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Ultimate Strength Analysis of Double Bottom Structure by "Idealized Structural Unit Method" †

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Abstract

In a previous paper[1], two of the authors have developed an effective method for the ultimate strength analysis of large size structures. The method is named as "Idealized Structural Unit Method". In the method, a large size element with idealized nonlinear character was necessary and an example element, "Girder Element", was developed.

In this paper[2], this method is applied to double bottom structures. The state of two-dimensional stress in inner bottom and the bottom shell plating is considered. Conditions for buckling of these plates are established and its postbuckling stiffness is determined. The condition for their ultimate strength and stiffness at the ultimate strength state are determined in connection with the girder element.

An example structure is analysed and the results of the analysis are presented.

KEY WORDS: (Ultimate Strength) (Limiting Strength) (Double Bottom) (Idealized Structural Unit Method) (Nonlinear Analysis)

1. Introduction

Double bottom structures are mainly used in ships, as well as marine and land structures. They are important principal structures. When a double bottom structure is subjected to external load, local buckling and/or local yielding may take place in elements of the structure when the external load reaches a certain magnitude. By these local failures, the stiffness of these elements usually decreases, but the structure may be able to stand further increase of the load. Redistribution of the internal forces may take place until the structure eventually collapses.

For ultimate strength analysis of such large complicated structures exhibiting such elastoplastic-large deflection behavior, the finite element method or the finite strip method may be applied. However, the structure should be divided into a large number of elements. The required computing time and storage size are impractical. The autors, in a previous paper, have proposed a solution to this problem[1], that is to divide the structure into the largest possible structural units and idealize its elasto-plastic-large deflection behavior, the resulting concise characteristics of these units are used and the load is applied in increments until the structure attains it ultimate strength. This method is referred to as the "Idealized Structural Unit Method". The "Girder Element" was developed in this way and it was shown that structures having girders as their primary supporting system can be analysed in extremely short time. In this paper the method is extended. The buckling and plastic nonlinear behaviors of rectangular flat plate unit subjected to plane stress condition, which is found in such structures as inner and outer bottom, are idealized. It is combined with the already developed girder element and the ultimate strength analysis of double bottom structures is performed.

2. The Girder Element

The details of the girder element are presented in a previous paper[1]. In the following it is simply represented.

The element is composed of a web and two different size flanges as shown in **Fig. 1**. Stiffeners are fitted to both ends of the web normal to the axis of the element.

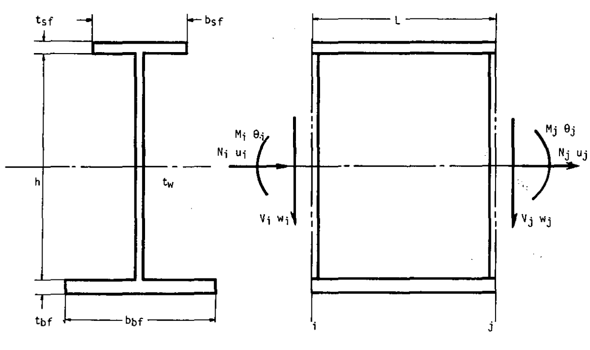

Fig. 1 Girder element and nodal forces

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Nodal points are set up at the mid-depth of the web at both ends i and j. Three components of nodal forces; axial force N, bending moment M and shearing force V act at each nodal point. The corresponding nodal displacements are axial displacement u, rotation θ and deflection w. Denoting the nodal forces by $\{R\}$ and the nodal displacements by $\{U\}$, they may be written as,

$$\{R\}^T = \{N_i,\ M_i,\ V_i,\ N_j,\ M_j,\ V_j\} \tag{1}$$

$$\{U\}^T = \{u_i,\ \theta_i, w_i,\ u_j,\ \theta_j,\ w_j\} \tag{2}$$

Using a dot to indicate increments, the kinematic behavior of the element may be expressed as

$$\{\dot{R}\} = [k_G]\ \{\dot{U}\} \tag{3}$$

where, $[k_G]$ ($[K]$ is used in ref.1)) is the stiffness matrix expressing the kinematic condition of the girder element. The elements of $[k_G]$ change according to the extent of local failures.

Buckling of the web is determined by the average internal forces occuring along the span. Buckling interaction relationship is expressed by Eq. (7) in ref. 1). When the shearing force, V, is smaller than the pure shear buckling force, V_{cr}, the postbuckling stiffness matrix is expressed by Eq. (23) in ref. 1), taking into account the effectiveness of the web. When V is larger than V_{cr}, the web is assumed to be unable to support normal stresses and the stiffness matrix is expressed by Eq. (24) in ref. 1).

The behavior of the element until it attains its limiting strength (ultimate strength or fully plastic strength) may be classified into three main categories:

i- The web buckles in the elastic range and the structure continues to support more load until it reaches its ultimate strength. This category may be sub-divided into two cases. The first is when the shearing force, V, is smaller than the pure shear buckling force, V_{cr}. The ultimate strength condition will be reached when at least the web and a flange at one end of the element yield. The interaction relationship in this case is expressed by Eqs. (29) through (32) in ref. 1). The second case is when V is greater than V_{cr}. A tension field develops in the web and the ultimate strength condition is attained by one of three different combinations of plastic collapse of flange and yielding of the web. The interaction relationship is expressed by Eq. (38) in ref. 1).

ii- The web buckles in the elasitc-plastic range mainly due to bending moment. Not much more load can be supported after buckling and this is considered a condition of ultimate strength. Interaction relationship is similar to category (i). However, different constants are given by Eq. (46) of ref. 1).

iii- The web does not buckle and the element attains its fully plastic strength at one or both nodal points. The interaction relationship is expressed by Eqs. (50) through (66) in ref. 1).

