Plasma–particulate interactions in nonuniform plasmas with finite flows

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The polarization force on a charged particulate or "dust" grain in a nonuniform plasma with finite ion flows and an external electric field is derived, based on a fluid approximation for the background plasma. This polarization force is proportional to the magnitude of the spatial gradient of the Debye length, and acts in the direction of decreasing Debye length. When the ion flow velocity is sufficiently large compared to ion thermal velocities, ions do not participate in the formation of sheaths around negatively charged particulates and the electron Debye length must be employed, since sheaths comprise only a deficiency of electrons. If the ion flow velocity is small, the contribution of the ion flow to the polarization force is proportional to the spatial gradient of the flow kinetic energy and is thus usually negligible. The expressions describing the plasma–particulate interaction may be applied to the modeling of contaminant behavior in materials-processing plasmas.

I. INTRODUCTION

If "mesoscopic" particles (i.e., particles that are small on the macroscopic scale but much larger than molecular sizes) are introduced into a plasma, they typically acquire negative charges due to the attachment of high-mobility plasma electrons. When the particle density is low, electrostatic interactions between particles will be negligible, and their transport is essentially governed by the interaction between individual particles and the ambient plasma.[1] When the density is sufficiently high, on the other hand, the interparticle potential energies may substantially exceed their thermal kinetic energies; the particle system then exhibits behavior similar to a liquid or the classical one-component plasma.[2–7]

Small charged particles or "particulates" are observed in a variety of plasma environments, ranging from the interstellar medium[8–10] to the low-pressure discharges widely used in processing semiconductor materials.[11–15] Our interest here is in the transport of charged particulates under typical glow discharge conditions, where they may experience strong electric fields, ion flows, and plasma density gradients.

The total force on a charged particulate in an unmagnetized plasma may be considered to compromise (i) the nominal electrostatic force $(-Q)\mathbf{E}_0$, where $-Q$ is the (negative) particulate charge and $\mathbf{E}_0$ is the external field; (ii) the polarization force, i.e., the electrostatic force due to any deformation of the Debye sheath around the particulate; (iii) the net plasma pressure force exerted on the particulate surface; and (iv) the ion drag force, i.e., the effect of Coulomb collisions with ions flowing past the particulate.

We have recently obtained an expression for the force on a particulate in the absence of plasma flows[1] that includes components (i)–(iii) above. In this paper, we extend our earlier study by investigating the plasma–particulate interaction in the presence of finite plasma flows. Since the sizes of particulates that we are concerned with are sufficiently small compared to the Debye length of the background plasma, we consider only the case of infinitesimal particulates in this paper. As in Ref. 1, we shall use a fluid model for the plasma, and thus the ion drag force—which arises from diffusion of the ion distribution function in velocity space due to Coulomb collisions—must be obtained separately from kinetic theories. The main issue addressed here is the question of how the polarization force obtained in Ref. 1 is modified by the presence of finite plasma flows.

Since plasma flows will contribute to the deformation of Debye sheaths surrounding particulates, the polarization force is expected to be dependent on the flow velocity and/or flow-velocity gradients. Unlike the ion drag force, the deformation of the sheath due to a finite plasma flow may be well described by a fluid model of the plasma. Based on the fluid approximation presented in the following section, we shall discuss the plasma–particulate interaction in detail, and systematically derive an expression for the total force exerted on the particulate.

II. FLUID MODEL OF THE PLASMA

Consider an unmagnetized plasma containing a particulate of negative charge $-Q(Q>0)$ at position $\mathbf{r}_p$. Assuming the particulate is of negligible size on the macroscopic scale, we employ a fluid model for the plasma[16] defined by the following equations:

$$\partial_t n_i + \nabla\cdot(n_i\mathbf{v}_i) = S_i, \tag{1}$$

$$\partial_t n_e + \nabla\cdot(n_e\mathbf{v}_e) = S_e, \tag{2}$$

$$m_i n_i(\partial_t\mathbf{v}_i + \mathbf{v}_i\cdot\nabla\mathbf{v}_i) = -k_B T_i\,\nabla n_i - q n_i\,\nabla\Phi - \nu_i m_i n_i \mathbf{v}_i - m_i S_i \mathbf{v}_i, \tag{3}$$

$$m_e n_e(\partial_t\mathbf{v}_e + \mathbf{v}_e\cdot\nabla\mathbf{v}_e) = -k_B T_e\,\nabla n_e + e n_e\,\nabla\Phi - \nu_e m_e n_e \mathbf{v}_e - m_e S_e \mathbf{v}_e, \tag{4}$$

$$-\epsilon_0\,\Delta\Phi = -Q\,\delta(\mathbf{r}-\mathbf{r}_p) + q n_i - e n_e, \tag{5}$$

where the subscripts i and e denote ion and electron quantities. Here n, $\mathbf{v}$, m, Φ, T, S, and ν represent density, flow velocity, mass, electric potential, temperature, particle source rate, and frequency of collisions with neutral species, respec-

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tively. Constant ion and electron temperatures are assumed, and ion and electron viscosities are neglected for the sake of simplicity.

In typical glow discharge or interstellar plasma environments, the sizes of particulates and their Debye sheaths are small compared to the ion mean-free path in the ambient plasma. For such systems, a fluid description of the plasma (the ions, especially) does not fully account for the interaction between a particulate and the background plasma. As noted in the previous section, for example, a fluid model cannot properly account for the ion drag force due to Coulomb scattering of ions by the charged particulate.

