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Preface

Productivity growth plays an essential role in both micro- and macro-economics, as it reflects the long-term improvements in production and operations at the firm, industry, and economy-wide levels. There is a wide variety of measures of productivity change, but the Malmquist-type indices are particularly noteworthy because of its widespread use in the literature on productivity. The essential characteristic of Malmquist-type indices is its dynamic view of efficiency, whereas the original efficiency analysis has been mostly static. Indeed, the efficiencies reflecting the performance of production activities are likely to change over time, and these changes have been considered as an important contribution to productivity growth. Therefore, the thesis covers both the theoretical and practical topics of efficiency and productivity analysis to estimate the productivity change with Malmquist-type indices.

The primary analyzing approach of this thesis is nonparametric in the sense that the measurement of the production frontier is entirely based on the observed input-output data. The thesis extends the theoretical and practical framework of two principle nonparametric methods involved: Data Envelopment Analysis (DEA) and Stochastic Nonparametric Envelopment of Data (StoNED). In particular, DEA is recognized as a modern mathematical programming method to deriving measures of efficiency and productivity change over time in the multi-input and multi-output production technology. The essential assumption of the traditional DEA models is its deterministic treatment of the production frontier, ignoring the statistical aspect of the data set. By contrast, StoNED is a regression-based method that imposes classical regression models of statistical noise into DEA. The use of a noise term makes it possible to estimate the production frontier under a stochastic setting. In practice, the choices of DEA and StoNED varies depending on whether the data have been measured correctly or not.

The thesis extends the theoretical frameworks of efficiency and productivity analysis in the following aspects: (1) a new scheme of allocative efficiency, which provides a comprehensive understanding of the sources of inefficiency in inputs and outputs, (2) a new Malmquist-type index termed profit-ratio change index, which gives a full picture of the sources of productivity change in the sense that the impact of allocative efficiency changes is incorporated, (3) a new panel-data model for estimating the Malmquist-type indices under stochastic noise, which addressed the issues of inconsistent inefficiency and measurement issues of intertemporal inefficiency. Further, the merits of the proposed methods and the validity of the evaluation results have been illustrated by analyzing the efficiency and productivity change of samples of 37 Japanese securities companies and 101 Japanese regional banks, respectively. These results provide realistic projections and policy implications for improving the productive performance.

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January 2020

List of frequently used symbols

x	Input vector
y	Output vector
$\bar{x}$	Input-spending vector
$\bar{y}$	Output-earnings vector
T	Production possibility set
$T_{\bar{x},\bar{y}}$	Value-based technology
TE^{I}, γ	Input-oriented technical efficiency
TE^{O}, ω	Output-oriented technical efficiency
TE^{GR}, θ	Graph measure of technical efficiency
$D^{I}(x, y), \phi$	Input distance function
$D^{O}(x, y), \varphi$	Output distance function
$\vec{D}(x, y, g, h), \eta$	Directional distance function
(g, h)	Direction vector
ε	Composite error that consists of inefficiency and random noise
u	Inefficiency
v	Random noise
$E(u), \mu$	Expected value of inefficiency
σ_u	Standard deviation of inefficiency
σ_v	Standard deviation of random noise
σ_ε	Standard deviation of composite error term
$\pi(x, y)$	Profit-ratio function
PE	Profit-ratio efficiency
AE	Allocative efficiency
λ	Intensity variable

MI	Malmquist-type index
TEC	Technical efficiency change, Catch-up
TC	Technical change, Frontier shift, Innovation
PI	Profit-ratio change index
PEC	Profit-ratio efficiency change
PTC	Change of profit-ratio boundary
AMI	Allocation Malmquist productivity index
AEC	Allocative efficiency change
ATC	Allocation-technical change
ε^{CNLS}	Estimator of the CNLS problem
δ	Estimator of the directional distance function
$\bar{M}$	Stochastic nonparametric estimation of the Malmquist-type index
EC	Efficiency (inefficiency) change
R	Set of real numbers
$m, (i = 1, \ldots, m)$	Dimension of the input (input-spending) vector, indexed by i
$s, (r = 1, \ldots, s)$	Dimension of the output (output-earnings) vector, indexed by r
$K, (k = 1, \ldots, K)$	Observing times at some period, indexed by k
$n, (j = 1, \ldots, n)$	Number of observations, indexed by j
o	Observation under evaluation
α	Intercept of tangent hyperplanes
β	Shadow prices (slope parameters) of the inputs
τ	Shadow prices (slope parameters) of the outputs

Contents

Chapter 1

Introduction

1.1 Introduction to efficiency and productivity analysis

The thesis focuses on the essential practical and theoretical problems of analyzing efficiency and productivity, including the measurement of allocative efficiency under specific behavioral objectives, the characterization of productivity with consideration for allocative efficiency, and the performance evaluation under stochastic noise. Both efficiency and productivity are widely used concepts in the fields of management science and economics as a means to evaluate the performance of production activities such as firms, government agencies, and nonprofit organizations. In practice, however, assessing efficiency or productivity can be difficult in some situations, especially when the benchmarks (standards) are unavailable or multiple performance metrics (i.e., inputs and outputs) are involved.

Consider bank branches, for example. The managers may be interested in knowing how efficiently their business processes operate concerning the use of inputs such as labor, deposits, and capital, and the outputs such as loans and securities. In such a situation, efficiency can be easily estimated if we have a priori information on the relationship among multiple inputs and multiple outputs (i.e., there is an available functional form that can fully characterize the business processes of bank branches). Unfortunately, such information is not always available. For this reason, efficiency is often considered as a result of relative comparisons among all observed production activities. Specifically, by comparing the current production activity with similar ones, we can empirically estimate a so-called efficient frontier that identifies all best practices. Since producers or managers are not required to operate on the efficient frontier, the deviations from the frontier are then explained as a natural measure of efficiency. A detailed description of efficiency is presented in Section 2.3. Further extensions of the concepts of efficiency are discussed in Sections 3.2 and 5.3.

A majority of literature on efficiency analysis has been static, which means any comparison through time is entirely ignored. However, the performance of production activities is likely to change over time. Moreover, considering a time component, it becomes possible to associate the changes in performance over time with productivity change (e.g., higher or lower productivity). Note that productivity is a static concept that compares the performance of production activities at a given point of time, while productivity change is a dynamic view of productivity. Although productivity change focuses on the differences in past performance, understanding recent historical trends of productivity and what has driven these can be important for policy-makers or regulators. For example, regulators can set reasonable expectations for future production plans by assessing what can be achieved through efficiency improvements and what can be achieved through changes of efficient frontier over time. On the other hand, analyzing productivity change can also help to examine the impact of a policy or managerial decisions over a long period. I provide a brief review of the productivity analysis in Section 2.4 and further clarify the difference between the concepts of efficiency and productivity. In Chapters 3 and 5, I address the theoretical issues of measuring productivity change and propose the solutions by introducing a profit-ratio change index and stochastic nonparametric estimation of Malmquist-type indices.

1.2 Brief review of methods

There are two main approaches for evaluating efficiency and productivity: deterministic nonparametric and stochastic parametric approaches. In the standard deterministic nonparametric approach, Data Envelopment Analysis (DEA, Charnes et al. [1]) has demonstrated its utility for measuring productive performance. Notably, DEA can be applied to a multi-input and multi-output production technology, which is based on theoretical axioms of production theory such as free disposability, convexity, and returns to scale (see also [2–5]). The significant properties of production technology are described in Section 2.2. It is also well known that the deterministic nonparametric approach commonly assumes away stochastic noise, suggesting that any deviations from the frontier (e.g., gauging the distance to the boundary of the production technology) can be considered as a measure of pure inefficiency. By contrast, Stochastic Frontier Analysis (SFA, Aigner et al. [6], Meeusen and van Den Broeck [7]), a general stochastic parametric approach, accounts for stochastic noise by treating all deviations from the frontier as aggregations of both inefficiency and noise. However, compared with the flexibility of nonparametric measurements, because SFA is a parametric methodology it relies heavily on an accurately pre-specified functional form for production technology.

Moving on, a growing number of theoretical studies attempt to combine the advantages of deterministic nonparametric and stochastic parametric approaches (see, e.g., [8–11]). These studies offer potential in terms of improved understandings of performance benchmarking. Stochastic Nonparametric Envelopment of Data (StoNED) introduced by Kuosmanen [12] has been variously applied in the literature. It has been shown that both DEA and SFA can be integrated into the StoNED framework (see also Kuosmanen and Johnson [13]). The unknown production frontier in StoNED is handled with convex nonparametric least squares (CNLS), which is a nonparametric regression technique proposed by Kuosmanen [14]. Based on the results of CNLS regression, efficiency analysis can be further performed with either parametric or nonparametric methods.

The primary theoretical approach of this thesis is based on nonparametric techniques: DEA and StoNED. In Section 2.5, I provide a short description of these techniques and further discuss the measurement issues for efficiency and productivity analysis.

1.3 Objectives of this thesis

The main objective of this thesis is to provide new nonparametric methodologies for the estimations of efficiency and productivity change. This objective is motivated by concerning the following three aspects:

(a) Given specific behavioral objectives, efficiency analysis can be performed regarding the optimal combinations (mix) of inputs and/or outputs, which leads to the concept of allocative efficiency. Note that the common used behavioral objectives are either cost minimization or revenue maximization. In practice, however, producers in profit-seeking organizations can be both cost minimizers and revenue maximizers. For example, consider the production activities whose underlying behavioral objectives are the maximization of profit ratio (the ratio of revenue to expenses). The conventional measures of allocative efficiency may not give a comprehensive understanding of the sources of inefficiency because the effect caused by the wrong mix of both revenue and expenses is not incorporated. Therefore, it is necessary to develop a new scheme of allocative efficiency.

(b) The Malmquist index is a useful tool for measuring productivity changes. Besides measures quantifying productivity changes, there are also various empirical studies investigating the drivers of productivity changes. In recent years, the decomposition of productivity changes into a technical efficiency change component and a technical change component using the Malmquist index has been widely used. However, the conventional construction of the Malmquist index ignores the impact of allocative efficiency, which has been proved to account for the changes in productivity in empirical applications. Thus, it is necessary to consider a new Malmquist-type index for incorporating the impact of allocative efficiency changes on productivity change.

(c) In the deterministic approach, efficiency is often quantified by gauging the distance to the production frontier. Meanwhile, the conventional Malmquist-type indices are constructed from the distance functions, which makes it possible to use the evaluated efficiencies to calculate the productivity change. However, in some situations, the stochastic approach may be preferable than the deterministic approach, especially when the evaluated efficiency is considered to be sensitive to mismeasurements or outliers. In the nonparametric techniques, StoNED allows us to assume the deviations from the unknown production frontier consist of both inefficiencies and noise in the data. Consequently, compared with the deterministic approach, the distance estimated from the cross-sectional StoNED model cannot be straightforwardly extended to construct a Malmquist-type index. Therefore, it is necessary to develop a new model to estimate the Malmquist-type indices under stochastic noise.

The secondary objective is to demonstrate the advantages of the proposed methods on empirical applications. Specifically, two types of decision-makers are considered: securities companies and regional banks. Considering both DEA and StoNED are methods for decision making, the policy implications of the empirical results are also discussed, respectively.

1.4 Outline of chapters

The summaries of the remaining chapters are provided below.

Chapter 2 provides the general theoretical background required for efficiency and productivity analysis. It begins by introducing the structure of multi-input and multi-output production technology. Some notable properties of production technology are illustrated in detail. Attention then moves to the concepts of efficiency and productivity. I outline a variety of efficiency measures and the definition of the Malmquist index, which is dealt with within the following chapters. Eventually, I introduce the main nonparametric techniques for analyzing efficiency and productivity.

Chapter 3 is devoted to the productivity analysis with consideration for allocative efficiency. The methodology is established on a value-based measure with due considerations to the imprecise price and the heterogeneity in physical inputs and physical outputs. In what follows, a new scheme of allocative efficiency in terms of profit-ratio maximization is firstly proposed. I show how this can be incorporated into a DEA model theoretically. A profit-ratio change index is developed correspondingly, which can be applied to panel data to measure productivity change and suitable for situations when producers desire to maximize revenue and minimize expenses simultaneously. To identify the drivers of changes in a profit-ratio change index, the index is further decomposed into profit-ratio efficiency change and change of profit-ratio boundary. An alternative decomposition of the profit-ratio change index is also proposed, which is the product of the Malmquist input-oriented productivity index and an allocation Malmquist productivity index.

Chapter 4 demonstrates the methodology in Chapter 3 by considering a sample of 37

Japanese securities companies observed from 2011 to 2015. To derive valuable information for organization management, the observed securities companies are categorized into six different groups based on their technical and allocative performance. Through such a categorization, the empirical results revealed the strengths and weaknesses of Japanese securities companies and identified the potential opportunities to improve current operations and management.

Chapter 5 is concerned with productivity analysis under a stochastic setting. Considering the presence of random noise in empirical data, a panel-data StoNED model is introduced for estimating the production technology in terms of the directional distance function. By virtue of using panel data, the inefficiency can be estimated consistently based on the residuals of the CNLS problem. The problem of using CNLS is that the intertemporal efficiency for constructing Malmquist-type indices cannot be assessed directly. To solve this issue, an estimator of the directional distance function is developed for analyzing the intertemporal efficiency. It is further extended to the estimation of Malmquist-type indices. A major feature of this approach is that it measures productivity changes over time while capturing both inefficiency and noise in a nonparametric multiple-input multiple-output setting.

Chapter 6 investigates the productive performance of a sample of 101 Japanese regional banks over two periods by applying the methodology developed in Chapter 5. To investigate the drivers of productivity change, the proposed Malmquist index is decomposed into components of efficiency change and technical change. The policy implications of the empirical results are also discussed.

Chapter 7 provides a further productivity analysis for the sample of 37 Japanese securities companies observed from 2011 to 2015. Considering the significant changes in business management that appeared in the Japanese securities industry around the year 2013, the methodology developed in Chapter 5 is adopted to estimate the productivity changes between the analyzing periods 2011-2013 and 2013-2015. Notably, I investigate the extent to which the outlier affects the estimation of inefficiency. I further discuss the policy implications of the estimated Malmquist index and examine the main drivers of productivity growth.

Chapter 8 concludes the whole thesis and provides several directions for future research.

Chapter 2

Theoretical background

2.1 Introduction

The objective of this chapter is to provide the necessary theoretical underpinnings for analyzing efficiency and productivity. Section 2.2 introduces the structure and some notable properties of production technology where multiple inputs are used to produce multiple outputs. Based on the production technology, efficiency and productivity are then described in Sections 2.3 and 2.4, respectively. Section 2.3 covers classical efficiency concepts, including the input-oriented, the output-oriented, and the graph measure of technical efficiencies, as well as more advanced concepts like the directional measure of inefficiency, the profit-ratio efficiency, and the allocative efficiency regarding the profit-ratio maximization. Section 2.4 is concerned with the measurement of productivity in terms of Malmquist-type indices. Attention then moves to the methods for analyzing efficiency and productivity. Section 2.5 summarized the basics of two nonparametric techniques: Data Envelopment Analysis (DEA) and Stochastic Nonparametric Envelopment of Data (StoNED). Section 2.6 concludes this chapter.

2.2 Production technology

All production processes are considered as a transformation of inputs into outputs. Production technology is a mathematical description of this transformation relationship and can be represented with either sets or functions. The choice of which depends on the analytical approaches and the purposes of productivity analysis. Especially, in productivity literature on nonparametric approaches (e.g., DEA), the production technology is almost represented with sets, while in those on parametric approaches (e.g., SFA), some specific production functions with unknown parameters are commonly used which include the Cobb-Douglas, the translog, and the generalized production function. However, it is worth noting that in some specific situations involving multiple outputs, distance functions are also useful as alternative functional representations of the production technology and can be handled with either nonparametric or parametric approaches. In what follows, I first consider the representation of the sets due to the purpose of modeling the production technology in a nonparametric approach. Alternative functional representations using distance functions will also be considered.

Denote $x \in R_+^m$ a nonnegative vector of inputs and $y \in R_+^s$ a nonnegative vector of outputs. The production technology defined with ***production possibility set*** is given by

$$T = \{(x, y) \in R_+^{m+s}: x \ \ can \ \ produce \ \ y\}. \quad (2.1)$$

Note that both the inputs and outputs in Eq. (2.1) are quantities without any random noise, and at this point, I do not account for any price information or assume any particular behavior such as cost minimization, revenue maximization, or other economic behaviors. Set T contains all technologically feasible combinations of input-output vectors, and thus it is also termed the ***graph*** of production technology. The thesis assumes T satisfies the following standard axioms (e.g., Shephard [15]; Färe and Primont [16]):

A.1. ***Boundedness***.

A.2. ***Closed set***.

A.3. ***Convexity***: If $(x, y) \in T$ and $(x', y') \in T$, then $(\lambda x + (1-\lambda)x', \lambda y + (1-\lambda)y') \in T$ for all $\lambda \in [0,1]$.

A.4. ***No free lunch***: If $y > 0$ and $(x, y) \in T$, then $x > 0$.

A.5. ***Free (or strong) disposability of inputs and outputs***: If $(x, y) \in T$, then $(x', y') \in T$ for $(x, -y) \leq (x', -y')$.

The above axioms are assumed to hold throughout this thesis. However, further assumptions such as ***returns to scale*** will be discussed as required. A.1, along with A.2, guarantees the existence of ***production frontier*** (the boundary of the production technology) which gives the maximum possible outputs that can be produced from the given level of inputs or, equivalently, the minimum possible inputs required for any given level of outputs. For example, if a production process only has a single output or an obtainable aggregate output of multiple outputs to be produced (e.g., $s = 1$), then the production frontier can be defined by using the function $g(x) = \max\{y: (x, y) \in T\}$. A.3 says any convex combination of two feasible production activities also belongs to the production technology. This axiom is one of the standard theoretical axioms of production theory in microeconomics (see [2–5]). For example, in the case of $s = 1$, A.3 implies a diminishing marginal rate of (technical) substitution. A.4 says positive outputs can always be produced by positive inputs. A.5 imposes the monotonicity of inputs and outputs. Specifically, it ensures that the production technology is monotonically increasing in inputs and monotonically decreasing in outputs. Figure 2.1 illustrates the production technology imposed with A.1~A.5 in the case of a single input and a single output.

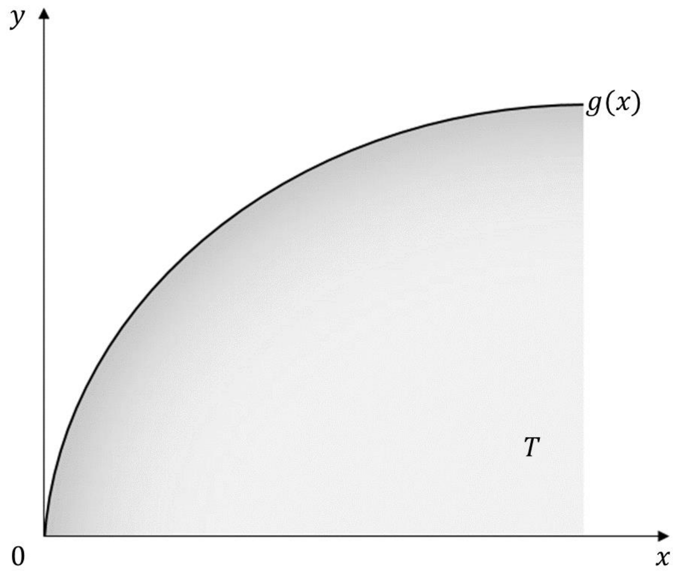


Figure 2.1 Illustration of production technology with a single input and single output

It is worth noting that A.5 can be relaxed under some specific situations. For example, one can impose the ***weak disposability of inputs and outputs*** in the sense that the underlying production technology is expected to be radially increasing in inputs and radially decreasing in outputs: $(x, y) \in T \Rightarrow (x, y) \in T$ for $\lambda \geq 1$ and $(x, y) \in T \Rightarrow (x, \lambda y) \in T$ for $0 \leq \lambda \leq 1$. Obviously, the strong disposability implies the weak disposability, however, the converse does not hold. In other words, the strong disposability allows the situation where inputs can be increased without decreasing any output, and also the situation where outputs can be decreased without increasing any input. By contrast, the weak disposability indicates that any increase in inputs or decrease in outputs comes at the cost of decreasing outputs or increasing inputs. Throughout the thesis, the strong disposability is assumed due to the following considerations: First, strong disposability allows for non-radial efficiency improvements, which will be considered in upcoming sections. Second, strong disposability is a sufficient condition (see, e.g., Färe et al. [17]) for the functional representation of production technology based on the ***directional distance function*** (DDF). Further details on DDF will be discussed in Chapter 5.

2.3 Efficiency

Recall that the production possibility set contains all technologically feasible combinations of input-output vectors. This implicitly says that producers may operate below or on the production frontier. Generally, if an input-output combination $(x_A, y_A) \in T$ is on the production frontier, the production activity is labeled ***technically efficient*** (or, $(x_A, y_A) \in T$ is ***weakly efficient*** if there remains slack in inputs or outputs) in the sense that decreasing any input or increasing any output is not possible without increasing any other input or decreasing any other output [18]. In contrast, production activity is ***technically inefficient*** if producers operate below the production frontier. Clearly, the concept of "***efficiency***" provides a relative comparison for all feasible input-output combinations, that is, efficiency means the extent to which each production activity differs from those that appeared on the production frontier. More straightforwardly, efficiencies are evaluated by comparing a production activity to the production frontier (or, more precisely, to the estimated production frontier (***best practices***) derived from the observations). Mathematically, efficiencies are generally expressed in either ***ratio*** or ***difference*** form. Both of these can be further discussed in ***radial*** and ***non-radial*** measures, depending on the purpose of efficiency analysis. The interest of radial measures is mainly on the achievement of the maximum equally proportional contraction in all inputs or the maximum equally proportional expansion of all outputs, while the non-radial measures allow for the situation where inputs and outputs are not changed by the same proportion. No matter which measure is used, however, it is essential to consider an economic hypothesis: ***returns to scale***. This is because the concept of returns to scale provides a characterization of the shape of the underlying production technology (i.e., a reasonable identification of returns to scale leads to a reasonable characterization of efficient production activities). For example, if there is not enough evidence to prove that the differences of operating scales among evaluated production activities relate directly with economies of scale, then it may be reasonable to consider that the underlying production technology exhibits ***constant returns to scale*** (CRS, that is, any feasible input-output combination can arbitrarily be scaled up or down; i.e., $T = kT, k > 0$), and the efficient production activities are of the ***most productive scale size*** [19]. Further details of returns to scale will be discussed as required. Our focus in evaluating efficiencies is on the following:

(a) The ***input-oriented technical efficiency*** (TE^I): TE^I is a radial measure that attempts to minimize inputs while producing at least the given outputs [1,20], which is defined as

$$TE^I = \inf_{\gamma}\{\gamma: (\gamma x, y) \in T, 0 < \gamma \leq 1\}, \tag{2.2}$$

where the superscript "I" denotes the input orientation.

(b) The ***output-oriented technical efficiency*** (TE^O): TE^O is also a radial measure whose objective is to maximize outputs while using no more than the observed level of any input [1,20], which is defined as

$$TE^O = \sup_{\omega}\{\omega: (x, \omega y) \in T, \omega \geq 1\}, \tag{2.3}$$

where the superscript "O" denotes the output orientation.

(c) The ***graph measure of technical efficiency*** (TE^{GR}): TE^{GR} is simultaneously both a radial input contraction and radial output expansion. Different from TE^I and TE^O, this measure follows a hyperbolic path to the production frontier and thus is also termed "graph hyperbolic measure" [21,22]. Formally, TE^{GR} is defined as

$$TE^{GR} = \inf_{\theta}\{\theta: (\theta x, \theta^{-1} y) \in T, 0 < \theta \leq 1\}, \tag{2.4}$$

where the superscript “GR” stands for graph hyperbolic measure.

The above measures are of ratio forms and can be simply grouped into radial measures. Let now $TE^{I} = \gamma^{*}$, $TE^{O} = \omega^{*}$, and $TE^{GR} = \theta^{*}$, we can illustrate their differences by using Figure 2.2.

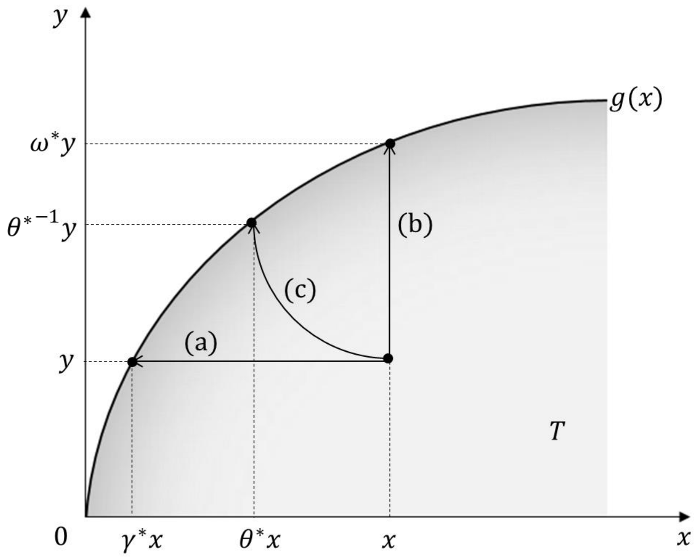


Figure 2.2 Input-oriented, output-oriented, and graph measure of technical efficiencies.

Measures (a) and (b) were first proposed by Debreu [23] and Farrell [20], and thus also referred to as “Farrell efficiencies” or “Debreu-Farrell measures of technical efficiency.” These measures have been proved to be the inverse of the ***Shephard distance functions***, which are given as

$$D^{I}(x, y) = \sup_{\phi}\{\phi: (x/\phi, y) \in T, \phi > 0\} = 1/TE^{I}, \tag{2.5}$$

$$D^{O}(x, y) = \inf_{\varphi}\{\varphi: (x, y/\varphi) \in T, \varphi > 0\} = 1/TE^{O}. \tag{2.6}$$

Note that $D^{I}(x, y)$ and $D^{O}(x, y)$ are termed ***input distance function*** and ***output distance function***, respectively (see, e.g., Färe and Primont [16]). As a more generalized approach of both Farrell efficiencies and Shephard distance functions, the ***directional distance function*** (DDF), which was proposed by Chambers et al. [24], can also be used to evaluate efficiencies:

$$\vec{D}(x, y, g, h) = \sup_{\eta}\{\eta : (x - \eta g, y + \eta h) \in T\} \text{ with } (g, h) \in R_{+}^{m+s}. \qquad (2.7)$$

This function simply measures how far a given input-output combination (x, y) can be projected onto the production frontier along with some given direction (g, h). It has been shown that either Farrell efficiencies or Shephard distance functions are, in principle, special cases of the directional distance function [25]. Specifically, $\vec{D}(x, y, g, h) = 1 - 1/D^{I}(x, y) = 1 - TE^{I}$ if we specify the direction vector as $(g, h) = (x, 0)$ and $\vec{D}(x, y, g, h) = 1/D^{O}(x, y) - 1 = TE^{O} - 1$ if $(g, h) = (0, y)$. However, it is also straightforward to see that the directional distance function is not limited to the input or the output orientation. Just like the graph measure of technical efficiency in Eq. (2.4), the directional distance function in Eq. (2.7) combines the ideas of input and output orientation by examining to what extent the actual inputs and outputs can be simultaneously improved. A major difference between Eqs. (2.4) and (2.7) is that the directional distance function has an additive nature (difference form), which allows for the potential for non-radial efficiencies.

(d) The directional measure of ***inefficiency*** (u): u is the distance quantified by scaling inputs and outputs to the production frontier in the direction vector $(g, h) \in R_{+}^{m+s}$. Formally,

$$u = \vec{D}(x, y, g, h). \qquad (2.8)$$

Figure 2.3 illustrates the relations of the inefficiency and some pre-assigned direction vector.

