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An Efficient Method for Estimation of Reduction of Welding Residual Stresses from Stress-Relief Annealing (Report I)†

–Development of Basic Equations for One-dimensional Stress State and Their Accuracy–

Keiji NAKACHO* and **Yukio UEDA****

Abstract

Stress-relief annealing (SR treatment) is often applied to relieve welding residual stresses. This treatment may be included in the fabrication process of pressure vessels, etc. This study aims at development of an efficient method as simple as hand calculation to estimate reduction of residual stresses of very thick welded joint by SR treatment. In this 1st report, an estimating method is developed for relaxation tests (in uni-axial stress state) at changing and constant temperatures. The stress relaxation phenomenon at the relaxation tests is considerably similar to that observed during SR treatment. In the next report, this method is extended to be applicable to SR treatment of very thick welded joints in multi-axial stress state.

KEY WORDS: (Simple Estimating Method) (Stress-Relief Annealing) (Welding) (Relaxation Test) (Transient Stress) (Residual Stress) (Thermal Elasitc-Plastic-Creep Analysis) (Thick Welded Joint)

1. Introduction

Stress-relief annealing (hereinafter called SR treatment) of which main purpose is to relieve welding residual stresses is included in the fabrication process of welded structures such as pressure vessels. Standard conditions for SR treatment are specified in JIS, ASME code, etc. However, they are not necessarily rational to the recent large-size structures constructed of high quality thick plates. The specified conditions seem to require to keep the structures at higher temperature for longer time than actually needed. One of the reasons for difficulty to specify rational conditions for SR treatment is that the effect of SR treatment on stress relief (that is, the change of welding residual stresses during SR treatment) is not fully known, especially for thick joints.

On the other hand, in accordance with the development of large computers, the finite element method has been adopted in various fields. This method enabled to develop the available theory1) of thermal elastic-plastic-creep analysis, and with the theory the change of stress in the welded joint of a very thick plate due to SR treatment was analyzed2). However, the theory and a computer program for the analysis are so complicated that it is not easy to perform the analysis even with a large computer. Furthermore, for the three-dimensional problem, very long CUP time is necessary for computation.

This study aims at development of an analytical method which facilitates the calculation of transient and residual stresses during SR treatment (as simple as hand calculation) in order to easily determine reasonable conditions for SR treatment. For this purpose, the various relations between stresses and strains during SR treatment are idealized and estimating equations are developed. In this report, estimating equations are formulated for relaxation tests at changing and constant temperatures (in uni-axial stress state) which present a phenomenon similar to the stress relaxation phenomenon during SR treatment. It is shown that the analytical results of simple calculations using these equations are highly accurate. In the next report, this method will be further developed to apply to the SR treatment in three-axial stress state. In this study, the theory is developed for the material of 2 1/4 C_r-1 M_0 steel which is widely used in structures like large pressure vessels. The same method may be applied to other various materials, too.

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2. Estimating Method of Transient and Residual Stresses during and after Relaxation Test at Changing or Constant Temperature

In the relaxation test at changing temperature, a certain amount of stress is given to the specimen in the axial direction at a room temperature, then the specimen is heated so as to examine the stress relaxation at a high temperature. At the heating stage, the loading condition of the specimen is adjusted so that additional thermal stress due to the increment of temperature should not be produced. In the relaxation test at constant temperature, the specimen is heated to a certain temperature and is subjected to a specified amount of stress. Keeping the temperature and the length of the specimen constant, the stress relaxation is observed. In this chapter, a simple estimating method is presented for such transient and residual stresses during and after relaxation tests as mentioned above.

2.1 Approximate equations to estimate relaxed stresses

To develop a simple estimating method of transient stress during the relaxation test and the resulting residual stress, approximate equations are formulated for uni-axial stress state (X-direction), since the loading condition of the specimen is uni-axial. In this study, it is assumed that no plastic deformation occurs during the test and the initial axial stress σ_x is positive (for the case of $\sigma_x < 0$, the same method may be applied, too.)

2.1.1 Basic equations

As any stress component other than the axial one is zero in the specimen, the relation between stress and strain can be expressed as

$$\sigma_x = E\,\epsilon_x^e \tag{1}$$

where σ_x : axial stress (kgf/mm^2)
ϵ_x^e : axial elastic strain
E : Young's modulus (kgf/mm^2)

In general, Young' s modulus shows temperature-dependency. **Figure 1** indicates the case of 2 1/4 C_r-1 M_0 steel[2] which is used here.

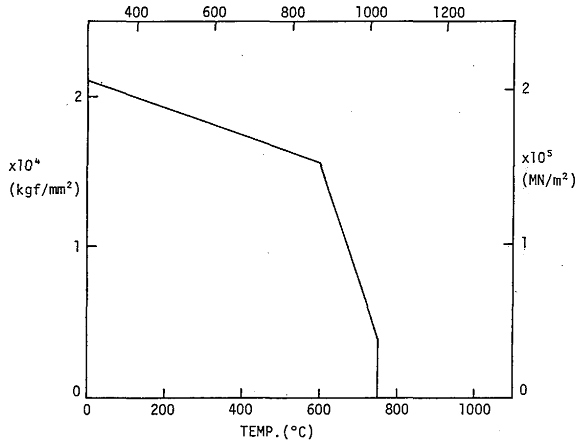


Fig. 1 Assumed modulus of elasticity in calculation

Then, the relation among strain components is taken into consideration. In the relaxation test at changing temperature, the specimen ends are moved along the axis (X-direction) for the amount of displacement which corresponds to the thermal strain. Therefore, during the relaxation tests at changing and constant temperatures, axial strain except thermal one is always equal to its initial value. That is, if no plastic deformation occurs, the amount of produced creep strain equals to that of reduced elastic strain. Accordingly, the relations among strain components and strain rate components can be expressed as follows.

