strain) perpendicular to the section, that is in x direction. Therefore, any deformation in x direction should be produced only by effective inherent strain ε^*_x. When the specimen is sufficiently long, the deformation by these inherent strains is also in the state of plane deformation in the middle portion and the accompanied stresses in x direction are self-equilibrating. On the other hand, the newly exposed cutting sections of R'-specimen, where $x=\pm L'/2$ and $y=\pm B'/2$, become free surfaces by this processing. Accordingly, the stresses acting on those sections are released. Naturally, the residual stresses are also relaxed at the measuring position in the middle of R'-specimen. When the length of R'-specimen is long enough, the effect of distribution pattern of the released stresses disappear and the stresses produce plane deformation.

3.1.2 Requisite size of R'-specimen and released stress $\{\sigma^C\}$ due to cutting from R-specimen

When R-specimen is cut into R'-specimen, a certain amount of stress $\{\sigma^C\}$ is released at the measuring position ($x=y=0$). Based on Saint-Venant's principle, assuming that $B'>t$, if the measuring position is located at an adequate distance, more than its thickness, from the end surface, changes of the stresses can be normalizd and regarded as to distribute linearly (Appendix). Consequently, in order to make the changes of the relaxed stress $\{\sigma^C\}$ linear in the thickness direction, the following dimensional ratios become requisite to R'-specimen.

$$L'>2B'>4t \qquad \text{for } L>B>t \qquad (2)$$

or $$L'>2t>4B'=4B \qquad \text{for } L>t>B \qquad (2)'$$

The relaxed stresses $\{\sigma^C\}$ of thus prepared R'-specimen at the measuring position ($x=y=0$) can be obtained by putting the observed values of stresses on the top and bottom surfaces (Fig. 1) into the equations below.

$$\begin{aligned} \sigma^c_x &= \sigma^c_{xU}+(\sigma^c_{xL}-\sigma^c_{xU})\,(z/t) \\ \sigma^c_y &= \sigma^c_{yU}+(\sigma^c_{yL}-\sigma^c_{yU})\,(z/t) \\ \sigma^c_z &= \tau^c_{yz}=\tau^c_{zx}=\tau^c_{xy}=0 \end{aligned} \qquad (3)$$

where, σ^c_{xu}, σ^c_{yu} : Relaxed stresses directly observed in x and y directions at $x=y=0$ (on the top surface).

σ^c_{xL}, σ^c_{yL} : Relaxed stresses directly observed in x and y directions at $x=y=0$ and $z=t$ (on the bottom surface).

The observed values of the stresses in the above expressions may be calculated from observed strains.

If R-specimen is very large, the machine processing becomes difficult, but gas cutting. In such a case, a specimen is furnished from the original R-specimen by gas cutting several centimeteres outside the supposed R'-specimen. Afterwards, R'-specimen of a fixed size is cut out by machine. These processings must be done to eliminate the thermal effect by gas cutting.

3.2 Simplified measuring methods of three dimensional stresses $\{\sigma^B\}$ in R'-specimen and its accuracy

3.2.1 Relation between effective inherent strains along the weld line and elastic strains in the same direction

Stresses produced by the effective inherent strain components $\{\varepsilon^*_y, \varepsilon^*_z, \gamma^*_{yz}\}$ in the transverse cross section are always self-equilibrating. Therefore, the plane strain state is produced at a section adequately distant from the end surfaces, and no strains are produced in the direction of the weld line. On the other hand, if R'-specimen has the required dimensional ratios, the total strains along the weld line $\{e_x\}$ in R'-specimen, which are produced by effective inherent strains along the weld line $\{\varepsilon^*_x\}$, are in the state of plane deformation in the cross section at middle of the weld line and thus constitute a plane surface. This total strains are given by the summation of the inherent strains $\{\varepsilon^*_x\}$ and the elastic strains $\{\varepsilon_x\}=\{\varepsilon_{xl}\}+\{\varepsilon_{xn}\}$ which can be observed as mentioned in 2.1, that is,

$$e_x=a_1+a_2y+a_3z=\varepsilon^*_x+\varepsilon_x \qquad (4)$$

Therefore,

$$\varepsilon^*_x=(a_1+a_2y+a_3z)-\varepsilon_x \qquad (4)'$$

where, a_1, a_2, a_3 : coefficients

Apparently, the first term of the above equation(4)', $(a_1+a_2y+a_3z)$, corresponds to the linear component of the inherent strains in the transverse cross section in R'-specimen, which make free expansion and are the non-effective inherent strains having no relation with residual stresses. Therefore, the effective inherent strains along the weld line in R'-specimen is simply expressed as the opposite sign of the elastic strains to be observed as follows.