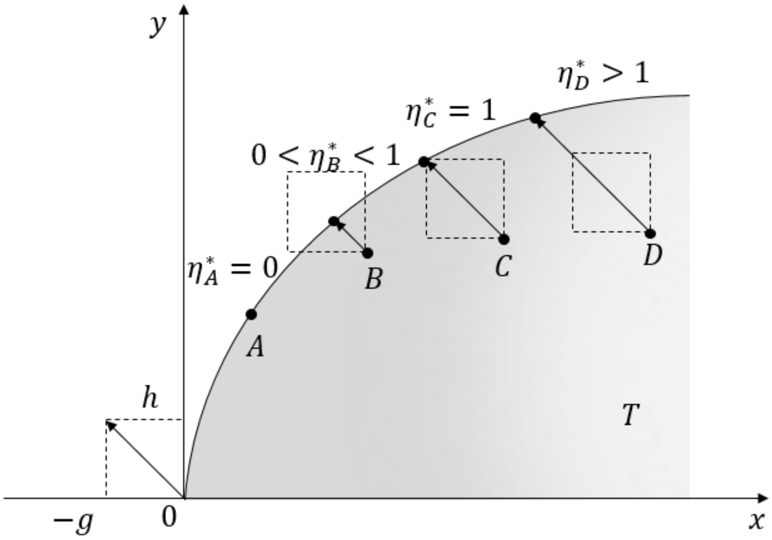


Figure 2.3 The directional measure of inefficiency.

The observed production activities are points A, B, C, and D. It can be seen that with any arbitrary given direction vectors, points on the production frontier are obtained as $u = \eta^* = 0$, which means no inefficiency (e.g., point A). Meanwhile, points under the production frontier are estimated with positive inefficiency, that is $u = \eta^* > 0$ (e.g., points B, C, and D). This relation was first explained by Chambers et al., [26] and can be formally described as $\vec{D}(x, y, g, h) \geq 0$ if and only if $(x, y) \in T$. Moreover, this relation also indicates that the directional distance function provides a complete characterization of the production technology, which will be discussed further in Chapter 4.

So far, no assumptions have been made about the behavioral objectives such as cost minimization, revenue maximization, and profit maximization, among others. However, it is possible to account for such ***behavioral objectives*** in the framework of efficiency analysis. The thesis considers explicitly the production activities whose underlying behavioral objectives are the maximization of profit ratio. Note that the term "***profit ratio***" is consistent with the concept of "***profitability***" which is commonly defined as the ratio of revenue to expenses [27–30], or the criterion "return to the dollar" proposed by Georgescu-Roegen [31]. The assumption of profit-ratio maximization provides a possibility for analyzing the economic frontier (i.e., the profit-ratio boundary. See further details in Chapter 3), which means it is possible to evaluate the profitability performance (e.g., ***profit-ratio efficiency***) of the production activities if their maximum achievable profit ratio is obtainable.

(e) The ***profit-ratio efficiency*** (PE): PE is a measure of the extent to which the actual profit ratio falls short of achieving the maximum profit ratio. Specifically,

$$PE = \frac{\pi(x, y)}{\pi(x^*, y^*)}, \tag{2.9}$$

where $\pi(\cdot)$ is a profit-ratio function that maximizes the ratio of revenue to expenses. $(x^*, y^*) \in T$ are the input-output combinations on the profit-ratio boundary and thus $\pi(x^*, y^*)$ represents the maximum profit ratio for the observed production activities.

As for the production activities who consider profit-ratio maximization, they are responsible not only for picking a technically efficient point on the production frontier but also for picking the right one to maximize the profit ratio. The latter is associated with ***allocative efficiency***.

(f) The ***allocative efficiency*** regarding profit-ratio maximization (AE): AE identifies the wrong mix in input-spending and output-earnings. If there is no wrong mix in both input-spending and output-earnings, the technically efficient point is the same one with the maximum profit ratio.

Note that the commonly used definition of allocative efficiency identifies the existence of the wrong mix in physical inputs and physical outputs, given the exact price information [20,23]. However, considering that the inaccurate price information can distort measures of allocative efficiency [32], the allocative efficiency is evaluated in a ***value-based technology*** in Chapter 3. In the case that one uses the data on volumes and prices of inputs and outputs to calculate the input-spending and output-earnings, the allocative efficiency identifies the wrong mix in input-spending and output-earnings rather than in the physical inputs and physical outputs. The concept of allocative efficiency will be further considered in Chapter 3.

2.4 Productivity

As explained in Section 2.3, the concept of "***efficiency***" provides a relative comparison for all production activities. Besides efficiency, another concept termed "***productivity***" can also be used to compare the performance of production activities at a given point of time. Although efficiency and productivity are closely related to each other, they are fundamentally different concepts. If the production process only involves a single input and a single output, then productivity can be simply defined as the ratio of the output to the input (i.e., output per unit of input), and sometimes is referred to as ***partial productivity***. In contrast to partial productivity, the measurement of productivity becomes more complicated in a multiple-input and multiple-output production technology because aggregation of inputs and outputs is required for the construction of productivity. Note that the ratio of aggregate output to aggregate input is also called ***total factor productivity***. Generally, the concept of productivity is related to the efficiency in the following sense: If the underlying production technology exhibits constant returns to scale, then all of the efficient production activities have the same score of productivity. However, this is not always true when decreasing or increasing returns to scale is appropriate. For example, if the underlying production technology exhibits decreasing returns to scale, then the score of productivity for the efficient production activities declines as more and more of the input is used. In other words, a production activity may be technically efficient but may still be able to improve its productivity by scale improvements.

Thus far, productivity has been considered at a given point of time. If panel data is available, then it is possible to examine the changes in productivity over time. For instance, the analysis of "***productivity change***" can be useful to examine the impact of policy or management decisions over a long period. It is also possible to examine the ***drivers of productivity change***. That is, one can examine whether the productivity growth was driven by efficiency improvements, or whether it was driven by scale improvements, or by technological improvements (i.e., there is an upward shift in the production technology). The thesis focuses on the ***Malmquist index*** [33] for measuring productivity change over time. Since the Malmquist index was first introduced in productivity literature by Caves et al. [34], there has been a great deal of interest in empirical studies quantifying productivity change. Consider two periods t and $t+1$, respectively. The input-oriented Malmquist index at period t is given as

$$M^t = \frac{D^t(x_{t+1}, y_{t+1})}{D^t(x_t, y_t)}, \tag{2.10}$$

where $D(\cdot)$ is the input distance function defined in Eq. (2.5). Note that the superscript "I" is dropped for simplicity (e.g., $D^{I,t}(x_t, y_t)$). This index compares two input-output combinations, (x_t, y_t) and (x_{t+1}, y_{t+1}), to a reference production technology at period t. Similarly, a comparison at period $t+1$ is

$$M^{t+1} = \frac{D^{t+1}(x_{t+1}, y_{t+1})}{D^{t+1}(x_t, y_t)}. \tag{2.11}$$

To avoid an arbitrary choice of a reference production technology, the ***input-oriented Malmquist index*** can be conveniently defined as the geometric mean of both the M^t and M^{t+1}. Formally,

$$M = \left[\frac{D^t(x_{t+1}, y_{t+1})}{D^t(x_t, y_t)} \times \frac{D^{t+1}(x_{t+1}, y_{t+1})}{D^{t+1}(x_t, y_t)}\right]^{\frac{1}{2}}. \tag{2.12}$$

Here, the terms $D^t(x_t, y_t)$ and $D^{t+1}(x_{t+1}, y_{t+1})$ are the measurements within the same period, while the terms $D^t(x_{t+1}, y_{t+1})$ and $D^{t+1}(x_t, y_t)$ are the intertemporal comparisons. As a consequence, M measures the productivity change between periods t and $t+1$. If M is greater, equal, or smaller than unity, the productivity shows, on average, decline, stagnation, or growth between periods t and $t+1$.

In recent years, the ***decomposition of productivity change*** into a technical efficiency change component and a technical change component using the Malmquist index has been widely used. The above definition of the input-oriented Malmquist index can be decomposed as follows:

$$M = \frac{D^{t+1}(x_{t+1}, y_{t+1})}{D^t(x_t, y_t)} \times \left[\frac{D^t(x_{t+1}, y_{t+1})}{D^{t+1}(x_{t+1}, y_{t+1})} \times \frac{D^t(x_t, y_t)}{D^{t+1}(x_t, y_t)}\right]^{1/2}. \tag{2.13}$$

The component outside the square brackets in Eq. (2.13) captures ***technical efficiency change*** (or Catch-up) between periods t and $t+1$, while the component inside the square brackets measures the ***shift of production frontier*** (technical change, or frontier shift, innovation) over time. In general, for both component indices in Eq. (2.13), more than 1 indicates regress, while equal to 1 and less than 1 show the status quo and progress, respectively. Figure 2.4 illustrates the input-oriented Malmquist index and its component indices with a single input and single output production technology,

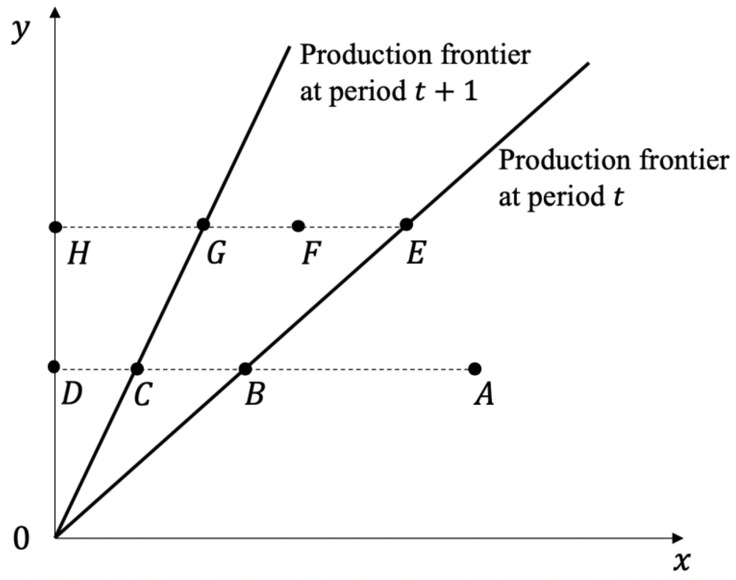


Figure 2.4 The concept of input-oriented Malmquist index.

It is worth noting that in Fig. 2.4, the constant returns to scale is implicitly assumed. It has been shown that a Malmquist index may not correctly measure productivity change when ***variable returns to scale*** (VRS) is assumed [35–37]. In Fig. 2.4, point A and point B are the same production activity obtained at period t and $t+1$, respectively. The efficiency change is then expressed as $(HF/HG)/\ (DA/DB)$, and the technical change is $[(HF/HE)/(HF/HG) \times (DA/DB)/(DA/DC)]^{1/2}$, which can be simplified as $[(HG/HE) \times (DC/DB)]^{1/2}$.

On the other hand, as argued by Maniadakis et al. [38], the Malmquist index may not give a full picture of the source of productivity change since the impact of allocative efficiency change is not incorporated. However, given the purpose of profit-ratio maximization, it is possible to measure the Malmquist-type indices with consideration for allocative efficiency. In this thesis, a ***profit-ratio change index*** is also proposed, which can be applied to panel data to measure productivity growth and suitable for situations when producers desire to maximize revenue and minimize expenses simultaneously. Further details will be discussed in Chapter 3.

2.5 Nonparametric techniques for efficiency and productivity analysis

Both the concepts of efficiency and productivity are established based on unknown production technology. Thus, the mathematical formulation of the production technology becomes a key issue for measuring efficiency and productivity. As mentioned in Chapter 1, this thesis focuses on two nonparametric techniques, that is, Data Envelopment Analysis (DEA) and Stochastic Nonparametric Envelopment of Data (StoNED). This section covers the basics and some additional material on DEA and StoNED.

In general, DEA integrates two general stages of a) formulating linear programming models for constructing a ***piecewise linear production technology***, and b) gauging the efficiency of each production activity (i.e., Decision-Making Units, DMUs) relative to the estimated production frontier. Recall that our production process transforms m inputs to s outputs. Suppose we observe the activities of n producers indexed by j, $j = 1, \dots, n$. Assuming the underlying production technology T satisfies A.1-A.5 in Section 2.2, a DEA representation of the production technology under constant returns to scale (CRS) is then formulated as

$$T^{DEA} = \left\{ (x, y) \in R_+^{m+s} : x_i \geq \sum_{j=1}^{n} \lambda_j x_{ij}, i = 1, \dots, m, \right. \\ \left. y_r \leq \sum_{j=1}^{n} \lambda_j y_{rj}, r = 1, \dots, s, \lambda_j \geq 0 \right\}, \tag{2.14}$$

where $\lambda_j \geq 0, j = 1, \dots, n$ is an ***intensity variable*** that enables us to scale up or down the observed input-output combinations (i.e., the observed production activities) to construct unobserved but feasible input-output combinations (i.e., the unobserved feasible production activities). Therefore, $\sum_{j=1}^{n} \lambda_j x_{ij}\ (i = 1, \dots, m)$ and $\sum_{j=1}^{n} \lambda_j y_{rj}\ (r = 1, \dots, s)$ are referred to as feasible inputs and outputs of virtual production activity. It is worth noting that $\sum_{j=1}^{n} \lambda_j x_{ij}\ (i = 1, \dots, m)$ and $\sum_{j=1}^{n} \lambda_j y_{rj}\ (r = 1, \dots, s)$ are the ***convex combination*** of observed inputs and outputs, respectively. That is if we set $k\lambda'_j = \lambda_j, k > 0$, then we have $\sum_{j=1}^{n} \lambda'_j x_{ij}\ (i = 1, \dots, m)$ and $\sum_{j=1}^{n} \lambda'_j y_{rj}\ (r = 1, \dots, s)$ where

$\sum_{j=1}^{n} \lambda'_j = 1$. In other words, virtual production activity is a convex combination of observed production activities. It is now clear that Eq. (2.14) implicitly assumes $T = kT, k > 0$ for the underlying production technology T, which implies constant returns to scale (see, Section 2.3). It is also possible to impose different assumptions on returns to scale such as non-increasing, non-decreasing, and variable returns to scale by adding the convexity constraints $\sum_{j=1}^{n} \lambda_j \leq 1$, $\sum_{j=1}^{n} \lambda_j \geq 1$, and $\sum_{j=1}^{n} \lambda_j = 1$, respectively. Further, Eq. (2.14) requires each producer to use at least one positive input to produce at least one positive output (such production activities are also referred to as ***semi-positive*** input-output combinations).

Note that Eq. (2.14) satisfies the ***minimum extrapolation principle***, which implies T^{DEA} is the smallest set that contains all observed production activities and meanwhile satisfies A.1 to A.5 and CRS. From the perspective of efficiency or productivity analysis, T^{DEA} provides a performance standard for all observed production activities in the sense that any production activity not on the estimated production frontier (i.e., the boundary of T^{DEA}) can be scaled against a convex combination of the observed production activities on a subset of the estimated production frontier. Here, a simple model of DEA is introduced for measuring the input-oriented technical efficiency (i.e., TE^I).

$$\begin{aligned} &\gamma^* = \min_{\gamma,\lambda} \gamma \\ &s.t. \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq \gamma x_{io}, i = 1, \ldots, m; \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, r = 1, \ldots, s; \\ &\lambda_j \geq 0, j = 1, \ldots, n, \end{aligned} \tag{2.15}$$

where the subscript "o" represents the production activity (observation) under evaluation. Since $\gamma = 1$, $\lambda_o = 1$, $\lambda_j = 0$ $(j \neq o)$ is a feasible solution to Eq. (2.15), the optimal solution denoted by θ^* is less than or equal to 1. On the other hand, the second constraint ensures that any λ_j is positive due to the assumption of semi-positive data. Hence, the first constraint implies θ^* is greater than 0. Putting this all together, we have $0 < \theta^* \leq 1$. Therefore, Eq. (2.15) is consistent with the definition of input-oriented technical efficiency in Eq. (2.2). Moreover, in the literature of DEA, Eq. (2.15) is also called the input-oriented CCR model (see, Charnes, Cooper, and Rhodes [1]). If $\theta^* = 1$, the evaluated production activity is on the estimated production frontier in the sense that the current level of inputs cannot be proportionally reduced. Otherwise, the evaluated production activity is underneath the estimated production frontier because $0 < \theta^* < 1$ indicates the current level of inputs can be proportionally reduced by a positive rate. Recall the concepts of efficiency described in Section 2.3. If we ignore the existence of slacks, observed production activity is evaluated to be technically efficient if $\theta^* = 1$ and to be technically inefficient if $0 < \theta^* < 1$.

The data generating process suggested by DEA implies that any deviation from the production frontier can be considered as an expression of inefficiency. Hence, the efficiency resulted from the DEA models is sensitive to mismeasurement or outliers. To address this issue, the thesis considers a stochastic nonparametric approach such as StoNED in which the data generating process assumes the deviations are the results of both inefficiency and noise. Similar to DEA, StoNED is a unified framework that combines a) the stochastic nonparametric estimation of an unknown production frontier and b) the efficiency analysis for each observed production activity.

To interpret the basic concepts of StoNED, a simple case is considered where the production process involves multiple inputs $x \in R_+^m$ and a single output $y \in R_+$. Suppose we observe the activities of n producers indexed by j, $j = 1, \dots, n$. Instead of a set representation of production technology, an unknown production function $f: R_+^m \to R_+$ is introduced, which gives the maximum possible output that can be produced from the given level of inputs. The statistical model for estimating the unknown f is formally given as

$$y_j = f(x_{ij}) + \varepsilon_j, i = 1, \dots, m, j = 1, \dots, n, \tag{2.16}$$

where $\varepsilon_j = v_j - u_j$ is a composite error term that consists of the stochastic noise term v_j and the nonnegative inefficiency term u_j. Specifically, the following assumptions are made for Eq. (2.16):

S.1. The stochastic noise terms v_j have an unknown symmetrical distribution with a zero mean and a finite variance $\sigma_v^2 < \infty$.

S.2. The nonnegative inefficiency terms $u_j \geq 0$ have an unknown asymmetric distribution with a positive expected value $\mu > 0$ and a finite variance $\sigma_u^2 < \infty$.

S.3. Terms v_j and u_j and hence $\varepsilon_j, (j = 1, \ldots, n)$ are homoscedastic (i.e., σ_v^2 and σ_u^2 are constant across all observed production activities) and statistically independent of each other (i.e., $\sigma_\varepsilon^2 = \sigma_v^2 + \sigma_u^2$).

S.4. Terms v_j and u_j and hence $\varepsilon_j, (j = 1, \ldots, n)$ are statistically independent of inputs x_{ij}, $(j = 1, \ldots, n)$.

In the StoNED framework, the unknown production function f is estimated with convex nonparametric least squares (CNLS), which is a nonparametric regression technique proposed by Kuosmanen [14]. However, due to the above assumptions, the expected value of the composite error term becomes $E(\varepsilon_j) = -E(u_j) = -\mu < 0$. Thus, applying the least squares estimation to Eq. (2.16) violates the Gauss-Markov properties. This issue can be resolved by rephrasing the model as $y_j = [f(x_{ij}) - \mu] + [\varepsilon_j + \mu], i = 1, \ldots, m, j = 1, \ldots, n$. The shape of the unknown production frontier is then estimated by the following ***convex nonparametric least squares*** (CNLS) problem:

$$\begin{aligned}
&\min_{\alpha,\beta,\varepsilon} \sum_{j=1}^{n} \left(\varepsilon_j^{CNLS}\right)^2 \\
&s.t. \\
&y_j = \alpha_j + \sum_{i=1}^{m} \beta_{ij} x_{ij} \quad + \varepsilon_j^{CNLS}, \forall j = 1, \ldots, n, \\
&\alpha_j + \sum_{i=1}^{m} \beta_{ij} x_{ij} \leq \alpha_z + \sum_{i=1}^{m} \beta_{iz} x_{iz}, \forall z, j = 1, \ldots, n, \\
&\beta_{ij} \geq 0, \forall i = 1, \ldots, m, j = 1, \ldots, n,
\end{aligned} \tag{2.17}$$

where ε_j^{CNLS} is an estimator of $\varepsilon_j + \mu$. Consistency of this estimator is proved by Seijo et al. [39] and Lim and Glynn [40]. The first constraint contains a set of linear regression equations where parameters α_j and β_{ij} define tangent hyperplanes to an unknown function $h(x_{ij}) = f(x_{ij}) - \mu$. Note that α_j and β_{ij} are specific to each production activity, and thus there are n different hyperplanes used for characterizing the unknown function. The second constraint imposes concavity by applying Afrait inequalities (see Afrait [41]). The Afrait inequalities ensure all hyperplanes not associated with j must be above j's hyperplane. The last constraint imposes the monotonicity for the unknown function. Based on the solutions to Eq. (2.17), it is possible to apply the minimum extrapolation principle (e.g., Eq. (2.14)) to estimate the smallest function $h(x_{ij}) = f(x_{ij}) - \mu$ that envelops all observed production activities. If we further estimate the expected inefficiency μ from the solution $\hat{\varepsilon}_j^{CNLS}$, the unknown production function f can be then restored by adding the estimated expected inefficiency $\hat{\mu}$ to the estimated function $\hat{h}^{CNLS}(x_{ij})$ as $\hat{f}(x_{ij}) = \hat{h}^{CNLS}(x_{ij}) + \hat{\mu}$.

It is worth noting that gauging the distance from an observed production activity to the estimated production frontier cannot be interpreted as the inefficiency because all observations are subject to noise in the stochastic setting. In the cross-sectional setting, one may use the JLMS estimator [42] to estimate the conditional mean $E(u_j|\varepsilon_j)$ by imposing further parametric assumptions for v_j and u_j. The JMLS estimator may be sufficient for the purpose of relative efficiency rankings. However, it cannot be used directly for further productivity analysis (e.g., the construction of the Malmquist index) since $E(u_j|\varepsilon_j)$ never approaches u_j as the number of observations approaches infinity. To solve this issue, the use of the panel data is considered in a fully nonparametric setting. Further details will be discussed in Chapter 5.

2.6 Concluding remarks

The chapter provides the necessary materials for analyzing efficiency and productivity. Based on the multi-input and multi-output production technology, I described the concepts of efficiency and productivity. Since the primary theoretical approach of this thesis is based on the nonparametric techniques, I also summarized the basics of Data Envelopment Analysis (DEA) and Stochastic Nonparametric Envelopment of Data (StoNED).

Chapter 3

Productivity changes regarding allocative efficiency

3.1 Introduction

The purpose of this chapter is to develop a new approach for measuring productivity change regarding profit-ratio maximization. Such performance analysis can be applied to profit-seeking organizations or industries where producers are both cost minimizers and revenue maximizers. A profit-ratio efficiency measure and Malmquist-type indices decompositions are also developed, which account for the contribution of allocative efficiency. The proposed approach is further extended to categorize observed production activities into six different groups based on their technical and allocative performance to derive valuable information for organization management.

In Chapter 2, I have introduced the theoretical basis of DEA. Using a DEA methodology, Färe et al. [43] developed a DEA-based Malmquist productivity index, which measures the productivity change between two periods and further applied it to empirical studies [44,45]. However, as argued by Maniadakis et al. [38]), the Malmquist index may not give a full picture of the source of productivity change since the impact of allocative efficiency change is not accounted for (see also Coelli et al. [46]). Maniadakis et al. [38] have developed a cost Malmquist productivity index applicable when producers are cost minimizers, and the firm-level input price data are available. Following the study of Maniadakis et al. [38], an allocation Malmquist productivity index with the underlying assumption of cost minimization is also proposed by Zhu et al. [47]. In this chapter, the purpose of profit-ratio maximization is considered when adopting a Malmquist-type index. The use of profit ratio is due to the following considerations: As noted by Georgescu-Roegen [31], the ratio of revenue to expenses (i.e., profit ratio) is independent of the scale of production, and thus it can be considered as an appropriate performance criterion on which to evaluate performance of activities of varying sizes. Furthermore, the use of profit ratio also simplifies the performance analysis even when some activities earn negative or zero profits, whereas the use of profit (which is commonly defined as the difference between revenue and expenses) may be problematic (Cooper et al. [48]).

Instead of using quantity data described in Chapter 2, the efficiency measures and Malmquist-type indices are developed by using a ***value-based measure*** [49–51]. A distinctive feature of the value-based measure is the use of all feasible input-spending and output-earnings, and it requires no direct knowledge of prices. Even when the prices are observable in some situations, as pointed out by Camanho and Dyson [52], the input and output prices in real-life markets are not exogenously given but can depend on negotiation. Therefore, the efficiency measures based on the fixed price assumption in DEA may be of limited use. Also, as argued by Fukuyama and Weber [32], the price data used for analyzing the efficiencies of financial institutions are usually synthetically constructed, which means it can distort measures of allocative efficiency. Another reason for applying a value-based measure is because of the consideration of heterogeneity in physical inputs and physical outputs. As argued by Sahoo et al. [49], if inputs or outputs are heterogeneous, the construction of factor-based production technology set in DEA becomes problematic. Since the value-based measure considers the price information and has a common unit of both inputs and outputs, a value-based technology set is used.

The current chapter is organized as follows. Section 3.2 introduces the basic concepts and notations used for deriving the allocative efficiency in terms of profit-ratio maximization. Section 3.3 defines the profit-ratio change index. Section 3.4 presents the decompositions of the profit-ratio change index as well as its component indices. Concluding remarks are given in the last section.

3.2 Allocative efficiency regarding profit-ratio maximization

This section is structured beginning with a description of a value-based technology and then presents the efficiency measures, which include the graph measure of technical efficiency, radial measures of technical efficiency, and profit ratio efficiency. Then I show how those efficiency measures can be used to derive the allocative efficiency regarding profit-ratio maximization.

Consider a set of n observations on production activities. The input-spending and output-earnings vectors of each observation, the jth producer $(j = 1, \dots, n)$, are denoted as $\bar{x}_j = (\bar{x}_{1j}, \dots, \bar{x}_{mj})' \in R_+^m$ and $\bar{y}_j = (\bar{y}_{1j}, \dots, \bar{y}_{sj})' \in R_+^s$, respectively. The superscript "$'$" denotes the transpose of vectors. Assume that the input-spending and output-earnings vectors are measured in a common monetary unit (e.g., dollars, cents, or pounds). According to Sahoo et al. [49], the value-based technology can be represented as

$$T_{\bar{x},\bar{y}} = \{(\bar{x}, \bar{y}) \in R_+^{m+s} : \bar{x} \ \ can \ \ produce \ \ \bar{y}\}. \tag{3.1}$$

In contrast to the production possibility set T defined in Eq. (2.1), $T_{\bar{x},\bar{y}}$ is a set that comprises all feasible input-spending and output-earnings vectors. That is, all inputs and outputs should be measured in monetary terms. Assuming A.1 to A.5 in Section 2.2, the DEA representation of $T_{\bar{x},\bar{y}}$ under constant returns to scale (CRS) is then given by

$$T_{\bar{x},\bar{y}}^{DEA} = \left\{ (\bar{x}, \bar{y}) \in R_+^{m+s} : \bar{x}_i \geq \sum_{j=1}^{n} \lambda_j \bar{x}_{ij}, i = 1, \dots, m, \right. \\ \left. \bar{y}_r \leq \sum_{j=1}^{n} \lambda_j \bar{y}_{rj}, r = 1, \dots, s, \lambda_j \geq 0 \right\}. \tag{3.2}$$

Relative to $T_{\bar{x},\bar{y}}^{DEA}$, the value-based measure of the input-oriented technical efficiency (TE^I), the output-oriented technical efficiency (TE^O), and the graph measure of technical efficiency (TE^{GR}) are defined as

$$TE^{I} = \inf_{\gamma}\{\gamma: (\gamma\bar{x}, \bar{y}) \in T_{\bar{x},\bar{y}}^{DEA}, 0 < \gamma \leq 1\}, \tag{3.3}$$

$$TE^{O} = \sup_{\omega}\{\omega: (\bar{x}, \omega\bar{y}) \in T_{\bar{x},\bar{y}}^{DEA}, \omega \geq 1\}, \tag{3.4}$$

$$TE^{GR} = \inf_{\theta}\{\theta: (\theta\bar{x}, \theta^{-1}\bar{y}) \in T_{\bar{x},\bar{y}}^{DEA}, 0 < \theta \leq 1\} \tag{3.5}$$

, respectively.