$$\epsilon_x^c = \epsilon_{xo}^e - \epsilon_x^e \tag{2}$$

where ϵ_x^c : axial creep strain
ϵ_{xo}^e : initial value of axial elastic strain ϵ_x^e (when $\epsilon_x^e = \epsilon_{xo}^e$, $\epsilon_x^c = 0$)

$$\dot{\epsilon}_x^c = -\,\dot{\epsilon}_x^e \tag{3}$$

where $\dot{\epsilon}_x^c$: axial creep strain rate (1/min.)
$\dot{\epsilon}_x^e$: axial elastic strain rate (1/min.)

The creep strain rate $\dot{\epsilon}_x^c$ in the above expression is different depending on the material of the specimen and the temperature range. As for the material of 2 1/4 Cr-1 M_0 steel, it is adequate to assume that it obeys the strain-hardening creep law if the temperature is below 575°C and the power creep law above 575°C[2]. They can be represented as follows.

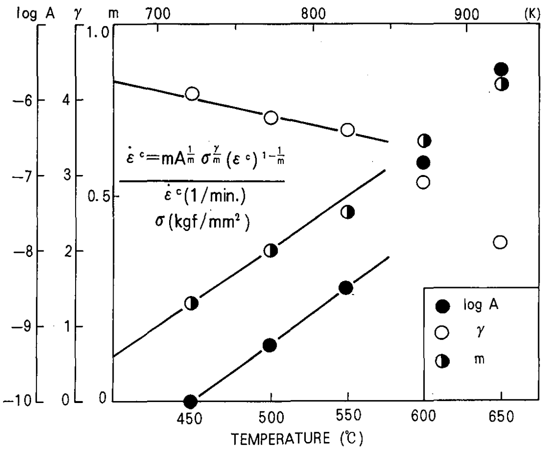
(i) Creep strain rate below 575°C (Strain-hardening creep law)

$$\dot{\epsilon}_x^c = mA^{1/m}(\sigma_x)^{\gamma/m}(\epsilon_x^c)^{1-1/m} \tag{4}$$

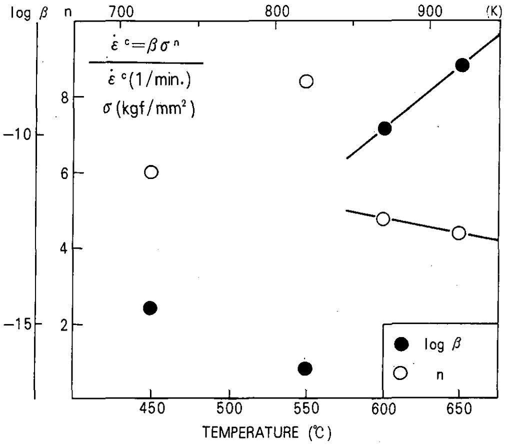
(ii) Creep strain rate above 575°C (Power creep law)

$$\dot{\epsilon}_x^c = \beta(\sigma_x)^n \tag{5}$$

m, A, γ and β, n in the above equations are creep constants. Their temperature-dependencies are shown in **Figs. 2(a)** and **(b)**[2].



(a) For the strain-hardening creep law



(b) For the power creep law

Fig. 2 Observed values of creep constants

Finally, the relation between temperature and time in the heating stage can be expressed as follows on the assumption that the heating rate is constant.

$$T = T_0 + vt \qquad (6)$$

where T : temperature (°C)
T_0 : initial temperature (room temperature) (°C)
v : heating rate (°C/min.)
t : time (min.)

Equations (1)–(6) expressed above are basic equations.

2.1.2 Development of approximate equations to estimate relaxed stresses

Obtaining elastic strain which decreases according to the increase of creep strain, transient and residual stresses produced in the specimen can be immediately calculated by Eq. (1). So, it is prerequisite to estimate the change of elastic strain produced in the specimen during heating and holding stages. Here, the following method is adopted to estimate the change of elastic strain.

According to the temperature range, either Eq. (4) or Eq. (5) is substituted for the creep strain rate in the left side of Eq. (3). Then, stress σ_x and creep strain ϵ_x^c are eliminated by Eqs. (1) and (2), so that an equation consisted of material properties (Young's modulus and creep constants), elastic strain and elastic strain rate is derived. This equation can be separated into a term of only material properties and that consisted of elastic strain rate, elastic strain and material properties. This rewritten equation is to be integrated by time. With the relation of dt=(1/v)dT given from Eq. (6), integration of the term of only material properties by time can be replaced with one by temperature. In this result, following equations are derived.

(i) In the temperature range below 575°C (In obedience to the strain-hardening creep law)

$$\int_{T_{ss}}^{T_{es}} mA^{1/m}E^{\gamma/m}dT = -v\int_{\epsilon_{xos}^e}^{\epsilon_x^e} (\epsilon_x^e)^{-\gamma/m}(\epsilon_{xo}^e - \epsilon_x^e)^{-1+1/m}d\epsilon_x^e \qquad (7)$$

where T_{ss} : temperature at which creep strain begins to be remarkably produced in obedience to strain-hardening creep law, that is, T_{ss} = 400°C.
T_{es} : temperature at which elastic strain ϵ_x^e is to be estimated.
ϵ_{xos}^e: initial axial elastic strain at $T = T_{ss}$, then $\epsilon_{xos}^e = \epsilon_{xo}^e$.