After the element has attained its limiting strength in any of the above mentioned categories, its stiffness matrix is derived on the plastic flow theory. Each interaction relationship is regarded as a plastic potential and resulting load-deflection relationships at the limiting strength state are given by Eqs. (43), (45) and (67) in ref. 1).

3. Idealization of Double Bottom Structure

3.1 Double bottom structure model

The double bottom structure in an actual ship is a complicated structure with bilge sections, different stiffeners, manholes etc.. However, in this analysis, it may be simplified as follows. First, the loss of strength and stiffness due to manholes existing in some members is covered by furnishing stiffeners and then such members may be replaced by equivalent flat plates. The round of the bilge is neglected and a square bilge is assumed. The double bottom structure, thus idealized, is composed, of floors and longitudinal girders arranged in a grillage structure covered with inner bottom and bottom shell plating as shown in **Fig. 2**. It may also be considered to be composed of parallelepipeds built up of four vertical

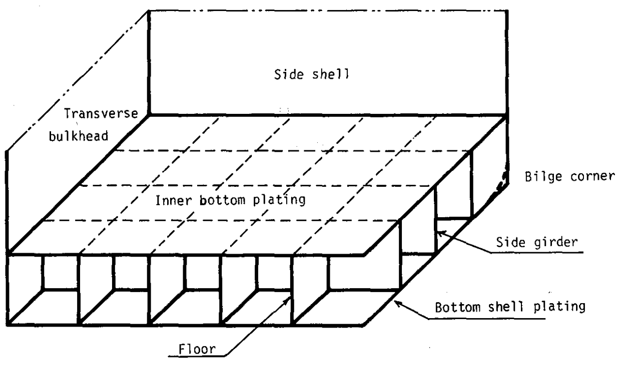

Fig. 2 Double bottom

walls (parts of two successive floors and parts of two successive longitudinal girders) and two horizontal plates (a part of bottom shell plating and a part of inner bottom plating). In the following this substructure is considered. The effect of two dimensional stress field acting in the two horizontal plates is superimposed on the girder element and the analysis is performed.

3.2 Behavior and idealization of structural unit

Considering the prescribed structural unit which is built up of 6 plates, inplane deformations are compatible at the nodal points, and out-of-plane deflections take place in each plate separately. The vertical edges of the four vertical webs are assumed to be simply supported and the horizontal edges fixed. All edges of the horizontal plates are assumed to be simply supported.

The structural unit is shown in **Fig. 3**. Nodal points are located at the four corners of each horizontal plate and at

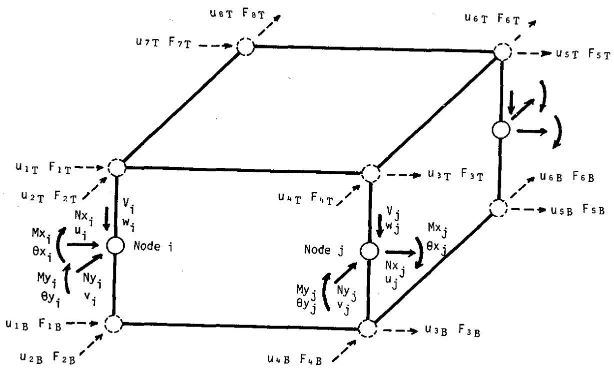

Fig. 3 Nodal forces and displacement of structural unit.

the mid-depth of both sides of each vertical web. Assuming the displacement distribution at both sides of each web to be linear, and utilizing the compatibility of displacements, the nodal displacements at the four corners of each horizontal plate may be expressed by those at the mid-depth of the web and can be eliminated. Thus the behavior of the structural unit may be described only in terms of those nodal displacements. In the following, the behavior of the structural unit will be dealt with the aid of the relationship of forces and displacements occuring at these nodal points. All external forces are replaced by equivalent nodal forces. Nodal force components at each nodal point are: axial forces Nx, Ny, bending moments Mx, My, and shearing force V. The corresponding nodal displacement components are: axial displacement u, v, rotation θx, θy and deflection w. Thus the nodal forces $\{F\}$ and the nodal displacements $\{U\}$ of the structural unit may be written as,

$$\{F\}^T = \{F_i, F_j, F_k, F_l\},$$

$$\{F_i\} = \{N_{xi}, N_{yi}, M_{xi}, M_{yi}, V_i\}, \text{etc.} \tag{4}$$

$$\{U\}^T = \{U_i, U_j, U_k, U_l\},$$

$$\{U_i\} = \{u_i, v_i, \theta_{xi}, \theta_{yi}, w_i\}, \text{etc.} \tag{5}$$

The contribution of the two horizontal plates, which are in a plane stress condition, is divided into two parts; one as flanges of girders, and the other as the rest of the contribution. The structural unit may then be considered as an assembly of girder elements (composed of webs and flanges) and the rest of the contribution of the two horizontal plates. Denoting the stiffness matrices of the girder elements by $[K_G]$ and that of the rest of the contribution of the horizontal plates by $[K_{PL}]$ the incremental relationship between the nodal forces and the nodal displacements may be written as,

$$\{\dot{F}\} = ([K_G] + [K_{PL}])\{\dot{U}\} = [K]\{\dot{U}\} \tag{6}$$

As external forces increase, the girder elements exhibit local failures. This is handled by changing $[K_G]$ of Eq. (6) into that corresponding to the modes of these local failures. On the other hand, horizontal plates may buckle and/or show local plastification. This affects not only $[K_{PL}]$ of Eq. (6) but also the effectiveness of these plates as flanges included in the composition of $[K_G]$.