The computational aspect of TE^{I}, TE^{O}, and TE^{GR} are provided as follows. The graph measure of technical efficiency (TE^{GR}) under CRS in Eq. (3.5) is calculated by the following programming problem [21,22]:

$$\begin{aligned} &\theta^{*} = \min_{\theta,\lambda} \theta \\ &s.t. \\ &\sum_{j=1}^{n} \lambda_j \bar{x}_{ij} \leq \theta \bar{x}_{io}, i = 1, \dots, m; \\ &\sum_{j=1}^{n} \lambda_j \bar{y}_{rj} \geq \theta^{-1} \bar{y}_{ro}, r = 1, \dots, s; \\ &\lambda_j \geq 0, j = 1, \dots, n, \end{aligned} \tag{3.6}$$

where $\bar{x}_{io}$ and $\bar{y}_{ro}$ are the i th input-spending and r th output-earnings for the evaluated production activity, respectively. The program Eq. (3.6) can be transformed into the equivalent linear programming problem below, by imposing $\gamma = \theta^2$ and $\mu_j = \theta\lambda_j$:

$$\begin{aligned} &\gamma^{*} = \min_{\gamma,\mu} \gamma \\ &s.t. \\ &\sum_{j=1}^{n} \mu_j \bar{x}_{ij} \leq \gamma \bar{x}_{io}, i = 1, \dots, m; \\ &\sum_{j=1}^{n} \mu_j \bar{y}_{rj} \geq \bar{y}_{ro}, r = 1, \dots, s; \\ &\mu_j \geq 0, j = 1, \dots n. \end{aligned} \tag{3.7}$$

Note that the solution γ^{*} of Eq. (3.7) is equivalent to the input-oriented technical efficiency (TE^{I}) defined in Eq. (3.3). Therefore, under CRS, the square of the graph measure of technical efficiency is equal to the input-oriented technical efficiency. In addition, the input-oriented technical efficiency is equal to the reciprocal of output-oriented technical efficiency if and only if $T^{DEA}_{\bar{x},\bar{y}}$ exhibits CRS [53,54]. Thus, the relations among TE^{I}, TE^{O}, and TE^{GR} can be represented as follows:

$$(TE^{GR})^2 = TE^{I} = \frac{1}{TE^{O}} \ . \tag{3.8}$$

Consider the production activities whose underlying behavioral objectives are the maximization of profit ratio. The following function is used to calculate the maximum profit ratio for the observed production activities:

$$\begin{aligned}\pi(\bar{x}^{*}_{io}, \bar{y}^{*}_{ro}) = \sup_{\bar{x}_i, \bar{y}_r, \lambda_j} \Bigg\{ &\pi(\bar{x}_i, \bar{y}_r) = \frac{\sum_{r=1}^{s} \bar{y}_r}{\sum_{i=1}^{m} \bar{x}_i} : \bar{x}_i = \sum_{j=1}^{n} \lambda_j \bar{x}_{ij} \,, \\ &\bar{y}_r = \sum_{j=1}^{n} \lambda_j \bar{y}_{rj} \,, \bar{x}_{io}, \geq \bar{x}_i, \bar{y}_{ro} \leq \bar{y}_r, \lambda_j \geq 0 \Bigg\},\end{aligned} \tag{3.9}$$

where $\pi(\bar{x}_i, \bar{y}_r) = \sum_{r=1}^{s} \bar{y}_r \,/\, \sum_{i=1}^{m} \bar{x}_i$ represents the profit-ratio function that maximizes the ratio of revenue to expenses, and $(\bar{x}_i, \bar{y}_r) \in T^{DEA}_{\bar{x},\bar{y}}$. This function ensures that, for the evaluated production activity, a specific level of maximum profit ratio can be observed relative to its input-spending and output-earnings. Note that Eq. (3.9) is a fractional programming problem. It can be transformed into the linear programming problem below, by introducing a positive scalar $\xi \in R_{++}$.

$$\begin{aligned}&\sum_{r=1}^{s} \hat{y}^{*}_{ro} = \max_{\hat{x}, \hat{y}, \hat{\lambda}, \xi} \sum_{r=1}^{s} \hat{y}_r \\ &s.t. \\ &\sum_{i=1}^{m} \hat{x}_i = 1; \\ &\xi \hat{x}_{io} \geq \hat{x}_i = \sum_{j=1}^{n} \hat{\lambda}_j \bar{x}_{ij} \,, i = 1, \dots, m; \\ &\xi \hat{y}_{io} \leq \hat{y}_r = \sum_{j=1}^{n} \hat{\lambda}_j \bar{y}_{rj} \,, r = 1, \dots, s; \\ &\hat{\lambda}_j \geq 0, j = 1, \dots, n,\end{aligned} \tag{3.10}$$

where $\hat{x}_i = \xi \bar{x}_i$, $\hat{y}_i = \xi \bar{y}_r$, $\hat{\lambda}_j = \xi \lambda_j$, $\xi > 0$. The relationship between the solution of

Eq. (3.9) and that of the program Eq. (3.10) is explained in Cooper et al. [48]: Let an optimal solution of the program Eq. (3.10) be $(\xi^*, \hat{x}_{io}^*, \hat{y}_{ro}^*, \hat{\lambda}_j^*)$. Since $\xi^* > 0$, the optimal solution of Eq. (3.9) can be obtained from $\bar{x}_{io}^* = \hat{x}_{io}^*/\xi^*$, $\bar{y}_{ro}^* = \hat{y}_{ro}^*/\xi^*$, and $\lambda_j^* = \hat{\lambda}_j^*/\xi^*$.

Given the maximum profit ratio $\pi(\bar{x}_{io}^*, \bar{y}_{ro}^*)$, a profit-ratio boundary for the evaluated production activity is defined as follows:

$$Iso\ \ \pi(\bar{x}_{io}^*, \bar{y}_{ro}^*) = \left\{(\bar{x}, \bar{y}) \in R_+^{m+s} : \frac{\sum_{r=1}^{s} \bar{y}_r}{\sum_{i=1}^{m} \bar{x}_i} = \pi(\bar{x}_{io}^*, \bar{y}_{ro}^*)\right\}. \tag{3.11}$$

Eq. (3.11) contains input-spending and output-earnings vectors that are feasible at the level of the maximum profit ratio $\pi(\bar{x}_{io}^*, \bar{y}_{ro}^*)$. Similar to Eq. (2.9) in Chapter 2, the value-based measure of profit-ratio efficiency is defined as

$$PE = \frac{\pi(\bar{x}_{io}, \bar{y}_{ro})}{\pi(\bar{x}_{io}^*, \bar{y}_{ro}^*)}, \tag{3.12}$$

which is a measure of the extent to which the actual profit ratio expressed in the numerator, falls short of achieving the maximum profit ratio expressed in the denominator. Eq. (3.12) satisfies $0 < PE \leq 1$.

Moving on, the above efficiency measures can be related to the measure of allocative efficiency. For this, I show that the profit-ratio efficiency PE is less than or equal to the input-oriented technical efficiency TE^I (that is the square of the graph measure of technical efficiency TE^{GR}) in the following sense:

Proposition: If TE^{GR} and TE^I are obtained from the programs Eqs. (3.6) and (3.7), respectively, and PE is defined as Eq. (3.12), then for any evaluated production activity,

$$PE \leq (TE^{GR})^2 = TE^I. \tag{3.13}$$

Proof. Let an optimal solution for the programs Eqs. (3.6) and (3.7) be (θ^*, λ_j^*) and (γ^*, μ_j^*), respectively. Then, $(\theta^* \bar{x}_{io}, {\theta^*}^{-1} \bar{y}_{ro}, \lambda_j^*)$ is feasible for the program Eq. (3.10). Hence, it follows that $\sum_{r=1}^{s} {\theta^*}^{-1} \bar{y}_{ro} / \sum_{i=1}^{m} \theta^* \bar{x}_{io} \leq \sum_{r=1}^{s} \bar{y}_{ro}^* / \sum_{i=1}^{m} \bar{x}_{io}^*$. This leads to $\frac{\sum_{r=1}^{s} \bar{y}_{ro} / \sum_{i=1}^{m} \bar{x}_{io}}{\sum_{r=1}^{s} \bar{y}_{ro}^* / \sum_{i=1}^{m} \bar{x}_{io}^*} \left(= \frac{\pi(\bar{x}_{io}, \bar{y}_{ro})}{\pi(\bar{x}_{io}^*, \bar{y}_{ro}^*)}\right) \leq {\theta^*}^2 = \gamma^*$. □

According to Eq. (3.13), the relationship between the profit-ratio efficiency and radial measures of technical efficiencies can be expressed as

$$\pi(\bar{x}_{io}^{*}, \bar{y}_{ro}^{*}) \geq \frac{1}{\gamma^{*}} \frac{\sum_{r=1}^{s} \bar{y}_{ro}}{\sum_{i=1}^{m} \bar{x}_{io}} \tag{3.14}$$

which can be rewritten as either

$$\pi(\bar{x}_{io}^{*}, \bar{y}_{ro}^{*}) \geq \frac{\sum_{r=1}^{s} \bar{y}_{ro}}{\sum_{i=1}^{m} (\gamma^{*} \bar{x}_{io})}, \tag{3.15}$$

or

$$\pi(\bar{x}_{io}^{*}, \bar{y}_{ro}^{*}) \geq \frac{\sum_{r=1}^{s} \left(\frac{1}{\gamma^{*}} \bar{y}_{ro}\right)}{\sum_{i=1}^{m} \bar{x}_{io}}. \tag{3.16}$$

The expression Eq. (3.15) is related to the input-oriented technical efficiency measure defined in Eq. (3.3). It becomes equality when there is no distortion in the actual input-spending mix. Similarly, the expression in Eq. (3.16) is related to the output-oriented technical efficiency measure in Eq. (3.4), and it becomes equality when there is no distortion in the actual output-earnings mix.

Figures 3.1 and 3.2 depict the state of one production activity when there are two inputs and two outputs, respectively. Figure 3.1 illustrates the expression Eq. (3.15), and Fig. 3.1 illustrates the expression Eq. (3.15).

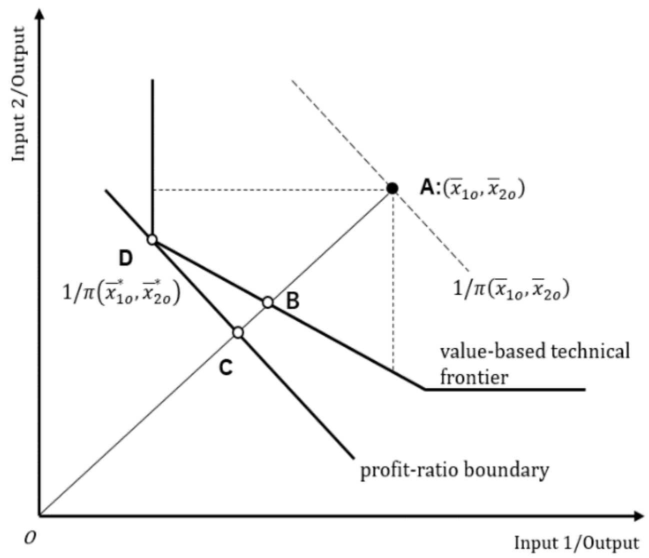


Figure 3.1 Illustration of the input-oriented allocative efficiency regarding profit-ratio maximization.

In Fig. 3.1, the output is fixed at its current level, and the interest is in input reductions. Point A is an evaluated production activity in the interior of the value-based technology. The dashed line passing through A represents the contour of the reciprocal of the profit ratio: $\bar{x}_1/\bar{y}+\bar{x}_2/\bar{y}(=1/(\bar{y}/(\bar{x}_1+\bar{x}_2))\ )=1/\ \pi(\bar{x}_{1o},\bar{x}_{2o})$. To illustrate the profit-ratio boundary for A, I alternatively depict the contour of the reciprocal of the maximum profit ratio in the left panel. Activity A achieves the maximum profit ratio when it is projected on the profit-ratio boundary (say at point D). Now consider the point C which is at the intersection of the profit-ratio boundary through D with the ray from the origin to A, we can obtain the profit-ratio efficiency of A as $0<OC/OA\leq 1$. In addition, we can also form the ratio $0<OB/OA\leq 1$ to obtain a radial measure of input-oriented technical efficiency. Given the input-oriented technical efficiency, we can obtain the projection of A as point B. However, in Fig. 3.1, the profit ratio of this projection can still be increased by moving from B to D along the value-based technical frontier. Since both C and D achieve the same level of profit ratio, we can determine the ratio $0<OC/OB\leq 1$ as a radial measure of "***input-oriented allocative efficiency***." This ratio represents the extent to which the technically efficient point B falls short of achieving the maximum profit ratio because of the wrong mix in the input-spending vectors. Relating all three of these efficiency concepts to each other, we have $OC/OA=(OB/OA)\times(OC/OB)$, which we can verbalize by saying that the profit-ratio efficiency is equal to the product of the input-oriented technical efficiency and the input-oriented allocative efficiency. Denote the input-oriented allocative efficiency as AE^I, we then have $PE=TE^I\times AE^I$.

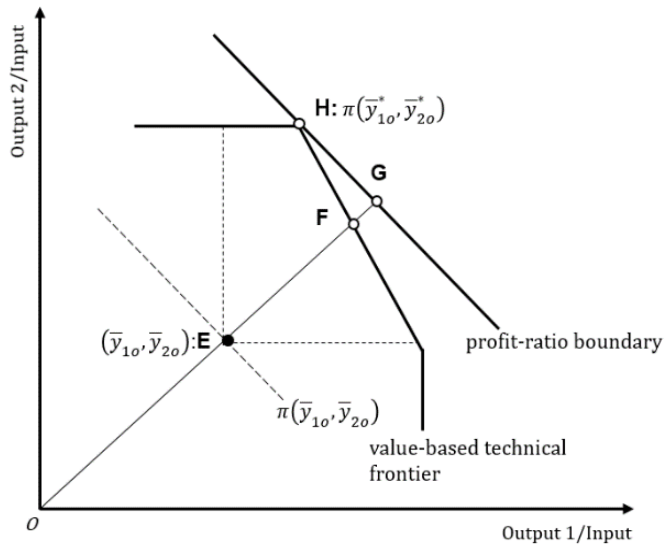


Figure 3.2 Illustration of the output-oriented allocative efficiency regarding profit-ratio maximization.

Similarly, for another production activity E in Fig. 3.2, the maximum profit ratio is at point H and the solid line passing through points H and G is the profit-ratio boundary that is associated with $\bar{y}_1/\bar{x} + \bar{y}_2/\bar{x} (= (\bar{y}_1 + \bar{y}_2)/\bar{x}) = \pi(\bar{y}_{1o}, \bar{y}_{2o})$. The profit-ratio efficiency of E is then obtained as $0 < OE/OG \leq 1$. We can also obtain a radial measure of output-oriented efficiency from the ratio $OF/OE \geq 1$. In addition, we can form the ratio $OG/OF \geq 1$ and call it a radial measure of "***output-oriented allocative efficiency***" because of failure to make the reallocations involved in moving from point F to H along the value-based technical frontier. As a result, we have $OE/OG = (1/(OF/OE)) \times (1/(OG/OF))$. This equation shows that profit-ratio efficiency is the product of the reciprocal of the output-oriented technical efficiency and the reciprocal of the output-oriented allocative efficiency. Let the output-oriented allocative efficiency be AE^O, then $PE = 1/(TE^O \times AE^O)$.

In brief, Figs. 3.1 and 3.2 explain the inequality in Eq. (3.14) maybe caused by either the wrong output-earnings mix or the wrong input-spending mix. However, note at this point that A and E are treated as two different production activities. Now consider both the input-oriented and output-oriented technical efficiencies for the same production activity. It is clear that under CRS, $AE^I = 1/AE^O$ because (a) $PE = TE^I \times AE^I$ and

$PE = 1/(TE^O \times AE^O)$, (b) for the same production activity, profit-ratio efficiency is unchangeable whether the interest is in the input-oriented measure or the output-oriented measure, and (c) under CRS, $TE^I = 1/TE^O$.

I next consider the situation where the allocative efficiency is caused by both the wrong output-earnings mix and the input-spending mix. To gain intuition, let us focus on the inequality $\sum_{r=1}^{s} \theta^{*^{-1}} \bar{y}_{ro} / \sum_{i=1}^{m} \theta^* \bar{x}_{io} \leq \sum_{r=1}^{s} \bar{y}_{ro}^* / \sum_{i=1}^{m} \bar{x}_{io}^*$ (see the proof in Eq. (3.13)) that is related to the graph measure of technical efficiency in Eq. (3.5). This inequality implies that the realization of the maximum profit ratio is not entirely guaranteed by only improving the graph measure of technical efficiency. Since the maximum profit ratio is evaluated by the program Eq. (3.10), as well as the optimal input-spending and output-earnings, the activities can achieve the maximum profit ratio by changing their actual input-spending and output-earnings mixes into the optimal ones. Therefore, the inequality $\sum_{r=1}^{s} \theta^{*^{-1}} \bar{y}_{ro} / \sum_{i=1}^{m} \theta^* \bar{x}_{io} \leq \sum_{r=1}^{s} \bar{y}_{ro}^* / \sum_{i=1}^{m} \bar{x}_{io}^*$ becomes equality when there is no distortion in both actual input-spending and out-earnings mix. In an analogous manner with the input- and output-oriented allocative efficiencies, we can determine $0 < \rho^* \leq 1$ satisfying $\sum_{r=1}^{s} \theta^{*^{-1}} \rho^{*^{-1}} \bar{y}_{ro} / \sum_{i=1}^{m} \theta^* \rho^* \bar{x}_{io} = \sum_{r=1}^{s} \bar{y}_{ro}^* / \sum_{i=1}^{m} \bar{x}_{io}^*$ as the estimated "***graph measure of allocative efficiency***." Let the notation of the graph measure of allocative efficiency be AE^{GR}, we then have $PE = (TE^{GR} \times AE^{GR})^2$. In addition, since (a) $PE = (TE^{GR} \times AE^{GR})^2$ and $PE = TE^I \times AE^I$, (b) profit-ratio efficiency is unchangeable whether the interest is in the input-oriented measure or the output-oriented measure, and (c) under CRS, $(TE^{GR})^2 = TE^I$, it is clear that under CRS, $AE^I = (AE^{GR})^2$.

As a result, the inequality Eq. (3.13) maybe caused by either the wrong output-earnings mix or the input-spending mix, or both. The relations discussed above are summarized below:

(i) $PE = TE^I \times AE^I$;

(ii) $PE = 1/(TE^O \times AE^O)$;

(iii) $PE = (TE^{GR} \times AE^{GR})^2$.

Because the assumption of CRS implies $TE^I = 1/TE^O = (TE^{GR})^2$, we then have $AE^I = 1/AE^O = (AE^{GR})^2$. Therefore, under CRS, the input-oriented allocative efficiency can be derived directly from either the output-oriented measure or the graph measure.

In the rest of this chapter, I focus on the input-oriented measure and drop the superscript "I" for simplicity. The output-oriented measure and the graph measure can be discussed analogously. Formally, given $PE = \pi(\bar{x}_{io}, \bar{y}_{ro})/\pi(\bar{x}_{io}^{*}, \bar{y}_{ro}^{*})$ in Eq. (3.12) and $TE = \gamma^{*}$ in Eq. (3.7), the (input-oriented) allocative efficiency regarding profit-ratio maximization is defined as

$$AE = \frac{\pi(\bar{x}_{io}, \bar{y}_{ro})}{\pi(\bar{x}_{io}^{*}, \bar{y}_{ro}^{*})}\frac{1}{\gamma^{*}} = \frac{PE}{TE}. \tag{3.17}$$

If there is neither the wrong output-earnings mix nor the wrong input-spending mix, that is $AE = 1$, then $PE = TE$, and vice versa.

Note that the commonly used definition of allocative efficiency [20,23] requires exact knowledge of prices, whereas the inaccurate information on prices can distort measures of allocative efficiency [32]. Suppose one has data of the physical inputs and physical outputs (that are both homogeneous), as well as data on input and output prices (that are accurate and may be different across activities). The allocative efficiency obtained using the commonly used definition identifies the existence of the wrong mix in physical inputs and physical outputs, given the price information. In contrast, since the allocative efficiency defined in Eq. (3.17) follows a value-based measure, the data should be in monetary terms (e.g., expenses term). In the case that one uses the data on volumes and prices of inputs and outputs to calculate the input-spending and output-earnings, the allocative efficiency in Eq. (3.17) identifies the wrong mix in input-spending and output-earnings rather than in the physical inputs and physical outputs. The scheme of allocative efficiency defined in a value-based measure was first considered by Tone [50] and subsequently extended by various authors [32,49,55].

3.3 A profit-ratio change index

This section describes a profit-ratio change index regarding profit-ratio maximization. A distinctive feature of this index is the use of profit-ratio boundary (see Eq. (3.11)) for measuring productivity change over time. Assume two periods t and $t+1$, respectively.

Denote the input-spending and output-earnings vectors of the evaluated production activity o in periods t and $t+1$ by $(\bar{x}_{io,t}, \bar{y}_{ro,t})$ and $(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})$, respectively. Let $\gamma^t(\bar{x}_{io,t}, \bar{y}_{ro,t})$ and $\gamma^{t+1}(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})$ be the input-oriented technical efficiencies obtained from Eq. (3.7). Relative to a value-based technology, the input-oriented Malmquist index is defined as:

$$MI^t = \frac{\gamma^t(\bar{x}_{io,t}, \bar{y}_{ro,t})}{\gamma^t(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})}, \tag{3.18}$$

$$MI^{t+1} = \frac{\gamma^{t+1}(\bar{x}_{io,t}, \bar{y}_{ro,t})}{\gamma^{t+1}(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})}, \tag{3.19}$$

$$MI = [MI^t \times MI^{t+1}\ ]^{1/2}. \tag{3.20}$$

Note that the conventional Malmquist index in Chapter 2 is based on quantity data (see Eqs. (2.10) -(2.12)) while Eqs. (3.18) - (3.20) are constructed with input-spending and output-earnings vectors. If MI is greater, equal, or smaller than unity, the productivity shows, on average, decline, stagnation, or growth between periods t and $t+1$.

The profit-ratio change index is defined in terms of the profit-ratio efficiency as follows:

$$PI^t = \frac{\pi^t(\bar{x}_{io,t}, \bar{y}_{ro,t})/\pi^t(\bar{x}^{*t}_{io,t}, \bar{y}^{*t}_{ro,t})}{\pi^t(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})/\pi^t(\bar{x}^{*t}_{io,t+1}, \bar{y}^{*t}_{ro,t+1})}, \tag{3.21}$$

$$PI^{t+1} = \frac{\pi^{t+1}(\bar{x}_{io,t}, \bar{y}_{ro,t})/\pi^{t+1}(\bar{x}^{*t+1}_{io,t}, \bar{y}^{*t+1}_{ro,t})}{\pi^{t+1}(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})/\pi^{t+1}(\bar{x}^{*t+1}_{io,t+1}, \bar{y}^{*t+1}_{ro,t+1})}, \tag{3.22}$$

$$PI = [PI^t \times PI^{t+1}\ ]^{1/2}. \tag{3.23}$$

Here, $\pi^t(\bar{x}^{*t}_{io,t}, \bar{y}^{*t}_{ro,t})$ and $\pi^{t+1}(\bar{x}^{*t+1}_{io,t+1}, \bar{y}^{*t+1}_{ro,t+1})$ are the measurements within the same period, while $\pi^t(\bar{x}^{*t}_{io,t+1}, \bar{y}^{*t}_{ro,t+1})$ and $\pi^{t+1}(\bar{x}^{*t+1}_{io,t}, \bar{y}^{*t+1}_{ro,t})$ are the intertemporal comparisons. The component PI^t in Eq. (3.21) measures the profit-ratio efficiency change regarding period t as the reference period. From Eq. (3.21), we see that the numerator is the profit-ratio efficiency of $(\bar{x}_{io,t}, \bar{y}_{ro,t})$ measured at period t, whereas the denominator is the profit-ratio efficiency of $(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})$ measured at period t. If the evaluated production activity has improved its profit-ratio efficiency from period t to $t+1$, the value of the numerator is less than that of the denominator, and therefore, PI^t is smaller than unity. Similarly, the component PI^{t+1} in Eq. (3.22) is the profit-ratio efficiency change regarding period $t+1$ as the reference period. To avoid an arbitrary choice of a reference period, the profit-ratio change index PI in Eq. (3.23) is defined by the geometric means of PI^t and PI^{t+1}. Here, PI measures the average change of profit-ratio efficiency between periods t and $t+1$. If the index is greater, equal, or smaller than unity, the change of profit-ratio efficiency over time shows, on average, decline, stagnation, or growth between periods t and $t+1$.

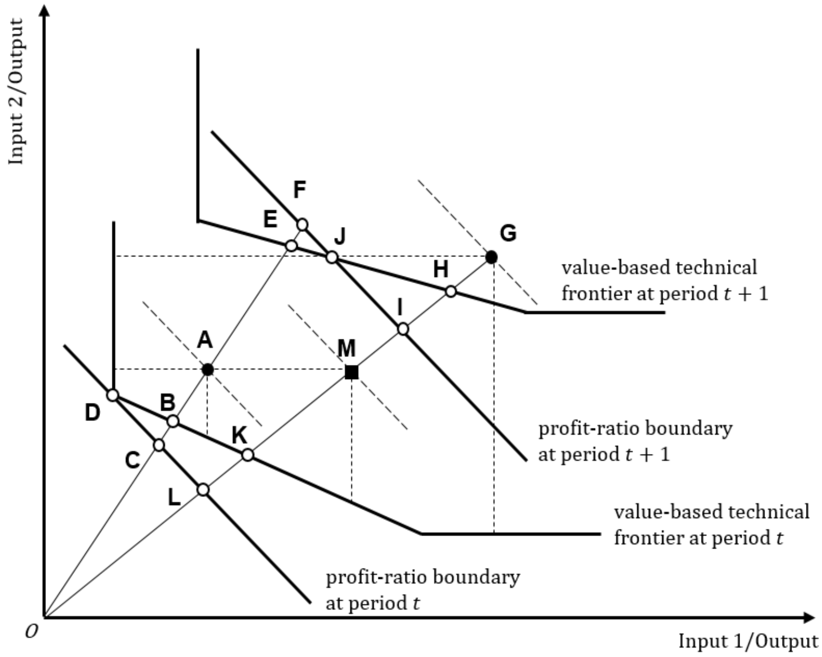


Figure 3.3 The concept of profit-ratio change index.

A simple two-inputs, one-output case is illustrated in Fig. 3.3 to clarify the differences between MI and PI. For the same evaluated production activity (A in period t and G in period $t+1$), its specific level of maximum profit ratios at periods t and $t+1$ are obtained at point D and J, respectively. Graphically, the profit-ratio change index is given by

$$PI = [PI^t \times PI^{t+1}]^{1/2} = \left[\frac{OC/OA}{OL/OG} \times \frac{OF/OA}{OI/OG}\right]^{1/2}, \qquad (3.24)$$

where C and L have the same profit ratio as D, as both points lie on the same profit-ratio boundary which is alternatively depicted as the contour of the reciprocal of the maximum profit ratio in Fig. 3.3. For the same reason, F and I also have the same profit ratio as J.

Similarly, MI is expressed as

$$MI = [MI^t \times MI^{t+1}]^{1/2} = \left[\frac{OB/OA}{OK/OG} \times \frac{OE/OA}{OH/OG}\right]^{1/2}. \quad (3.25)$$

Note that in Fig. 3.3, the value-based technical frontier of period $t+1$ does not encompass the activity A. This implies the intertemporal comparison term $\pi^{t+1}(\bar{x}_{io,t}^{*t+1}, \bar{y}_{ro,t}^{*t+1})$ does not have a feasible solution to Eq. (3.9), and $\gamma^{t+1}(\bar{x}_{io,t}, \bar{y}_{ro,t})$ does not have a feasible solution to the program Eq. (3.7). In such cases, by following the literature of the DEA-based Malmquist productivity index [56], a super efficiency evaluation [57] is adopted to calculate the profit-ratio efficiency in Eq. (3.12), i.e., the profit-ratio efficiency of A measured at period $t+1$ is obtained as $OF/OA > 1$, and the input-oriented technical efficiency is $OE/OA > 1$.