Recently, Northrop and Birmingham[17] have obtained the total interaction, including the ion drag force, between a microscopic dust grain and a *uniform* background plasma, based on the kinetic theories developed by Hubbard[18] and Kihara and Aono.[19] Because of the plasma uniformity, however, the polarization force vanishes in their system: the total interaction between a particulate and a uniform plasma is due to the ion drag force and "plasma collective effects" (which together may be considered to include pressure forces).

We are concerned here with nonuniform plasmas, for which a kinetic treatment of the system is substantially more complex, and thus the simpler system defined by Eqs. (1)–(5) is a reasonable alternative to study. As noted in the preceding section, deformations of the sheath due to density gradients and finite plasma flows are well described by this fluid model (which is a generalization of the model used in Ref. 1 to estimate the polarization force in the absence of plasma flows).

III. EQUILIBRIUM AND ELECTROSTATIC PERTURBATION

We consider first a steady-state (i.e., $\partial_t=0$), unperturbed, nonuniform plasma that contains no particulates. From Eqs. (1)–(5), the set of equations governing the equilibrium state is

$$\nabla\cdot(n_{i0}\mathbf{v}_{i0})=S_i, \tag{6}$$

$$\nabla\cdot(n_{e0}\mathbf{v}_{e0})=S_e, \tag{7}$$

$$m_i n_{i0}\mathbf{v}_{i0}\cdot\nabla\mathbf{v}_{i0}=-k_B T_{i0}\,\nabla n_{i0}-qn_{i0}\,\nabla\Phi_0 - \nu_i m_i n_{i0}\mathbf{v}_{i0}-m_i S_i \mathbf{v}_{i0}, \tag{8}$$

$$m_e n_{e0}\mathbf{v}_{e0}\cdot\nabla\mathbf{v}_{e0}=-k_B T_{e0}\,\nabla n_{e0}+en_{e0}\,\nabla\Phi_0 - \nu_e m_e n_{e0}\mathbf{v}_{e0}-m_e S_e \mathbf{v}_{e0}, \tag{9}$$

$$-\epsilon_0\,\Delta\Phi_0=qn_{i0}-en_{e0}. \tag{10}$$

Here the subscript 0 denotes the unperturbed (i.e., equilibrium) state.

In this paper we are not directly concerned with solving Eqs. (6)–(10). Instead, we shall assume that an equilibrium plasma state satisfying these equations is given, and then determine how the introduction of a charged particulate perturbs the system. Denoting the perturbed density, potential, and flow velocity by $\tilde{n}$, ϕ, and $\tilde{\mathbf{v}}$, respectively, we may write $n=n_0+\tilde{n}$, $\Phi=\Phi_0+\phi$, and $\mathbf{v}=\mathbf{v}_0+\tilde{\mathbf{v}}$. Assuming that these perturbations are small, we now linearize Eqs. (1)–(5) with respect to them.

From Eq. (3), for example, we obtain

$$m_i n_{i0}\,\partial_t\tilde{\mathbf{v}}_i+m_i n_{i0}[(\mathbf{v}_{i0}\cdot\nabla)\tilde{\mathbf{v}}_i+(\tilde{\mathbf{v}}_i\cdot\nabla)\mathbf{v}_{i0}]+m_i\tilde{n}_i(\mathbf{v}_{i0}\cdot\nabla)\mathbf{v}_{i0} =-k_B T_i\,\nabla\tilde{n}_i-qn_{i0}\,\nabla\phi-q\tilde{n}_i\,\nabla\Phi_0 -\nu_i m_i(n_{i0}\tilde{\mathbf{v}}_i+\tilde{n}_i\mathbf{v}_{i0}).$$

Eliminating the term $(\mathbf{v}_{i0}\cdot\nabla)\mathbf{v}_{i0}$ from the above by using Eq. (8), we obtain Eq. (12) below. Similarly, from Eq. (4) and (9) with $m_e\to 0$, we obtain

$$-k_B T_e\nabla\left(\frac{\tilde{n}_e}{n_{e0}}\right)+e\,\nabla\phi=0.$$

Upon integration, this yields the Boltzmann relation for the electron density given by Eq. (14) below.

The linearized equations for the perturbed quantities are thus given by

$$\partial_t\tilde{n}_i+\nabla(n_{i0}\tilde{\mathbf{v}}_i+\tilde{n}_i\mathbf{v}_{i0})=0, \tag{11}$$

$$m_i\,\partial_t\tilde{\mathbf{v}}_i=-\frac{1}{\beta_i}\,\nabla\left(\frac{\tilde{n}_i}{n_{i0}}\right)-q\,\nabla\phi+m_i(\mathbf{v}_{i0}\cdot\nabla\tilde{\mathbf{v}}_i+\tilde{\mathbf{v}}_i\cdot\nabla\mathbf{v}_{i0}) -\nu_i m_i\tilde{\mathbf{v}}_i, \tag{12}$$

$$-\epsilon_0\,\Delta\phi=-Q\,\delta(\mathbf{r}-\mathbf{r}_p)+q\tilde{n}_i-e\tilde{n}_e, \tag{13}$$

$$\tilde{n}_e=e\beta_e n_{e0}\phi. \tag{14}$$

Here $\beta_i=1/k_B T_i$ and $\beta_e=1/k_B T_e$. Note that the equation determining the electron flow velocity $\mathbf{v}_e$ is decoupled from the system (11)–(14).

In the case of zero ion flow velocity, Eq. (12) gives the Boltzmann relation $\tilde{n}_i=-q\beta_i n_{i0}\phi$ for the ions, and the system reduces to that discussed in Ref. 1. In other words, Eqs. (11)–(14), which include the effects of finite plasma flows, represent a generalization of the model used in Ref. 1.