(ii) In the temperature range above 575°C (In obedience to the power creep law)

$$\int_{T_{sp}}^{T_{ep}} \beta E^n dT = -v\int_{\epsilon_{xop}^e}^{\epsilon_x^e} (\epsilon_x^e)^{-n}d\epsilon_x^e \qquad (8)$$

where T_{sp} : temperature at which creep strain begins to be produced in obedience to the power creep law, that is, T_{sp} = 575°C.
T_{ep} : temperature at which elastic strain ϵ_x^e is to be estimated.
ϵ_{xop}^e: initial axial elastic strain at $T = T_{sp}$.

A difficult problem is included in practising the integrations in Eqs. (7) and (8). Although material properties (Young's modulus and creep constants) during heating stage vary according to temperature, the left sides of Eqs. (7) and (8) can be integrated considering the temperature-dependencies of material properties because they are to be integrated by temperature (generally, numerical integration is conducted). As for the right sides of Eqs. (7) and (8), elastic strain changes according to production of creep strain, while creep constants included in the integral terms of elastic strain vary in the domain of integration. These creep constants can be evaluated with temperature, having no relation with elastic strain. Accordingly, the right sides of Eqs. (7) and (8) cannot be integrated as far as creep constants show temperature-dependencies. In Figs. 2(a) and (b), creep constants m, γ and n are far less temperature-dependent than creep constants A and β. If these m, γ and n are assumed to be constant in the domain of integration (for example, their values at the average temperature in the domain of integration are used), the right sides of Eqs. (7) and (8) can be integrated to give approximate values (these constants are assumed to be constant also in the left sides). The right side of Eq. (7) in which creep strain rate obeys the strain-hardening creep law has no simple analytical solution. Nevertheless, if elastic strain ϵ_x^e in the integral equation is normalized being divided by its initial value ϵ_{xos}^e, the equation can be transformed into one excluding initial elastic strain. The result is represented by Eq. (9). On the other hand, the right side of Eq. (8) for the power creep law has a simple analytical solution. As is obvious from Fig. 2 (b), $n \fallingdotseq 1$ then. With this condition, Eq. (10) is derived.

(i) In the temperature range below 575°C (In obedience to the strain-hardening creep law)

$$m\int_{T_{ss}}^{T_{es}} A^{1/m}E^{\gamma/m}dT = -v\,(\epsilon_{xos}^e)^{(1-\gamma)/m} \times \int_1^{{}_n\epsilon_x^e} ({}_n\epsilon_x^e)^{-\gamma/m}(1-{}_n\epsilon_x^e)^{1/m-1}d_n\epsilon_x^e \tag{9}$$

where ${}_n\epsilon_x^e$: normalized axial elastic strain, that is,

$${}_n\epsilon_x^e = \epsilon_x^e/\epsilon_{xos}^e$$

(ii) In the temperature range above 575°C (In obedience to the power creep law)

$$\int_{T_{sp}}^{T_{ep}} \beta E^n dT = -\frac{v}{1-n}\{(\epsilon_x^e)^{1-n} - (\epsilon_{xop}^e)^{1-n}\} \tag{10}$$

Equations (9) and (10) can be further transformed as follows.

(i) In the temperature range below 575°C

$$-\frac{m}{v}(\epsilon_{xos}^e)^{(\gamma-1)/m}\int_{T_{ss}}^{T_{es}} A^{1/m}E^{\gamma/m}dT$$

$$= \int_1^{{}_n\epsilon_x^e} ({}_n\epsilon_x^e)^{-\gamma/m}(1-{}_n\epsilon_x^e)^{1/m-1}d_n\epsilon_x^e \tag{11}$$

(ii) In the temperature range above 575°C

$$-\frac{1-n}{v}\int_{T_{sp}}^{T_{ep}} \beta E^n dT + (\epsilon_{xop}^e)^{1-n} = (\epsilon_x^e)^{1-n} \tag{12}$$

In the left sides of the above equations the integrations are possible, besides the preceding multipliers are the known coefficients. Therefore, using Eq. (11), the change of normalized elastic strain can be calculated in the way to decide the domain of integration in the right side to equilibrate the integrated value to the calculated value of the left side. The stress can be obtained by multiplying this normalized strain by the initial strain and Young's modulus E. Using Eq. (12), elastic strain which equilibrate the value of the right side to the value of the left side can be directly obtained. Details of these calculations are shown in the following section. The equations to approximately obtain elastic strains in the heating stage are thus developed.

A similar method is applied to the relaxation test at constant temperature which corresponds to the holding stage of SR treatment. When 2 1/4 C_r·1 M_0 steel is used in pressure vessel etc., the holding temperature of SR treatment is generally above 575°C. Therefore, only the case the material obeys the power creep law is discussed. The equation is developed basically in the same manner as Eq. (10) for the stage of changing temperature. Necessary equations are only Eqs. (1), (3) and (5). Since temperature is constant here, integration by temperature turns to one by time and material properties are constant in the domain of integration, which makes integration easy. The following equation is derived (provided $n \fallingdotseq 1$).

$$\epsilon_x^e = \{(\epsilon_{xopc}^e)^{1-n} + (n-1)\beta E^n t\}^{1/(1-n)} \tag{13}$$

where ϵ_{xopc}^e : initial axial elastic strain at the beginning of relaxation test at constant temperature

t : time from the beginning of relaxation test at constant temperature

Thus, the integral equations to estimate the change of elastic strain in the heating and holding stages of relaxation test are formulated.