To further clarify the differences between PI and MI, let us consider a (virtual) point M in Fig. 3.3 that lies on the ray from the origin to G. It is clear that M and G have the same mix in both inputs and outputs (i.e., a proportional change in both inputs and outputs will not change their mixes). If we *temporally* treat the point M and the point G as the same activity at periods t and $t+1$, respectively, graphically we will have PI for this activity given by

$$PI = \left[\frac{OL/OM}{OL/OG} \times \frac{OI/OM}{OI/OG}\right]^{\frac{1}{2}} = \frac{OG}{OM}, \quad (3.26)$$

and MI given by

$$MI = \left[\frac{OK/OM}{OK/OG} \times \frac{OH/OM}{OH/OG}\right]^{\frac{1}{2}} = \frac{OG}{OM}. \quad (3.27)$$

Eqs. (3.26) and (3.27) indicate that PI and MI have the same value when there is no average change in the mix of inputs and outputs over time. To illustrate this difference, let us consider the component $(OL/OM)/(OL/OG)$ in Eq. (3.26). This component measures the profit-ratio efficiency change regarding the period t as the reference period. Its numerator (OL/OM) represents the profit-ratio efficiency of M measured at period t and can be decomposed into $(OK/OM) \times (OL/OK)$ (see Eq. (3.17)). Here, OK/OM and OL/OK represent the (input-oriented) technical and allocative efficiencies of M measured at period t, respectively. Similarly, we have the decompositions of the denominator as $OL/OG = (OK/OG) \times (OL/OK)$, where OK/OG and OL/OK represent, respectively, the (input-oriented) technical and allocative efficiencies of G measured at period t. Combing the decompositions of both the numerator and denominator, we then have

$$\frac{OL/OM}{OL/OG} = \frac{OK/OM}{OK/OG} \times \frac{OL/OK}{OL/OK}. \tag{3.28}$$

This makes it clear that the (input-oriented) allocative efficiency is identical at points M and G regarding the period t as the reference period. The second component $(OI/OM)/(OI/OG)$ in Eq. (3.26) can be discussed in an analogous manner. Therefore, when there is no average change in allocative efficiencies over time, PM has the same value as MI.

Returning now to a more general case in Fig. 3.3 that activity A in period t and G in period $t+1$ are the same activity. Similar to the illustration of PI in Eq. (3.24) or MI in Eq. (3.25), the average change in the (input-oriented) allocative efficiency over time (AMI) is given by

$$AMI = [AMI^t \times AMI^{t+1}]^{1/2} = \left[\frac{OC/OB}{OL/OK} \times \frac{OF/OE}{OI/OH}\right]^{1/2}. \tag{3.29}$$

In this thesis, Eq. (3.29) is called the "***allocation Malmquist productivity index***." Just as with the definition of PI and MI, the components AMI^t and AMI^{t+1} in Eq. (3.29) measure the allocative efficiency change regarding the periods t and $t+1$ as the reference period, respectively. To avoid an arbitrary choice of a reference period, AMI is defined by the geometric means of AMI^t and AMI^{t+1}. In addition, if AMI is greater, equal, or similar than unity, the allocative efficiency change over time shows, on average, decline, stagnation, or growth between periods t and $t+1$.

Combing Eqs. (3.24), (3.25), and (3.29), we have the following equity:

$$PI = MI \times AMI. \tag{3.30}$$

Eq. (3.30) implies that PI accounts for the impact of the average change in allocative efficiency over time while MI does not. This difference is further discussed in Section 3.4.

3.4 Decompositions of the profit-ratio change index

This section develops an alternative decomposition of the profit-ratio change index and further clarifies the differences between PI and MI. The decomposition proposed can be further used to identify the drivers of the profit-ratio change over time.

The conventional Malmquist index can be rearranged to show that it is equivalent to the product of a technical efficiency change (or Catch-up) and a technical change (or Frontier shift, innovation) [37,44,45,56]. The profit-ratio change index (PI) can be decomposed into the sources of productivity change in a similar way. The decomposition is formally stated as

$$PI = \frac{\pi^t(\bar{x}_{io,t}, \bar{y}_{ro,t})/\pi^t(\bar{x}^{*t}_{io,t}, \bar{y}^{*t}_{ro,t})}{\pi^{t+1}(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1})/\pi^{t+1}(\bar{x}^{*t+1}_{io,t+1}, \bar{y}^{*t+1}_{ro,t+1})} \times \left[\frac{\pi^t(\bar{x}^{*t}_{io,t+1}, \bar{y}^{*t}_{ro,t+1})}{\pi^{t+1}(\bar{x}^{*t+1}_{io,t+1}, \bar{y}^{*t+1}_{ro,t+1})} \times \frac{\pi^t(\bar{x}^{*t}_{io,t}, \bar{y}^{*t}_{ro,t})}{\pi^{t+1}(\bar{x}^{*t+1}_{io,t}, \bar{y}^{*t+1}_{ro,t})}\right]^{1/2} = PEC \times PTC. \quad (3.31)$$

The component outside the square brackets in Eq. (3.31) captures "profit-ratio efficiency change (PEC)" between periods t and $t+1$. Since the profit-ratio efficiency (PE) compares the actual profit ratio with the maximum profit ratio that lies on the profit-ratio boundary, the term "profit-ratio efficiency change (PEC)" indicates whether the evaluated production activity is getting close to the profit-ratio boundary or not. Returning to Fig. 3.3, PEC is graphically represented by $PEC = (OC/OA)/(OI/OG)$. In this case, $PEC < 1$, $PEC > 1$, $PEC = 1$ imply the profit-ratio efficiency, respectively, progress, regress, and constant between periods t and $t+1$.

On the other hand, the component inside the square brackets consists of two ratios: The first ratio compares maximum profit ratios of the period $t+1$ activity with respect to profit-ratio boundaries of periods t and $t+1$. Similarly, the second ratio inside the brackets compares maximum profit ratios of the period-t activity with respect to profit-ratio boundaries of periods t and $t+1$. Hence, the geometric mean of those two ratios measures the average shift of profit-ratio boundary from the period t to $t+1$, and is referred to as "change of profit-ratio boundary (PTC)." In Fig. 3.3, PTC is illustrated as

$PTC = \left[\frac{OI/OG}{OL/OG} \times \frac{OF/OA}{OC/OA}\right]^{1/2} = \left[\frac{OI}{OL} \times \frac{OF}{OC}\right]^{1/2}$. Graphically, PTC is the average change in maximum profit ratios over two periods. In this case, $PTC < 1$ indicates an improvement in the average change of maximum profit ratios (progress in the profit-ratio boundary) while $PTC > 1$ and $PTC = 1$ indicate, on average, regress and constant change of maximum profit ratios, respectively.

The component index PEC can be further decomposed into the (input-oriented) allocative (AEC) and technical efficiency change (TEC) as follows:

$$\begin{aligned} PEC &= \frac{\left(\pi^t\left(\bar{x}_{io,t}, \bar{y}_{ro,t}\right)/\pi^t\left(\bar{x}^{*t}_{io,t}, \bar{y}^{*t}_{ro,t}\right)\right)/\ \gamma^t\left(\bar{x}_{io,t}, \bar{y}_{ro,t}\right)}{\left(\pi^{t+1}\left(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1}\right)/\pi^{t+1}\left(\bar{x}^{*t+1}_{io,t+1}, \bar{y}^{*t+1}_{ro,t+1}\right)\right)/\gamma^{t+1}\left(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1}\right)} \\ &\times \frac{\gamma^t\left(\bar{x}_{io,t}, \bar{y}_{ro,t}\right)}{\gamma^{t+1}\left(\bar{x}_{io,t+1}, \bar{y}_{ro,t+1}\right)} \\ &= AEC \times TEC. \end{aligned} \tag{3.32}$$

As discussed in Section 3.3, allocative efficiency captures the distortion in the mix of input-spending and/or output-earnings relative to the optimum mix (determined by the profit-ratio efficiency). Therefore, the first component AEC in Eq. (3.32) identifies whether the distortion suggested by allocative efficiency is diminishing or increasing from the period t to $t+1$. In Fig.3.3, AEC is represented as $AEC = \frac{(OC/OA)/(OB/OA)}{(OI/OG)/(OH/OG)} = \frac{OC/OB}{OI/OH}$. The remaining part in Eq. (3.32) is called "Catch-up," which indicates whether the evaluated production activity is getting closer to the value-based technical frontier or not. Reference to Fig. 3.3, TEC is $TEC = (OB/OA)/(OH/OG)$.

The component index PTC can be further decomposed as follows:

$$
\begin{aligned}
&PTC \\
&= \left[\frac{\gamma^{t+1}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})}{\gamma^{t}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})} \times \frac{\gamma^{t+1}(\bar{x}_{io,t},\bar{y}_{ro,t})}{\gamma^{t}(\bar{x}_{io,t},\bar{y}_{ro,t})}\right]^{1/2} \\
&\times \left[\frac{\left(\pi^{t+1}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})/\pi^{t+1}(\bar{x}^{*t+1}_{io,t+1},\bar{y}^{*t+1}_{ro,t+1})\right)/\gamma^{t+1}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})}{\left(\pi^{t}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})/\pi^{t}(\bar{x}^{*t}_{io,t+1},\bar{y}^{*t}_{ro,t+1})\right)/\gamma^{t}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})}\right. \\
&\times \left.\frac{\left(\pi^{t+1}(\bar{x}_{io,t},\bar{y}_{ro,t})/\pi^{t+1}(\bar{x}^{*t+1}_{io,t},\bar{y}^{*t+1}_{ro,t})\right)/\gamma^{t+1}(\bar{x}_{io,t},\bar{y}_{ro,t})}{\left(\pi^{t}(\bar{x}_{io,t},\bar{y}_{ro,t})/\pi^{t}(\bar{x}^{*t}_{io,t},\bar{y}^{*t}_{ro,t})\right)/\gamma^{t}(\bar{x}_{io,t},\bar{y}_{ro,t})}\right]^{1/2} \\
&= TC \times ATC.
\end{aligned} \tag{3.33}
$$

The first bracket in Eq. (3.33) is referred to as “Frontier shift or technical change (TC).” It captures the shift of the value-based technical frontier between periods t and $t+1$. As shown in Fig. 3.3, this term is expressed as $TC = \left[\frac{OH/OG}{OK/OG} \times \frac{OE/OA}{OB/OA}\right]^{1/2} = \left[\frac{OH}{OK} \times \frac{OE}{OB}\right]^{1/2}$. The second square bracket in Eq. (3.33) consist of four component ratios that follow the definition of allocative efficiency in Eq. (3.17). This term will be referred to as “allocation-technical change (ATC).” According to the definition of the profit-ratio function in Eq. (3.9), we have the expressions $\pi^{t+1}(\bar{x}_{io,t},\bar{y}_{ro,t}) = \pi^{t}(\bar{x}_{io,t},\bar{y}_{ro,t})$ and $\pi^{t+1}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1}) = \pi^{t}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})$. Therefore, the second square bracket of Eq. (3.33) can be further simplified as

$$
\begin{aligned}
ATC = \Bigg[&\frac{\pi^{t}(\bar{x}^{*t}_{io,t+1},\bar{y}^{*t}_{ro,t+1})\gamma^{t}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})}{\pi^{t+1}(\bar{x}^{*t+1}_{io,t+1},\bar{y}^{*t+1}_{ro,t+1})\ \gamma^{t+1}(\bar{x}_{io,t+1},\bar{y}_{ro,t+1})} \\
&\times \frac{\pi^{t}(\bar{x}^{*t}_{io,t},\bar{y}^{*t}_{ro,t})\gamma^{t}(\bar{x}_{io,t},\bar{y}_{ro,t})}{\pi^{t+1}(\bar{x}^{*t+1}_{io,t},\bar{y}^{*t+1}_{ro,t})\gamma^{t+1}(\bar{x}_{io,t},\bar{y}_{ro,t})}\Bigg]^{1/2}.
\end{aligned} \tag{3.34}
$$

The four technical efficiencies in Eq. (3.34) construct the term TC which is the shift of the value-based technical frontier. The maximum profit ratios, on the other hand, construct the term PTC that is the change of profit-ratio boundary. Hence, we have $ATC = PTC/TC$. It is clear that the term ATC captures the residual change of profit-ratio boundary from the period t to $t+1$. Furthermore, since the components of ATC follow the definition of allocative efficiency, this residual change reflects the contribution of relative changes of the input-spending and/or output-earnings mix on changes of the maximum profit ratio. In Fig.3.3, this term is

$$ATC = \left[\frac{(OI/OG)/(OH/OG)}{(OL/OG)/(OK/OG)} \times \frac{(OF/OA)/(OE/OA)}{(OC/OA)/(OB/OA)}\right]^{1/2}$$
$$= \left[\frac{OI/OH}{OL/OK} \times \frac{OF/OE}{OC/OB}\right]^{1/2}. \qquad (3.35)$$

The component indices mentioned above can be rearranged as follows:

$$\begin{aligned} PI &= PEC \times PTC \\ &= (AEC \times TEC) \times (TC \times ATC) \\ &= (TEC \times TC) \times (AEC \times ATC) \\ &= MI \times AMI. \end{aligned} \qquad (3.36)$$

Eq. (3.36) shows that the allocation Malmquist productivity index (AMI) is equal to the product of the allocative efficiency change (AEC) and the allocation-technical change (ATC). In general, for any indices or component indices mentioned above, more than 1 indicates regress, while equal to 1 and less than 1 show the status quo and progress, respectively.

3.5 Concluding remarks

Chapter 2 introduced the input-oriented, the output-oriented, and the graph measure of technical efficiencies, as well as the profit-ratio efficiency. Based on these efficiencies, the concept of allocative efficiency regarding profit-ratio maximization is developed. Compared with the conventional efficiency concepts, the new efficiency can be used to identify the wrong mix in both input spending and output earnings.

A profit ratio change index is also proposed in this chapter. It can be applied to panel data to measure productivity growth and suitable for situations when producers desire to maximize revenue and minimize cost simultaneously. To identify the drivers of changes in a profit-ratio change index, the index is decomposed into profit-ratio efficiency change and change of profit-ratio boundary. Furthermore, the profit-ratio efficiency change is decomposed into technical and allocative efficiency change and change of profit-ratio boundary into technical change and allocation technical change. An alternative decomposition is also provided, where the profit-ratio change index is decomposed into the Malmquist input-oriented productivity index and an allocation Malmquist productivity index. The decompositions suggest the method gives a comprehensive understanding of the source of productivity change. As a consequence, the proposed index accounts for the impact of the average change in allocative efficiency over time, while the Malmquist input-oriented productivity index does not. Further, it makes a difference in identifying the drivers of productivity change, whether we account for allocative efficiency or not.

Chapter 4

Productivity changes of Japanese securities companies

4.1 Introduction

There is limited literature in DEA for analyzing the productive performance of securities companies. Examples of recent studies include Fukuyama and Weber [32], Zhang et al. [58], and Zhu et al. [47]. In this section, the efficiency measures and the profit-ratio change index in Chapter 3 are applied to a sample of 37 Japanese securities companies observed from 2011 to 2015. The data were gathered from annual securities reports as published by each securities company. Those annual reports can be found from the investor relations library of each company's homepage or Japan Securities Dealers Association (JSDA).

The securities companies in Japan relied heavily on the brokerage business until the latter half of the 1990s regarding both revenues and business volumes. During this period, the "Big Four" securities (Nomura, Daiwa, Nikko, and Yamaichi) gained a large share of the securities market as many small and medium-sized securities companies were affiliated to them. However, due to the bursting of the bubble economy in 1989, the structure of Big Four oligopoly broke up, and the reforms and deregulation have proceeded in securities markets, e.g., banks were allowed to operate the securities business, and the types of securities businesses became increasingly diverse. As pointed out by Fukuyama and Weber [32], these reforms and deregulation will likely impact the competitive structure and efficiency of financial services in Japan. Because the Japanese securities companies play an important role as intermediaries in the securities markets, it is necessary to analyze their efficiencies, especially the allocative efficiency that relates to the diverse securities businesses. The detailed analysis and the projections regarding allocative efficiency are provided in Section 4.3.

Today, the Japanese securities companies are facing a big challenge in their management under uncertain economic conditions and business environment. According to the annual reports of JSDA, the number of Japanese securities members in JSDA totaled 253 companies at the end of the fiscal year 2016 (excluding foreign securities members). However, since 1997 in which Yamaichi Securities collapsed, there have been about 220 Japanese securities newly entering the securities markets while about 230 exiting due to voluntary dissolution, merger, or other reasons. Given the severe external environment, it is necessary to analyze the productive performance for both the industry and the individual level of Japanese securities companies. On the other hand, significant changes in business management appeared around the year 2013. According to the Fact Books of JSDA, the number of net assets of investment trusts has been sluggish since the financial crisis of 2008. However, it increased rapidly by 27.4% year on year by the end of 2013 and has grown steadily ever since. Hence, the Japanese securities companies tend to focus more on the asset management business since 2013. Considering the differences in business management, the analysis of productivity change is separated into the years 2011-2013 and 2013-2015. The detailed analysis is provided in Section 4.4.

Also, according to JSDA, the observed 37 companies can be separated into four groups, which consist of five major securities companies, seven online brokers, seven bank-affiliated securities companies, and eighteen other integrated securities companies. However, when benchmarking the individual productive performance, this commonly used categorization may not directly reflect the differences in productive performance because it is based on multi-criteria (e.g., the bank-affiliated securities companies are separated with the major ones based on their capital scale but with the online brokers because of their differences in the trading platform). Therefore, in Section 4.4.2, a new categorization for Japanese securities companies is provided based on their productive performance.

4.2 Selection of inputs and outputs

The sample excludes the securities companies for which the data are missing and covers 14.6% of members in JSDA. The selection of 37 companies is based on their securities businesses: According to Financial Instruments and Exchange Act (enforced in September 2007) in Japan, the principal businesses that securities companies are authorized to operate are primarily divided into brokerage, dealing, underwriting and selling businesses by the type of services. A securities company may operate some or all the principal businesses. It may also undertake other businesses that require notification to the authorities, such as the investment management business. Therefore, to keep the homogeneity assumption when adopting a DEA methodology, the thesis only considers 37 of the 253 securities companies that operate all the above businesses.

Sources of expenses for securities companies consist of: (a) Trading related expenses; (b) Personal expenses; (c) Office expenses; (d) Real estate-related expenses; (e) Depreciation expenses; (f) Sundry taxes expense; and (g) Other expenses. Here, (a) ~ (c) represent the economic and human resources of securities companies, while (d) ~ (g) are related to fixed capital assets. Since it is less meaningful to consider the adjustments in (d) ~ (g) regarding the efficiency measures discussed in Chapter 3, (a) ~ (c) are confirmed as input variables. Besides, according to the financial summary (Tokyo Stock Exchange, Inc.) between 2011 and 2015, it is clear that (a) ~ (c) are the primary sources of expenses for the Japanese securities industry (see Fig.4.1 for details).

Inputs:

Input 1 ($\bar{x}_1$): Trading related expenses

Input 2 ($\bar{x}_2$): Personal expenses

Input 3 ($\bar{x}_3$): Office expenses

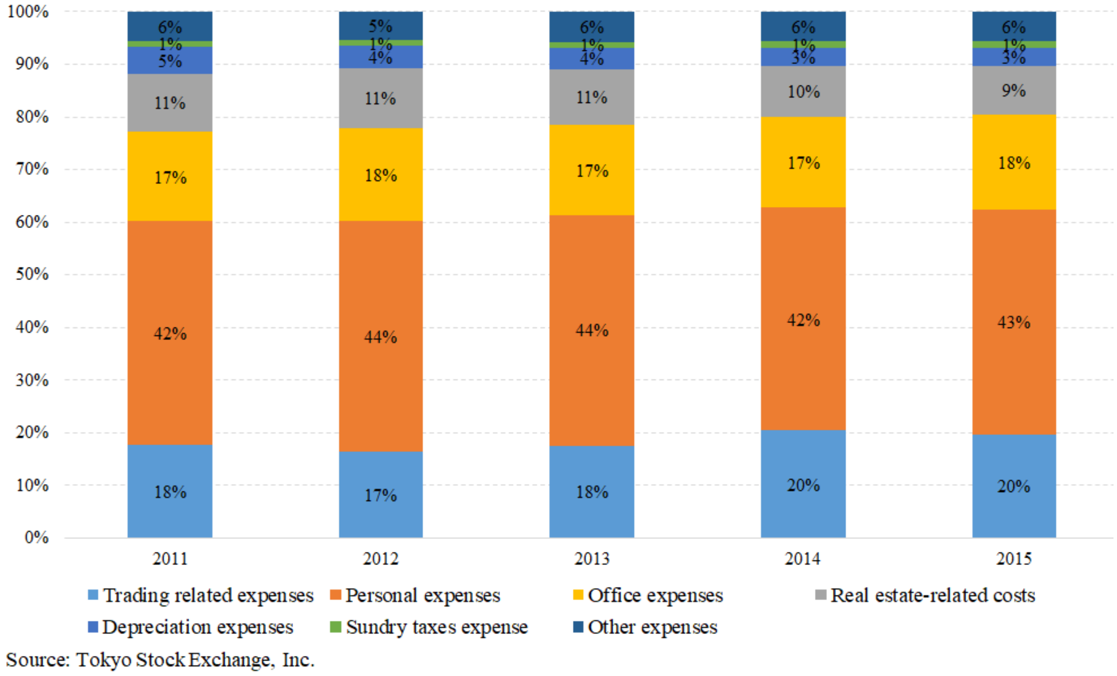


Figure 4.1 The cost structure of the Japanese securities industry.

On the other hand, sources of revenue for securities companies that are associated with the securities businesses (see Fig.4.2 for details) include (a) Brokerage income; (b) Trading income; (c) Underwriting and Selling income; and (d) Other income. The term "Other income" is the commission income of the investment management business. Financial income consisting of interest or dividends is also considered as a source of revenue. However, due to a lack of adequate accounting records of financial income, only (a) ~ (d) are considered as output variables.

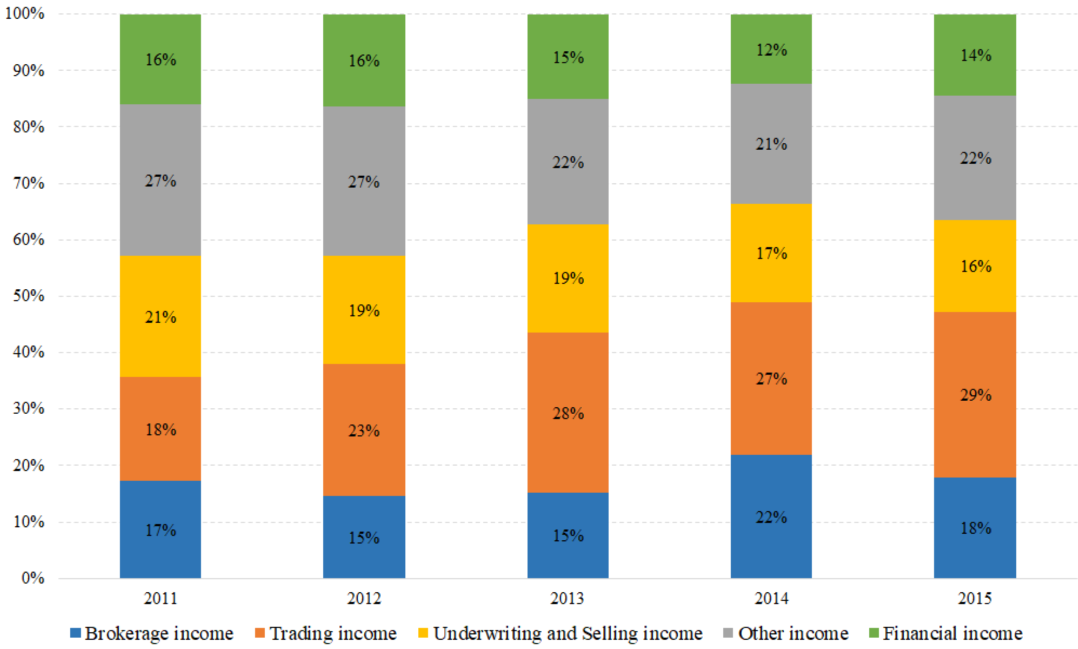
Outputs:

Output 1 ($\bar{y}_1$): Brokerage income

Output 2 ($\bar{y}_2$): Trading income

Output 3 ($\bar{y}_3$): Underwriting and Selling income

Output 4 ($\bar{y}_4$): Other income



Source: Tokyo Stock Exchange, Inc.

Figure 4.2 The revenue structure of the Japanese securities industry.

Note that all inputs and outputs are measured in the same unit (yen) but analyzed separately. Since each of the input or output terms has different sources of income or expenses, it is meaningful to make them distinct when estimating the allocative efficiency of the financial sources given a qualified adjusted way.

One might also consider the use of an aggregated input and aggregated output. Since the allocative efficiency is not evaluable in such cases, the adoption of the proposed index can be used to classify the differences between the profit-ratio change index and the Malmquist input-oriented productivity index. In Section 4.5, the profit-ratio change index is shown to be reduced to the Malmquist input-oriented productivity index under a single input and single output, while under the multiple inputs and multiple outputs, it can be considered as an extension of the Malmquist input-oriented productivity index that takes into consideration the effects of allocative efficiency over time.

4.3 Results of profit-ratio, technical and allocative efficiencies

Table A-1 in Appendix A shows the results of profit-ratio, (input-oriented) technical, and allocative efficiencies in 2011, 2013, and 2015. This section aims to provide examples of projection for Japanese securities companies and further illustrate the allocative efficiency described in Section 3.2.

A detailed description of analysis steps can be described as follows:

Step 1 Solve the linear programming problem Eq. (3.17) to evaluate the (input-oriented) technical efficiency (TE). The graph measure of technical efficiency (TE^{GR}) is then derived by calculating the square root of TE.

Step 2 Solve the linear programming problem Eq. (3.10) to evaluate the maximum profit ratio $\pi(\bar{x}_{io}^{*}, \bar{y}_{ro}^{*})$. The profit-ratio efficiency (PE) is then calculated by using Eq. (3.12).

Step 3 Apply Eq. (3.17) to evaluate the (input-oriented) allocative efficiency (AE). The graph measure of allocative efficiency (AE^{GR}) can be derived by calculating the square root of AE.

From Table A-1, it can be seen that the activities with the most efficient profit-ratio efficiency (PE =1) obtain the most efficient technical and allocative efficiencies (e.g., activity B6). Meanwhile, the profit-ratio efficiencies PE are less than or equal to the technical efficiencies TE, which is consistent with the proposition in Section 3.2. Regarding the scores of technical efficiency TE, for example, activity B10 achieved full efficiency marks in 2013, whereas it fell short in the profit-ratio efficiency score (TE =1.000, PE =0.630). According to the discussions in Section 3.2, this result may be due to the wrong mix in either inputs or outputs, or both. Specifically, in the year 2013, activity B10 had the current input mix $\bar{x} =$ (478,672,379) and output mix $\bar{y} =$(103,301,814,652), while the optimal mix $\bar{x}^{*}$ was (478, 672, 233.613) and $\bar{y}^{*}$ was (147.749, 728.555, 1156.807, 652). Note that the optimal input-mix and output-mix can be obtained from Eq. (3.7), and for simplicity, the optimal solution is truncated to the three decimal places. For example, the real optimized $\bar{y}_1^{*}$ for B10 in 2013 is 147.748450191852. Hence, for activity B10, the wrong mix appeared in both input mix and output mix. According to Eq. (3.17), AE was obtained as 0.630. As an adjustment plan for B10, it needs to reduce $\bar{x}_3$ (office expenses), meanwhile, increase $\bar{y}_1$ (brokerage income), $\bar{y}_2$ (trading income) and $\bar{y}_3$ (underwriting and selling income).

On the other hand, for the activities which have worse technical efficiencies, e.g., O15 in the year 2011 (TE =0.969), improving technical efficiency does not guarantee the achievement of the maximum profit ratio. In the year 2011, O15 had the current inputs and outputs $\bar{x}$ =(4322,1943,4826) and $\bar{y}$ =(9750,1789,657,2054): According to the graph measure of technical efficiency TE^{GR} in Eq. (3.6), the current inputs of O15 should be reduced to the level (4254.969, 1912.866, 4751.152) and the current outputs should be increased to the level (9903.597, 1817.183, 667.350, 2086.358). Note that TE^{GR} =0.984 can be obtained by calculating the square root of TE. However, the profit ratio after improved TE^{GR} resulted in 1.326, whereas the maximum profit ratio obtained from Eq. (3.10) was 1.889. This gap is due to the existence of allocative efficiency. Using Eq. (3.10), the optimal input mix $\bar{x}^*$ of O15 was obtained as (4132.236, 1943, 169.081) and the optimal output mix $\bar{y}^*$ was (10072.01, 1789, 657, 2054). Compared with the current inputs and outputs of O15, it is seen that the wrong mix existed in both inputs and outputs. To achieve the maximum profit ratio, O15 needs to reduce $\bar{x}_1$ (trading related expenses) and $\bar{x}_3$ (office expenses), meanwhile, increase $\bar{y}_1$ (brokerage income).

4.4 Results of the profit-ratio change index and its component indices

The results of the profit-ratio change index PI and its component indices are summarized in Tables A-2 and A-3 (Appendix A). Tables A-2 and A-3 also report the cases that the intertemporal comparison terms of PI had infeasible solutions. In the following, the results are discussed at the overall industry level and an individual level, respectively.