IV. STEADY-STATE SOLUTIONS

We now solve Eqs. (11)–(14) in a steady state to obtain the Debye sheath surrounding a particle under the influence of an external field Ψ_0, a density gradient ∇n_0, and an ion flow $\mathbf{v}_{i0}$. Since plasma perturbations due to the charged particulate are confined within a small volume (the Debye sheath), it is natural to assume that the spatial variations of perturbed quantities are much larger than those of the corresponding equilibrium quantities, in the sense that, for example, $|\nabla\tilde{n}|\gg|\nabla n_0|$.

To make the system of equations more tractable, we select a local coordinate system by setting $\mathbf{r}_p=\mathbf{0}$ and choosing the z axis parallel to the direction of gradients of equilibrium quantities (we assume for simplicity that the ion flow velocity and the gradients of all equilibrium quantities, such as n_0, $\mathbf{v}_0$, and Ψ_0, are parallel). Then, to accuracy $\mathcal{O}(\delta)$, where $\delta=\lambda_D/L$, with L being a representative macroscopic scale (e.g., the density gradient scale $|\nabla\ln n_{i0}|^{-1}$), the equilibrium quantities in Eqs (11)–(14) will depend only on z, and are given by $n_{i0}(z)=n_{i0}(0)+n'_{i0}(0)z$, $\mathbf{v}_{i0}(z)=v_{i0}(z)\hat{\mathbf{z}}$

$=[v_{i0}(0)+v'_{i0}(0)z]\hat{\mathbf{z}},\ldots$, etc., where the primes denote derivatives with respect to z and $\hat{\mathbf{z}}$ is a unit vector in the z direction.

Using the following normalized variables:

$$\boldsymbol{\rho}=\frac{\mathbf{r}}{\lambda_e},\quad \zeta=\frac{z}{\lambda_e},\quad \tilde{\nabla}=\lambda_e\nabla,\quad \mathbf{u}=\frac{\mathbf{v}_i}{v_{i,\mathrm{th}}},$$

$$u_0=\frac{v_{i0}}{v_{i,\mathrm{th}}},\quad u_0'=\frac{\lambda_e v'_{i0}}{v_{i,\mathrm{th}}},\quad n=\frac{\tilde{n}_i}{n_{i0}(0)},\quad \psi=q\beta_i\phi,$$

$$\mu=\frac{\lambda_e n'_{i0}(0)}{n_{i0}(0)},\quad \mu_e=\frac{\lambda_e n'_{e0}(0)}{n_{e0}(0)},$$

$$\alpha=\frac{\kappa_i^2}{\kappa_e^2},\quad Q^*=-\frac{q\beta_i Q}{\epsilon_0\lambda_e},$$

where $\mathbf{r}$ is the position vector in the new local coordinate system, $v_{i,\mathrm{th}}=(k_\mathrm{B}T_i/m_i)^{1/2}$ is the ion thermal velocity, $\lambda_e=\kappa_e^{-1}=[\epsilon_0/e^2\beta_e n_{e0}(0)]^{1/2}$ and $\kappa_i^{-1}=[\epsilon_0/q^2\beta_i n_{i0}(0)]^{1/2}$ are the electron and ion Debye lengths evaluated at the particulate position $\mathbf{r}=\mathbf{0}$, we may rewrite Eqs. (11)–(13) in the steady state as

$$\tilde{\nabla}\cdot\mathbf{u}+u_0\frac{\partial n}{\partial\zeta}=-\mu\tilde{\nabla}\cdot(\zeta\mathbf{u})-u_0'\frac{\partial}{\partial\zeta}(\zeta n), \tag{15}$$

$$\begin{aligned}\tilde{\nabla}n+\tilde{\nabla}\psi+u_0\frac{\partial}{\partial\zeta}\mathbf{u}&\\ =\mu n\hat{\mathbf{z}}-\mu\zeta\,\tilde{\nabla}\psi+(u_0'+\mu u_0)\zeta\frac{\partial}{\partial\zeta}\mathbf{u}&\\ -u_0'u_z\hat{\mathbf{z}}-\frac{\nu_i\lambda_e}{v_{i,\mathrm{th}}}(1+\mu\zeta)\mathbf{u},&\end{aligned} \tag{16}$$

$$\tilde{\Delta}\psi-\psi+\alpha n=-Q^*\delta(\boldsymbol{\rho})+\mu_e\zeta\psi. \tag{17}$$

Here u_z is the z component of $\mathbf{u}$ and $\tilde{\Delta}=\tilde{\nabla}^2$ is the normalized Laplacian. Note that μ, μ_e, and u_0' are $\mathcal{O}(\delta)$.

In the last term of Eq. (16), the coefficient is $\nu_i\lambda_e/v_{i,\mathrm{th}}\simeq\lambda_e/\lambda_\mathrm{mfp}\ll 1$, where λ_mfp denotes the mean-free path for collisions between ions and neutral species (which is usually large compared to the electron Debye length). In other words, the effects of collision between ions and neutral species may be ignored for dynamics on the scale of the Debye length. Thus, the last term of Eq. (16) (i.e., the term proportional to ν_i) will be dropped henceforth.