3.2.1 Horizontal plate stiffness component

Regarding each horizontal plate as one element, four temporary nodal points may be established at the corners (**Fig. 3**). Nodal forces $\{f\}$ and nodal displacements $\{u\}$ are composed of inplane components,

$$\{u_{PL}\}^T = \{u_T \ u_B\}$$

$$\{u_T\}^T = \{u_{1T} \ u_{2T} \cdots u_{8T}\},$$

$$\{u_B\}^T = \{u_{1B} \ u_{2B} \cdots u_{8B}\} \tag{7}$$

$$\{f_{PL}\}^T = \{f_T \ f_B\}$$

$$\{f_T\}^T = \{f_{1T} \ f_{2T} \cdots f_{8T}\},$$

$$\{f_B\}^T = \{f_{1B} \ f_{2B} \cdots f_{8B}\} \tag{8}$$

The stiffness matrix of this element, $[k_{PL}]$, may be derived by integrating over the plate and the resulting matrix may be divided into two parts,

$$\begin{aligned}[k_{PL}] &= \int [P]^T [D] [P] \, dV \\ &= \int [P]^T [D_1] [P] \, dV + \int [P]^T [D_2] [P] \, dV \\ &= [k_1] + [k_2]\end{aligned} \tag{9}$$

where, $[P]$ is displacement-strain matrix, and $[D]$ is stress-strain matrix.

$$[D] = [D_1] + [D_2], \quad [D_1] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$[D_2] = \frac{E}{1-\nu^2}\begin{bmatrix} 0 & \nu & 0 \\ \nu & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$[k_1]$ corresponds to normal stress components and can be included in $[k_G]$ by suitable choice of breadths of flanges of the girder elements. On the other hand, $[k_2]$ is the stiffness component due to the effect of Poisson's ratio in an inplane stress condition and due to inplane shear. This component, after the following treatment, is reduced to $[K_{PL}]$ of Eq. (6) which is added to the girder elements stiffness matrix to compose the stiffness matrix of the structure.

Denoting the stiffness matrices $[k_2]$ of the inner bottom and bottom shell plating, which are evaluated by Eq. (9), by $[k_{2T}]$ and $[k_{2B}]$, nodal displacements by $\{u_T\}$ and $\{u_B\}$, and nodal force components for the stiffness component by $\{f_T\}$ and $\{f_B\}$, the stiffness equations may be written in the incremental form as,

$$\{\dot{f}_T\} = [k_{2T}]\{\dot{u}_T\}, \quad \{\dot{f}_B\} = [k_{2B}]\{\dot{u}_B\} \tag{10}$$

Equations (10) may be assembled as,

$$\{\dot{f}_{PL}\} = \begin{Bmatrix} \dot{f}_T \\ \dot{f}_B \end{Bmatrix} = \begin{bmatrix} k_{2T} & 0 \\ 0 & k_{2B} \end{bmatrix}\begin{Bmatrix} \dot{u}_T \\ \dot{u}_B \end{Bmatrix}$$

$$= [\bar{k}_{PL}]\{\dot{u}_{PL}\} \tag{11}$$

The nodal forces and nodal displacements at the nodal points established at the four corners of the top and bottom plates, are not independent of those at the nodal points at the mid-depth of the webs. They may be related to each other through the assumption that the edges of the webs remain straight after deformation. If $[C_f]$ is the transformation matrix relating nodal forces and $[C_u]$ is that relating nodal displacements, it may be written that

$$\{\dot{\bar{F}}\} = [C_f]\{\dot{f}_{PL}\}, \quad \{\dot{\bar{U}}\} = [C_u]\{\dot{u}_{PL}\} \tag{12}$$

As the relation $[C_u]^{-1} = [C_f]^T$ is valid, $\dot{\bar{F}}$ equals $\dot{F}$ with shearing force terms, $\dot{V}$, removed and $\dot{\bar{U}}$ equals $\dot{U}$ with deflection terms, $\dot{w}$, removed,

$$\{\dot{\bar{F}}\} = \{\dot{\bar{F}}_i\ \dot{\bar{F}}_j\ \dot{\bar{F}}_k\ \dot{\bar{F}}_l\},$$
$$\{\dot{\bar{F}}_i\} = \{\dot{N}_{xi}\ \dot{N}_{yi}\ \dot{M}_{xi}\ \dot{M}_{yi}\}, \text{etc.} \tag{13}$$
$$\{\dot{\bar{U}}\} = \{\dot{\bar{U}}_i\ \dot{\bar{U}}_j\ \dot{\bar{U}}_k\ \dot{\bar{U}}_l\},$$
$$\{\dot{\bar{U}}_i\} = \{\dot{u}_i\ \dot{v}_i\ \dot{\theta}_{xi}\ \dot{\theta}_{yi}\}, \text{etc}$$

From Eqs. (11) and (12)

$$\{\dot{\bar{F}}\} = [C_f]\ [\bar{k}_{PL}]\ [C_u]^{-1}\{\dot{\bar{U}}\} \tag{14}$$
$$= [C_f]\ [\bar{k}_{PL}]\ [C_f]^T\{\dot{\bar{U}}\} = [\bar{K}_{PL}]\{\dot{\bar{U}}\}$$

Rearranging the matrix $[\bar{K}_{PL}]$, derived as before, to suit the total nodal displacements of the structural unit and enlarging it to a 20 x 20 matrix, the matrix $[K_{PL}]$ of Eq. (6) may be obtained.