$$\varepsilon^*_x=-\varepsilon_x=-(\varepsilon_{xl}+\varepsilon_{xn}) \qquad (5)$$

where, ε_{xl} : linear component elastic strains, which is obtained from the observed strains on the top and bottom surfaces of L_l-specimen when L_l-specimen is machined.

ε_{xn} : nonlinear component of elastic strains.

As a result, it is no more necessary to estimate the linear component of inherent strains in x direction, ε^*_{xl}, using the observation equation (5), and the point (i) for simplicity stated in Chap.2 is resolved.

3.2.2 Analysis of three dimensional stresses $\{\sigma^B\}$

As in the above, if the cross section of R'-specimen to be measured is in the state of plane deformation under the influence of the inherent strains ε^*_x, the relation beween the effective inherent strains along the weld line, ε^*_x, and the elastic strains ε_x in the same direction can be given by Eq.(5), by which it is apparent that the plane strain state (total strain in x direction $e_x = \varepsilon_x^* + \varepsilon_x = 0$) is satisfied.

Accordingly, the three dimensional stresses $\{\sigma^B\}$ are in plane strain state and can be easily analyzed by a finite element computer program as two dimensional elastic problem which is reproduced by imposing the inherent strains in Eq.(5). As the result, the point (ii) for simplification explained in Chap.2 is resolved.

As is obvious from the above argument, no approximation is introduced in the method except the assumption set up at the beginning of this paper. Then, the method is correct in the framework of elasticity. Therefore, if the original object satisfies the condition of plane deformation, $L>2B$, of itself, the cutting in 3.1 is naturally unnecessary.

3.2.3 Number of L_i-specimens for measurement and accuracy of estimated stresses $\{\sigma^B\}$ — Proposal of nL_y method—

To start with, the relation between the effective inherent strains along the weld line, ε_x^*, and their production, three dimensional stresses $\{\sigma^B\}$, is considered.

Generally, $\{\sigma^B\}$ produced at the point P in an object is not only greatly dependent on the magnitude of ε_x^* at the point, but also be under the influence of ε_x^* even in other parts because the object should be satisfied the equilibrium and compatibility conditions. As for the influence of ε_x^* in other parts on point P, ε_x^* in the vicinity have a great influence since strains are produced so as to satisfy the compatibility condition, however ε_x^* in the remote part is considered to influence a little and that as the balanced stresses on the whole cross section.

Under the state of plane strain ($e_x=0$) which is studied in this chapter, stresses produced by the effective inherent strains ε_x^* are considered to have their effect on those in other parts because the stresses should be equilibrating keeping $e_x=0$. For this reason, if the inherent strains at point P and in its vicinity are attained, the three dimensional stresses $\{\sigma^B\}$ are supposed to be estimated with considerable degree of accuracy.

This idea will be applied to estimate the three dimensional residual stress component $\{\sigma^B\}$ along z-axis, which is assumed as the measuring object in this paper. For practical application, it is considered that $\{\sigma^B\}$ can be obtained with high accuracy by the method in 3.2.2, performing an analysis in which ε_x^* is attained by obseving the elastic strains $\varepsilon_x = \varepsilon_{xl} + \varepsilon_{xn}$ in n pieces of L_i-specimens containing z-axis and other neighboring ones(three to four pieces) (Fig.1(d)) ignoring effective inherent strains ε_x^* in other parts. Here, this measuring method is named nL_y method and when n is equal to 3, the method is called $3L_y$ method.

For investigation of the accuracy of nL_y method, the following numerical experiment was performed. First of all, the distribution of residual inherent strains in a multipass welded joint (B=200 m, t=50mm) which was measured in Ref.2) is expressed by the following equation.

$$\varepsilon_x^* = [-2180+960\ \sin\{(z-25)\pi/30\}]\times10^{-6} \cdot \exp\{-(y/18)^2/2\} \quad (6)$$

(y,z) *in mm*

(Inherent strains here are residual inherent strains, therefore Eq.(5) cannot be applied.)