To solve Eqs. (15)–(17), we use the Fourier transformation defined by

$$\hat{f}(\mathbf{k})=\int f(\boldsymbol{\rho})\exp(i\mathbf{k}\cdot\boldsymbol{\rho})d\boldsymbol{\rho},$$

where $f(\boldsymbol{\rho})$ is an arbitrary function and the integration is over all space. Note that under this transformation, $\nabla f\to -i\mathbf{k}\hat{f}$, $\zeta f\to -i\,d\hat{f}/dk_z$, and $\delta(\boldsymbol{\rho})\to 1$. The Fourier transformation of Eqs. (15)–(17) is then readily obtained as

$$\mathbf{k}\cdot\hat{\mathbf{u}}+u_0k_z\hat{n}=i\mu\mathbf{k}\cdot\frac{d\hat{\mathbf{u}}}{dk_z}+iu_0'k_z\frac{d\hat{n}}{dk_z}, \tag{18}$$

$$\begin{aligned}\mathbf{k}\hat{n}+\mathbf{k}\hat{\psi}+u_0k_z\hat{\mathbf{u}}=i\mu\hat{n}\hat{z}-iu_0'u_z\hat{\mathbf{z}}+i\mu\frac{d}{dk_z}(\mathbf{k}\hat{\psi})&\\ -i(u_0'+\mu u_0)\frac{d}{dk_z}(k_x\hat{u}),&\end{aligned} \tag{19}$$

$$(k^2+1)\hat{\psi}-\alpha\hat{n}=Q^*+i\mu_e\frac{d}{dk_z}\hat{\psi}. \tag{20}$$

Since the system is symmetric about the z axis, we may choose a unit vector $\hat{\mathbf{e}}_\perp$ orthogonal to $\hat{\mathbf{z}}$, such that $\mathbf{u}=u_\perp\hat{\mathbf{e}}_\perp+u_z\hat{\mathbf{z}}$ and $\mathbf{k}=k_\perp\hat{\mathbf{e}}_\perp+k_z\hat{\mathbf{z}}$. Equations (18)–(20) may then be written in matrix form as

$$M\boldsymbol{\xi}=iA_1\boldsymbol{\xi}+iA_2\frac{d}{dk_z}\boldsymbol{\xi}+\mathbf{b}, \tag{21}$$

where

$$\boldsymbol{\xi}=\begin{bmatrix}\hat{u}_\perp\\ \hat{u}_z\\ \hat{n}\\ \hat{\psi}\end{bmatrix}\quad\text{and}\quad \mathbf{b}=\begin{bmatrix}0\\0\\0\\Q^*\end{bmatrix}.$$

Also,

$$M=\begin{bmatrix}k_\perp & k_z & u_0k_z & 0\\ u_0k_z & 0 & k_\perp & k_\perp\\ 0 & u_0k_z & k_z & k_z\\ 0 & 0 & -\alpha & 1+k^2\end{bmatrix},$$

$$A_1=\begin{bmatrix}0 & 0 & 0 & 0\\ -(u_0'+\mu u_0) & 0 & 0 & 0\\ 0 & -(2u_0'+\mu u_0) & \mu & \mu\\ 0 & 0 & 0 & 0\end{bmatrix},$$

and

$$A_2=\begin{bmatrix}\mu k_\perp & \mu k_z & u_0'k_z & 0\\ -(u_0'+\mu u_0)k_z & 0 & 0 & \mu k_\perp\\ 0 & -(u_0'+\mu u_0)k_z & 0 & \mu k_z\\ 0 & 0 & 0 & \mu_e\end{bmatrix},$$

where $k=|\mathbf{k}|$. Note that each nonzero entry of A_1 and A_2 is $\mathcal{O}(\delta)$.

The solution to Eq. (21) may be given in the form $\boldsymbol{\xi}=\boldsymbol{\xi}^{(0)}+\boldsymbol{\xi}^{(1)}$, where $\boldsymbol{\xi}^{(0)}=\mathcal{O}(1)$ and $\boldsymbol{\xi}^{(1)}=\mathcal{O}(\delta)$. The zeroth-order solution $\boldsymbol{\xi}^{(0)}$ to Eq. (21) then evidently satisfies $M\boldsymbol{\xi}^{(0)}=\mathbf{b}$, or, from Eqs. (18)–(20).

$$\mathbf{k}\cdot\hat{\mathbf{u}}^{(0)}+u_0k_z\hat{n}^{(0)}=0, \tag{22}$$

$$\mathbf{k}\hat{n}^{(0)}+\mathbf{k}\hat{\psi}^{(0)}+u_0k_z\hat{\mathbf{u}}^{(0)}=0, \tag{23}$$

$$(k^2+1)\hat{\psi}^{(0)}-\alpha\hat{n}^{(0)}=Q^*. \tag{24}$$

Solving the above equations, we obtain the zeroth-order potential,

$$\hat{\psi}^{(0)}(\mathbf{k})=Q^*\left(k^2+1+\frac{\alpha}{1-u_0^2\cos^2\theta}\right)^{-1}, \tag{25}$$

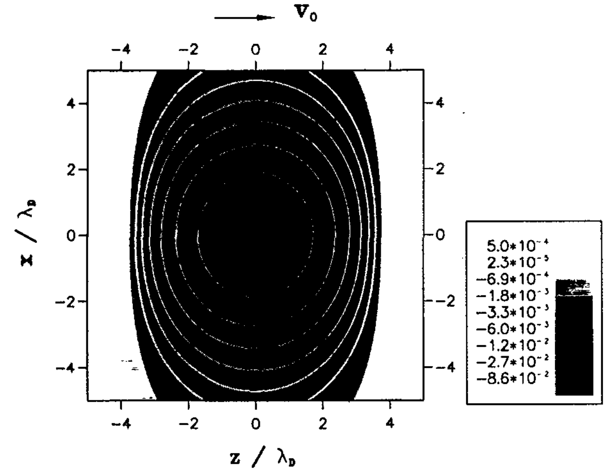

FIG. 1. The potential contours around an infinitesimal particle (located at the origin) obtained from Eq. (27). Here $u_0^2=0.1$ and $\kappa_D=\kappa_i$ are assumed. The potential is normalized by $Q\kappa_D/a\pi\epsilon_0$. Note that the contours are on a logarithmic scale.