The main steps for analyzing the profit-ratio change index are summarized below:

Step 1 Apply Eq. (3.31) to evaluate the profit-ratio efficiency change (PEC) and the change of profit-ratio boundary (PTC). The profit-ratio change index is then derived by calculating the product of PEC and PTC.

Step 2 Apply Eq. (2.12) to evaluate the (input-oriented) technical efficiency change (TEC) and the frontier shift (TC). The (input-oriented) Malmquist index is then derived by calculating the product of TEC and TC.

Step 3 Apply Eq. (3.23) and Eq. (3.33) to evaluate the (input-oriented) allocative efficiency change (AEC) and the allocation-technical change (ATC), respectively. The (input-oriented) allocation Malmquist productivity index (AMI) is then derived by calculating the product of AEC and ATC.

4.4.1. At the overall industry level

In order to identify the drivers of productivity change of Japanese securities companies, I summarized the geometric means of PI and its decompositions in Table 4.1 (see the decimal numbers) and further expressed those decimal numbers in the form of percentage change by subtracting unity from them.

Table 4.1. The geometric means of PI and its decompositions

	PI	PEC	PTC	MI	TEC	TC	AMI	AEC	ATC
2011-2013	0.793 (20.7%)	0.941 (5.9%)	0.842 (15.8%)	0.860 (14.0%)	0.939 (6.1%)	0.916 (8.4%)	0.922 (7.8%)	1.003 (-0.3%)	0.919 (8.1%)
2013-2015	0.897 (10.3%)	1.001 (-0.1%)	0.896 (10.4%)	0.907 (9.3%)	1.010 (-1.0%)	0.898 (10.2%)	0.989 (1.1%)	0.991 (0.9%)	0.999 (0.1%)

Note:

a. $PI = MI \times AMI; MI = TEC \times TC; AMI = AEC \times ATC.$

b. $PI = PEC \times PTC; PEC = TEC \times AEC; PTC = TC \times ATC.$

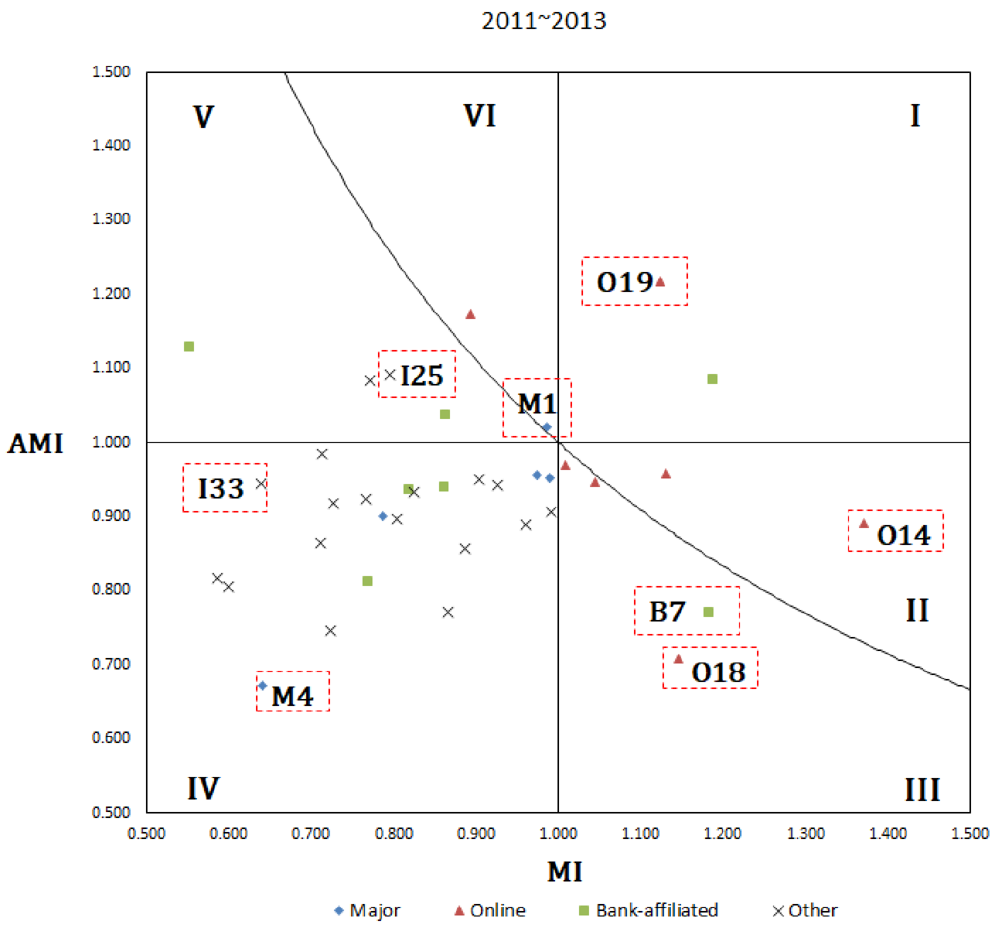
The thesis assumes that the behavioral objectives of securities companies are the maximization of the profit ratio. Hence, from the viewpoint of the sustainable development of the Japanese securities industry, the profit-ratio change index PI needs to be progressed. Since we have obtained two alternative approaches for decomposing

PI, $PI = MI \times AMI$ is firstly considered to identify the drivers of changes in PI. For 2011-2013, the results indicate that the average growth rate of PI (20.7%) was higher than that of MI (14.0%) because AMI had an average growth rate of 7.8%. On the other hand, for 2013-2015, AMI only improved at an average rate of 1.1%. The positive impact of AMI caused the average growth rate of PI (10.3%) greater than that of MI (9.3%). This explains the importance of considering allocative efficiency when analyzing the productivity of the Japanese securities industry: the progress of AMI have a positive impact on PI. Furthermore, considering $MI = TEC \times TC$ and $AMI = AEC \times ATC$, the results indicate that the shift of the value-based technical frontier TC was more influential than the technical efficiency change TEC to the MI for both two periods. The results also imply that the progress of AMI was mainly attributed to the term ATC.

Secondly, the decomposition $PI = PEC \times PTC$ is used to identify the main drivers of PI. For 2011-2013, the profit-ratio efficiency change PEC increased at an average rate of 5.9%, and the change of profit-ratio boundary PTC had an average growth rate of 15.8%. However, for 2013-2015, the decomposition shows that PEC decreased by 0.1%, and PTC progressed at an average rate of 10.4%. Since the fluctuations in PTC were greater than those in PEC for both two periods, PTC can be considered as the main driver that causes PI progress. Furthermore, considering $PEC = TEC \times AEC$, it is clear that, for 2011-2013, the progress of PEC was mainly attributed to the improvement of technical efficiency (TEC increased by 6.1%), although there was an allocative efficiency regress suggested by geometric mean of 1.003 (the negative sign of its percentages change shows that the allocative efficiency dropped by 0.3%). On the other hand, PEC from 2013 to 2015 regressed by 0.1%. This is mainly due to the regress in TEC. Similarly, according to the decomposition $PTC = TC \times ATC$, the results show that the progress of PTC was mainly caused by the progress of TC for both periods.

4.4.2. At an individual level

In order to benchmark the evaluated activities at an individual level, an applicable approach (Fig.4.1) is further provided according to the different performances suggested by PI, MI, and AMI.



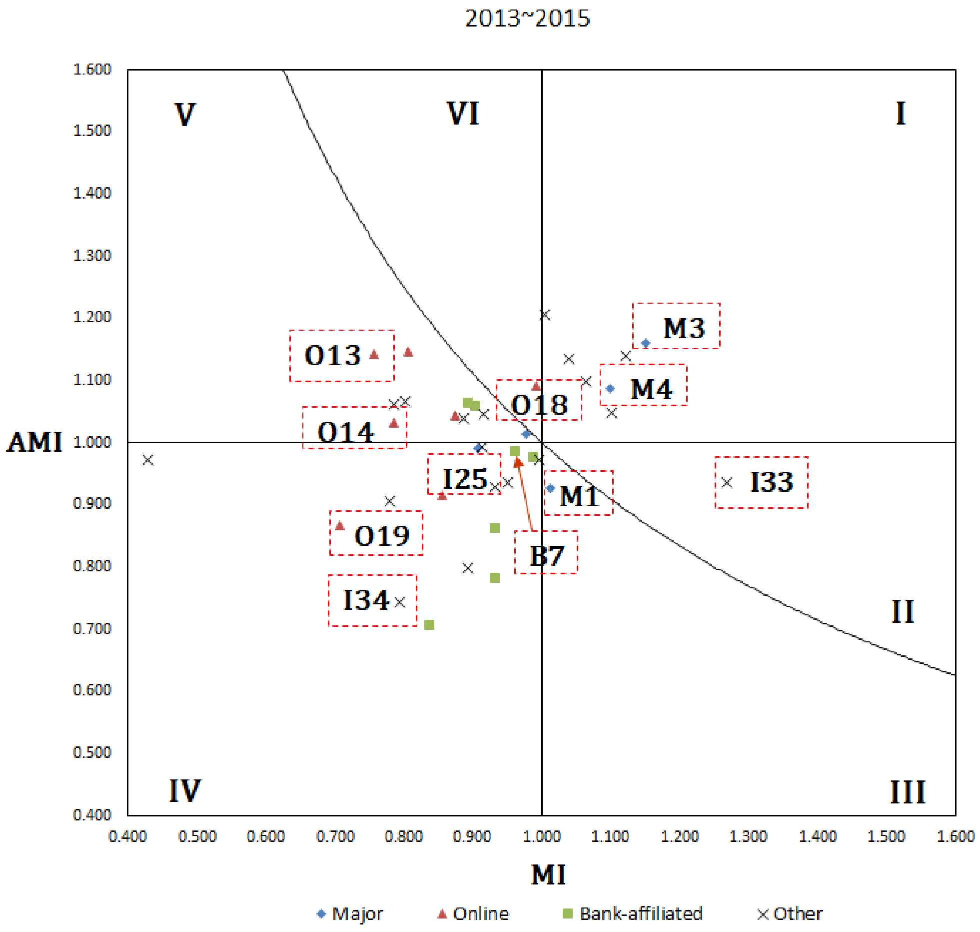


Figure 4.1 The results of PI, MI, and AMI at an individual level.

Figure 4.1 shows the results for the sample of Japanese securities companies. The detailed results can be found in table A-2 and A-3, Appendix A. As already discussed in Section 4.1, the commonly used categorization (major, online, bank-affiliated, and other integrated securities companies) may not directly reflect the differences in the productive performance. Thus, the observed activities are divided into six different groups regarding their performance evaluated by PI, MI, and AMI. Specifically, the horizontal axis represents the index MI, and the vertical axis represents the index AMI. The index PI can be represented as a hyperbolic curve passing through the point (1,1) since it is the product of MI and AMI. The basic evaluate of MI suggests that the region I, II, and III show bad productive performance ($MI > 1$) and the region IV, V and VI show good productive performance ($MI < 1$). In contrast, Fig.4.1 further identifies the activities that need to focus on the management of allocative efficiency over time.

As can be seen from Fig. 4.1, the indices PI, MI, and AMI in the region I are greater than unity, indicating there is no observable productivity growth. Therefore, the activities located in this region can be benchmarked as “the bad performance.” Since the fluctuations in PI, MI, and AMI get larger as the activities are getting away from the origin, we can identify the one with “the worst practice” by calculating the furthest distance from the origin. According to Fig.4.1, we have observed that O19 in the period 2011-2013 and M3 in the period 2013-2015 were “the worst practice,” respectively.

In region II, both PI and MI are greater than unity and thus indicate regress, whereas AMI shows progress ($AMI < 1$). PI shows regress because the regress of MI is rigorous enough to offset the progressive effect of AMI (the percentage change of MI is greater than that of AMI). The activities in this region (e.g., O14 in period 2011-2013; I33 in period 2013-2015) need to focus more on the production management for the purpose of improving MI.

Conversely, in region III, the regressive effect of MI is not enough to offset the progressive effect brought by AMI (the percentage change of MI is less than that of AMI). As a result, the index PI shows progress. Although the activities in this region (e.g., B7, O18 in period 2011-2013; M1 in period 2013-2015) show better performances than those in the region II, there still is much room to improve the estimated value of MI.

The region IV is referred to as “the good performance” region as the indices PI, MI and AMI are less than unity. We can further identify “the best practice” by finding out the one with the closest distance from the origin. According to Fig. 3, we have obtained that M4 in the period 2011-2013 and I34 in the period 2013-2015 were “the best practice,” respectively.

The activities in the region V have regressed according to AMI (>1) but progressed according to both PI (<1) and MI (<1). The progress of PI is mainly due to the progress of MI as MI has a more influential effect than AMI (the percentage change of MI is greater than that of AMI). Compared with “the good performance” region, the activities in the region V (e.g., I25 in period 2011-2013; O13 in period 2013-2015) should pay attention to the index AMI which is affected by both allocative efficiency change and allocation-technical change.

The region VI also shows that the activities have regressed according to the index AMI (>1). However, compared with the region V, PI shows regress because the progressive effect brought by MI (<1) is not enough to cover the regress of AMI (the percentage change of MI is less than that of AMI). To improve PI, the activities in this region (e.g., M1 in period 2011-2013; O18 in period 2013-2015) must focus more on their management of allocative efficiency over time.

In summary, the activities in the region II and III should keep the level of AMI and focus on improving MI which is affected by both the technical efficiency change and the shift of the value-based technical frontier. For example, B7 was in region III for 2011-2013, while it successfully improved itself to the region of "the good performance" in the period 2013-2015 due to the progress of MI. This does not mean that the change of AMI is not important: O18 was obtained in the region III for 2011-2013, and it improved its value of MI in the latter period. However, it could not be evaluated as "the good performance" because of the regress of AMI (AMI =1.092 for 2013-2015, see more details in Appendix A). On the other hand, the activities in the region V and VI need to keep the level of MI and pay attention to AMI which is affected by both the allocative efficiency change and allocation-technical change. The producers or managers have to make the right decisions to improve allocative efficiency over time. As already discussed in Section 3.2, allocative efficiency can be achieved by reconsidering the resources mix of the input-spending and output-earnings so as to maximize the profit ratio. For example, I25 (for 2011-2013) succeeded in improving itself to the region of "the good performance" in the period 2013-2015 due to the progress of AEC (AEC =0.983 in the latter period, see more details in Appendix A). However, improving the allocative efficiency does not mean that the production management is not important: the activity M1 (for 2011-2013) improved its AMI in the latter period, but it still could not be evaluated as "the good performance" due to the regress of MI. Also, the activities in the region I should pay attention to both MI and AMI. Figure 4.1 further shows that the one benchmarked as "the worst practice" can get into the region of good performance by improving both MI and AMI (e.g., O19 was evaluated as the good performance for 2013-2015 whereas the worst practice for 2011-2013). Similarly, the one benchmarked as "the best practice" can also be dropped into the region of bad performance as long as it could not keep the level of both MI and AMI (e.g., M4 was evaluated as the best practice for 2011-2013 whereas the bad performance for 2013-2015). Therefore, the activities in the region IV should at least keep the current level of the progress for both MI and AMI.

4.5 Comparisons with the results of an aggregated input and aggregated output

This section further clarifies the differences between the profit-ratio change index PI and the Malmquist input-oriented productivity index MI. As already discussed in Section 4.2, one might consider the use of an aggregated input and aggregated output. In such cases, the aggregated input is the total expenses, which summarize the inputs 1, 2, and 3. The aggregated output is the total revenue defined as the sum of output 1 to output 4.

Using the aggregated input and output variable, the profit-ratio change index can be adopted to analyze the productive performance of Japanese securities companies. However, because there is no longer any observable allocative inefficiency in the single input and single output setting, the wrong mix for either total expenses or total revenue is not evaluable anymore. The definition of allocative efficiency in Eq. (3.17) then becomes a unity in the sense that we have no means to evaluate it. In fact, the maximum profit ratio defined in Eq. (3.9) is of "most productive scale size" under CRS (Banker et al. [59]) and is identical to all the evaluated activities for a single input and single output case. This indicates that the profit-ratio efficiency PE in Eq. (3.12) is equivalent to the technical efficiency TE in Eq. (3.7) when using an aggregated input and output. Furthermore, the component indices of the profit-ratio change index, AEC in Eq. (3.32) and ATC in Eq. (3.33), also become unity either. Therefore, in the single input and single output setting, the definition of the profit-ratio change index in Eq. (3.23) will be reduced to the Malmquist input-oriented productivity index in Eq. (3.20).

Table 4.2 presents the results of the aggregated input and output case (Case (a)). The details can be found in Table A-4 and A-5 (Appendix A). To further clarify the interpretation of the index PI and its decompositions, I also added the results shown in

Table 4.1 (Case (b)). Note that the decimal numbers are the geometric means of indices, and the percentage changes are calculated by subtracting unity from the decimals.

Table 4.2. Comparisons with the results of an aggregated input and aggregated output

2011-2013	PI	MI	PEC	TEC	PTC	TC	AMI	AEC	ATC
Case (a)	0.831 (16.9%)		1.125 (-12.5%)		0.739 (26.1%)		1.000 (0.0%)	1.000 (0.0%)	1.000 (0.0%)
Case (b)	0.793 (20.7%)	0.941 (5.9%)	0.842 (15.8%)	0.860 (14.0%)	0.939 (6.1%)	0.916 (8.4%)	0.922 (7.8%)	1.003 (-1.3%)	0.919 (8.1%)

2013-2015	PI	MI	PEC	TEC	PTC	TC	AMI	AEC	ATC
Case (a)	0.931 (6.9%)		0.846 (15.4%)		1.100 (-10.0%)		1.000 (0.0%)	1.000 (0.0%)	1.000 (0.0%)
Case (b)	0.897 (10.3%)	1.001 (-0.1%)	0.896 (10.4%)	0.907 (9.3%)	1.010 (-1.0%)	0.898 (10.2%)	0.989 (1.1%)	0.991 (0.9%)	0.999 (0.1%)

Note: Case (a) represents the results calculated by the aggregated input (total expenses) and output (total revenue), while Case (b) is the case of three inputs and four outputs in Table 4.1.

As can be seen in Table 4.2, the geometric means of PI in Case (a) (which is equivalently the index MI) are close to those of PI in Case (b) for both periods 2011-2013 and 2013-2015. In Case (a), the profit-ratio boundary overlapped the value-based technical frontier, and hence there were no more residual shifts of the profit-ratio boundary (ATC =1.000) or the changes of allocative efficiency (AEC =1.000). However, in Case (b), the profit-ratio boundary needs not to be at the same level as the value-based technical frontier. This ensures that the allocative efficiency is observable in the multiple inputs and multiple outputs cases. Therefore, the difference between PI (or MI) in Case (a) and PI in Case (b) should be interpreted by the existence of the observable allocative efficiency. When the allocative efficiency is observable (e.g., Case (b)), its effects on the productivity change over time are aggregated into the index AMI. As a result, the proposed index PI should be interpreted as the one that captures the average effects of both technical and allocative efficiency over time. Furthermore, although the allocative efficiency is observable in Case (b), the index MI under this circumstance does not consider the potential effects of changing the input and output mix of the evaluated activities regarding their maximum possible profit ratios. Therefore, the proposed index

PI can be considered as an extension of the conventional MI as it takes into consideration the effects of allocative efficiency over time.

4.6 Concluding remarks

This chapter focuses on the productive performance of 37 Japanese securities companies to demonstrate the methods proposed in Chapter 3. Based on the results of profit-ratio, technical, and allocative efficiencies, I explained the situation where the wrong mix appeared in both input mix and output mix. Some examples of projection for Japanese securities companies are provided by considering the allocative efficiency regarding profit-ratio maximization. On the other hand, the results of PI, MI, and AMI are used to characterize Japanese securities companies according to their different productive performance over time. Because the characterization reveals the strengths and weaknesses of securities companies, it is useful to help them to find their unique positions under the competitive business environment.

To clarify the differences between the proposed PI and the conventional MI, I further considered a special case that an aggregated input is used to produce an aggregated output. Based on the results, I concluded that PI is an extension of MI as it accounts for the impacts of allocative efficiency over time.

Chapter 5

Productivity analysis under stochastic noise

5.1 Introduction

The purpose of this chapter is to develop a new model for estimating Malmquist-type indices under a stochastic setting. Since existing approaches are mostly based on either DEA or SFA, I seek to apply the StoNED approach to estimate Malmquist-type indices. There are few studies attempting to combine both the StoNED approach and the estimation of Malmquist indices: for example, Cheng et al. [60] estimated a Malmquist-type index by utilizing a cost frontier, and Zhou [61] considered an aggregatable input variable for estimating the input-oriented Malmquist index. However, both of them are restricted to a single-input and multiple-output setting. In fact, most existing models in StoNED have been presented in either a single-input or a single-output setting. To allow for a multiple-input and multiple-output production technology in StoNED, Kuosmanen and Johnson [62] (see also Layer et al. [63]) have recently proposed a cross-sectional model by adopting the directional distance function (Chambers et al. [26]). Once the production frontier is estimated, it is possible to gauge the distance for each observed production activity. However, as mentioned before, measuring the distance to the production frontier in a stochastic setting provides a measure of an aggregation of both inefficiency and noise. Therefore, the distance estimated from the cross-sectional StoNED model cannot be straightforwardly extended to construct a Malmquist-type index.

This chapter contributes to the literature by suggesting a panel-data StoNED model for estimating Malmquist-type indices in a multiple-input and multiple-output setting. Compared with conventional Malmquist-type indices, the index proposed herein can account for the impact of stochastic noise in a nonparametric setting. More specifically, I elaborate on the use of directional distance functions to allow for a multiple-input and multiple-output production technology and further suggest the use of panel data in efficiency analysis. Note that panel data for efficiency analysis provide several significant advantages over conventional cross-sectional data, leading to estimates with more desirable statistical properties (e.g., Kumbhakar and Lovell [64], Hsiao [65,66]).

In what follows, I first consider the estimation of the multi-input and multi-output production frontier and then perform the inefficiency analysis for constructing the Malmquist-type indices. Specifically, in Section 5.2, a panel-data StoNED model is proposed, which identifies the reference points on the production frontier. In Section 5.3, I approximate a piecewise linear frontier based on these estimated reference points and develop a consistent estimator of inefficiency. An input-oriented Malmquist index defined with the estimated inefficiencies is then presented in Section 5.4. Concluding remarks are provided in Section 5.5.

5.2 A panel-data model for multi-input and multi-output production frontier

In this section, the cross-sectional model of Kuosmanen and Johnson [62] is extended to estimate the directional distance function in a panel-data setting. Recall that the directional distance function suggested by Chambers et al. [26] provides an alternative characterization of production technology as well as a measure of inefficiency. Thus, in the estimation procedure, I first estimate the production frontier with the directional distance function and then perform the inefficiency analysis accordingly.

Consider a balanced panel where n firms are observed multiple times, indexed by subscript k, $(k = 1, \dots, K)$. Suppose that the producer of firm j, $(j = 1, \dots, n)$ uses m-dimensional inputs $x_{jk} = \left(x_{1jk}, x_{2jk}, \dots, x_{mjk}\right)' \in R_+^m$ to produce s-dimensional outputs $y_{jk} = \left(y_{1jk}, y_{2jk}, \dots, y_{sjk}\right)' \in R_+^s$. The observed data point $\left(x_{jk}, y_{jk}\right)$ is assumed to differ from its reference point $\left(x_{jk}^*, y_{jk}^*\right)$ in the exogenously given direction $(\mathrm{g}, \mathrm{h}) \in R_+^{m+s}$ by both inefficiency and noise:

$$\left(x_{jk}, y_{jk}\right) = \left(x_{jk}^* + \varepsilon_{jk}\mathrm{g}, y_{jk}^* - \varepsilon_{jk}\mathrm{h}\right), \forall j, k, \tag{5.1}$$

where $\varepsilon_{jk} = u_j + v_{jk}$ is a composite error term that consists of firm-specific inefficiency u_j and random noise v_{jk}. Specifically, the following assumptions are incorporated:

(a) The inefficiency components u_j are independent of observing times k;

(b) The inefficiency components u_j have an unknown asymmetric distribution with a positive mean μ and a finite constant variance;

(c) The noise components v_{jk} are uncorrelated random variables and have an unknown symmetrical distribution with zero mean and finite constant variance;

(d) u_j and v_{jk} are independent of (x_{jk}^*, y_{jk}^*).

It has been shown in the cross-sectional setting that Eq. (5.1) satisfies $\vec{D}(x_j, y_j, \mathrm{g}, \mathrm{h}) = \varepsilon_j, \forall j$. The same logic can be applied straightforwardly to the panel-data setting, which is now expressed by $\vec{D}(x_{jk}, y_{jk}, \mathrm{g}, \mathrm{h}) = \varepsilon_{jk}, \forall j, k$. This relation provides a regression equation for estimating the directional distance function. To apply the CNLS regression, I define the conditional mean distance function as $\vec{D}(x_{jk}, y_{jk}, \mathrm{g}, \mathrm{h}) - E(\varepsilon_{jk}), \forall j, k$. Given pre-assigned directions (g, h) and observations $(x_{jk}, y_{jk}), \forall j, k$, the CNLS problem is stated as

$$
\begin{aligned}
&\min_{\alpha,\beta,\tau,\varepsilon} \sum_{k=1}^{K} \sum_{j=1}^{n} \left(\varepsilon_{jk}^{CNLS}\right)^2 \\
&s.t. \\
&\sum_{r=1}^{s} \tau_{rjk} y_{rjk} = \alpha_{jk} + \sum_{i=1}^{m} \beta_{ijk} x_{ijk} + \varepsilon_{jk}^{CNLS}, \forall j = 1, \dots, n;\ \ k = 1, \dots, K \\
&\alpha_{jk} + \sum_{i=1}^{m} \beta_{ijk} x_{ijk} - \sum_{r=1}^{s} \tau_{rjk} y_{rjk} \le \alpha_{zl} + \sum_{i=1}^{m} \beta_{izl} x_{ijk} - \sum_{r=1}^{s} \tau_{rzl} y_{rjk}, \\
&\qquad \forall j, z = 1, \dots, n;\ \ k, l = 1, \dots, K \\
&\sum_{i=1}^{m} \beta_{ijk} \mathrm{g} + \sum_{r=1}^{s} \tau_{rjk} \mathrm{h} = 1, \forall j = 1, \dots, n;\ \ k = 1, \dots, K \\
&\beta_{ijk} \ge 0, \forall i = 1, \dots, m;\ \ j = 1, \dots, n;\ \ k = 1, \dots, K \\
&\tau_{rjk} \ge 0, \forall r = 1, \dots, s;\ \ j = 1, \dots, n;\ \ k = 1, \dots, K
\end{aligned}
\tag{5.2}
$$

where ε_{jk}^{CNLS} is an estimator of $E(\varepsilon_{jk}) - \varepsilon_{jk} = \mu - (u_j + v_{jk})$ for the jth firm observed in the rth time. Eq. (5.2) defines a least-squares problem that minimizes the sum of squares of the composite error terms. The first constraint contains a set of linear regression equations where parameters α, β, and τ define a specific tangent hyperplane to an unknown directional distance function for the jth firm observed in the kth time. The second constraint imposes concavity on the directional distance function by applying Afrait inequalities (see Afrait [41]). Due to the duality between the directional distance function and the profit function, as shown in Chambers et al. [26], parameters β and τ can be further interpreted as nonnegative shadow prices for inputs and outputs. The third constraint ensures the translation property of the directional distance function by normalizing the shadow prices with the direction (g, h). The last two constraints impose monotonicity relative to all inputs and outputs. It is worth noting that the use of firm-specific directions in the CNLS estimator may violate the global convexity of the production technology, as argued by Layer et al. [63]. Hence, at the stage of estimating the production frontier, the direction (g, h) in Eq. (5.2) is specified as a pre-assigned vector that is common across all firms.

Given the CNLS estimates $\hat{\varepsilon}_{jk}^{CNLS}$, the average performance of firm j is computed as

$$\bar{\varepsilon}_j^{CNLS} = \frac{1}{K}\sum_{k=1}^{K} \hat{\varepsilon}_{jk}^{CNLS}. \tag{5.3}$$

Note that $\bar{\varepsilon}_j^{CNLS}$ captures impacts of the inefficiency u_j, the noise v_{jk}, and the positive mean μ. However, as the number of observing times K increases, the impacts of noise can be effectively averaged out. Therefore, Eq. (5.3) can be used as a measure of firm-specific effect.