Afterward, if the relaxation test proceeds into the cooling stage like SR treatment, Eqs. (11) and (12) developed for the heating stage can be applied to this stage because the sign of heating rate v was not specified in the development of the equations. Accordingly, $v > 0$, $T_{ss} < T_{es}$

and $T_{sp} < T_{ep}$ in the heating stage, while $v < 0$, $T_{ss} > T_{es}$ and $T_{sp} > T_{ep}$ in the cooling stage.

2.2 Estimating method of transient and residual stresses

In the preceding section, Eqs. (11), (12) and (13) were formulated to estimate the change of elastic strain (due to production of creep strain) in the heating, holding and cooling stages. Based on these equations, estimating method of transient and residual stresses during and after relaxation tests at changing and constant temperatures is to be discussed.

2.2.1 Tabulation of integrated results in approximate equations

Estimating procedure of transient and residual stresses will be described in 2.2.2. Since integrations in Eqs. (11) and (12) are prerequisite for the actual calculation, the terms of material properties are integrated by temperature in the left sides of Eqs. (11) and (12) and the term of normalized elastic strain and creep properties is integrated by normalized elastic strain in the right side of Eq. (11). The domains of integration of material properties correspond with the temperature ranges in which respective creep laws are applied. That is, the strain-hardening creep law is applied between 400°C ~ 575°C and the power creep law 575°C ~ 700°C. On the other hand, the domain of integration of normalized elastic strain is from 1 to 0. **Tables 1 ~ 3** indicate the results. Initial elastic strain and heating rate are not included in these integrations. Therefore, if these three kinds of integrations are performed for a single time with a single set of material properties, the results can be applied to any initial stress and heating rate. Instructions for these integrations are described in the appendix.

2.2.2 Procedure of estimation

Using the Tables prepared in 2.2.1, transient and residual stresses resulted from relaxation tests are estimated in the following procedure. The flow chart is shown in **Fig. 3**.

To start with, stress when the temperature in the heating stage is in the range of the strain-hardening creep law (below 575°C) is estimated. When the temperature is T_{es}, the integrated value I_{ps} in the left side of Eq. (11) for the temperature T_{es} can be obtained from Table 1. With the already known initial elastic strain ϵ^e_{xos} and heating rate v, the integrated value I_s of normalized elastic strain can be determined from Eq. (11) so that the both sides of the equation are equal. Then, the magnitude of normalized elastic strain, ${}_n\epsilon^e_x$, can be known from Table 3, using the integrated value I_s. Multiplying elastic strain ϵ^e_x given from the normalized elastic strain ${}_n\epsilon^e_x$ and initial one ϵ^e_{xos} to Young's modulus E at the temperature T_{es}, stress σ_x can be obtained.

Table 1 Integration of the left side (the term I_{ps}) of Eq. (11)

T_{es}	I_{ps}	T_{es}	I_{ps}	T_{es}	I_{ps}
401	1.2884E+15	461	3.1953E+18	521	6.7715E+20
403	4.2382E+15	463	3.8227E+18	523	8.0926E+20
405	7.7656E+15	465	4.5726E+18	525	9.6713E+20
407	1.1983E+16	467	5.4691E+18	527	1.1557E+21
409	1.7027E+16	469	6.5408E+18	529	1.3812E+21
411	2.3058E+16	471	7.8218E+18	531	1.6506E+21
413	3.0269E+16	473	9.3531E+18	533	1.9725E+21
415	3.8893E+16	475	1.1183E+19	535	2.3571E+21
417	4.9204E+16	477	1.3371E+19	537	2.8168E+21
419	6.1532E+16	479	1.5986E+19	539	3.3660E+21
421	7.6274E+16	481	1.9112E+19	541	4.0223E+21
423	9.3900E+16	483	2.2849E+19	543	4.8064E+21
425	1.1497E+17	485	2.7315E+19	545	5.7434E+21
427	1.4017E+17	487	3.2654E+19	547	6.8629E+21
429	1.7030E+17	489	3.9035E+19	549	8.2005E+21
431	2.0632E+17	491	4.6661E+19	551	9.7988E+21
433	2.4939E+17	493	5.5777E+19	553	1.1708E+22
435	3.0089E+17	495	6.6672E+19	555	1.3989E+22
437	3.6245E+17	497	7.9693E+19	557	1.6715E+22
439	4.3606E+17	499	9.5257E+19	559	1.9972E+22
441	5.2407E+17	501	1.1385E+20	561	2.3863E+22
443	6.2928E+17	503	1.3608E+20	563	2.8512E+22
445	7.5507E+17	505	1.6265E+20	565	3.4066E+22
447	9.0545E+17	507	1.9441E+20	567	4.0702E+22
449	1.0852E+18	509	2.3236E+20	569	4.8629E+22
451	1.3001E+18	511	2.7771E+20	571	5.8100E+22
453	1.5571E+18	513	3.3191E+20	573	6.9414E+22
455	1.8643E+18	515	3.9668E+20	575	8.2930E+22
457	2.2315E+18	517	4.7409E+20		
459	2.6705E+18	519	5.6660E+20		

(1.0000E+05 $= 1.0000 \times 10^5$)

where

$$I_{ps} \equiv \int_{T_{ss}}^{T_{es}} A(T)^{\frac{1}{m}} E(T)^{\frac{Y}{m}} dT , \quad T_{ss} = 400 \ (°C)$$

Table 2 Integration of the left side (the term I_{pp}) of Eq. (12)