where $\cos\theta=k_z/k$. In dimensional form, the equation above becomes

$$\frac{\lambda_e^3\hat{\psi}^{(0)}(\mathbf{k})}{q\beta_i}=-\frac{Q}{\epsilon_0}\left(\bar{k}^2+\kappa_e^2+\frac{\kappa_i^2}{1-u_0^2\cos^2\theta}\right)^{-1},$$

where $\bar{k}=\kappa_e k$. Note that, when $u_0<1$, the ion Debye length $\lambda_i=\kappa_i^{-1}$ is effectively replaced by $\lambda_i(1-u_0^2\cos^2\theta)^{1/2}$. Namely the sheath thickness around the particle is reduced most along the z axis (i.e., $\theta=0$ and π).

In coordinate space, the sheath potential is given to lowest order by

$$\phi^{(0)}(\mathbf{r})=\frac{1}{q\beta_i(2\pi)^3}\int\hat{\psi}^{(0)}(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{r}/\lambda_e)d\mathbf{k}. \quad (26)$$

In particular, if $u_0\ll1$, this potential becomes

$$\phi^{(0)}(\mathbf{r})=-\frac{Q}{4\pi\epsilon_0 r}\exp(-\kappa_D r)+\frac{Q\kappa_i v_{i0}^2}{16\pi\epsilon_0 v_{i,\mathrm{th}}^2}\left\{\left(1-\frac{z^2}{r^2}\right)\right.$$
$$\times\exp(-\kappa_D r)-\frac{8}{\kappa_D^3 r^3}\left(1-\frac{3z^2}{r^2}\right)\left[1-\exp(-\kappa_D r)\right.$$
$$\left.\left.\times\left(1+\kappa_D r+\frac{1}{2}\kappa_D^2 r^2\right)\right]\right\}, \quad (27)$$

where $r=|\mathbf{r}|$ and $\kappa_D=(\kappa_i^2+\kappa_e^2)^{1/2}$. The derivation of this expression is given in Appendix A. The first term of Eq. (27) represents the well-known screened Coulomb potential. Figure 1 illustrates Eq. (27) for $u_0^2=(v_{i,0}/v_{i,\mathrm{th}})^2=0.1$, and $\kappa_i/\kappa_D=1$.

The force exerted on the particulate by the electrostatic field is given by

$$\mathbf{F}=(-Q)\mathbf{E}|_{\mathbf{r}=0}=Q\nabla\left(\phi(\mathbf{r})-\frac{-Q}{4\pi\epsilon_0 r}\right)\bigg|_{\mathbf{r}=0}-Q\mathbf{E}_0(\mathbf{0}), \quad (28)$$

$$=\frac{-iQ}{\lambda_e q\beta_i(2\pi)^3}\int d\mathbf{k}\,\mathbf{k}\left(\hat{\psi}(\mathbf{k})+\frac{Q}{\epsilon_0 k^2}\right)-Q\mathbf{E}_0(\mathbf{0}). \quad (29)$$

The term $-Q/4\pi\epsilon_0 r$ in Eq. (28) represents the self-energy of the particulate, which must be subtracted out in the force calculation. Note that the Fourier transform of $1/4\pi\rho$ is $1/k^2$.

By substituting Eq. (25) into Eq. (29), we find that the force $\mathbf{F}^p=\mathbf{F}+Q\mathbf{E}_0(\mathbf{0})$ due to the lowest-order sheath field potential $\phi^{(0)}(\mathbf{r})$ vanishes, i.e., $\mathbf{F}^p=\mathbf{0}$, since the integral over the polar angle θ vanishes. Indeed, as may be easily seen from Eq. (26), the lowest-order sheath potential $\phi^{(0)}$ is symmetric about the x-y plane—$\phi^{(0)}(\mathbf{r})$ is invariant under the map $z\to-z$. In other words, the Debye sheath does *not* become polarized in the presence of a uniform ion flow $\mathbf{v}_{i0}$, although the sheath may be somewhat squeezed in the z direction, as depicted in Fig. 1. No charge separation is associated with this symmetric deformation of the Debye sheath, and thus no net electrostatic force is exerted on the particulate. The first term $\mathbf{F}^p$ of Eq. (29) may be called a polarization force, since it becomes nonzero only when there is a charge separation (i.e., polarization) of the sheath surrounding the particulate.

Uniform ion flows thus exert no net force on particulates in the fluid approximation. In kinetic treatments, however, Coulomb collisions between the flowing ions and particulates cause an asymmetry in the ion distribution function and give rise to a nonzero ion drag force.[20] (The situation here is somewhat different from the well-known d'Alembert paradox in fluid dynamics, which states that two-dimensional, incompressible, irrotational, inviscid flows exert no drag forces.) In our fluid model, although the size of the particulate is infinitesimal, we nevertheless assume that it is a "small" macroscopic object immersed in the plasma. Under such conditions, the force due to the nonuniformity of the background plasma—such as the polarization force—becomes the dominant force exerted on the particulate. We therefore proceed to higher-order calculations.