In the next place, in order to measure stresses $\{\sigma^B\}$ under the state of plane deformation which is produced by giving these residual inherent strains to the object, the finite element analysis is made taking advantage of symmetry of the inherent strain distribution with respect to z-axis (stresses $\{\sigma^B\}$ attained here is regarded as the exact one). The modulus of elasticity is taken as E=21000kgf/mm^2 and the Poisson's ratio as ν=0.3.

Nextly, L_i-specimens are cut out from the center line and its vicinity of the object. Assuming that the above correct elastic strains ε_x are observed on the necessary number (n pieces) of L_i-specimens, the effective inherent strains ε_x^* are detemined to be the same as ε_x with their opposite sign. In other L_i-specimens, no strains assumed to be observed. Then $\{\sigma^B\}$ along z-axis in the portion where y=0 is

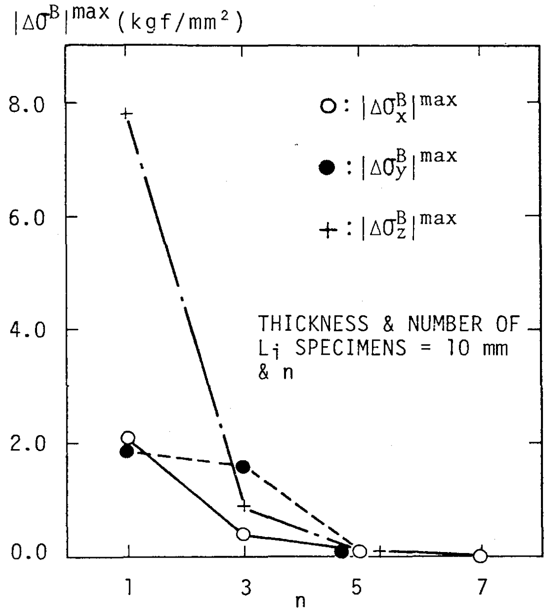


Fig. 2 Estimation error due to nL_y method for measurement of the stresses $\{\sigma^B\}$ at y=0
($|\Delta\sigma^B|^{max}$: Max.error, n : Number of L_i specimens)

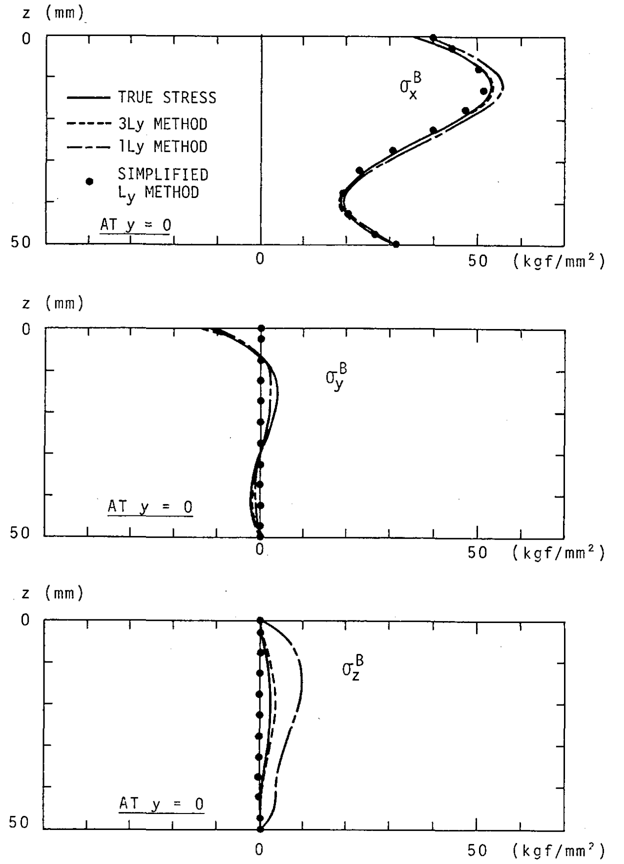


Fig. 3 True and estimated stresses $\{\sigma^B\}$ with aid of approximate measuring method