3.2.2 Condition of elastic buckling and post-buckling stiffness matrix of horizontal plates.

Inner bottom and bottom shell plating of the double bottom of a ship, are subjected to serveral kinds of loads. Besides the local distributed lateral loads, inplane uniform compressive load from bilge to bilge due to water pressure, longitudinal unifrom compressive (or tensile) load mainly due to vertical bending moment and inplane bending due to horizontal bending moment act on these plates. The resulting stress distribution is represented by the solid

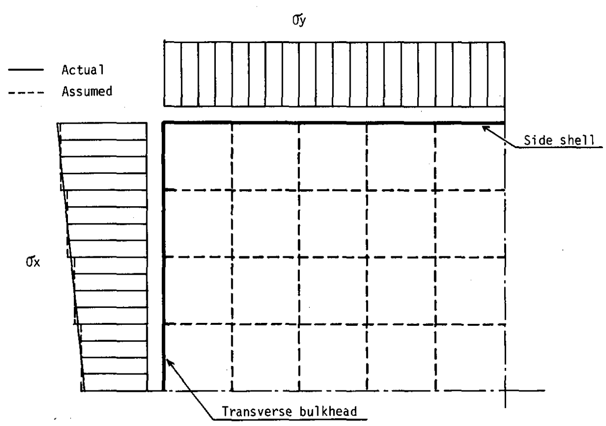

Fig. 4 Stress distributions along boundaries of model

line in **Fig. 4**. However, the slope of the stress distribution due to the above mentioned inplane bending is usually small and may be approximated to the broken line in the same figure.

On the other hand shearing stress in the horizontal plates is small as long as torsional moment on the ship structure is not excessively large, because the inplane shear stiffness of the structure is high. This small shearing stress may be assumed to have no effect on buckling. Thus, buckling of the horizontal plates may be analysed as a problem under two dimensional inplane compression. The buckling wave shape may be taken as the fundamental one. On this basis, a stress function may be adopted, and buckling condition be obtained together with post-buckling effective section coefficient. Post-buckling stiffness may then be evaluated.

Now consider a rectangular plate (a x b x t) subjected to inplane uniform compression along its boundaries in x and y directions, **Fig. 5**. The average compressive stresses σ_x and σ_y in the respective directions may be taken as

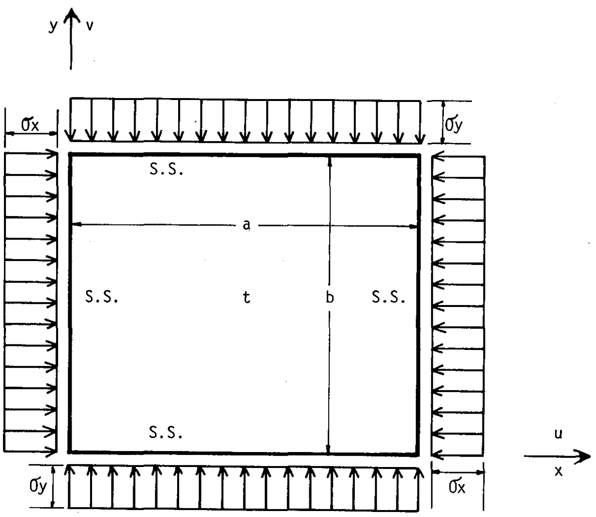

Fig. 5 Rectangular plate subjected to compression in two directions

parameters indicating the magnitudes of the external loads. According to the finite displacement theory with a stress function F and deflection w,

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{15}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{t}{D}\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right) \tag{16}$$

Deflection, w, may be assumed as follows, satisfying the boundary conditions,

$$w = w_m \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \tag{17}$$

Solving Eqs. (15), (16) and (17), the stress function F and the deflection at the center of the rectangular plate w_m may be obtained as,

$$F = -\sigma_x\frac{y^2}{2} - \sigma_y\frac{x^2}{2} + \frac{E \cdot w_m^2}{32} \cdot \left(\frac{a^2}{b^2}\cos\frac{2\pi x}{a} + \frac{b^2}{a^2}\cos\frac{2\pi y}{b}\right) \tag{18}$$

$$w_m^2 = \frac{16a^2b^2}{E\pi^2\left[\left(\frac{b}{a}\right)^2 + \left(\frac{a}{b}\right)^2\right]} \cdot \left[\frac{\sigma_x}{a^2} + \frac{\sigma_y}{b^2} - \frac{D \cdot \pi^2}{t}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2\right] \tag{19}$$

Buckling takes place when $w_m = 0$. Denoting buckling interaction function of a horizontal plate by $\bar{\Gamma}_B$, the buckling condition may be written as,

$$\bar{\Gamma}_B = \frac{\sigma_x}{a^2} + \frac{\sigma_y}{b^2} - \frac{D \cdot \pi^2}{t}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2 = 0 \tag{20}$$