T_{ep}	I_{pp}	T_{ep}	I_{pp}	T_{ep}	I_{pp}
577	2.8825E+09	619	2.5238E+11	661	2.3693E+12
579	6.1080E+09	621	2.8373E+11	663	2.6128E+12
581	9.7162E+09	623	3.1848E+11	665	2.8793E+12
583	1.3751E+10	625	3.5701E+11	667	3.1706E+12
585	1.8261E+10	627	3.9970E+11	669	3.4888E+12
587	2.3302E+10	629	4.4696E+11	671	3.8362E+12
589	2.8932E+10	631	4.9927E+11	673	4.2149E+12
591	3.5219E+10	633	5.5713E+11	675	4.6276E+12
593	4.2237E+10	635	6.2109E+11	677	5.0767E+12
595	5.0068E+10	637	6.9175E+11	679	5.5651E+12
597	5.8803E+10	639	7.6978E+11	681	6.0957E+12
599	6.8542E+10	641	8.5588E+11	683	6.6714E+12
601	7.9396E+10	643	9.5084E+11	685	7.2955E+12
603	9.1488E+10	645	1.0555E+12	687	7.9712E+12
605	1.0495E+11	647	1.1707E+12	689	8.7021E+12
607	1.1994E+11	649	1.2976E+12	691	9.4917E+12
609	1.3662E+11	651	1.4372E+12	693	1.0343E+13
611	1.5516E+11	653	1.5906E+12	695	1.1261E+13
613	1.7578E+11	655	1.7591E+12	697	1.2250E+13
615	1.9869E+11	657	1.9441E+12	699	1.3312E+13
617	2.2414E+11	659	2.1470E+12	700	1.3872E+13

(1.0000E+05 $= 1.0000 \times 10^5$)

where

$$I_{pp} \equiv \int_{T_{sp}}^{T_{ep}} \beta(T)\, E(T)^n\, dT , \quad T_{sp} = 575 \ (°C)$$

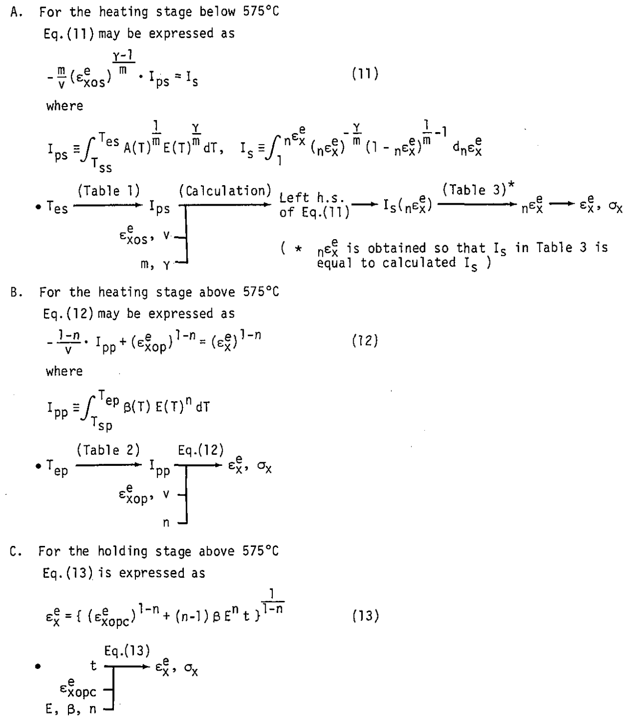
Table 3 Integration of the right side (the term I_s) of Eq. (11)

${}_n\varepsilon_x^e$	$-I_s$	${}_n\varepsilon_x^e$	$-I_s$	${}_n\varepsilon_x^e$	$-I_s$
0.99	1.3052E-06	0.66	5.6983E-01	0.33	1.7724E+03
0.98	9.4448E-06	0.65	7.0337E-01	0.32	2.4468E+03
0.97	3.1066E-05	0.64	8.6831E-01	0.31	3.4077E+03
0.96	7.4075E-05	0.63	1.0722E+00	0.30	4.7908E+03
0.95	1.4816E-04	0.62	1.3248E+00	0.29	6.8035E+03
0.94	2.6531E-04	0.61	1.6379E+00	0.28	9.7670E+03
0.93	4.4032E-04	0.60	2.0269E+00	0.27	1.4186E+04
0.92	6.9147E-04	0.59	2.5108E+00	0.26	2.0866E+04
0.91	1.0413E-03	0.58	3.1141E+00	0.25	3.1112E+04
0.90	1.5175E-03	0.57	3.8676E+00	0.24	4.7083E+04
0.89	2.1543E-03	0.56	4.8110E+00	0.23	7.2408E+04
0.88	2.9936E-03	0.55	5.9948E+00	0.22	1.1333E+05
0.87	4.0866E-03	0.54	7.4841E+00	0.21	1.8083E+05
0.86	5.4963E-03	0.53	9.3628E+00	0.20	2.9472E+05
0.85	7.2996E-03	0.52	1.1739E+01	0.19	4.9173E+05
0.84	9.5908E-03	0.51	1.4755E+01	0.18	8.4202E+05
0.83	1.2485E-02	0.50	1.8594E+01	0.17	1.4843E+06
0.82	1.6123E-02	0.49	2.3498E+01	0.16	2.7032E+06
0.81	2.0678E-02	0.48	2.9786E+01	0.15	5.1080E+06
0.80	2.6359E-02	0.47	3.7877E+01	0.14	1.0065E+07
0.79	3.3427E-02	0.46	4.8331E+01	0.13	2.0817E+07
0.78	4.2195E-02	0.45	6.1897E+01	0.12	4.5534E+07
0.77	5.3052E-02	0.44	7.9579E+01	0.11	1.0638E+08
0.76	6.6473E-02	0.43	1.0273E+02	0.10	2.6885E+08
0.75	8.3040E-02	0.42	1.3321E+02	0.09	7.4728E+08
0.74	1.0346E-01	0.41	1.7354E+02	0.08	2.3365E+09
0.73	1.2864E-01	0.40	2.2721E+02	0.07	8.4822E+09
0.72	1.5963E-01	0.39	2.9905E+02	0.06	3.7443E+10
0.71	1.9780E-01	0.38	3.9581E+02	0.05	2.1593E+11
0.70	2.4478E-01	0.37	5.2701E+02	0.04	1.8345E+12
0.69	3.0262E-01	0.36	7.0617E+02	0.03	2.8760E+13
0.68	3.7385E-01	0.35	9.5264E+02	0.02	1.3795E+15
0.67	4.6162E-01	0.34	1.2944E+03	0.01	1.0164E+18