estimated by the finite elment analysis under the state of plane strain. As in **Figure 2**, the maximum absolute values of error, $|\Delta\sigma^B|^{max}$, made by the simplification of the measuring method is shown for each stress component in relation to the number of L_i-specimens cut out, n. Concerning the location of L_i-specimens, for example, L_i-specimens for $n=3$ of this case are cut out from the place where $y=0$ and $y=\pm 10mm$. As is obvious from the illustration, the error of the estimated stresses rapidly decreases as the number of L_i-specimens in which elastic strains ε_x are observed increases. When the number of L_i-specimens becomes 7, the estimated value of $\{\sigma^B\}$ at $y=0$ coincides with the exact value. For practical use, three L_i-specimens are considered to be adequate to attain necessary accuracy because the error following the simplification turns out to be $2kgf/mm^2$ at the most. Moreover, in the case where the stresses in z direction is unnecssary, one L_i-specimen which contains the measuring position to be observed is sufficient for labor saving. Because even though the estimated stresses in z direction, σ_z^B, are not reliable, other stress components such as σ_x^B and σ_y^B are estimated with reasonable accuracy when one L_i-specimen is cut out.

In **Figure.3**,the stress distribution of $\{\sigma^B\}$ estimated by the simplified method and its real distribution are compared in the cases where one L_i-specimen and three L_i-specimens are used. As is known from the figure, it is recognized that every value except the estimated value of σ_z^B is very close to the exact and is fairly accurate. As the result, the point (iii) in Chap.2 is considered to be resolved.

3.2.4 Proposal of Simple L_y method (Simplification of $1L_y$ method)

In the preceding sections so far, the simplified measuring method based on the theory of elasticity has been developed in order to measure three dimensional welding residual stresses with least difficulty and with sufficient accuracy. However, the method developed above finally requires a numericl analysis or the two dimensional finite element method in order to evaluate stresses $\{\sigma^B\}$.

In contrast with this, a measuring method to estimate $\{\sigma^B\}$ by hand calculation is considered to be practically useful on occasions even though it lacks sufficient accuracy. Therefore, a simpler measuring method which does not require an analysis by the finite element method will be proposed in this section.

When $1L_y$ method is used, the effective inherent strains is considered to exist only in one L_i-specimen. However, this specimen and the neighboring specimens are constrained each other through the compatibility condition. This requires two dimensional stress analysis. If the compatibility between L_i-specimens is ignored, this analysis is unnecessary and $1L_y$ method is simplified. This is named Simple L_y method.

In this method it is considered that stresses $\{\sigma^B\}$ in the state of plane deformation which is produced by inherent strains along the weld line are produced only by elastic strains ε_x in the same direction and in the uniaxial stress state in x direction. Therefore, in this measuring method, only one L_i-specimen is cut out from the part to be measured and stresses $\{\sigma^B\}$ are given by the equations below.

$$\sigma_x^B = E\varepsilon_x = E(\varepsilon_{xl}+\varepsilon_{xn}) \tag{7}$$

$$\sigma_y^B=\sigma_z^B=\tau_{yz}^B=\tau_{zx}^B=\tau_{xy}^B=0$$

In comparison with the example of the numerical experiment stated in the previous section, the estimated stresses $\{\sigma^B\}$ by this measuring method are indicated as "●" in Fig.3. Judging from the figure, serious errors are produced in σ_y^B and σ_z^B (in this example, about $10kgf/mm^2$) although σ_x^B is considerably accurate. As the result, this simplified method is considered to be a very effective method because of its simplicity in case only residual stresses in x direction are necessary.

3.3 Procedures of measurement of three dimensional welding residual stresses by nL_y method and Simple L_y method

3.3.1 Cutting process of specimens and procedure of measurement of elastic strains

The processes of cutting and observation of strains according to the simplified measuring methods described before are arranged in order as follows.

Cutting should be done in such a way as to produce no plastic strains. Especially, R-specimen should be cut off step by step from the end parts as far as possible so as to relax restraint gradually in order to prevent initiation of plastic deformation. It is also necessary to control the rise of temperature caused by cutting (by experience the highest temperature is 50℃ to the utmost).