To benchmark the performance of each firm to best practice, Eq. (5.3) is normalized to the nonnegative inefficiency by using the following definition (see Schmidt and Sickles [67]):

$$\hat{u}_j = \max_{z \in \{1,\dots,n\}} \bar{\varepsilon}_z^{CNLS} - \bar{\varepsilon}_j^{CNLS}. \tag{5.4}$$

Eq. (5.4) is consistent if there is a strictly positive probability of observing a fully efficient firm (see Park and Simar [68]). The positive μ can be then estimated as $\hat{\mu} = \frac{1}{n}\sum_{j=1}^{n} \hat{u}_j = \max \bar{\varepsilon}_j^{CNLS}$ (note that the CNLS estimates are known to sum to zero, i.e., $\sum_{k=1}^{K}\sum_{j=1}^{n} \hat{\varepsilon}_{jk}^{CNLS} = 0$). As a consequence, the estimated reference points on the production frontier are obtained as $(\hat{x}_{jk}, \hat{y}_{jk}) = (x_{jk} - (\hat{\varepsilon}_{jk}^{CNLS} + \hat{\mu})\mathrm{g}, y_{jk} + (\hat{\varepsilon}_{jk}^{CNLS} + \hat{\mu})\mathrm{h}), \forall j, k$.

5.3 Estimation of within and intertemporal efficiencies

Although Eq. (5.4) provides an estimator of inefficiency for firm j, it cannot be used for assessing intertemporal efficiency (i.e., estimating the inefficiency of firm j at period t to the production technology at period $t+1$). The following estimator of directional distance function is then considered as a general measure of inefficiency.

Relative to the estimated reference points $(\hat{x}_{jk,t}, \hat{y}_{jk,t}), \forall j, k$ at period t, it is possible to construct an $(m \times nK)$ matrix of optimal inputs and an $(s \times nK)$ matrix of optimal outputs. The estimator of the distance for the jth firm observed in the kth time at period t is defined as

$$\begin{aligned} &\delta_{jk}^{t}(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t}) = \max_{\delta,\lambda} \delta_{jk} \\ &s.t. \\ &\sum_{d=1}^{nk} \lambda_d \hat{x}_{d,t} \leq x_{jk,t} - \delta_{jk} g_{jk,t}, \\ &\sum_{d=1}^{nk} \lambda_d \hat{y}_{d,t} \geq y_{jk,t} + \delta_{jk} h_{jk,t}, \\ &\sum_{d=1}^{nk} \lambda_d = 1, \\ &\lambda_j \geq 0, \end{aligned} \tag{5.5}$$

where $j = 1, \dots, n, k = 1, \dots K, d = 1, \dots, nK$. To assess efficiency and productivity, a piecewise linear production frontier is estimated relative to the reference points resulting from the CNLS problem. Note that the notation (g, h) are pre-assigned common directions for estimating the underlying production technology (see, Eq. (5.2)) while the notation $(g_{jk,t}, h_{jk,t})$ in Eq. (5.5) are allowed to be firm-specific depending on the purpose of the efficiency analysis. In other words, once the production frontier is estimated, one can compute the distance from any observed point $(x_{jk,t}, y_{jk,t})$ to the production frontier in any direction $(g_{jk,t}, h_{jk,t})$ by using Eq. (5.5). Eq. (5.5) implicitly assumes variable returns to scale. By dropping the restrictions $\sum_{d=1}^{nk} \lambda_d = 1$, the assumption of constant returns to scale can be imposed. It is also worth noting that Eq. (5.5) differs from the DEA formulation of the directional distance function (see Chambers et al. [26]) in the following sense. In the DEA formulation, the directional distance function is calculated using a deterministic approach, while in Eq. (5.5), the directional distance function results in a random variable because $(x_{jk,t}, y_{jk,t})$ are subject to noise and inefficiency by assumption.

Denote now the optimal solutions to Eq. (5.5) by $\delta_{jk}^{*t}(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t}), \forall j, k$. Analogous to Eq. (5.3), the average distance of firm j at period t is defined as

$$\bar{\delta}_j^t(x_{j,t}, y_{j,t}, g_{j,t}, h_{j,t}) = \frac{1}{K}\sum_{k=1}^{K} \delta_{jk}^{*t}(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t}). \tag{5.6}$$

Eq. (5.6) provides an estimator of inefficiency for firm j at period t in the sense that the noise term will be effectively canceled out as $K \to \infty$. Because it is measured within the same period, Eq. (5.6) is also referred to as the within inefficiency.

To assess intertemporal inefficiencies, let $\delta_{jk}^{t+1}(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t})$ be an estimator of the distance for the jth firm observed in the kth time at period t against the production technology at period $t+1$. Similar to Eq. (5.5), $\delta_{jk}^{t+1}(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t})$ is computed from the following linear programming problem:

$$
\begin{aligned}
&\delta_{jk}^{t+1}\left(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t}\right) = \max_{\delta,\lambda} \delta_{jk} \\
&s.t. \\
&\sum_{d=1}^{nk} \lambda_d \hat{x}_{d,t+1} \leq x_{jk,t} - \delta_{jk} g_{jk,t}, \\
&\sum_{d=1}^{nk} \lambda_d \hat{y}_{d,t+1} \geq y_{jk,t} + \delta_{jk} h_{jk,t}, \\
&\sum_{d=1}^{nk} \lambda_d = 1, \\
&\lambda_j \geq 0,
\end{aligned}
\tag{5.7}
$$

where $j = 1, \dots, n, k = 1, \dots K, d = 1, \dots, nK$.

Given the optimal solutions $\delta_{jk}^{*t+1}\left(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t}\right), \forall j, k$ to Eq. (5.7), the intertemporal average distance of firm j is expressed as

$$
\bar{\delta}_j^{t+1}\left(x_{j,t}, y_{j,t}, g_{j,t}, h_{j,t}\right) = \frac{1}{K} \sum_{k=1}^{K} \delta_{jk}^{*t+1}\left(x_{jk,t}, y_{jk,t}, g_{jk,t}, h_{jk,t}\right). \tag{5.8}
$$

Eq. (5.8) provides an estimator of intertemporal inefficiency for firm j at a mixed period in the sense that the noise term will be effectively canceled out as $K \to \infty$. Similarly, we can also evaluate firm j at period $t + 1$ relative to the production technology at period t by replacing the notations t and $t + 1$ in Eqs. (5.7) - (5.8) with each other.

5.4 Stochastic nonparametric estimation of Malmquist-type indices

This section provides an example of estimating the input-oriented Malmquist index based on the efficiencies defined in Section 5.3. Similar procedures can also be applied straightforwardly to the output-oriented Malmquist index as well as the Malmquist-Luenberger indices (Chung et al. [69]). Recall the definition of the input-oriented Malmquist index in Eq. (2.12). It has been shown that the input distance function and the directional distance function are related by $1/D(x, y) = 1 - \vec{D}(x, y, g, h)$ if we specify the direction vector as $(g, h) = (x, 0)$ [25]. Therefore, the input-oriented Malmquist index in Eq. (2.12) can be subsequently written as

$$M = \left[\frac{1 - \vec{D}^t(x_t, y_t, x_t, 0)}{1 - \vec{D}^t(x_{t+1}, y_{t+1}, x_{t+1}, 0)} \times \frac{1 - \vec{D}^{t+1}(x_t, y_t, x_t, 0)}{1 - \vec{D}^{t+1}(x_{t+1}, y_{t+1}, x_{t+1}, 0)}\right]^{\frac{1}{2}}. \tag{5.9}$$

Note that both Eqs. (2.12) and (5.9) can be measured in the deterministic approach, indicating that any distance between the observations and the production frontier can be treated as a measure of inefficiency. However, in practice, the observed data may be affected by both inefficiency and random noise, which makes it imprecise to interpret the estimated distance as a measure of inefficiency. If we apply the estimator of the directional distance function in Eqs. (5.5) and (5.7) directly to the definition Eq. (5.9), index M will result in random variables. The following definition is then considered as a stochastic nonparametric estimation of an input-oriented Malmquist index for firm j:

$$\bar{M}_j = \left[\frac{1 - \bar{\delta}_j^t\left(x_{j,t}, y_{j,t}, x_{j,t}, 0\right)}{1 - \bar{\delta}_j^t\left(x_{j,t+1}, y_{j,t+1}, x_{j,t+1}, 0\right)} \times \frac{1 - \bar{\delta}_j^{t+1}\left(x_{j,t}, y_{j,t}, x_{j,t}, 0\right)}{1 - \bar{\delta}_j^{t+1}\left(x_{j,t+1}, y_{j,t+1}, x_{j,t+1}, 0\right)}\right]^{\frac{1}{2}}. \tag{5.10}$$

Eq. (5.10) is derived relative to the within and intertemporal efficiencies of firm j defined in Eqs. (5.6) and (5.8). Similar to the input-oriented Malmquist index in Eq. (2.12), $\bar{M}_j < 1$ indicates progress in productivity from period t to $t + 1$, while $\bar{M}_j = 1$ and $\bar{M}_j > 1$ indicate no change and regressing productivity, respectively.

It is also possible to decompose the Malmquist index defined in Eq. (5.10) as follows:

$$\bar{M}_j = \frac{1 - \bar{\delta}_j^t\left(x_{j,t}, y_{j,t}, x_{j,t}, 0\right)}{1 - \bar{\delta}_j^{t+1}\left(x_{j,t+1}, y_{j,t+1}, x_{j,t+1}, 0\right)} \times \left[\frac{1 - \bar{\delta}_j^{t+1}\left(x_{j,t+1}, y_{j,t+1}, x_{j,t+1}, 0\right)}{1 - \bar{\delta}_j^t\left(x_{j,t+1}, y_{j,t+1}, x_{j,t+1}, 0\right)} \times \frac{1 - \bar{\delta}_j^{t+1}\left(x_{j,t}, y_{j,t}, x_{j,t}, 0\right)}{1 - \bar{\delta}_j^t\left(x_{j,t}, y_{j,t}, x_{j,t}, 0\right)}\right]^{\frac{1}{2}}. \tag{5.11}$$

The first term measures the efficiency change (EC) between periods t and $t+1$. A value of EC less (greater) than unity indicates improved (declining) efficiency, and a value of EC equal to unity indicates no change in the efficiency. The second term captures the shift in the production frontier over time and is therefore referred to as technical change (TC). The frontier progresses if $TC < 1$, remains the same if $TC = 1$, and regresses if $TC > 1$.

5.5 Concluding remarks

This chapter provided a stochastic nonparametric estimation of an input-oriented Malmquist index. It is worth noting that the approach can also be applied straightforwardly to output-oriented Malmquist indices as well as Malmquist-Luenberger indices. Compared with the conventional input-oriented Malmquist index, the proposed index has the following features:

(a) It measures productivity change over time using a nonparametric approach.
(b) It is capable of dealing with multi-input and multi-output production technology.
(c) It allows for the presence of random noise and meanwhile captures inefficiencies.

Specifically, to account for the impact of random noise, a nonparametric regression technique, CNLS, is adopted to estimate multi-input and multi-output production technology characterized by a directional distance function. To evaluate inefficiencies, I first considered an estimator of inefficiency based on the residuals of the CNLS problem. By virtue of using panel data, inefficiency can be estimated consistently without imposing distributional assumptions. However, intertemporal efficiency cannot be assessed directly. To address this issue, I considered the estimator of the directional distance function and then extended it to the estimation of Malmquist-type indices.

Chapter 6
Productivity analysis of Japanese regional banks

6.1 Introduction

The performance of regional banks has drawn a great deal of interest among researchers as well as regulators (e.g., Barros et al. [70], Fukuyama and Matousek [71], Fukuyama and Weber [72–74], Mamatzakis et al. [75]). However, most previous studies have applied either stochastic parametric estimation (e.g., SFA) or the deterministic nonparametric approach (e.g., DEA). Few studies have analyzed the banking sector in a StoNED framework (a rare example being Eskelinen and Kuosmanen [76]). This chapter investigates the productive performance of a sample of 101 Japanese regional banks over two periods by applying the stochastic nonparametric estimation of the input-oriented Malmquist index proposed in Section 5.4. The first and second periods cover 2008 to 2012 and 2013 to 2017, respectively. Both panels are balanced. The dataset is drawn from the financial statements of all banks, as published by the Japanese Bankers Association (JBA).

Japanese regional banks serve the diverse financial needs of individual customers, small- and medium-sized companies, and local governments, playing a primary role in the country's regional finance. These banks can be divided into two groups, namely, regional banks I (members of the Regional Banks Association of Japan) and regional banks II (members of the Second Association of Regional Banks). According to data from the Financial Services Agency in the fiscal year 2017, there were 64 banks classified as regional banks I and 40 banks classified as regional banks II. After eliminating those banks lacking adequate accounting records, the sample in this thesis contains 63 regional banks I and 38 regional banks II. Most regional banks II were referred to as mutual banks before they converted to regional banks, starting in 1989, under the Banking Law. Since both types of regional banks have the same scope of the business, I simply use the term "regional banks" to refer to them. However, due to different financial supports in regional banks I and II, their efficiency levels may differ from each other (see, [71]). Therefore, I also investigate the differences between regional banks I and II.

In exploring the performance of regional banks, it is important to acknowledge and consider prevailing political and macroeconomic contexts because these may affect bank management practices. After the 2008 global financial crisis, the Bank of Japan (BOJ) introduced significant monetary policy initiatives, including Quantitative-Qualitative Easing (QQE) in April 2013 to address inflation expectations and push down long-term interest rates. In practice, however, regional banks have been struggling to maintain profitability amid the persistently low-interest rates.

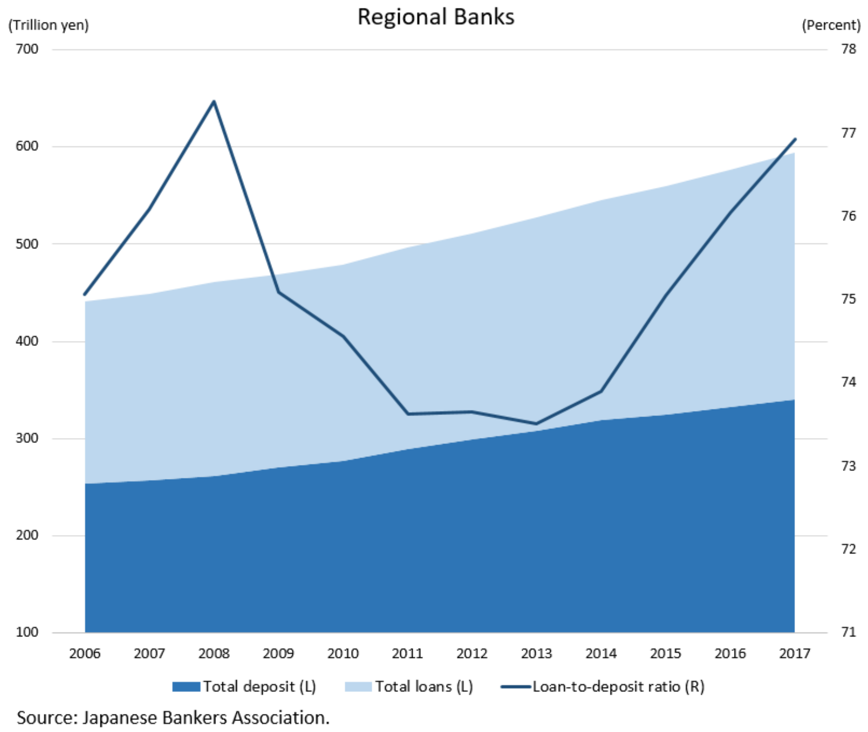


Figure 6.1 Total loans, total deposits, and the loan-to-deposit ratio of regional banks.

Figure 6.1 shows the loan-to-deposit ratio of regional banks during 2006-2017; a declining trend is evident from 2008, but this switched to an increasing trend starting in 2013. According to the Financial System Report released by the BOJ in April 2018, regional banks are facing heated competition because of the low-interest rates, and have been actively increasing the number of loans to low-return borrowers across a wide range of industries. Meanwhile, deposits have remained relatively low compared to loans due to the expansion of QQE with low deposit rates. On the other hand, long-term low deposit rates may have a positive contribution to the increase in interest income. However, the primary interest income (including interest on loans and securities), as shown in Fig.6.2, has been decreasing in practice since 2008 due to the higher decline in lending rates compared to deposit rates.

Based on the above discussion, it is necessary to estimate the productive performance of regional banks either from the perspective of profitability management or policy analysis. Moreover, it is clear that different political and economic conditions have different impacts on the productive performance of regional banks. Therefore, the periods 2008-2012 and 2013-2017 are considered separately to empirically estimate the productivity of regional banks over a long-term period.

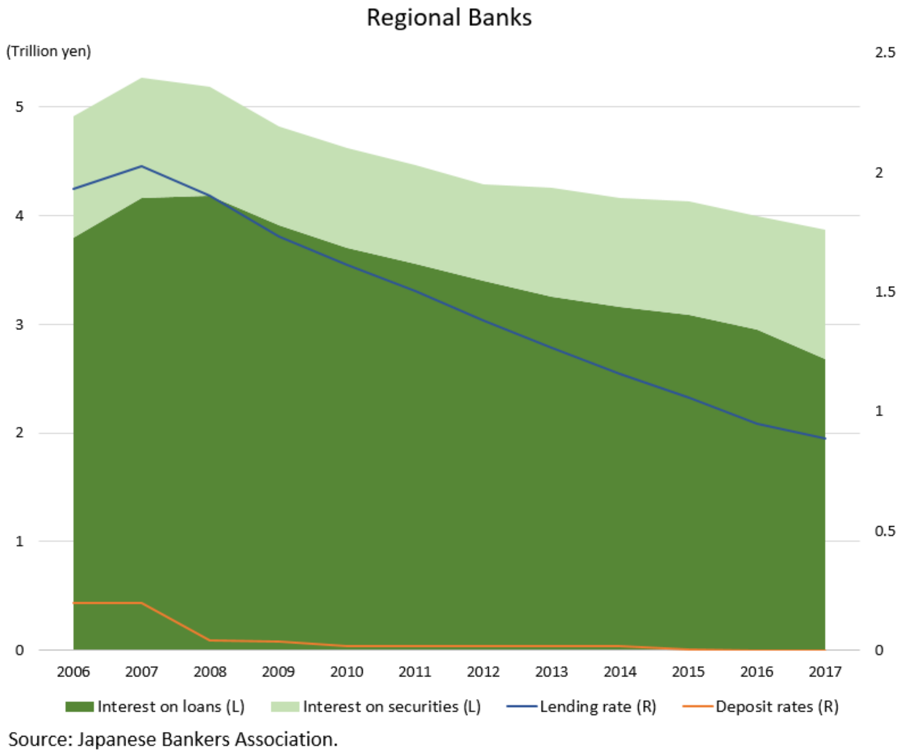


Figure 6.2 Growth of primary resources of revenue amid persistently low-interest rates.

6.2 Selection of inputs and outputs

To characterize the inputs and outputs of regional banks, an intermediation approach is applied, which considers regional banks as financial intermediaries between savers and investors (see, e.g., [71,77,78]). As shown in Table 6.1, I assume regional banks use three inputs to produce two outputs. According to the 2018 Financial System Report, compared to real demand, there may be an excessive number of employees and offices in the banking sector due to the declining population. Thus, the number of employees (labor) and real estate (capital) should be considered as sources of inefficiency in the production process of regional banks. Further, based on Figs.6.1 and 6.2, I also specify deposits as an input, and loans and securities as outputs. Note that loans on the balance sheet contain both performing and non-performing loans. Nevertheless, this chapter focuses only on performing loans because, as argued by Boussemart et al. [79], the assumption of weak disposability of the good and bad outputs jointly is difficult to justify provided all regional banks properly evaluate all loan applications (see also [80]).

Table 6.1. Definitions of inputs and outputs

	Variables	Definitions
x^1	Labor	Number of employees
x^2	Deposits	Deposits
x^3	Capital	Asset value of premises and real estate
y^1	Loans	Performing loans
y^2	Securities	Asset value of securities investments

6.3 Results of stochastic nonparametric estimation of the Malmquist index

Appendix B shows the results of the proposed stochastic nonparametric estimation of an input-oriented Malmquist index. The essential steps of the estimation can be described as follows:

Step 1 Solve the CNLS problem in Eq. (5.2) to estimate $\hat{\varepsilon}_j^{CNLS}$. Note that the scaling vectors $\mathrm{g} = (1,1,1)$ and $\mathrm{h} = (1,1)$ are specified for all observing years as the common pre-assigned direction, so that the underlying production technology allows for simultaneous unit contraction in inputs and expansion in outputs.

Step 2 Apply Eq. (5.3) to evaluate the average performance of each regional bank ($\bar{\varepsilon}_j^{CNLS}$). The nonnegative consistent inefficiency ($\hat{u}_j$) is then computed by adopting Eq. (5.4). Calculate the reference points according to $(\hat{x}_{jk}, \hat{y}_{jk}) = (x_{jk} - (\hat{\varepsilon}_{jk}^{CNLS} + \hat{\mu})\mathrm{g}, y_{jk} + (\hat{\varepsilon}_{jk}^{CNLS} + \hat{\mu})\mathrm{h}), \forall j, k.$

Step 3 Calculate the (input-oriented) within inefficiency by solving Eqs. (5.5) and (5.6). Calculate the (input-oriented) intertemporal inefficiency by solving Eqs. (5.7) and (5.8). Apply Eq. (5.11) to estimate the (input-oriented) efficiency change (EC) and the technical change (TC). The stochastic nonparametric estimation of the input-oriented Malmquist index is then derived by calculating the product of EC and TC.

Table 6.2 reports descriptive statistics for the estimated coefficients of the CNLS problem. Due to the pre-specified directions, the average estimated coefficients $\hat{\beta}$ and $\hat{\tau}$ for each period are summed to unity. Further, as explained in Section 5.2, parameters α, β, and τ characterize the shape of the boundary of the underlying production technology, as they specify tangent hyperplanes to the directional distance function. Since there is a little variation in the estimated coefficients $\hat{\beta}$ and $\hat{\tau}$, it is reasonable to assume that the underlying production technology exhibits constant returns to scale when we proceed to estimate productivity growth. Meanwhile, both estimated coefficients $\hat{\beta}$ and $\hat{\tau}$ for 2013-2017 exhibit more variation than those for 2008-2012.

Therefore, it is meaningful to infer the difference in production technologies over the two periods.

Table 6.2. Descriptive statistics of the estimated coefficients

	Mean	St. dev.	Min	Max
(2008-2012)				
$\hat{\alpha}$	-537.10228	594.18758	-2183.04860	4500.43787
$\hat{\beta}^1$	0.99724	0.02955	0.34129	0.99912
$\hat{\beta}^2$	0.00002	0.00032	0	0.00703
$\hat{\beta}^3$	0.00182	0.02913	0	0.65010
$\hat{\tau}^1$	0.00045	0.00047	0.00036	0.00800
$\hat{\tau}^2$	0.00047	0.00011	0.00000	0.00249
(2013-2017)				
$\hat{\alpha}$	223.77785	15981.55023	-2195.17121	358377.19030
$\hat{\beta}^1$	0.99507	0.04558	0	0.99926
$\hat{\beta}^2$	0.00006	0.00027	0	0.00240
$\hat{\beta}^3$	0.00393	0.04361	0.00000	0.95433
$\hat{\tau}^1$	0.00050	0.00204	0	0.04568
$\hat{\tau}^2$	0.00044	0.00019	0.00000	0.00216

Notes: Units of deposits, capital, loans, and securities are in a million yen.

Productivity growth can be estimated based on the results of the CNLS problem by simply applying Eq. (5.10). Figure 6.3 reports empirical results of the proposed input-oriented Malmquist index under the assumption of constant returns to scale. The vertical axis represents the frequency of the estimated Malmquist index, and the horizontal axis represents the interval. As can be seen from the figure, a majority of regional banks had improved their productivity over the entire period. There was no evident difference in productivity change between regional banks I and II. Specifically, 17 of 63 (27%) regional banks I and 10 of 38 (26.3%) regional banks II have regressed in their productivity. Overall, the estimated input-oriented Malmquist index grew at an average of 2% from 2008-2012 to 2013-2017.

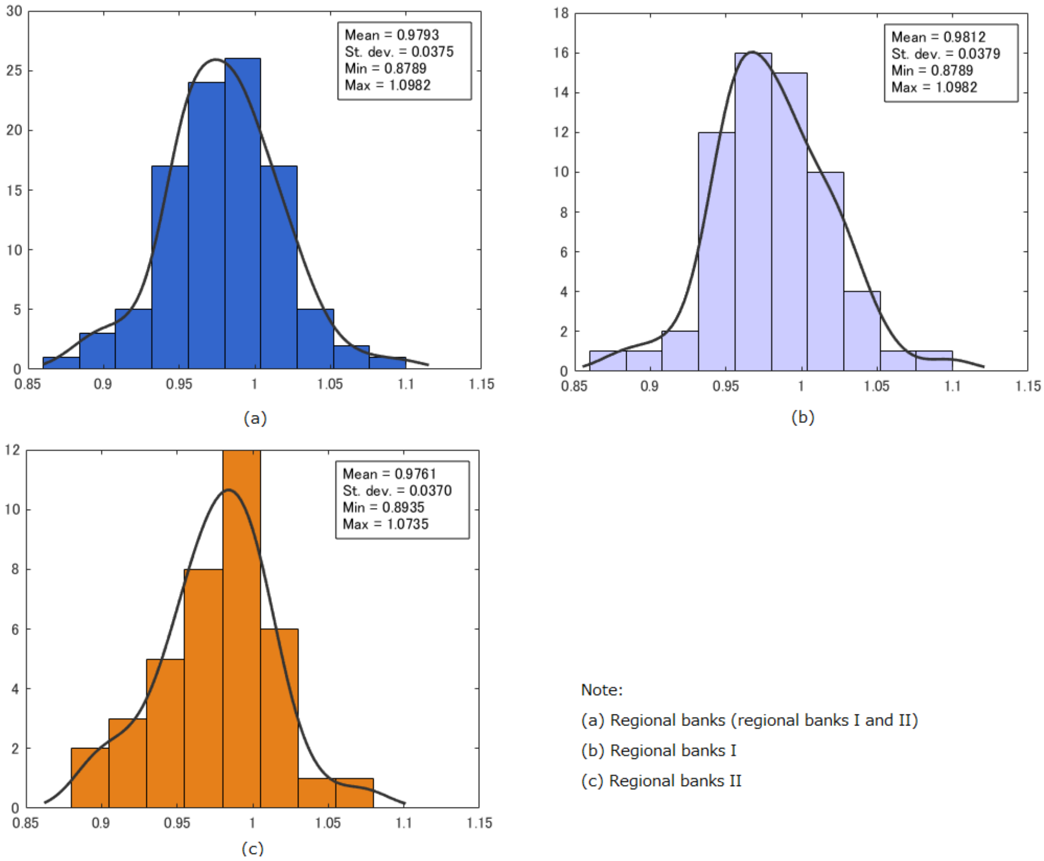


Figure 6.3 Empirical results of stochastic nonparametric estimation of an input-oriented Malmquist index.

To further investigate the drivers of productivity growth, I categorized the decomposed components of the proposed input-oriented Malmquist index into a two-dimensional table, as shown in Fig.6.4 (see Appendix B for details). According to this decomposition, the productivity growth of regional banks is mainly attributable to technical progress. This could be related to political and macroeconomic changes (i.e., ongoing market expansion due to the heated competition spurred by low-interest rates, as discussed in Section 6.1). On the other hand, the estimated values of efficiency change are evenly distributed on the two sides of unity, and this applies to both regional banks I and II. As shown in Fig.6.4, 32 of 63 (50.8%) regional banks I and 23 of 38 (60.5%) regional banks II exhibited efficiency improvements. Further, the average efficiency change of all regional banks is close to 1 (0.9986), which means average efficiency remained unchanged over the entire period. To understand this, note that regional banks have suffered a decline in primary interest income since 2008. Unchanged efficiency would suggest the overall effects of managerial improvement are offset by the effects of low profitability. As a consequence, continuous productivity growth in regional banks would require efficiency improvements. From the perspective of operational management, regional banks should review their personnel assignments and fixed investments to improve average efficiencies. From the perspective of profitability management, regional banks should not only keep increasing loans to low-return borrowers but also increasingly diversify investments to improve their interest income, so that continuous productivity growth is achievable.