V. FORCES DUE TO PLASMA NONUNIFORMITY

As shown in the previous section, the effects of plasma nonuniformity are contained in the first-order solution $\xi^{(1)}$ to Eq. (21)—a uniform plasma with finite flows, as represented by the zeroth-order solution $\xi^{(0)}$, does not exert a polarization force on the particulate. Therefore, we now solve Eq. (21) to $\mathcal{O}(\delta)$ to obtain the nonzero polarization force. The algebraic manipulations in this section were mostly performed using the AXIOM system.[21]

By solving $M\xi^{(0)}=\mathbf{b}$, or Eqs. (22)–(24), we obtain

$$\xi^{(0)}=\frac{1}{P}\begin{bmatrix}k_\perp k_z u_0\\ k_z^2 u_0\\ k^2\\ k_z^2u_0^2-k^2\end{bmatrix}, \quad (30)$$

where

$$P=-(1+k^2)(k^2-k_z^2u_0^2)-\alpha k^2.$$

Then, from Eq. (21), the first-order solution $\boldsymbol{\xi}^{(1)}$ satisfies

$$M\boldsymbol{\xi}^{(1)}=iA_1\boldsymbol{\xi}^{(0)}+iA_2\frac{d}{dk_z}\boldsymbol{\xi}^{(0)}.$$

Substituting Eq. (30) into the equation above and inverting the matrix M, we readily obtain the solution $\boldsymbol{\xi}^{(1)}$.

In particular, the first-order potential perturbation may be given as

$$\hat{\psi}^{(1)}(\mathbf{k})=iQ^*\frac{\mu\hat{\psi}_\mu+\mu_e\hat{\psi}_{\mu e}+u_0'\hat{\psi}_u}{\{[k_z^4+(k_\perp^2+1)k_z^2]u_0^2-k^2(k^2+\alpha+1)\}^3}, \tag{31}$$

where

$$\hat{\psi}_\mu=2\alpha k_zk^6-\alpha k_zk^2[6k_z^4+(3k_\perp^2-2\alpha-2)k_z^2-3k_\perp^4-(5\alpha+3)k_\perp^2]u_0^2+\alpha k_z^3[4k_z^4+(5k_\perp^2-2)k_z^2+k_\perp^4+k_\perp^2]u_0^4. \tag{32}$$

$$\hat{\psi}_{\mu e}=2k_zk^6-2k_zk^2(3k_z^4+3k_\perp^2k_z^2-\alpha k_\perp^2)u_0^2+2k_z^3(3k_z^4+3k_\perp^2k_z^2-\alpha k_\perp^2)u_0^4-2k_z^7u_0^6, \tag{33}$$

$$\hat{\psi}_u=2\alpha k_zk^4(k^2+\alpha+1)u_0-2\alpha k_z^3k^2(k^2+1)u_0^3. \tag{34}$$

It is neither easy nor practical to use the full solution for $\hat{\psi}^{(1)}(\mathbf{k})$ given by Eqs. (31)–(34) to estimate the polarization force. Instead, we shall consider two important limiting cases, in which the expression for the polarization force provides clearer physical insight and concise formulas that may be used in macroscopic (e.g., fluid or particle-in-cell) simulations to determine overall particulate transport characteristics in the plasma.

VI. SMALL FLOW LIMIT

In order to elucidate the relation between the complete solution obtained in the preceding section and that given in Ref. 1, we first take the limit of a small ion flow (i.e., $u_0\ll 1$) in Eq. (31):

$$\hat{\psi}^{(1)}(\mathbf{k})=-iQ^*\left(\frac{2(\alpha\mu+\mu_e)k_z}{(k^2+\alpha+1)^3}+u_0'u_0\frac{2\alpha k_z}{k^2(k^2+\alpha+1)^2}+\mathscr{O}(u_0^2)\right). \tag{35}$$

Here we have assumed that $\delta=\lambda_D/L\simeq u_0\ll 1$ and $u_0'=\mathscr{O}(\delta)$, rather than $u_0'=\mathscr{O}(\delta u_0)$ (i.e., the ion flow velocity is assumed to have a steep gradient near the particle), to clarify the effects of the ion flow gradient. Note that $\alpha\mu+\mu_e=\mathscr{O}(\delta)$.

The inverse Fourier transform of Eq. (35) yields

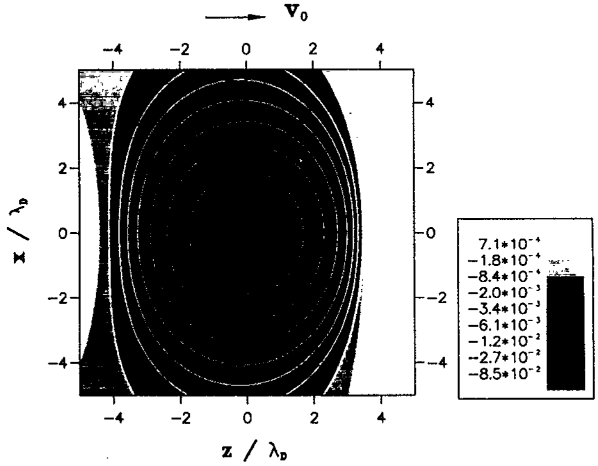

FIG. 2. The total potential $\phi^{(0)}+\phi^{(1)}$ obtained from Eqs. (27) and (36). To emphasize the ion flow effects, we have assumed $\sigma=0$ and $v_{i,0}'=v_{i,0}/L>0$, with $\lambda_D/L=0.3$. The other parameters are the same as those in Fig. 1. Note that the contours are on a logarithmic scale.