Next, using Eq. (18) together with the stress-strain relationship and strain-deformation relationship, displacements u and v for the post-buckling may be evaluated as,

$$u = -\frac{a}{E}\left[a_1\sigma_x + a_3\sigma_y - 2a_4\frac{D}{t}\left(\frac{\pi}{a}\right)^2\right],$$

$$v = -\frac{b}{E}\left[a_3\sigma_x + a_2\sigma_y - 2a_4\frac{D}{t}\left(\frac{\pi}{b}\right)^2\right] \tag{21}$$

where,

$$a_1 = \frac{3+\lambda^4}{1+\lambda^4},\quad a_2 = \frac{1+3\lambda^4}{1+\lambda^4},\quad a_3 = -\nu + \frac{2\lambda^2}{1+\lambda^4},$$

$$a_4 = \frac{(1+\lambda^2)^2}{1+\lambda^4},\quad \lambda = \frac{a}{b}$$

The relationship between inplane compressive strains e and average stresses σ may be represented in the incremental form as,

$$\dot{e}_x = \frac{a_1}{E}\dot{\sigma}_x + \frac{a_3}{E}\dot{\sigma}_y,\quad \dot{e}_y = \frac{a_3}{E}\dot{\sigma}_x + \frac{a_2}{E}\dot{\sigma}_y \tag{22}$$

The shear stiffness of the rectangular plate may decrease, more or less, after buckling. As its reduction is expected to be small, in this analysis, the shear stiffness is taken of the same value as before buckling. Thus, the average stress-strain relationship of the horizontal plates on this stage may be written as,

$$\begin{Bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\tau}_{xy} \end{Bmatrix} = [D^B] \begin{Bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{\gamma}_{xy} \end{Bmatrix} = \begin{bmatrix} d_1 & d_3 & 0 \\ d_3 & d_2 & 0 \\ 0 & 0 & d_4 \end{bmatrix} \begin{Bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{\gamma}_{xy} \end{Bmatrix} \qquad (23)$$

where

$$d_1 = \frac{E \cdot a_2}{a_1 a_2 - a_3^2}, \quad d_2 = \frac{E \cdot a_1}{a_1 a_2 - a_3^2}, \quad d_3 = \frac{-Ea_3}{a_1 a_2 - a_3^2},$$

$$d_4 = \frac{E}{2(1+\upsilon)}$$

In similar to 3. 2. 1, $[D^B]$ may be divided into two parts

$$[D^B] = [D_1^B] + [D_2^B], \qquad (24)$$

where

$$[D_1^B] = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [D_2^B] = \begin{bmatrix} 0 & d_3 & 0 \\ d_3 & 0 & 0 \\ 0 & 0 & d_4 \end{bmatrix}$$

$[D_1^B]$ indicates the effectiveness of the horizontal plate as a flange, that is d_1 and d_2 being the effective moduli of elasticity of the plate in the x and y directions, respectively.

The post-buckling stiffness matrix of the horizontal plates $[k^B]$ may then be derived by substituting $[D^B]$ of Eq. (23) for $[D]$ in Eq. (9).

$$[k^B] = \int [P]^T [D^B][P]dV = \int [P]^T [D_1^B][P]dV + \int [P]^T [D_2^B][P]dV = [k_1^B] + [k_2^B] \qquad (25)$$

The stiffness equation, Eq. (6), after buckling of the horizontal plates becomes as follows,

$$\{\dot{F}\} = [K^B]\{\dot{U}\} = ([K_G^B] + [K_{PL}^B])\{\dot{U}\} \qquad (26)$$

where $[K_G^B]$ is evaluated with the effective moduli of elasticity d_1 and d_2, of the flanges of the girders element in the x and y directions respectively. $[K_{PL}^B]$ is evaluated by the procedure which was explained in 3. 2. 1, introducing $[k_2^B]$ of Eq. (25) instead of $[k_2]$ in Eqs. (9) to (14).

3.3 Limiting strength interaction relationship of the structural unit.

In this work, although the contribution of the horizontal plates is rigorously taken into account in the strength analysis of the structural units, the structure is basically treated as a grillage composed of girder elements. The limiting strength of a cross-section may be obtained by performing necessary modification on the limiting strength interaction relationship of the girder element. Local failures are decided according to nodal forces acting at that cross-section whether or not they satisfy the modified condition.

When the acting vertical shearing force is smaller than the pure shear buckling force of the web of girder element, the limiting strength of the horizontal plates, acting as flanges, may be attained by plastification, due to inplane loads, or by reaching its ultimate strength (collapsing strength) after buckling.

If the limiting strength interaction relationship of a horizontal plate subjected to inplane biaxial compression is obtained as in **Fig. 6**, the contribution of the horizontal plates as the flanges to the girder elements may be divided

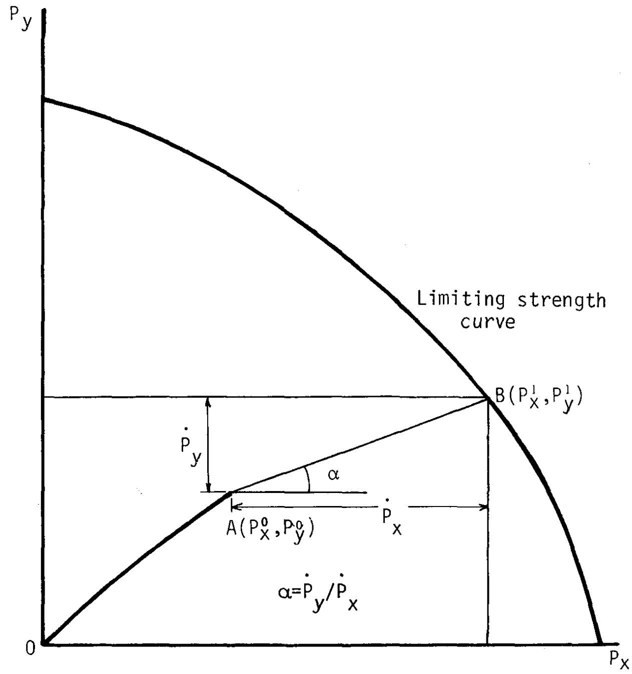

Fig. 6 Carrying capacity of plate at its strength limit

in the x and y directions in the following manner. Since the behavior of structural unit is linearized for each load increment, the ratio of increments of internal forces ($\dot{P}_x$, $\dot{P}_y$) acting in the horizontal plate can be evaluated (for example $\dot{P}_y/\dot{P}_x = \alpha$). Then point B at which the limiting strength curve is satisfied may be anticipated at the last increment of loads. The ordinates of this point, P_x^1 and P_y^1, represent the components in the x and y directions. Usually each horizontal plate acts as the flanges to two parallel girder elements in each direction. Thus one half of the plate acts as the flange on one side of the girder element in each direction.