– Cutting process and procedure of observation of strains (Fig.1) –

(a) Double axis gages in x and y directions are attached on the top and bottom surfaces of R-specimen at the measuring position ($x=y=0$) (Fig. 1 (a)).

(b)R'-specimen is cut in such a way as to give the dimensional ratios which satisfy Eq.(2). The stresses, σ^c_{xU}, σ^c_{yU}, σ^c_{xL}, and σ^c_{yL}, released by this cutting are observed by strain gages attached on the top and bottom surfaces (Fig.1(b)).

(c) If additional L_i-specimens are to be cut out, for improvement of measurement accuracy, from other positions than that where gages are already attached in (a), the strain gages in x direction are attached on the top and bottom surfaces of each L_i-specimen. Afterward, T-specimen and n pieces of L_i-specimen are cut out and the stresses released by this cutting are observed only on the top and bottom surfaces of L_i-specimens (ε^U_{xl}, ε^L_{xl}).

(b) T-specimen is machined from R'-specimen.

(e) Strain gages of uni-axis and bi-axes (of tri-axes in the case of observing τ^{AO}_{yz}) on sliced T-specimen and L_i-specimen are respectively attached. Then the specimens are cut and sliced. From T-specimen, elastic strains $\{\varepsilon^{AO}\}$ and from L_i-specimens, ε_{xn} are observed.

3.3.2 Procedure of stress analysis and accuracy of estimation of three dimensional residual stresses

Using the released and elastic strains which have been observed in the previous section, stress analysis is carried out as in the following process.

– Procedure of analysis –

(f) The released stresses on the top and bottom surfaces which have been observed in the above (b) are put into Eq.(3) in order to obtain the released stresses $\{\sigma^C\}$.

(g) From residual strains, $\{\varepsilon^{AO}\}$, in T-specimen which have been observed in (e) the residual stresses in plane stress state, $\{\sigma^{AO}\}$, may be calculated and these stresses may be transferred to plane strain state in order to estimate stresses $\{\sigma^A\}$ in R'-specimen.

(h) The released strains on the top and bottom surfaces, which have been observed in (c), are assumed to distribute linearly in L_i-specimens and ε_{xl} is calculated. The elastic strains ε_{xn} which remain in L_i-specimen have been observed in (d). The summation of ε_{xl} and ε_{xn} provides the inherent strains along the weld line, ε^*_x, as indicated by Eq.(5) (regarding the unobserved inherent strains ε^*_x in other L_i-specimens than n pieces as zero).

(i) When nL_y method is applied, the effective inherent strains ε^*_x which have been observed in (h) are used in order to estimate $\{\sigma^B\}$ in the state of plane deformation by the aid of finite element method. When Simple L_y method is applied, $\{\sigma^B\}$ is estimated by Eq.(7).

As the result of above-mentioned analyses, the required three dimensional welding residual stresses $\{\sigma\}$ in the middle section of the weld line are given by the following equation.

$$\{\sigma\} = \{\sigma^A\} + \{\sigma^B\} + \{\sigma^C\} \tag{8}$$

Finally, the surface stresses of the three dimensional residual stress distribution which has been estimated in Eq.(8) and the values directly observed by the strain gages on the top and bottom surfaces of the specimen in (a) are compared. In this way, the reliability of the estimated values is directly investigated.

The simplified measuring methods newly proposed in this paper, which assume the state of plane deformation, makes no theoretical approximation for estimation of stresses $\{\sigma^A\}$ and $\{\sigma^C\}$. Therefore, theoretical error may be induced in the process of simplification of estimation of $\{\sigma^B\}$. According to the simple numerical experiment using the model of a multipass weld joint which contains the idealized residual inherent strain distribution along the actual weld line, the value of $\{\sigma^B\}$ estimated by $3L_y$ method in which three L_i-specimens are used contains errors of only $2kgf/mm^2$ at the most regarding each stress component. Therefore, this is considered to be a highly accurate simplified measuring method. As to $1L_y$ method in which only one L_i-specimen is used, the errors contained in the estimated value of $\{\sigma^B\}$ except for that of σ_z^B are $2kgf/mm^2$ at the most. Therefore, this method is useful for the measurement of σ_x and σ_y alone. On the other hand, Simple L_y method is considered to be useful to measure σ_x alone.