$$\begin{aligned}\phi^{(1)}(\mathbf{r})&=\frac{1}{q\beta_i}\psi^{(1)}(\mathbf{r})\\&=\frac{1}{q\beta_i(2\pi)^3}\int\hat{\psi}^{(1)}(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{r}/\lambda_e)d\mathbf{k}\\&=\frac{Q\sigma z}{16\pi\epsilon_0\kappa_D}\exp(-\kappa_Dr)+\frac{v_{i,0}v_{i,0}'\kappa_i^2Qz}{2\pi v_{i,\mathrm{th}}^2\kappa_D^4r^3}\\&\quad\times[1-\exp(-\kappa_Dr)(1+\kappa_Dr+\tfrac{1}{2}\kappa_D^2r^2)],\end{aligned} \tag{36}$$

where $\kappa_D=(\kappa_i^2+\kappa_e^2)^{1/2}$ as before, and $\sigma=d\kappa_D^2(z)/dz$. Details of the above calculation are given in Appendix B. The first term in Eq. (36) represents the potential perturbation due to the density gradient (i.e., the gradient of the Debye length $\lambda_D=\kappa_D^{-1}$), while the second is due to the gradient of the ion flow velocity. Hence, polarizations of the Debye sheath represented by the first and second terms of Eq. (36) may be called the density-gradient and ion-flow-gradient polarizations, respectively. In the case of zero ion flow or flow gradient, Eq. (36) evidently reduces to the first-order potential perturbation given in Ref. 1.

Figure 2 shows the total potential (except for that of the applied electric field $\mathbf{E}_0$), i.e., $\phi^{(0)}+\phi^{(1)}$, derived from Eqs. (27) and (36). The particle is located at the origin. To emphasize the ion flow effects, we have assumed $\sigma=0$ and $v_{i,0}'=v_{i,0}/L>0$, with $\lambda_D/L=0.3$. The other parameters are the same as those in Fig. 1. Note that, since $v_{i,0}'>0$ (and thus $v_{i,0}$, is larger in the region $z>0$ than in $z<0$), the sheath thickness is smaller for $z>0$ than for $z<0$.

As remarked in Sec. IV, the zeroth-order solution $\hat{\psi}^{(0)}(\mathbf{k})$ does not contribute to the polarization force. Therefore, from Eq. (29), the z component of the polarization force $\mathbf{F}^p$ may be obtained from

$$F_z^p=\frac{-iQ}{\lambda_e q\beta_i(2\pi)^3}\int k_z\hat{\psi}^{(1)}(\mathbf{k})d\mathbf{k}$$
$$=\frac{\sigma Q^2}{16\pi\epsilon_0\kappa_\mathrm{D}}[1+\mathcal{O}(\epsilon^2)]+\frac{v_{i0}v_{i0}'}{v_{i,\mathrm{th}}^2}\frac{Q^2\kappa_i^2}{12\pi\epsilon_0\kappa_\mathrm{D}}[1+\mathcal{O}(\epsilon^2)], \tag{37}$$

where $\epsilon=u_0=v_{i0}/v_{i,\mathrm{th}}$ [the derivation of Eq. (37) is given in Appendix B]. As in Eq. (36), the first and second terms of Eq. (37) represent the density-gradient and ion-flow-gradient polarization forces, respectively. Again, in the case $\epsilon=0$, the above polarization force agrees with that given in Ref. 1.

To obtain the total force, one needs to also consider the contribution of pressure forces. As in Ref. 1, however, a direct calculation of the plasma pressure shows that the pressure force on an infinitesimal particle is zero. Therefore, aside from the ion drag force that needs to be obtained from kinetic theories, the total force $\mathbf{F}$ is given by the sum of the direct electrostatic force $(-Q)\mathbf{E}_0$ and the polarization force given by Eq. (37). In vector form, we may write

$$\mathbf{F}=-Q\mathbf{E}_0-\frac{\nabla\lambda_\mathrm{D}}{\lambda_\mathrm{D}^2}\frac{Q^2}{8\pi\epsilon_0}[1+\mathcal{O}(\epsilon^2)]$$
$$+\frac{\nabla|\mathbf{v}_{i0}|^2}{v_{i,\mathrm{th}}^2}\frac{Q^2\lambda_\mathrm{D}}{24\pi\epsilon_0\lambda_i^2}[1+\mathcal{O}(\epsilon^2)]. \tag{38}$$

Here we have used $\sigma=-2\lambda_\mathrm{D}^{-3}\,d\lambda_\mathrm{D}/dz$. Note that, in Eq. (38), all plasma quantities are evaluated at the position of the particulate.

Under typical plasma flow conditions, the flow velocity gradient v_{i0}' is expected to be of order v_{i0}/L (where L is the macroscopic length scale), rather than $v_{i0}'=v_{i0}/\lambda_\mathrm{D}$, as assumed above. In this case $u_0'=\mathcal{O}(\delta\epsilon)$, where $\delta=\lambda_\mathrm{D}/L$ and $\epsilon=v_{i0}/v_{i,\mathrm{th}}$, and therefore $|\nabla\lambda_\mathrm{D}|=\mathcal{O}(\delta)$ and $\lambda_\mathrm{D}|\nabla\mathbf{v}_{i,0}^2|/\mathbf{v}_{i,\mathrm{th}}^2=\mathcal{O}(\delta\epsilon^2)$. Then Eq. (38) may be written as

$$\mathbf{F}=-Q\mathbf{E}_0-\frac{Q^2}{8\pi\epsilon_0}\frac{\nabla\lambda_\mathrm{D}}{\lambda_\mathrm{D}^2}+\mathcal{O}(\delta\epsilon^2), \tag{39}$$

where the first two terms are the same as the total force obtained in Ref. 1. In other words, if the ion flow velocity is small compared to the ion thermal velocity, the ion-flow-gradient polarization force is $\mathcal{O}(\delta\epsilon^2)$, whereas the density polarization force is $\mathcal{O}(\delta)$. The force expression given by the first two terms of Eq. (39) is therefore a good approximation for a nonuniform plasma with small ion flows.