(1.0000E+05 =1.0000 × 10^5)

where

$$I_s \equiv \int_1^{{}_n\varepsilon_x^e} ({}_n\varepsilon_x^e)^{-\frac{\gamma}{m}} (1 - {}_n\varepsilon_x^e)^{\frac{1}{m}-1} \, d\,{}_n\varepsilon_x^e$$



A. For the heating stage below 575°C
Eq.(11) may be expressed as

$$-\frac{m}{v}(\varepsilon_{xos}^e)^{\frac{\gamma-1}{m}} \cdot I_{ps} = I_s \qquad (11)$$

where

$$I_{ps} \equiv \int_{T_{ss}}^{T_{es}} A(T)^{\frac{1}{m}} E(T)^{\frac{\gamma}{m}} dT, \quad I_s \equiv \int_1^{{}_n\varepsilon_x^e} ({}_n\varepsilon_x^e)^{-\frac{\gamma}{m}} (1 - {}_n\varepsilon_x^e)^{\frac{1}{m}-1} \, d\,{}_n\varepsilon_x^e$$

(* ${}_n\varepsilon_x^e$ is obtained so that I_s in Table 3 is equal to calculated I_s)

B. For the heating stage above 575°C
Eq.(12) may be expressed as

$$-\frac{1-n}{v} \cdot I_{pp} + (\varepsilon_{xop}^e)^{1-n} = (\varepsilon_x^e)^{1-n} \qquad (12)$$

where

$$I_{pp} \equiv \int_{T_{sp}}^{T_{ep}} \beta(T)\, E(T)^n \, dT$$

C. For the holding stage above 575°C
Eq.(13) is expressed as

$$\varepsilon_x^e = \{ (\varepsilon_{xopc}^e)^{1-n} + (n-1)\,\beta\, E^n\, t \}^{\frac{1}{1-n}} \qquad (13)$$

Fig. 3 Flow chart of calculation by simple estimating method

In the next place, stress when temperature is heated up to the range of the power creep law (above 575°C) is estimated. Preceding to this, elastic strain ε_{xop}^e at 575°C when temperature enters the range of the power creep law needs to be estimated. This elastic strain ε_{xop}^e can be given by substituting 575°C into the temperature T_{es} in the above-mentioned procedure. Notating the temperature when heating terminates in the range of the power creep law as T_{ep}, the integrated value I_{pp} in the left side of Eq. (12) can be obtained from Table 2. With the already known heating rate v and initial elastic strain ε_{xop}^e in the temperature range of the power creep law, elastic strain ε_x^e at the required temperature T_{ep} can be calculated by Eq. (12).