In this paper, L_y method is set as a base because the simplified measuring methods of three dimensional residual stresses in z direction in a portion of a specimen is mentioned. If the distribution of three dimensional residual stresses in y direction in a portion of a specimen is required, L_z method becomes the base so that the theory will be developed essentially in the same manner.

4. Conclusion

In this paper the simplified measuring methods of three dimensional residual stresses in a certain part of a three dimensional residual stress distribution which is produced at a welding joint of a thick plate and uniform along one direction (the weld line) has been discussed. And the new simplified measuring methods, which are nL_y method and Simple L_y method, are developed.

Main results obtained are as follows.

(1) By cutting the original R-specimen ($L>B>t$), a new smaller specimen, R'-specimen ($L'>B'>t$), is furnished, in which the state of plane deformation is intentionally produced. According to the Saint-Venant principle, R'-specimen is produced by this cutting under the condition of $L'>2B'>4t$.

(2) By utilizing the state of plane deformation of thus produced R'-specimen, three dimensional residual stresses in a portion of a thick plate can be comparatively easily estimated.

(3) In order to maintain necessary accuracy of measured values to every stress component, nL_y method has been developed. This method needs application of a numerical analysis of plane state problems, e.g. the finite element method. $3L_y$ method by which three L_i-specimens are cut out is accurate enough for practical use. If the accuracy of stress components, σ_x and σ_y alone, are necessary, $1L_y$ method is sufficient. The measurement error of the respective methods by simplification is proved to be $2kgf/mm^2$ at the most.

(4) Simple L_y method which enables measurement of three dimensional residual stresses by hand calculation alone has been proposed. The accuracy of stress σ_x is practically adequate. The simplification error contained in σ_y turns to be 5 to $10kgf/mm^2$ and that in σ_z is considered to be about $5kgf/mm^2$.

(5) The above-mentioned methods show a way to simplify the basic theory of measurment. Other methods may be developed according to the characteristics of the stress distribution of the respective measuring objects.

References

1) Y.Ueda, K. Fukuda, K. Nakacho and S. Endo:A New Measuring Method of Residual Stresses with the Aid of Finite Element Method and Reliability of Estimated Values, Journal of SNAJ (the Society of Naval Architects of Japan), 138 (1975), 525-533 (in Japanese).

2) Y. Ueda, K. Fukuda and M. Tanigawa: New Measuring Method of 3-Dimensional Residual Stresses Based on Theory of Inherent Strain, Journal of SNAJ, 145 (1979), 203-211 (in Japanese).

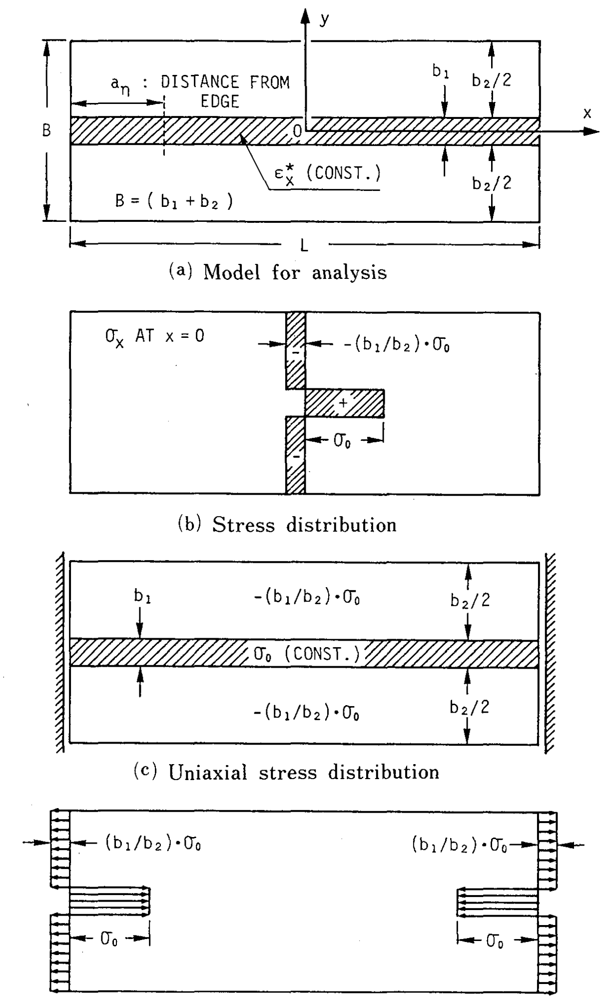
3) Y. Ueda, K. Fukuda and M. Fukuda: A Measuring Theory of Three Dimensional Residual Stresses in Long Welded Joints, Journal of JWS (the Japan Welding Society), 49-12 (1980), 845-853 (in Japanese).