On the other hand, when a web exhibits shear buckling, a tension field develops in the web. In addition to the biaxial inplane load acting on the flanges, a lateral distributed load comes into action. Limiting strength is reached when three plastic hinges are formed in the flange. When buckling does not take place in the horizontal plates, the inplane forces affect directly the formation of plastic hinges. The effect of the inplane force in the same direction of bending stresses appears in the form of interaction relationship of plastic hinges. This relationship is determined by the yield stress of the material which is influenced by the inplane force normal to the bending stress. When the horizontal plate buckles, it is supposed that the flanges have no more supporting capacity to the tension field prevailed in the web.

Taking these points into consideration, the effect of the horizontal plates on the limiting strength of girder elements can be evaluated.

The above-mentioned limiting strength of the horizontal plates may be obtained as follows. First, fully plastic strength may be easily calculated since the horizontal plates are subjected to uniformly distributed biaxial stress. Tresca's yield condition is adopted in this work. On the other hand, for post-buckling ultimate strength of the horizontal plates a series of analyses by the finite element method was performed. Utilizing its results, the following equation for the interaction relationship was obtained, **Fig. 7**.

$$\Gamma_u = P_x^2 + P_y^2 + P_x \cdot P_y - P_{ult}^2 = 0 \tag{27}$$

In this equation, P_x and P_y are the forces acting in the x and y directions, and P_{ult} is the ultimate strength in uniaxial inplane compression. In this work Karman's equation is adopted to calculate P_{ult}. Denoting the buckling coefficient of a plate by k, the yield stress of the material by σ_Y and the plate thicknesss by t, P_{ult} may be expressed as,

$$P_{ult} = t^2 \sqrt{k \cdot \pi^2 E\sigma_Y / 12(1-\nu^2)} \tag{28}$$

In **Fig. 7**, some results of the elasitc-plastic large deflection analysis for a square plate (500 x 500 x 4.5 mm) and a rectangular plate (500 x 650 x 4.5 mm) with simply supported boundaries are represented by ○ and △, respectively. In both cases Eq. (27) represents a very good approximation.

As examples, the limiting strength interaction relationships of the square and rectangular plates derived above are presented in **Fig. 8**. At points of intersection of the plastic strength curve (the yield condition) and the buckling curve, plastic buckling takes place and the ultimate strength curve should pass through these points. The parts of the limiting strength curve between these points and points of the ultimate strength in uniaxial compression are approximated into straight lines.