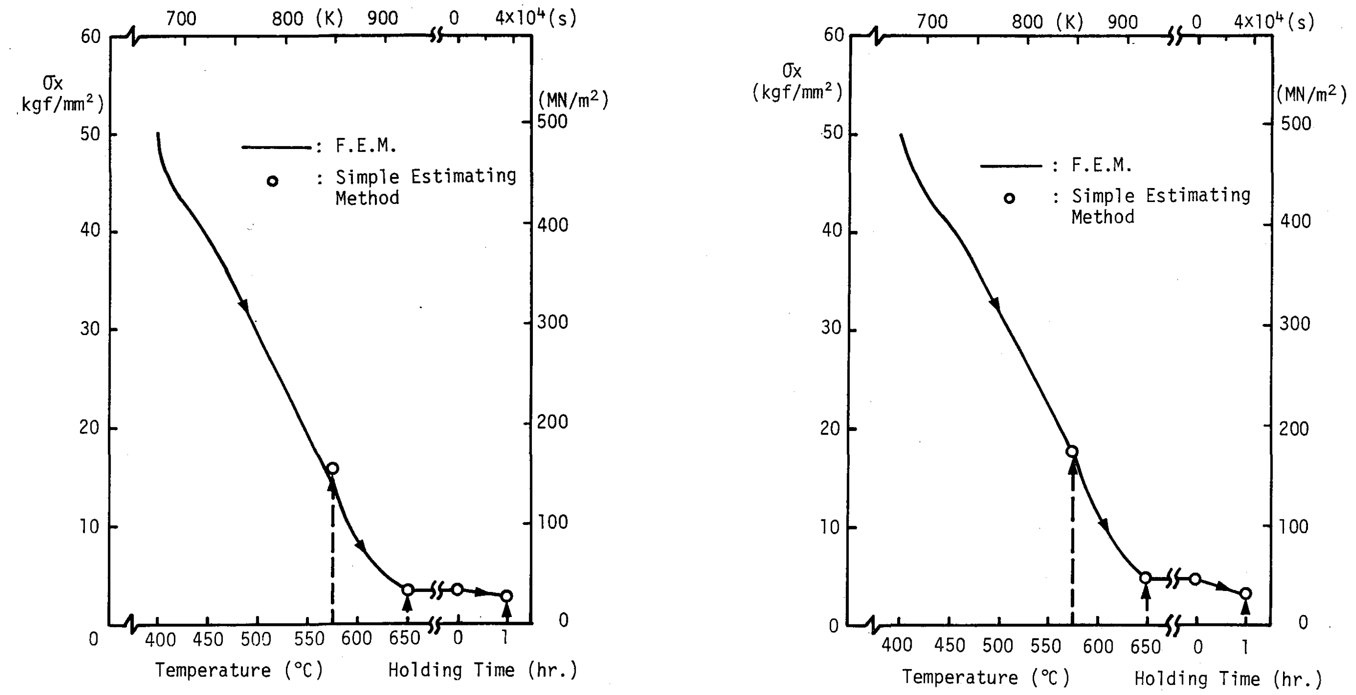
When the temperature of the specimen is held at T_{ep}, elastic strain in the holding stage can be calculated by Eq. (13). The change of elastic strain in the following cooling stage can be estimated by calculating in the reverse order of the heating stage. In another words, the calculation in the temperature range of the power creep law precedes the one in the strain-hardening creep law. Tables 1 and 2 are available as they are. Nevertheless, temperatures T_{ss}, T_{sp} and T_{es}, T_{ep} need to be treated conversely. In using the tables, T_{es}, T_{ep} are the temperature when each creep law begins to be applied and T_{ss} and T_{sp} are the temperatures at the end of each temperature range.

Consequently, transient and residual stresses during and after relaxation tests at changing and constant temperatures are estimated. The analytical method developed here has the following characteristics. The thermal elastic-plastic-creep analysis based on the finite element method which has been represented in Ref. 1, etc. has wide application and accuracy but requires incremental calculations, so that the whole stages from heating through holding to cooling need to be consecutively calculated. On the other hand, integral equations are used in this method. Therefore, stress at any temperature (or time) in each domain of integration can be directly estimated by extremely simple calculations.

3. Accuracy of the Estimating Method

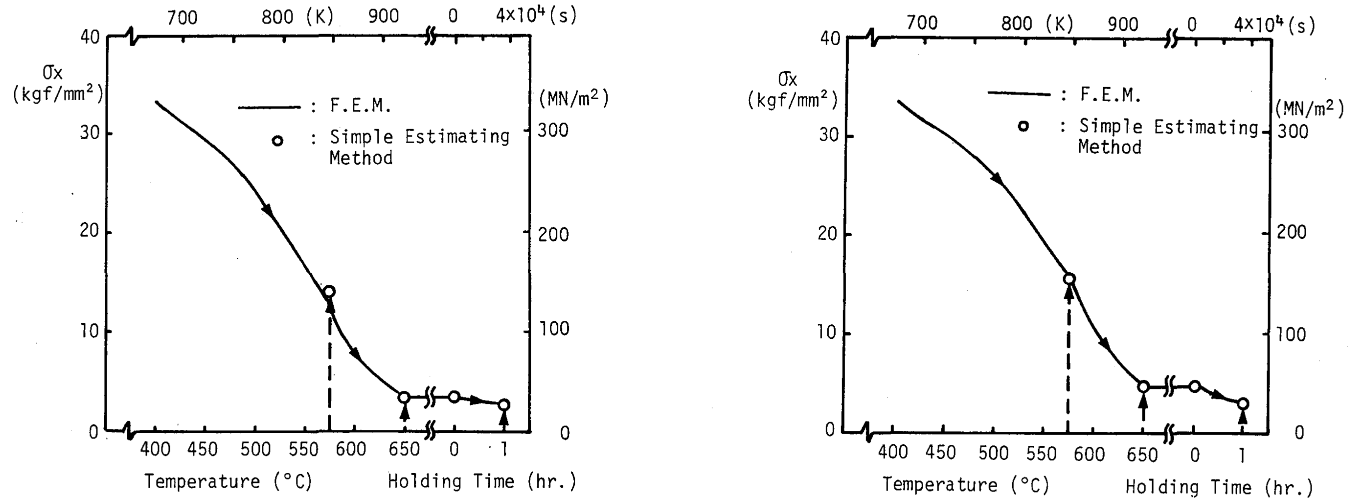
In order to investigate the applicability of the estimating method mentioned in the preceding chapter, the changes of stress during the relaxation tests at changing and constant temperatures under various conditions (for different initial stress and heating rate) are calculated by this method and compared with the accurate solutions obtained by the finite element method.