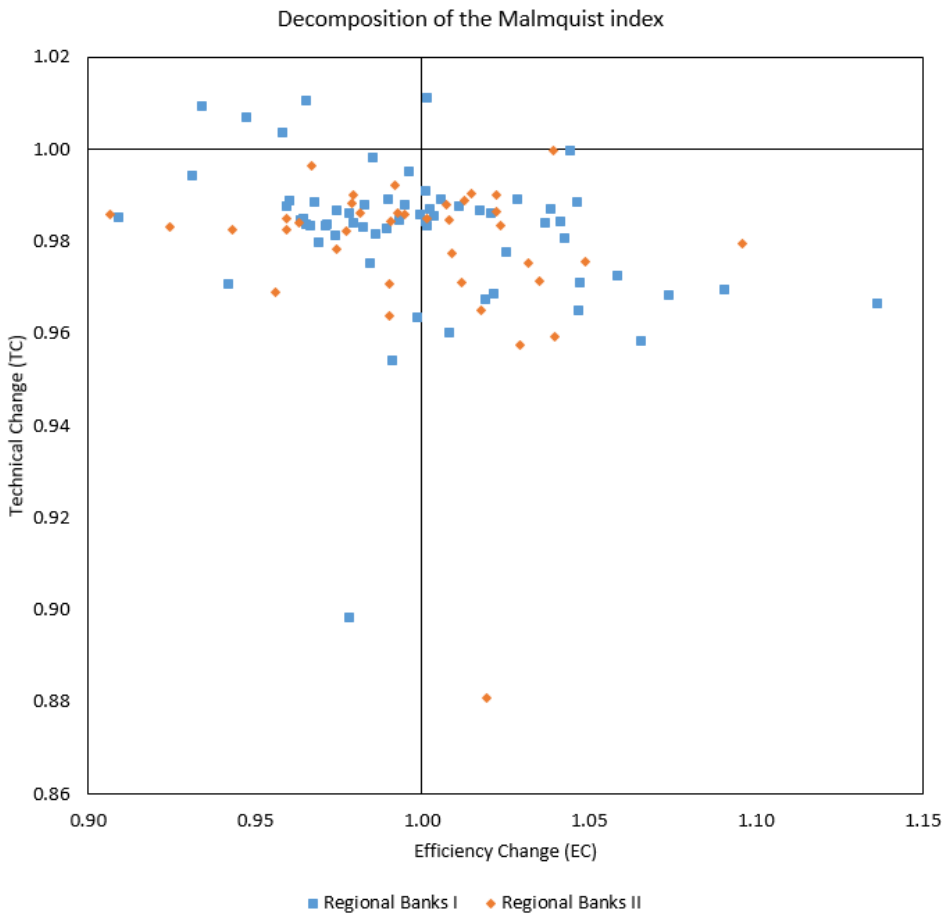


Figure 6.4 Results of decomposing the proposed input-oriented Malmquist index.

6.4 Concluding remarks

The methodology developed in Chapter 5 is applied to a sample of 101 Japanese regional banks over two periods. Note that the proposed index can be used for two or more than two periods. The estimated Malmquist index shows an increase in productivity change for both regional banks I and II. To investigate the drivers of productivity growth, I further decomposed the proposed index into components of efficiency change and technical change. The decomposition results show that the main factor contributing to productivity growth was technical progress, which provides a consistent interpretation of the estimated coefficients of the CNLS problem. The empirical results can reasonably be interpreted in terms of political and macroeconomic changes. Thus, the merits of the proposed index and the validity of the results have been illustrated.

Chapter 7

A further study of Japanese securities companies under stochastic noise

7.1 Introduction

In Chapter 4, the profit-ratio change index is demonstrated in terms of a sample of 37 Japanese securities companies observed from 2011 to 2015. Considering the significant changes in business management that appeared in the Japanese securities industry around the year 2013, the analysis is separated into the productivity change between the years 2011 and 2013 and the productivity change between the years 2013 and 2015. The empirical results indicate that the productivity of Japanese securities companies progressed for each period.

On the other hand, as already discussed in Chapter 5, different political and economic conditions may have different impacts on productive performance. Therefore, it would also be interesting to investigate the differences in productivity due to the changes in business management (i.e., the Japanese securities companies tended to focus more on the asset management business since 2013, see further details in Chapter 4). Since the primary focus for each period is the average productive performance of securities companies, I hereby adopt the stochastic nonparametric estimation of the Malmquist index to estimate the productivity changes between the analyzing periods 2011-2013 and 2013-2015. Specifically, I consider the same sample of 37 Japanese securities companies and the same selection of inputs and outputs, as reported in Section 4.2. To estimate the average productive performance before and after the year 2013, I collected the panel data of two analyzing periods from annual securities reports as published by each securities company. The first analyzing period covers the years 2011, 2012, and 2013, and the second one covers 2013, 2014, and 2015.

The rest of this chapter is organized as follows. Section 7.2 discusses the empirical results. Section 7.3 concludes the chapter. An appendix of the detail results is provided in the last section.

7.2 Empirical results

This section is divided into three parts. The first part provides the economic interpretation of the coefficients estimated from the panel-data model; the second part examines the results of the estimated inefficiencies; the last part summarizes the results of the stochastic nonparametric estimation of the input-oriented Malmquist index.

7.2.1 The interpretation of the estimated coefficients

To allow for the simultaneous unit contraction in inputs and expansion in outputs, I specify the direction vectors $g = (1,1,1)$ and $h = (1,1)$ for all observing years as the common pre-assigned direction when adopting the panel-data model in Section 5.2. The descriptive statistics of the estimated parameters $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\tau}$ in the two periods are reported in Table 7.1.

Table 7.1. Descriptive statistics of the estimated coefficients

	Mean	St. dev.	Min	Max
(2011-2013)				
$\hat{\alpha}$	1276.31044	7885.43320	-3699.294811	80952.44237
$\hat{\beta}^1$	0.30229	0.26452	0	0.84494
$\hat{\beta}^2$	0.13703	0.16991	0	0.68886
$\hat{\beta}^3$	0.20206	0.22682	0	0.70063
$\hat{\tau}^1$	0.08568	0.11621	0	0.99570
$\hat{\tau}^2$	0.11750	0.10034	0.00000	0.43425
$\hat{\tau}^3$	0.11184	0.12511	0.00000	0.49967

$\hat{\tau}^4$	0.04360	0.04322	0.00000	0.24026
(2013-2015)				
$\hat{\alpha}$	2508.85972	13255.29478	-3863.53292	137299.52707
$\hat{\beta}^1$	0.31297	0.27632	0	0.85622
$\hat{\beta}^2$	0.15080	0.18438	0	0.63350
$\hat{\beta}^3$	0.22703	0.24717	0	0.83953
$\hat{\tau}^1$	0.06616	0.07623	0	0.48147
$\hat{\tau}^2$	0.09953	0.08954	0.00000	0.45064
$\hat{\tau}^3$	0.08986	0.10552	0	0.51667
$\hat{\tau}^4$	0.05365	0.05888	0.00000	0.32041

According to the duality between the directional distance function and the profit function, coefficients β and τ can be interpreted as the marginal products of input-spending and output-earnings, respectively. Note that all inputs and outputs of Japanese securities companies have the same unit (yen), and thus the inefficiency is estimated on monetary sale. For example, increasing the trading related expenses ($\bar{x}_1$) by one yen increases inefficiency by 30.2% yen, on average, across all observed securities companies over the period 2011-2013. Further, recall that parameters α, β, and τ of the panel-data model characterize the shape of the boundary of the underlying production technology. As shown in Table 7.1, the estimated coefficients $\hat{\beta}$ and $\hat{\tau}$ for 2013-2015 exhibit more variation than those for 2011-2013, implying that production technology changes over two periods. Section 7.2.3 further investigates the extent of changes in production technologies by adopting the stochastic nonparametric estimation of the input-oriented Malmquist index.

7.2.2 Results of the estimated inefficiencies

Based on the estimated CNLS residuals, the inefficiency term of each securities company is derived by using Eq. (5.4) in Chapter 5. The results of the average inefficiencies and the standard deviations of the inefficiency and noise are summarized below.

Table 7.1. Results of the average inefficiencies and the standard deviations of the inefficiency and noise

	(2011-2013)	(2013-2015)
$\hat{\mu}$	802.66805	742.40660
$\hat{\sigma}_u$	344.78232	203.25239
$\hat{\sigma}_v$	424.10025	397.33423

As can be seen from Table 7.1, the average inefficiency $\hat{\mu}$ in the securities industry decreased over time. Meanwhile, there is a significant variation of the inefficiency over two periods in the sense that $\hat{\sigma}_u$ in the period 2011-2013 is larger than that in the period 2013-2015. In contrast, the differences in $\hat{\sigma}_v$ over two periods are quite small, which indicates that the noises for both periods behave nearly the same.

To further evaluate the results of the estimated inefficiencies, I next consider if there might be a problem with outliers (i.e., the one whose relative performance difference is extreme). A simple way to identify the outliers is to use the scatterplot matrix, which is symmetric about its diagonal and shows the relationships (linear correlations) between multiple variables. The scatterplot matrices for 37 Japanese securities companies with three inputs and four outputs are shown in Figs.7.1 and 7.2. The diagonals of those two figures plot univariate histograms of the input and output variables in two periods, respectively. It is clear that there are 3 points above all the other points as on the upper right of each figure. Indeed, these points are the same securities company (M1) who has more massive inputs and outputs than the others over the period 2011-2015. To see whether the outlier has a considerable influence on the or not, I eliminate M1 and re-estimate the panel-data model in Section 5.2.

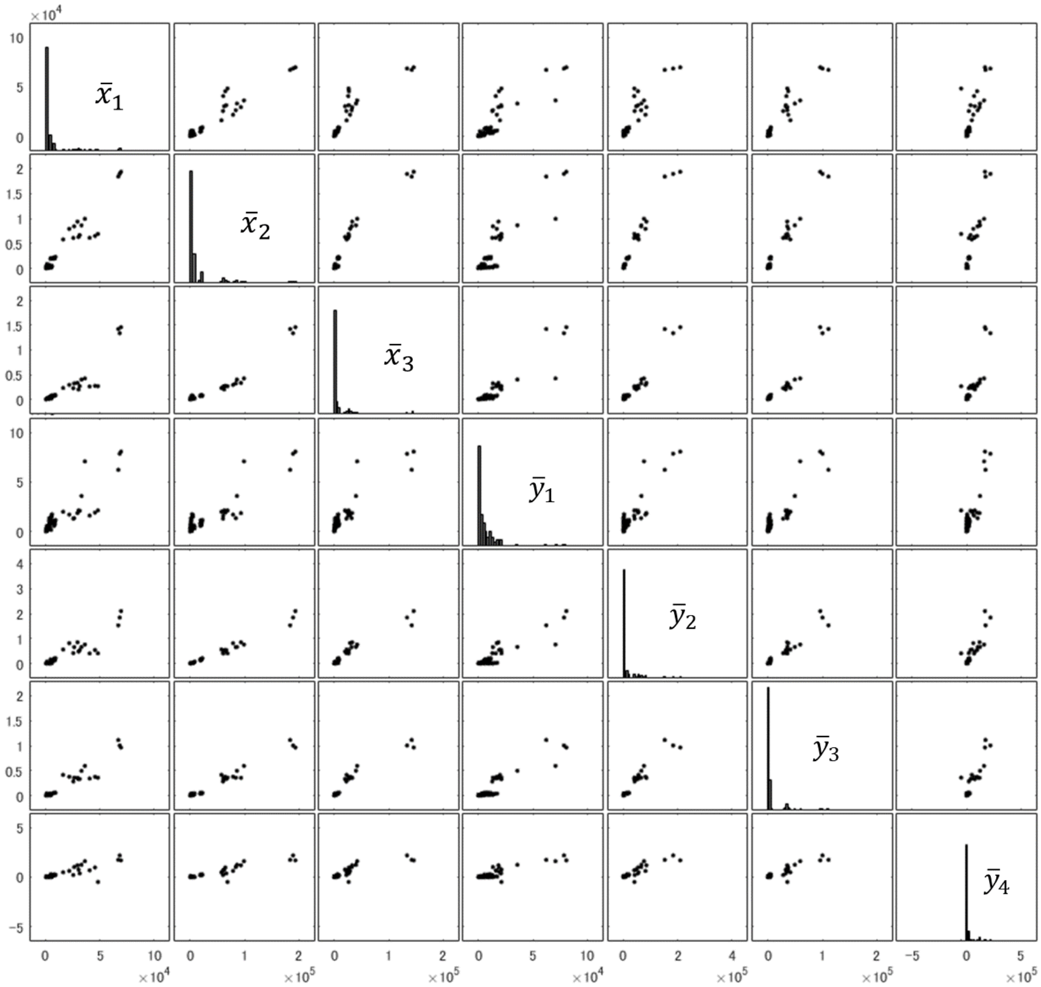


Figure 7.1 Scatterplot matrix of the data in the period 2011-2013.

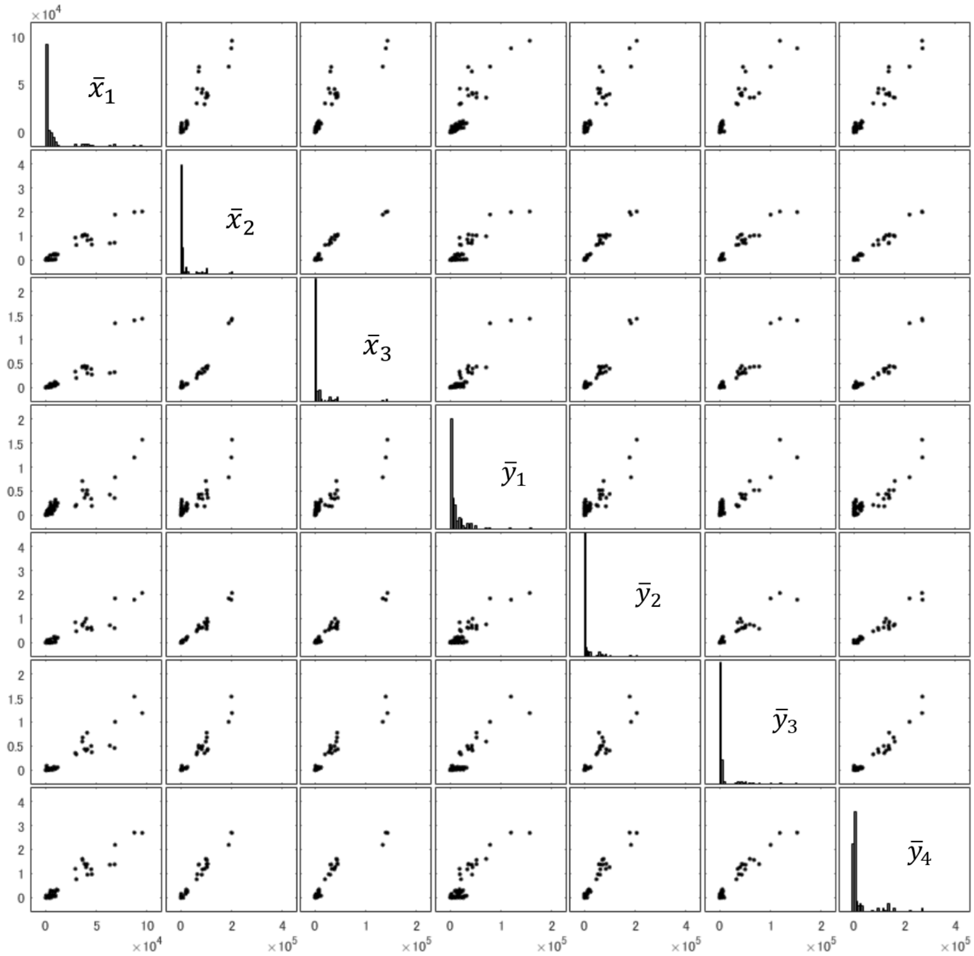


Figure 7.2 Scatterplot matrix of the data in the period 2013-2015.

The re-estimated results of the average inefficiencies and the standard deviations of the inefficiency and noise are summarized below.

Table 7.2. Re-estimated results of the average inefficiencies and the standard deviations of the inefficiency and noise

	(2011-2013)		(2013-2015)	
	37 observations	36 observations	37 observations	36 observations
$\hat{\mu}$	802.66805	799.93347	742.40660	741.75461
$\hat{\sigma}_u$	344.78232	353.75853	203.25239	202.35733
$\hat{\sigma}_v$	424.10025	425.00023	397.33423	363.17758

In Table 7.2, we can see that $\hat{\mu}$, $\hat{\sigma}_u$, and $\hat{\sigma}_v$ estimated without the outlier are nearly the same as the original values. To further investigate the extent to which the outlier affects the estimation of inefficiency, I calculated the rank correlation of the original estimated inefficiencies (where the inefficiency score of M1 is eliminated) and the re-estimated inefficiencies. As a result, the Spearman's rank correlation coefficient is 0.99990 in the period 2011-2013 and 0.99750 in the period 2013-2015. That is, the same ranking of inefficiencies for securities companies is observed, no matter whether the outliers are eliminated or not. Indeed, the same can be said of the estimated Malmquist index as well. The rank correlation of the Malmquist index based on the original inefficiencies and the one based on the re-estimated inefficiencies is 0.99995. These results indicate that, in this empirical application, the ranks of the efficiencies estimated in the panel-data model (as well as the Malmquist index to be discussed in Section 7.3) are unaffected by the outliers. Considering that the outlier (M1) may reflect the innovation in business management from which other securities companies would want to learn, the productivity analysis proceeds with the original dataset for 37 securities companies with three inputs and four outputs.

7.2.3 Results of the stochastic nonparametric estimation of the Malmquist index

Figure 7.3 visualizes the empirical results of the stochastic nonparametric estimation of an input-oriented Malmquist index (see Appendix C for more details). As can be seen from the figure, most Japanese securities companies had improved their productivity from the period 2011-2013 to the period 2013-2015. Specifically, 33 of 37 (89.2%) securities companies have progressed in their productivity. Overall, the estimated input-oriented Malmquist index implies the productivity of Japanese securities companies grew at an average of 19.7% after 2013, where the paradigm shifts in the asset management business occurred.

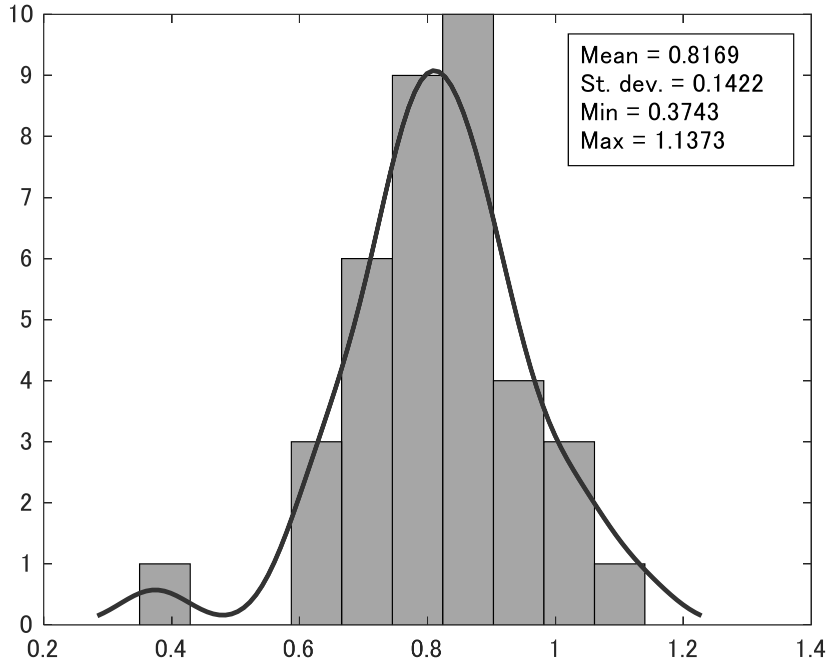


Figure 7.3 Empirical results of stochastic nonparametric estimation of an input-oriented Malmquist index.

The estimated input-oriented Malmquist index can be decomposed into a technical efficiency change component and a technical change component. As shown in Fig.7.4, the productivity growth in the securities industry was mainly attributable to the technical progress, which had an average growth rate of 23.5%. This could be related to the paradigm shifts in the asset management business over two periods and explains why the average inefficiency in the securities industry improved from the period 2011-2013 to the period 2013-2015. On the other hand, the (input-oriented) efficiency change regressed at an average rate of 5.0%, indicating the average (input-oriented) performance of the observed securities companies declined after the business management changed. In summary, the managers of Japanese securities companies should pay attention to their operational and managerial performance in inputs to catch up with the changes in business management.

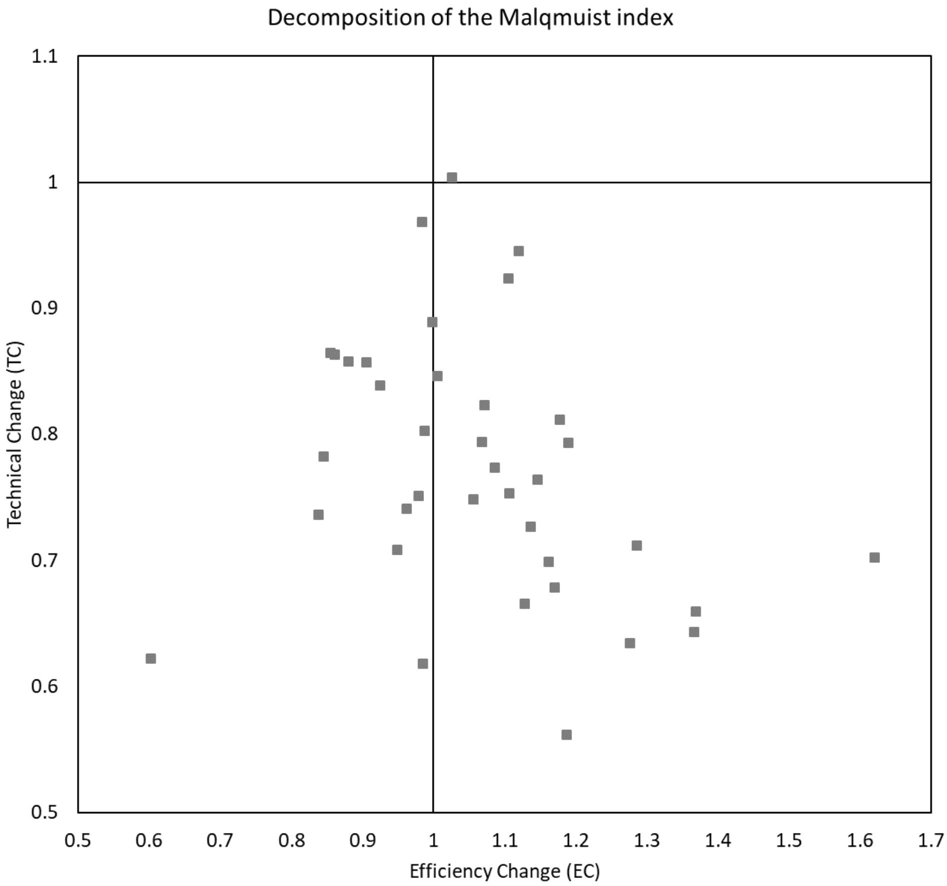


Figure 7.4 Results of decomposing the proposed input-oriented Malmquist index.

7.3 Concluding remarks

This chapter investigated the productivity changes of Japanese securities companies from the period 2011-2013 to the period 2013-2015. Specifically, the stochastic nonparametric estimation of the input-oriented Malmquist index proposed in Chapter 5 is applied. According to the analysis of estimated residuals and inefficiencies, there is a significant variation in inefficiencies over time whereas the noise behaves nearly the same for both periods. To see whether the outliers have a significant influence on the estimation of inefficiency, I re-estimated the panel-data model proposed in Chapter 5 and calculated the rank correlation of the original and the re-estimated inefficiencies. The results found that the ranks of the efficiencies are unaffected by the outlies. Considering that the outlier may reflect the innovation in business management from which other securities companies would want to learn, the productivity analysis has proceeded with the original dataset for 37 securities companies. The empirical results indicate that the productivity of Japanese securities companies progressed over two periods, and the main driver of productivity growth was the technical progress.

Chapter 8

Conclusions

The thesis provided two types of nonparametric methodologies for analyzing efficiency and productivity change. Based on the theory of DEA, a new efficiency concept is developed: allocative efficiency regarding profit-ratio maximization. The derived efficiency is then used to construct a novel comprehensive productivity index: a profit-ratio change index. Note that the DEA approach assumes away stochastic noise. Therefore, based on the theory of StoNED, a stochastic nonparametric estimation of Malmquist-type indices is also proposed to account for the impact of noise. Empirical applications are provided for demonstrating the proposed methods. Specifically, the contributions of this thesis are summarized as follows:

(a) A new scheme of allocative efficiency is developed. The allocative efficiency regarding profit-ratio maximization is suitable for performance evaluations in which producers desire to maximize revenue and minimize expenses simultaneously. It provides a comprehensive understanding of the sources of inefficiency, that is, the wrong input mix, the wrong output mix, and the wrong mix in both inputs and outputs. Further, the price information is not necessary because a value-based technology set is incorporated.

(b) A new Malmquist-type index is proposed, which accounts for the impact of allocative efficiency changes on productivity change. The profit-ratio change index can be applied to panel data to measure productivity change and suitable for profit-seeking organizations or industries. The index can be decomposed into the conventional Malmquist index and an allocation Malmquist index. Since the latter evaluates the impact of allocative efficiency changes on productivity change, the decomposition suggests the profit-ratio change index gives a comprehensive understanding of the sources of productivity change.

(c) A new model for estimating the Malmquist-type indices under stochastic noise is suggested. The proposed stochastic nonparametric estimation of Malmquist-type indices can be used to measure the productivity change in a stochastic setting. It is also capable of dealing with multi-input and multi-output production technology.

(d) In Japan, both securities companies and regional banks face significant challenges in their management under uncertain economic conditions and a competitive business environment. Using the above nonparametric methods, I evaluated the productive performances of Japanese securities companies and regional banks, respectively. I also investigated the drivers of productivity change by applying the decomposition of Malmquist-type indices. These results provide realistic projections and policy implications for improving the productive performance.

Despite the fact that the performance evaluation plays an essential role in the fields of management science and economics, assessing efficiency and productivity can be difficult in some situations, especially when the prior information on the production function is unavailable or multiple inputs and multiple outputs are involved. By addressing essential practical and theoretical problems in the measurement of allocative efficiency and performance evaluation under stochastic noise, the proposed methods of this thesis contributed to the nonparametric evaluation of efficiency and productivity in the multi-input and multi-output setting. In other words, those methods can be applied to a wide range of production activities that transform multiple inputs to multiple outputs, and the prior information on the relationship of inputs and outputs is not required. The proposed methods provide potent tools for decision-makers, regulators, or policy-makers. Through the performance evaluations, valuable information such as realistic projections and policy implications can be easily derived from the evaluated results, which is required for continuous performance improvement.

In future research, I would like to develop a StoNED-based estimation of the profit-ratio change index. It would also be attractive to consider alternative representations of multi-input and multi-output production technology using Shephard distance functions. I have done some theoretical works in this field, which may provide useful insights for future researches.

Appendices

Appendix A: Empirical results in Chapter 4

Note:

(a) M1~M5 are the major securities companies, B6~B12 are the bank-affiliated securities companies, O13~O19 are the online brokers, and I20~I37 are the other integrated securities companies.

(b) As discussed in Section 2.3, the intertemporal comparison terms of PI ($\pi^{t}\left(\bar{x}^{*t}_{io,t+1}, \bar{y}^{*t}_{ro,t+1}\right)$ and $\pi^{t+1}\left(\bar{x}^{*t+1}_{io,t}, \bar{y}^{*t+1}_{ro,t}\right)$) may have infeasible solutions. Such cases are reported in Tables A-3 and A-3, and alternatively used a super efficiency evaluation (Andersen and Petersen [57]) to calculate the profit-ratio efficiency in Eq. (8). The values with "*" represent the cases that $\pi^{t}\left(\bar{x}^{*t}_{io,t+1}, \bar{y}^{*t}_{ro,t+1}\right)$ were infeasible, implying the technology in time period t does not encompass the evaluated company in period $t+1$, and the values with "**" represent the cases that $\pi^{t+1}\left(\bar{x}^{*t+1}_{io,t}, \bar{y}^{*t+1}_{ro,t}\right)$ were infeasible, implying the technology in time period $t+1$ does not encompass the evaluated company in period t.