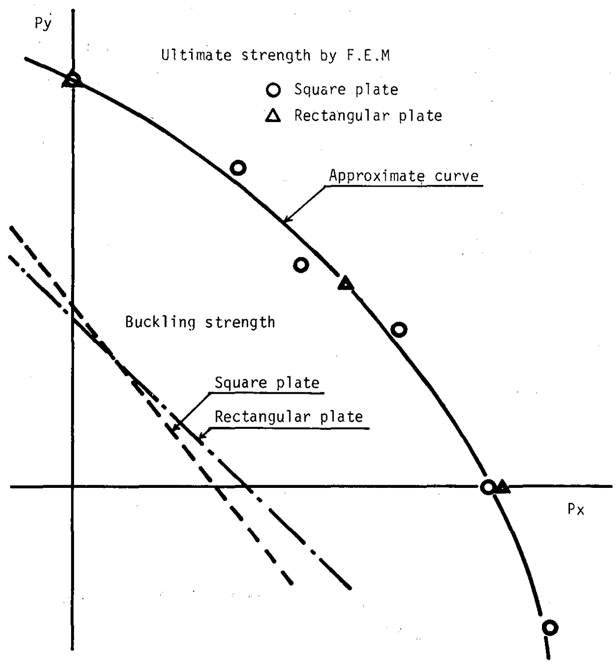

Fig. 7 Ultimate strength of rectangular plate

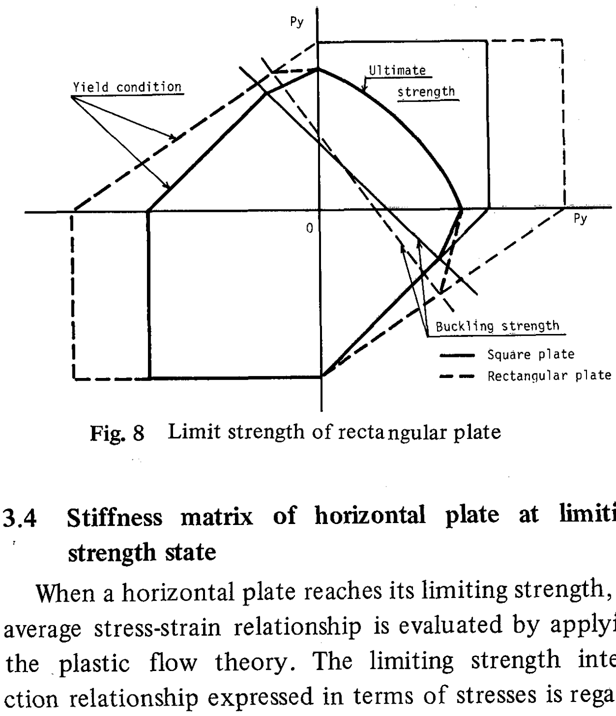

Fig. 8 Limit strength of rectangular plate

3.4 Stiffness matrix of horizontal plate at limiting strength state

When a horizontal plate reaches its limiting strength, its average stress-strain relationship is evaluated by applying the plastic flow theory. The limiting strength interaction relationship expressed in terms of stresses is regarded as a plastic potential, for example, Γ_u for ultimate strength. Denoting the average stress-strain matrix by $[D^L]$, in similar to Eq. (23), $[D^L]$ is divided into two parts.

$$[D^L] = \begin{bmatrix} d_1^L & d_3^L & 0 \\ d_3^L & d_2^L & 0 \\ 0 & 0 & d_4 \end{bmatrix} = \begin{bmatrix} d_1^L & 0 & 0 \\ 0 & d_2^L & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & d_3^L & 0 \\ d_3^L & 0 & 0 \\ 0 & 0 & d_4 \end{bmatrix}$$

$$= [D_1^L] + [D_2^L] \tag{29}$$

As seen from the components of $[D_1^L]$, the effective moduli of elasticity of the plate as the flange to the girder element decrease to d_1^L and d_2^L in the x and y directions, respectively. With these moduli, the stiffness of the girder element can be modified. The resulting new stiffness matrix is denoted by $[K_G^L]$.

In parallel with this, substituting $[D_2^L]$ for $[D]$ in Eq. (9), the stiffness matrix $[k_2^L]$ of the horizontal plate may be calculated as

$$[k_2^L] = \int [P]^T [D_2^L] [P] \, dV \tag{30}$$

For the two horizontal plates, the stiffness matrix $[\bar{k}_{PL}^L]$ can be calculated in the same way as shown in Eqs. (10) and (11). Applying the coordinate transformation to $[\bar{k}_{PL}^L]$, the stiffness matrix $[K_{PL}^L]$ will be obtained as explained in 3.2.1.

Consequently, the stiffness equation of the structural unit becomes as follows,

$$\{\dot{F}\} = [K^L]\{\dot{U}\} = ([K_G^L] + [K_{PL}^L])\{\dot{U}\} \tag{31}$$

4. Procedure of Analysis

Since the behavior of a structure is nonlinear, the incremental method is applied. First, the equilibrium equation of the structure may be established by the assembly of the elastic stiffness matrices of the original elements, imposing the loading and boundary conditions. The smallest load increment which just satisfies the condition of local failure is applied. The stiffness matrix of the element where the local failure has taken place is replaced by that corresponding to the new kinematical condition. The equilibrium equation of the structure is corrected and the calculations for the next load increment are performed. The analysis is continued by repeating this operation until the limiting strength state of the structure is obtained.

5. Example of Analysis

A double bottom structure of 8.0 m in width, 10.0 m in length and 1.0 m in depth, is subject to uniformly distributed loads acting on the bilges, at bulkheads and on inner bottom. The floors and girders are spaced at 1.0 m and the structure is considered fixed at the bulkheads (**Fig. 9**). The results of the analysis are given in **Fig. 10** in which the load-deflection curve together with the sequence of local failures are shown. Local failures start with buckling of the inner bottom plating at its central part. While it is expanding in all directions, plastification and buckling at the webs near the boundaries start to occur. As the load continues to increase, plastification proceeds at the webs near the boundaries until the final collapse. The computations are performed on the Kyoto University FACOM 230-75 and consumed 56.3 seconds for cpu.