Appendix Theoretical Study on the Effect of Free Edges upon Stress Distribution

A1 Object for Study

In a long weld joint in which the residual stresses distribute uniformly along the weld line, the stresses are self-equilibrating in a middle transverse section and constitute plane deformation. When the specimen is cut for measurement of the residual stresses, the stresses acting on the newly exposed section are released. In this case, if the specimen is sufficiently long, there is no disturbance of the stresses distribution at the middle section by release of the stresses at the exposed surfaces, since the effect of the self-equilibrating stresses is diminishing in the way to the middle section according to the Saint-Venant principle. The necessary length of the specimen for the above phenomena is theoretically studied.

The model of theoretical analysis is a thin plate ($L>B>t$) which is at the state of plane stress (**Figure.A1**). A weld is laid along the center line and the residual stresses in the plate are produced by inherent strain ε_x^* which is constant over the length L within a breadth b_1.



(a) Model for analysis

(b) Stress distribution

(c) Uniaxial stress distribution

(d) Load condition acting on the both ends

Fig. A1 Procedures of theoretical analysis for end effects

A2 Model for Analysis and Result of Analysis

When the model (Fig.A1(a)) is sufficiently long, the residual stresses distribute uniformly in cross sections of the middle part, which are at uni-axial stress state, and the stress σ_x vanishes at the ends (Fig.A1(b)). Accordingly, the actual stress distribution (Fig.A1(b)) may be represented by superposition of those stress distributions shown in Figs.A1(c) and (d). The former is uniform stress distribution and the same as that occurs in the middle section of the model (Fig.A1(a)). The latter may be reproduced by applying the restraint stresses at the ends to a free stressed plate of the same size as the original one. In reference to these stress distributions, the theoretical study on the effect of the released stresses at the edges (which is simply called the end effect hereafter) may be equivalent to study theoretically the effect of the applied stresses at the edges on the middle portion of the model.

When the stresses shown in Fig.A1(d) are applied to the edges of the plate, the induced stresses at any point (x, y) may be obtained by the same analysis method as used in Ref.A2. The largest component of the stresses is the axial one $\{\sigma_x\}$ and this distribution is expressed in the following non-dimensional form with respect to the maximum stress at edges,

$$\sigma'_x(x',y') = -\sigma_x/\sigma_0 = \sum_{m=1}^{\infty}\left(\frac{2}{m\pi}\sin m\pi b_1'\right)\{2\cos 2m\pi y'/(\sinh 2m\pi L' + 2m\pi L')\} \cdot [\{\sinh m\pi L' + m\pi L'\cosh m\pi L'\}\cosh 2m\pi x' - 2m\pi x'\sinh m\pi L' \cdot \sinh 2m\pi x'] \tag{A-1}$$

were

$$b_1' = b_1/B,\ L' = L/B,\ x' = x/B,\ y' = y/B$$

A3 Requisite Dimensional Ratio to Diminish the End Effect

The end effect may be studied at the relation of the non-dimensional stress $\{\sigma'_x\}$ and the distance from the edges.

The largest stress in a cross section appears along the center line ($y=0$). Denoting this by $\sigma'_x(x',0) = \eta$ $(0 \leq \eta \leq 1)$, the distance of a section from the edge, a_η, will be obtained.