Table A-1. Results of profit-ratio, technical and allocative efficiencies in 2011, 2013, and 2015

Activities	2011			2013			2015		
	PE	TE	AE	PE	TE	AE	PE	TE	AE
M1	1.000	1.000	1.000	0.749	0.975	0.768	0.751	0.969	0.775
M2	1.000	1.000	1.000	1.000	1.000	1.000	0.748	0.850	0.881
M3	1.000	1.000	1.000	1.000	1.000	1.000	0.708	0.932	0.759
M4	0.368	0.505	0.729	0.656	0.754	0.869	0.746	0.794	0.939
M5	0.656	0.734	0.895	0.671	0.795	0.844	0.681	0.785	0.867

B6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
B7	1.000	1.000	1.000	1.000	1.000	1.000	0.768	0.809	0.949
B8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
B9	1.000	1.000	1.000	0.633	0.915	0.691	0.615	0.859	0.716
B10	0.455	0.651	0.698	0.630	1.000	0.630	1.000	1.000	1.000
B11	0.691	0.923	0.749	0.624	0.871	0.717	0.665	0.948	0.701
B12	0.596	0.766	0.777	0.619	0.973	0.636	1.000	1.000	1.000
O13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
O14	1.000	1.000	1.000	0.790	0.914	0.865	0.757	0.878	0.862
O15	0.680	0.969	0.702	0.710	0.889	0.798	0.684	0.816	0.839
O16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
O17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
O18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
O19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
I20	0.675	0.796	0.849	0.730	0.899	0.812	1.000	1.000	1.000
I21	0.769	0.845	0.910	0.810	0.938	0.863	0.849	0.965	0.881
I22	0.892	0.994	0.897	1.000	1.000	1.000	0.802	0.973	0.824
I23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
I24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
I25	0.619	0.692	0.895	0.534	0.744	0.718	0.554	0.759	0.730
I26	0.825	0.907	0.909	1.000	1.000	1.000	0.794	0.909	0.873
I27	0.701	0.861	0.815	0.587	0.762	0.770	0.719	0.934	0.770
I28	0.689	0.872	0.789	0.729	0.917	0.795	0.729	0.897	0.812
I29	0.730	0.893	0.818	0.557	0.713	0.781	0.702	0.947	0.742
I30	0.465	0.655	0.710	0.766	0.930	0.823	0.508	0.803	0.633
I31	0.576	0.735	0.783	0.603	0.814	0.741	1.000	1.000	1.000
I32	0.563	0.753	0.747	1.000	1.000	1.000	0.626	0.955	0.655
I33	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
I34	0.488	0.738	0.662	0.560	0.951	0.589	1.000	1.000	1.000
I35	0.526	0.745	0.706	1.000	1.000	1.000	0.467	0.642	0.728
I36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
I37	0.595	0.861	0.691	1.000	1.000	1.000	1.000	1.000	1.000
Mean	0.799	0.889	0.885	0.837	0.939	0.884	0.834	0.930	0.890
G.Mean	0.769	0.878	0.876	0.817	0.935	0.873	0.816	0.926	0.882

SD	0.210	0.136	0.123	0.180	0.088	0.134	0.170	0.091	0.122
Min	0.368	0.505	0.662	0.534	0.713	0.589	0.467	0.642	0.633
Max	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A-2. Results of the profit-ratio change index and its component indices from 2011 to 2013 (the case of three inputs and four outputs)

Activities	$PI = PEC \times PTC$			$MI = TEC \times TC$			$AMI = AEC \times ATC$		
	PI	PEC	PTC	MI	TEC	TC	AMI	AEC	ATC
M1	1.006	1.335	0.754	0.986	1.025	0.962	1.021	1.302	0.784
M2	0.932	1.000	0.932	0.975	1.000	0.975	0.956	1.000	0.956
M3	0.941	1.000	0.941	0.989	0.998	0.990	0.952	1.002	0.951
M4	0.429	0.562	0.765	0.640	0.670	0.955	0.671	0.838	0.800
M5	0.709	0.977	0.725	0.787	0.923	0.853	0.901	1.059	0.850
B6	0.895	1.000	0.895	0.862	1.000	0.862	1.039	1.000	1.039
B7	0.911**	1.000	0.911**	1.182	1.000	1.182	0.771	1.000	0.771
B8	0.811	1.000	0.811	0.861	1.000	0.861	0.942	1.000	0.942
B9	1.288	1.581	0.815	1.187	1.092	1.087	1.085	1.447	0.750
B10	0.624	0.722	0.864	0.767	0.651	1.178	0.813	1.108	0.734
B11	0.766	1.106	0.693	0.818	1.059	0.773	0.936	1.045	0.896
B12	0.624	0.962	0.649	0.552	0.788	0.701	1.131	1.221	0.926
O13	1.048	1.000	1.048	0.894	1.000	0.894	1.173	1.000	1.173
O14	1.222	1.266	0.965	1.371	1.094	1.253	0.891	1.157	0.771
O15	0.990	0.958	1.034	1.045	1.090	0.959	0.947	0.879	1.078
O16	0.977	1.000	0.977	1.008	1.000	1.008	0.969	1.000	0.969
O17	1.084**	1.000	1.084**	1.131	1.000	1.131	0.958	1.000	0.958
O18	0.812**	1.000	0.812**	1.146	1.000	1.146	0.708	1.000	0.708
O19	1.367**	1.000	1.367**	1.124	1.000	1.124	1.217	1.000	1.217
I20	0.707	0.925	0.764	0.766	0.885	0.865	0.923	1.045	0.883
I21	0.701	0.950	0.738	0.712	0.901	0.791	0.984	1.055	0.933
I22	0.771*	0.892	0.864*	0.825	0.994	0.830	0.934	0.897	1.041
I23	0.873*	1.000	0.873*	0.926	1.000	0.926	0.943	1.000	0.943
I24	0.859*	1.000	0.859*	0.904	1.000	0.904	0.951	1.000	0.951
I25	0.868	1.160	0.749	0.795	0.931	0.854	1.092	1.246	0.876

I26	0.836	0.825	1.014	0.771	0.907	0.850	1.085	0.909	1.193
I27	0.899	1.195	0.753	0.991	1.130	0.877	0.907	1.058	0.858
I28	0.759	0.945	0.804	0.886	0.952	0.931	0.858	0.993	0.864
I29	0.854	1.312	0.651	0.960	1.251	0.767	0.889	1.048	0.849
I30	0.483	0.608	0.795	0.600	0.705	0.851	0.806	0.863	0.934
I31	0.667	0.955	0.698	0.727	0.903	0.805	0.917	1.058	0.867
I32	0.478	0.563	0.850	0.586	0.753	0.778	0.816	0.747	1.093
I33	0.605*	1.000	0.605*	0.640	1.000	0.640	0.945	1.000	0.945
I34	0.616	0.872	0.706	0.712	0.776	0.918	0.865	1.124	0.770
I35	0.539	0.526	1.025	0.723	0.745	0.970	0.746	0.706	1.057
I36	0.722	1.000	0.722	0.805	1.000	0.805	0.898	1.000	0.898
I37	0.667	0.595	1.121	0.866	0.861	1.006	0.770	0.691	1.115
Mean	0.820	0.967	0.855	0.879	0.948	0.926	0.930	1.013	0.928
G.Mean	0.793	0.941	0.842	0.860	0.939	0.916	0.922	1.003	0.919
SD	0.216	0.219	0.157	0.188	0.132	0.141	0.123	0.148	0.130
Min	0.429	0.526	0.605	0.552	0.651	0.640	0.671	0.691	0.708
Max	1.367	1.581	1.367	1.371	1.251	1.253	1.217	1.447	1.217

Table A-3. Results of the profit-ratio change index and its component indices from 2013 to 2015 (the case of three inputs and four outputs)

Activities	$PI = PEC \times PTC$			$MI = TEC \times TC$			$AMI = AEC \times ATC$		
	PI	PEC	PTC	MI	TEC	TC	AMI	AEC	ATC
M1	0.938	0.998	0.940	1.013	1.007	1.006	0.926	0.991	0.935
M2	1.194	1.336	0.893	1.098	1.177	0.933	1.087	1.135	0.957
M3	1.336	1.412	0.946	1.151	1.074	1.071	1.160	1.315	0.883
M4	0.899	0.879	1.022	0.908	0.950	0.956	0.990	0.925	1.070
M5	0.991	0.986	1.005	0.977	1.012	0.965	1.015	0.974	1.042
B6	0.965**	1.000	0.965**	0.988	1.000	0.988	0.977	1.000	0.977
B7	0.947	1.302	0.727	0.960	1.236	0.776	0.986	1.053	0.936
B8	0.956	1.000	0.956	0.903	1.000	0.903	1.059	1.000	1.059
B9	0.804	1.028	0.782	0.932	1.065	0.875	0.862	0.965	0.894
B10	0.591	0.630	0.937	0.836	1.000	0.836	0.706	0.630	1.121
B11	0.950	0.939	1.012	0.893	0.919	0.972	1.064	1.022	1.041

B12	0.729	0.619	1.177	0.931	0.973	0.957	0.783	0.636	1.230
O13	0.863	1.000	0.863	0.756	1.000	0.756	1.142	1.000	1.142
O14	0.810	1.044	0.776	0.784	1.041	0.754	1.033	1.003	1.029
O15	0.782	1.038	0.754	0.856	1.091	0.785	0.915	0.952	0.961
O16	0.925	1.000	0.925	0.806	1.000	0.806	1.147	1.000	1.147
O17	0.912	1.000	0.912	0.874	1.000	0.874	1.045	1.000	1.045
O18	1.083	1.000	1.083	0.992	1.000	0.992	1.092	1.000	1.092
O19	0.613*	1.000	0.613*	0.707	1.000	0.707	0.868	1.000	0.868
I20	0.713	0.730	0.976	0.893	0.899	0.994	0.798	0.812	0.983
I21	0.969	0.953	1.016	0.996	0.973	1.023	0.973	0.980	0.993
I22	1.179	1.247	0.946	1.040	1.027	1.012	1.134	1.213	0.935
I23	0.905	1.000	0.905	0.912	1.000	0.912	0.992	1.000	0.992
I24	0.957	1.000	0.957	0.916	1.000	0.916	1.045	1.000	1.045
I25	0.867	0.964	0.900	0.933	0.980	0.951	0.930	0.983	0.946
I26	1.152	1.259	0.915	1.101	1.100	1.000	1.047	1.145	0.915
I27	0.707	0.816	0.866	0.779	0.816	0.956	0.907	1.001	0.907
I28	0.890	1.000	0.890	0.951	1.022	0.930	0.936	0.979	0.957
I29	0.834	0.793	1.052	0.785	0.753	1.042	1.062	1.053	1.009
I30	1.280	1.506	0.850	1.122	1.158	0.969	1.141	1.300	0.877
I31	0.416	0.603	0.690	0.428	0.814	0.526	0.972	0.741	1.312
I32	1.168**	1.598	0.731**	1.065	1.047	1.016	1.097	1.526	0.719
I33	1.187**	1.000	1.187**	1.268	1.000	1.268	0.936	1.000	0.936
I34	0.590	0.560	1.054	0.794	0.951	0.835	0.743	0.589	1.262
I35	1.209	2.142	0.565	1.003	1.558	0.644	1.206	1.374	0.877
I36	0.920	1.000	0.920	0.887	1.000	0.887	1.038	1.000	1.038
I37	0.855	1.000	0.855	0.802	1.000	0.802	1.066	1.000	1.066
Mean	0.921	1.037	0.907	0.920	1.017	0.908	0.997	1.008	1.005
G.Mean	0.897	1.001	0.896	0.907	1.010	0.898	0.989	0.991	0.999
SD	0.205	0.297	0.136	0.147	0.129	0.133	0.119	0.186	0.118
Min	0.416	0.560	0.565	0.428	0.753	0.526	0.706	0.589	0.719
Max	1.336	2.142	1.187	1.268	1.558	1.268	1.206	1.526	1.312

Table A-4. Results of the profit-ratio change index and its component indices from 2011 to 2013 (the case of an aggregated input and aggregated output)

Activities	$PI = PEC \times PTC$			$MI = TEC \times TC$			$AMI = AEC \times ATC$		
	PI	PEC	PTC	MI	TEC	TC	AMI	AEC	ATC
M1	0.915	1.238	0.739	0.915	1.238	0.739	1.000	1.000	1.000
M2	0.829	1.122	0.739	0.829	1.122	0.739	1.000	1.000	1.000
M3	0.922	1.247	0.739	0.922	1.247	0.739	1.000	1.000	1.000
M4	0.449	0.608	0.739	0.449	0.608	0.739	1.000	1.000	1.000
M5	0.739	1.000	0.739	0.739	1.000	0.739	1.000	1.000	1.000
B6	0.855	1.157	0.739	0.855	1.157	0.739	1.000	1.000	1.000
B7	0.849	1.149	0.739	0.849	1.149	0.739	1.000	1.000	1.000
B8	0.797	1.078	0.739	0.797	1.078	0.739	1.000	1.000	1.000
B9	1.089	1.473	0.739	1.089	1.473	0.739	1.000	1.000	1.000
B10	0.683	0.924	0.739	0.683	0.924	0.739	1.000	1.000	1.000
B11	0.905	1.225	0.739	0.905	1.225	0.739	1.000	1.000	1.000
B12	0.689	0.932	0.739	0.689	0.932	0.739	1.000	1.000	1.000
O13	1.022	1.382	0.739	1.022	1.382	0.739	1.000	1.000	1.000
O14	1.033	1.398	0.739	1.033	1.398	0.739	1.000	1.000	1.000
O15	0.932	1.261	0.739	0.932	1.261	0.739	1.000	1.000	1.000
O16	0.995	1.346	0.739	0.995	1.346	0.739	1.000	1.000	1.000
O17	1.082	1.464	0.739	1.082	1.464	0.739	1.000	1.000	1.000
O18	0.819	1.108	0.739	0.819	1.108	0.739	1.000	1.000	1.000
O19	1.014	1.372	0.739	1.014	1.372	0.739	1.000	1.000	1.000
I20	0.786	1.064	0.739	0.786	1.064	0.739	1.000	1.000	1.000
I21	0.804	1.088	0.739	0.804	1.088	0.739	1.000	1.000	1.000
I22	0.858	1.160	0.739	0.858	1.160	0.739	1.000	1.000	1.000
I23	0.876	1.185	0.739	0.876	1.185	0.739	1.000	1.000	1.000
I24	0.885	1.198	0.739	0.885	1.198	0.739	1.000	1.000	1.000
I25	0.939	1.270	0.739	0.939	1.270	0.739	1.000	1.000	1.000
I26	0.774	1.047	0.739	0.774	1.047	0.739	1.000	1.000	1.000
I27	0.920	1.244	0.739	0.920	1.244	0.739	1.000	1.000	1.000
I28	0.790	1.070	0.739	0.790	1.070	0.739	1.000	1.000	1.000
I29	0.862	1.166	0.739	0.862	1.166	0.739	1.000	1.000	1.000
I30	0.606	0.820	0.739	0.606	0.820	0.739	1.000	1.000	1.000

I31	0.823	1.114	0.739	0.823	1.114	0.739	1.000	1.000	1.000
I32	0.694	0.939	0.739	0.694	0.939	0.739	1.000	1.000	1.000
I33	0.604	0.817	0.739	0.604	0.817	0.739	1.000	1.000	1.000
I34	0.862	1.166	0.739	0.862	1.166	0.739	1.000	1.000	1.000
I35	0.828	1.121	0.739	0.828	1.121	0.739	1.000	1.000	1.000
I36	0.890	1.204	0.739	0.890	1.204	0.739	1.000	1.000	1.000
I37	0.768	1.039	0.739	0.768	1.039	0.739	1.000	1.000	1.000
Mean	0.843	1.140	0.739	0.843	1.140	0.739	1.000	1.000	1.000
G.Mean	0.831	1.125	0.739	0.831	1.125	0.739	1.000	1.000	1.000
SD	0.135	0.182	0.000	0.135	0.182	0.000	0.000	0.000	0.000
Min	0.449	0.608	0.739	0.449	0.608	0.739	1.000	1.000	1.000
Max	1.089	1.473	0.739	1.089	1.473	0.739	1.000	1.000	1.000

Table A-5. Results of the profit-ratio change index and its component indices from 2013 to 2015 (the case of an aggregated input and aggregated output)

Activities	$PI = PEC \times PTC$			$MI = TEC \times TC$			$AMI = AEC \times ATC$		
	PI	PEC	PTC	MI	TEC	TC	AMI	AEC	ATC
M1	0.880	0.800	1.100	0.880	0.800	1.100	1.000	1.000	1.000
M2	1.184	1.076	1.100	1.184	1.076	1.100	1.000	1.000	1.000
M3	1.026	0.932	1.100	1.026	0.932	1.100	1.000	1.000	1.000
M4	0.926	0.842	1.100	0.926	0.842	1.100	1.000	1.000	1.000
M5	1.010	0.918	1.100	1.010	0.918	1.100	1.000	1.000	1.000
B6	0.874	0.794	1.100	0.874	0.794	1.100	1.000	1.000	1.000
B7	0.916	0.833	1.100	0.916	0.833	1.100	1.000	1.000	1.000
B8	0.945	0.858	1.100	0.945	0.858	1.100	1.000	1.000	1.000
B9	0.774	0.703	1.100	0.774	0.703	1.100	1.000	1.000	1.000
B10	0.674	0.612	1.100	0.674	0.612	1.100	1.000	1.000	1.000
B11	0.975	0.886	1.100	0.975	0.886	1.100	1.000	1.000	1.000
B12	0.893	0.812	1.100	0.893	0.812	1.100	1.000	1.000	1.000
O13	0.891	0.810	1.100	0.891	0.810	1.100	1.000	1.000	1.000
O14	0.876	0.796	1.100	0.876	0.796	1.100	1.000	1.000	1.000
O15	0.864	0.785	1.100	0.864	0.785	1.100	1.000	1.000	1.000
O16	0.827	0.752	1.100	0.827	0.752	1.100	1.000	1.000	1.000

O17	0.949	0.862	1.100	0.949	0.862	1.100	1.000	1.000	1.000
O18	1.184	1.076	1.100	1.184	1.076	1.100	1.000	1.000	1.000
O19	1.039	0.944	1.100	1.039	0.944	1.100	1.000	1.000	1.000
I20	0.903	0.821	1.100	0.903	0.821	1.100	1.000	1.000	1.000
I21	0.896	0.814	1.100	0.896	0.814	1.100	1.000	1.000	1.000
I22	1.184	1.076	1.100	1.184	1.076	1.100	1.000	1.000	1.000
I23	0.910	0.827	1.100	0.910	0.827	1.100	1.000	1.000	1.000
I24	0.886	0.805	1.100	0.886	0.805	1.100	1.000	1.000	1.000
I25	0.891	0.809	1.100	0.891	0.809	1.100	1.000	1.000	1.000
I26	0.976	0.887	1.100	0.976	0.887	1.100	1.000	1.000	1.000
I27	0.872	0.792	1.100	0.872	0.792	1.100	1.000	1.000	1.000
I28	0.988	0.898	1.100	0.988	0.898	1.100	1.000	1.000	1.000
I29	0.892	0.811	1.100	0.892	0.811	1.100	1.000	1.000	1.000
I30	1.177	1.070	1.100	1.177	1.070	1.100	1.000	1.000	1.000
I31	0.491	0.447	1.100	0.491	0.447	1.100	1.000	1.000	1.000
I32	1.153	1.048	1.100	1.153	1.048	1.100	1.000	1.000	1.000
I33	1.283	1.166	1.100	1.283	1.166	1.100	1.000	1.000	1.000
I34	0.914	0.831	1.100	0.914	0.831	1.100	1.000	1.000	1.000
I35	0.987	0.897	1.100	0.987	0.897	1.100	1.000	1.000	1.000
I36	0.907	0.824	1.100	0.907	0.824	1.100	1.000	1.000	1.000
I37	0.881	0.801	1.100	0.881	0.801	1.100	1.000	1.000	1.000
Mean	0.943	0.857	1.100	0.943	0.857	1.100	1.000	1.000	1.000
G.Mean	0.931	0.846	1.100	0.931	0.846	1.100	1.000	1.000	1.000
SD	0.148	0.134	0.000	0.148	0.134	0.000	0.000	0.000	0.000
Min	0.491	0.447	1.100	0.491	0.447	1.100	1.000	1.000	1.000
Max	1.283	1.166	1.100	1.283	1.166	1.100	1.000	1.000	1.000

Appendix B: Empirical results in Chapter 6

	$\bar{M}$	EC	TC		$\bar{M}$	EC	TC
No.1	1.0128	1.0016	1.0112	No.52	0.9488	0.9637	0.9845
No.2	0.9475	0.9593	0.9876	No.53	1.004	1.0174	0.9867
No.3	0.9537	0.9472	1.0069	No.54	1.0175	1.0287	0.9892
No.4	0.9892	1.0037	0.9855	No.55	1.0249	1.0413	0.9842
No.5	0.9495	0.9653	0.9836	No.56	0.9549	0.971	0.9833
No.6	0.9496	0.9692	0.9798	No.57	0.985	1.0017	0.9834
No.7	0.9261	0.9313	0.9943	No.58	0.9494	0.9601	0.9889
No.8	0.9681	1.0083	0.9601	No.59	0.9556	0.9713	0.9838
No.9	0.9637	0.9794	0.984	No.60	0.9945	1.0055	0.9891
No.10	0.8789	0.9782	0.8984	No.61	0.983	0.995	0.9879
No.11	1.0982	1.1363	0.9665	No.62	0.96	0.9843	0.9753
No.12	0.9567	0.9677	0.9886	No.63	1.0171	1.0473	0.9711
No.13	0.9561	0.9742	0.9814	No.64	0.9862	1.009	0.9774
No.14	0.9678	0.9861	0.9815	No.65	0.8935	0.9064	0.9858
No.15	0.9913	0.9961	0.9951	No.66	1.0064	1.0319	0.9753
No.16	0.9615	0.9743	0.9868	No.67	0.9089	0.9244	0.9832
No.17	1.0025	1.0253	0.9778	No.68	1.0014	1.0128	0.9888
No.18	0.9616	0.9583	1.0034	No.69	1.039	1.0393	0.9997
No.19	1.0295	1.0585	0.9726	No.70	1.0064	1.0235	0.9833
No.20	1.0211	1.0655	0.9584	No.71	0.9264	0.9561	0.9689
No.21	0.978	0.9933	0.9846	No.72	0.898	1.0195	0.8808
No.22	0.9658	0.9824	0.9831	No.73	0.9678	0.9815	0.9861
No.23	0.9984	1.011	0.9875	No.74	0.9698	0.9795	0.9901
No.24	0.9142	0.9418	0.9707	No.75	1.0082	1.0221	0.9864
No.25	1.057	1.0904	0.9694	No.76	0.9633	0.9669	0.9963
No.26	1.0062	1.0205	0.986	No.77	1.0118	1.0221	0.99
No.27	0.9791	0.9898	0.9892	No.78	1.0053	1.0352	0.9712
No.28	0.9921	1.0011	0.991	No.79	0.975	0.9907	0.9842

No.29	1.04	1.074	0.9684	No.80	1.0047	1.0147	0.9902
No.30	1.0228	1.0428	0.9808	No.81	0.9805	0.9947	0.9857
No.31	0.9896	1.0216	0.9687	No.82	0.9792	0.9929	0.9862
No.32	0.9427	0.934	1.0094	No.83	0.9839	0.9917	0.9921
No.33	0.9853	0.9995	0.9858	No.84	0.9613	0.9901	0.9708
No.34	0.9758	0.9655	1.0107	No.85	1.0735	1.0959	0.9796
No.35	0.9707	0.9826	0.9879	No.86	0.9951	1.0073	0.9879
No.36	0.9855	1.0012	0.9844	No.87	0.9266	0.9433	0.9823
No.37	0.9645	0.978	0.9862	No.88	0.9924	1.008	0.9845
No.38	1.0439	1.0443	0.9996	No.89	0.9427	0.9595	0.9824
No.39	1.0251	1.0385	0.9871	No.90	0.9863	1.0014	0.985
No.40	0.9834	0.9853	0.9981	No.91	0.9825	1.0117	0.9711
No.41	0.962	0.9986	0.9633	No.92	0.9534	0.9745	0.9784
No.42	0.9857	1.0188	0.9675	No.93	0.9453	0.9596	0.985
No.43	0.8957	0.9092	0.9851	No.94	0.9819	1.0175	0.965
No.44	0.9502	0.9663	0.9833	No.95	0.9602	0.9775	0.9823
No.45	1.0201	1.0368	0.9839	No.96	0.9545	0.9903	0.9638
No.46	1.0101	1.0468	0.965	No.97	0.9974	1.0399	0.9592
No.47	0.9725	0.9895	0.9828	No.98	0.9857	1.0295	0.9574
No.48	1.0345	1.0465	0.9885	No.99	1.0231	1.0487	0.9756
No.49	0.9894	1.0025	0.987	No.100	0.9479	0.9634	0.9839
No.50	0.9499	0.9643	0.985	No.101	0.9672	0.9788	0.9881
No.51	0.9454	0.991	0.954				

Appendix C: Empirical results in Chapter 7

	Dataset with outliers			Dataset without outliers		
	$\bar{M}$	EC	TC	$\bar{M}$	EC	TC
M1	0.95241	0.98328	0.96861			
M2	1.02119	1.10541	0.92381	1.02307	1.10410	0.92661
M3	1.03003	1.02634	1.00360	1.03121	1.04323	0.98848
M4	0.77551	0.92495	0.83844	0.77437	0.92445	0.83765
M5	0.88779	0.99875	0.88890	0.88746	1.00260	0.88516
B6	0.91517	1.28568	0.71182	0.91541	1.28647	0.71157
B7	0.81130	1.16147	0.69851	0.81046	1.16025	0.69852
B8	0.84771	1.06816	0.79361	0.84589	1.05435	0.80229
B9	1.13732	1.62026	0.70194	1.13690	1.61919	0.70214
B10	0.73536	0.97922	0.75096	0.73332	0.97716	0.75046
B11	0.88194	1.07181	0.82286	0.88063	1.06696	0.82536
B12	0.74318	0.86087	0.86329	0.74185	0.85771	0.86492
O13	0.71242	0.96170	0.74080	0.71200	0.96108	0.74084
O14	0.94400	1.18994	0.79332	0.93903	1.18003	0.79577
O15	0.80993	1.27672	0.63438	0.80972	1.27531	0.63492
O16	0.66680	1.18761	0.56147	0.66655	1.18707	0.56151
O17	0.90229	1.36903	0.65907	0.90211	1.37137	0.65782
O18	1.05878	1.11955	0.94572	1.05907	1.11967	0.94588
O19	0.87899	1.36613	0.64341	0.88143	1.36648	0.64504
I20	0.75532	0.88049	0.85784	0.75263	0.88148	0.85383
I21	0.77572	0.90495	0.85719	0.77393	0.90761	0.85271
I22	0.85061	1.00506	0.84633	0.85093	1.00473	0.84693
I23	0.87555	1.14585	0.76411	0.87590	1.14942	0.76203
I24	0.79047	1.05585	0.74866	0.78996	1.05906	0.74591
I25	0.66126	0.84546	0.78213	0.66212	0.84501	0.78357
I26	0.83285	1.10608	0.75297	0.83301	1.10596	0.75320
I27	0.84059	1.08660	0.77359	0.83850	1.08489	0.77290

I28	0.82628	1.13712	0.72664	0.82630	1.13640	0.72712
I29	0.73921	0.85513	0.86444	0.73911	0.85496	0.86450
I30	0.79329	0.98798	0.80294	0.79298	0.99148	0.79980
I31	0.37432	0.60168	0.62212	0.37563	0.60891	0.61689
I32	0.79393	1.17078	0.67812	0.79390	1.17113	0.67790
I33	0.95571	1.17724	0.81183	0.95568	1.17658	0.81225
I34	0.60833	0.98494	0.61763	0.60815	0.98468	0.61761
I35	0.75117	1.12831	0.66575	0.75241	1.13685	0.66184
I36	0.61701	0.83782	0.73645	0.61702	0.83791	0.73639
I37	0.67236	0.94933	0.70825	0.67236	0.94889	0.70858
Mean	0.81692	1.06534	0.77193	0.81281	1.06787	0.76580
G.Mean	0.80330	1.04991	0.76512	0.79919	1.05224	0.75951
SD	0.14221	0.18339	0.10401	0.14230	0.18478	0.09922
Min	0.37432	0.60168	0.56147	0.37563	0.60891	0.56151
Max	1.13732	1.62026	1.00360	1.13690	1.61919	0.98848

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