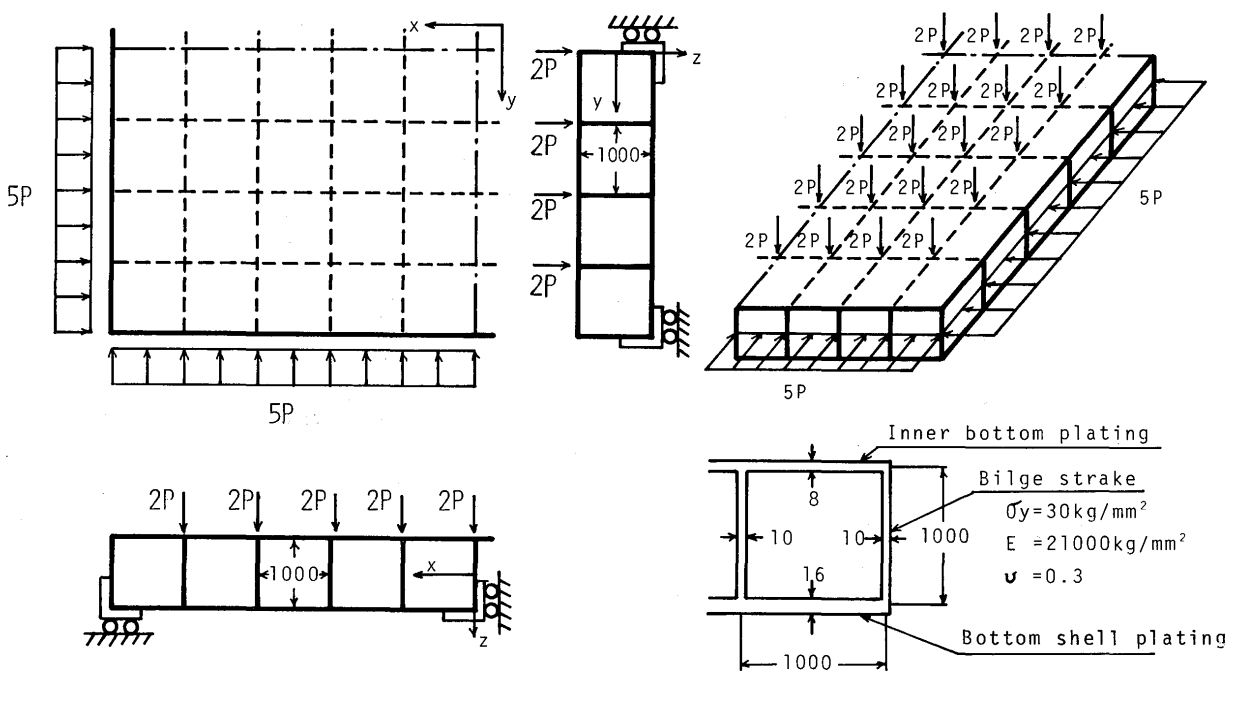

Fig. 9 Loading and supporting conditions of model

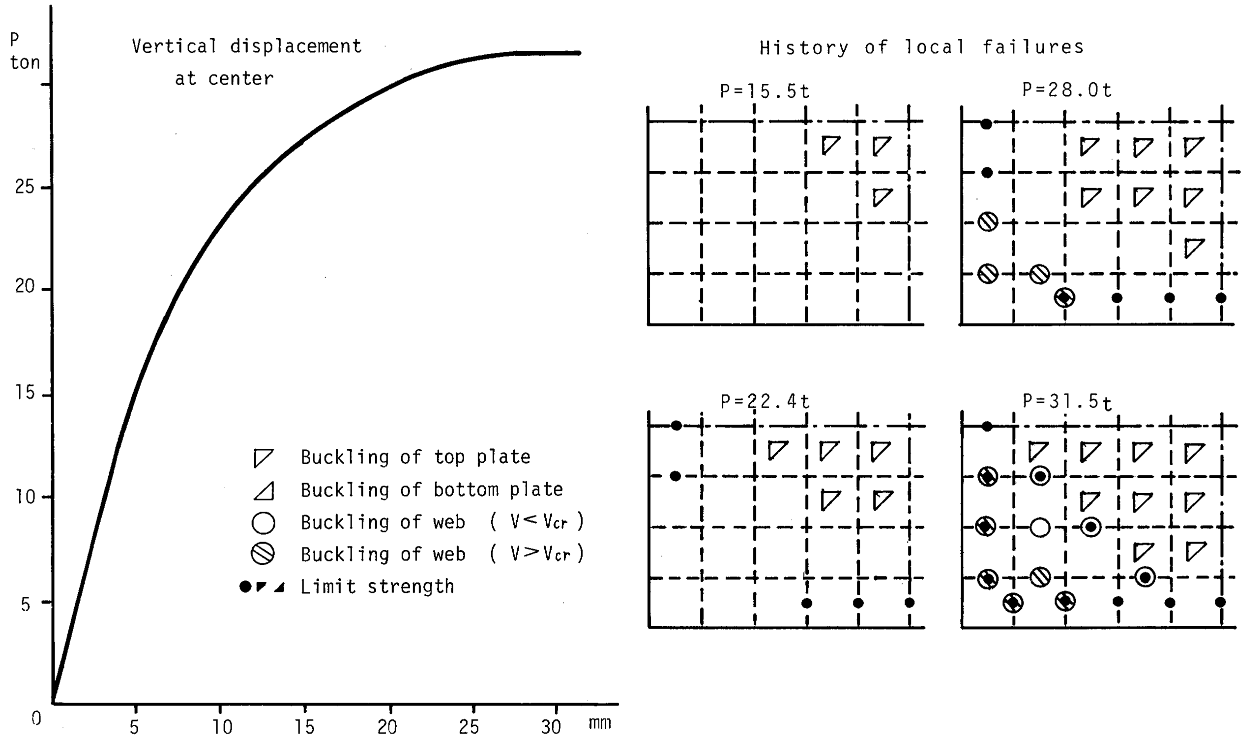

Fig. 10 History to collapse of model

6. Conclusions

As a means of ultimate strength analysis of ship-like large size structures, the authors have proposed the "Idealized Structural Unit Method", which utilizes large size elements incorporating the elastic-plastic large deflection nature of the structural components. The "Girder Element" was developed as one of such large size elements. The scope of application of this element is not limited to just girder structures, but it is possible to apply to other structures. In this work a "flat plate element" is developed and an ultimate strength analysis of the double bottom structures is performed. As shown in the example of analysis, the results exhibit the capability of the method to trace detail different phenomena of local buckling and local plastification within a very short computing time. The validity of the proposed method of analysis is once more confirmed.

References

1) Y. Ueda and M. H. Rashed, "An Ultimate Transverse Strength Analysis of Ship Structures", J. Soc. Naval Arch. of Japan, Vol. 136 (1974), (in Japanese). Selected Papers from J. S. N. A. J., Vol. 13 (1975), (in English).
2) Y. Ueda, M. H. Rashed and M. Katayama, "Ultimate Strength Analysis of Double Bottom Structure by Idealized Structural Unit Method", J. Soc. Naval Arch. of Japan, Vol. 138 (1975), (in Japanease).