Various combinations of initial stress σ_{x0} and heating rate v are conditioned. Three magnitudes of 20, 40 and 60



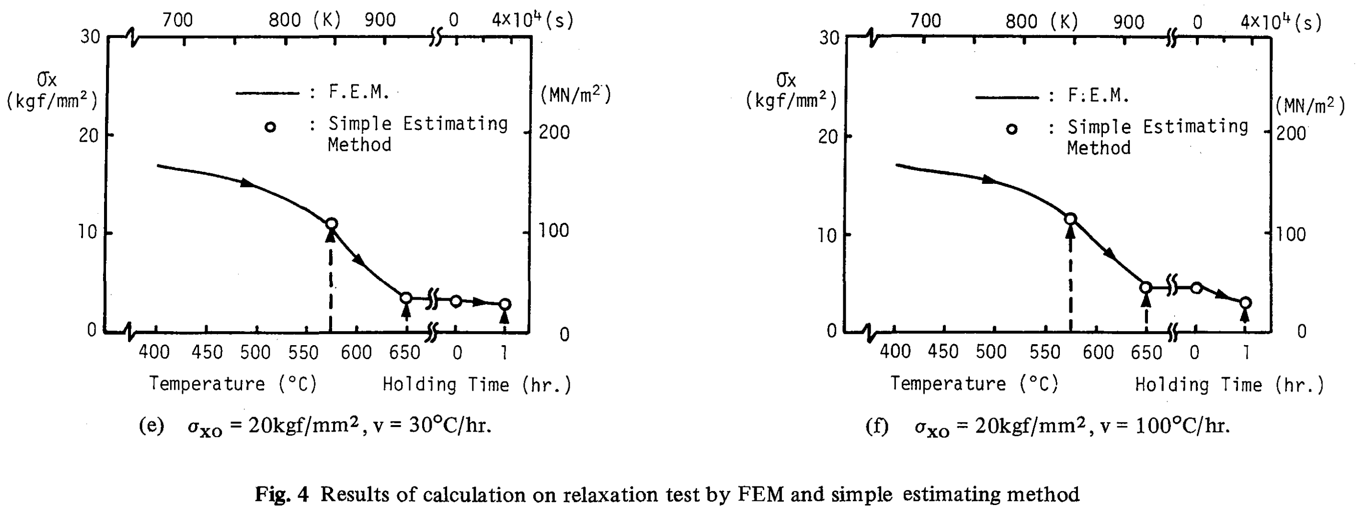
(a) σ_{xo} = 60kgf/mm², v = 30°C/hr.

(b) σ_{xo} = 60kgf/mm², v = 100°C/hr.



(c) σ_{xo} = 40kgf/mm², v = 30°C/hr.

(d) σ_{xo} = 40kgf/mm², v = 100°C/hr.



(e) σ_{xo} = 20kgf/mm², v = 30°C/hr.

(f) σ_{xo} = 20kgf/mm², v = 100°C/hr.

Fig. 4 Results of calculation on relaxation test by FEM and simple estimating method kgf/mm^2 are considered for σ_{x0} and two levels of 30 and 100°C/hr for v, which are slow because this kind of method will be applied to SR treatment of very thick welded joints in the series of this study. The holding temperature is set to be 650°C and the holding time to be 1 hour. Stresses are evaluated at three connecting points of calculations; (1) at the temperature when the creep law of the material changes from the strain-hardening creep law to the power creep law, (2) at the end of the heating stage and (3) at the end of the holding stage. The results are shown by mark "○" in **Figs. 4 (a) ~ (f)**. In the same figures, the accurately calculated results by the finite element method in consideration of temperature-dependencies of every material properties are shown by solid lines.

In this method, three of the creep constants of which temperature-dependencies are relatively little are assumed to take fixed values in each domain of integration. As is shown in Fig. 4, the results coincide well with the solutions by the finite element method under every conditions. This implies that the above assumption decreases little the accuracy of the results estimated by this method, that is, the highly accurate estimation for transient and residual stresses during relaxation tests can be given by this method. Although the changes of stress shown in Fig. 4 are only for the heating and the holding stages, an accurate solution for the succeeding cooling stage can be also obtained by this method.

4. Conclusion

The purpose of this study is to develop an analytical method to easily estimate (such as by a hand calculation) transient and residual stresses during and after SR treatment of very thick joints. In this 1st report, for relaxation tests at changing and constant temperatures which show a similar phenomenon to the stress relief by SR treatment, an analytical method is developed and its applicability is investigated. As the result, following conclusion is attained. In this method, previously calculating three simple numerical integrations of a single material, transient and residual stresses during and after relaxation tests at changing and constant temperatures can be simply and accurately estimated under every conditions for initial stress, heating rate and holding temperature.

Generally, in a structure or a structural member which undergoes SR treatment, stress state at any point is in multi-axial state and the changes of total strains at the point do not necessarily coincide with thermal strains at the point. In the next report, in order to estimate the changes of stresses due to thus conditioned SR treatment, the analytical method treated in this report will be extended and the approximate equations will be developed.

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Appendix: Integrations of terms of material properties and normalized elastic strain

In regard to integration of the terms of material properties by temperature, m, γ and n are assumed to be constant in each domain of integration but the other creep constants and Young's modulus are considered to be dependent on temperature. For this reason, the left sides of Eqs. (11) and (12) cannot be analytically integrated. Instead, they are numerically integrated. Since m, γ and n which are assumed to be constant vary approximately linearly with temperature, their values at 487.5°C or 612.5°C are used. The changes of these temperatures have little influence on accuracy of this method.

The domains of integration of these material properties by temperature are set two; 400°C ~ 575°C where the strain-hardening creep law is applied and 575°C ~ 700°C where the power creep law applied. Below 400°C, produced creep strain is so little as to be neglected. 575°C is considered to be the temperature at which the creep property of the material changes. The domain of integration for the power creep law is set within 700°C because the highest temperature treated in this research is that degree.

On the other hand, the domain of integration of the term of normalized elastic strain is set from 1 to 0. However, it is difficult to numerically integrate exactly from 1 to 0. Therefore, actual integration is conducted from 0.999 to 0.001. The influence of removal of minor portions from the both ends of the domain is negligible.