In the case of $L/B = 3.0$, changing the ratio of B/b_1, the corresponding necessary distances, $a_{0.1}$, $a_{0.05}$, and $a_{0.025}$, to η being 0.1, 0.05 and 0.025 were calculated by Eq.(A-1) and the relation between $2a_\eta/b_1$ and B/b_1 is represented in **Figure A2.** According to Ref.A3,

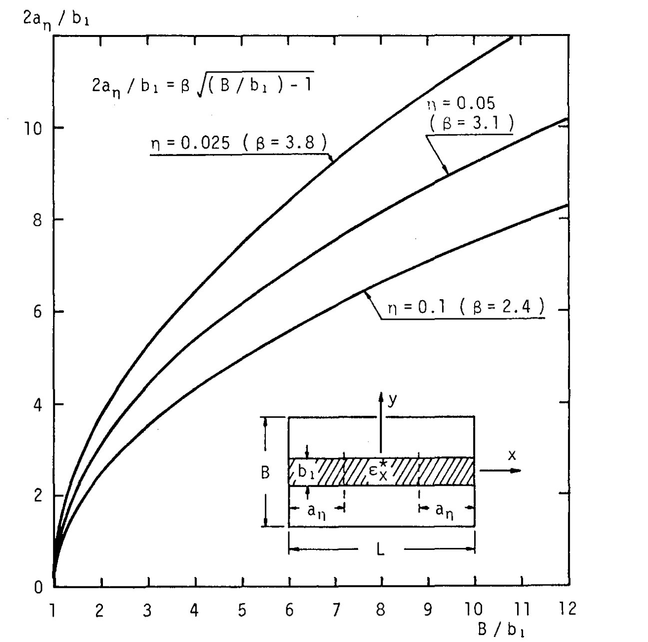


Fig. A2 Necessary distance from both ends, by which stress σ'_x decreases to $\sigma'_x = \eta$

the relation between a_η and B/b_1 may be approximately expressed by Eq.(A-2)

$$2a_\eta = \beta\sqrt{b_1(B-b_1)} \qquad \text{(A-2)}$$

In the above, the value of β was determined by curve fitting method for a conservative side. For example, it takes the values of 2.4, 3.1 and 3.8 for $\eta=0.1$, 0.05 and 0.025, respectively.

Next, when η becomes 0.05, the end effect is regarded as to vanish here. The necessary distance for this state, $a_{0.05}$, will be calculated in the following.

As $\beta=3.1$ for $\eta=0.05$, this value is introduced into Eq.(A-2) and the expression for $a_{0.05}$ is

$$2a_{0.05}/\beta = 3.1\sqrt{1/(B/b_1)-1/(B/b_1)^2} \qquad \text{(A-3)}$$

In most practical cases of welded plates, it is considered the $B/b_1=2$. Then, assuming $B/b_1=2$, Eq.(A-3) yields

$$2a_{0.05}/B = 1.55 \qquad \text{(A-4)}$$

As the result, the necessary condition under which the end effect may vanish at the middle section is

$$L > 2a_{0.05}\,(=1.55B) \qquad \text{(A-5)}$$

Furthermore, the effect of the side edges on the stress distribution at the center line of weld may be studied by the completely same way as the above. The result of analysis indicates such necessary breadth of the plate, by which the pattern of stress distribution along the side edges is nomalized, as

$B > 2t$

The results are summarized as follows,

$$\left.\begin{array}{ll} L \geq 2B \geq 4t & \text{for } L > B > t \\ L \geq 2t \geq 4B & \text{for } L > t > B \end{array}\right\} \qquad \text{(A-6)}$$

The condition of Eq.(A-5) implies that the end effect diminishes to 5% and this is practically sufficient to constitute plane deformation. If η is intended to be 0.025 for the case of B/b_1 being 2, as a more severe case, the necessary length is found to be larger than $1.9B$ from Eq.(A-2).

References on Appendix

A1) S.P. Timoshenco and J.N. Goodier: Theory of Elasticity, McGraw-Hill, 3rd Edition (1979), 53-59.

A2) Y. Ueda, K. Fukuda and Y.C. Kim: Restraint Intensities of a Slit in a Finite Plate, Trans. JWRI(Welding Research Institute of Osaka University), Vol. 7, No1. (1978), 11-16, and Journal of JWS, 48-10 (1979), 851-859 (in Japanese).

A3) Y. Ueda and K. Fukuda: Method of Analysis of Inherent Stress with the Aid of Principle of Min. Potential Energy, Preprints of the National Meeting of JWS, No.16 (Spring 1975), 301-302 (in Japanese).