VII. LARGE FLOW LIMIT

In typical glow discharges used in industrial applications, particulates become "trapped" at the plasma–sheath boundary—i.e., the boundary between the bulk plasma and the sheath adjacent to the electrode.[14] Ions in the presheath region are accelerated toward the plasma–sheath boundary and the ion flow velocity v_{i0} reaches the sound (or Bohm) velocity $c_s=(k_\mathrm{B}T_e/m_i)^{1/2}$. Since $T_e\gg T_i$ under typical conditions, the ion flow velocity exceeds the ion thermal velocity, i.e.,$v_{i0}\simeq c_s\gg v_{i,\mathrm{th}}$.

In the large ion flow limit we set $u_0\gg1$ in Eqs. (31)–(34) and retain only the lowest-order terms in the expansion parameter $1/u_0$. This gives

$$\hat{\psi}^{(1)}(\mathbf{k})=-2iQ^*\mu_e\frac{k_z}{(k^2+1)^3}+\mathcal{O}\left(\frac{\delta}{u_0^2}\right), \tag{40}$$

where $\mu_e=\mathcal{O}(\delta)$. Note that the lowest-order term is independent of u_0.

As in Eq. (36), the inverse Fourier transform of Eq. (40) gives the potential $\phi^{(1)}$, to lowest order, as

$$\phi^{(1)}(\mathbf{r})=\frac{Q\sigma_e z}{16\pi\epsilon_0\kappa_e}\exp(-\kappa_e r),$$

where $\sigma_e=-2\lambda_e^{-3}\,d\lambda_e/dz$. As in Eq. (37), the z component of the polarization force may be obtained from Eq. (40). To lowest order, we have

$$F_z^p=\frac{\sigma_e Q^2}{16\pi\epsilon_0\kappa_e}.$$

Since the pressure force is zero for an infinitesimal particulate, the total force in the presence of a large ion flow may be written in vector form as

$$\mathbf{F}=-Q\mathbf{E}_0-\frac{Q^2}{8\pi\epsilon_0}\frac{\nabla\lambda_e}{\lambda_e^2}+\mathcal{O}(\epsilon^{*2}), \tag{41}$$

where $\epsilon^*=1/u_0=v_{i,\mathrm{th}}/v_{i0}$. Note, again, that, in the limit of strong ion flows, the ion-flow-gradient polarization force vanishes to lowest order.

Comparing expressions (39) and (41) for the opposite limits of small and large ion flows, we see that they differ only in the Debye length that is used: in the small-flow limit, λ_D is used, whereas λ_e is appropriate to the large-flow limit. This reflects the fact that, in the absence of ion flows, the size of the Debye sheath surrounding a particulate is given by the "characteristic" Debye length $\lambda_\mathrm{D}=(1/\lambda_i^2+1/\lambda_e^2)^{-1/2}$, while in the presence of large ion flows it is given by just the electron Debye length λ_e.

The physical reason for this is that when the ion flow is large compared to the ion thermal velocity (i.e., $u_{i0}\gg v_{i,\mathrm{th}}$), ions cannot form a sheath, and the sheath comprises a deficiency of electrons. In this case, the ions are "blown away" by the fast ion flow, and the ion density profile around the particulate becomes almost uniform. In typical glow discharges, however, plasma flow velocities are generally small compared to the *electron* thermal velocity $v_{e,\mathrm{th}}=(k_\mathrm{B}T_e/m_e)^{1/2}$, so that the electron sheath, of dimension λ_e, is hardly affected by the plasma flow.

VIII. CONCLUDING REMARKS

In this paper we have extended earlier results[1] to obtain the total force $\mathbf{F}$ exerted on a charged particulate in a nonuniform plasma under the influence of finite ion flows. We have rigorously demonstrated, in the context of a fluid approximation of the plasma, that the expression for the polarization force given in Ref. 1 is a good approximation, unless the ion flow velocity is comparable to the ion thermal velocity.

From Eqs. (41) and (39), we may write the total force on a particulate as

$$\mathbf{F}=-Q\mathbf{E}_0-\frac{Q^2}{8\pi\epsilon_0}\frac{\nabla\lambda}{\lambda^2}+(\text{ion drag force}), \tag{42}$$

where λ represents the thickness of the Debye sheath around the particulate. The first term is the electrostatic force on the particulate charge—Q exerted by the external electric field $\mathbf{E}_0$, while the second term is the polarization force, i.e., the force due to the polarized Debye sheath. Note that $\mathbf{E}_0$ and λ in Eq. (42) are to be evaluated at the particulate position.

In the fluid approximation, polarization of Debye sheaths may be caused by density gradients and/or ion flow gradients. As discussed in Secs. VI and VII, however, the ion-flow-gradient polarization force is typically small and may be neglected to lowest order, as shown in Eq. (42).

If the ion flow velocity is small compared to the ion thermal velocity, i.e., $v_{i0} \ll v_{i,\text{th}}$, the sheath thickness is given by the characteristic Debye length, $\lambda_D=(1/\lambda_i^2+1/\lambda_e^2)^{-1/2}$, and λ_D should be used for λ in Eq. (42). Note that if $T_e \gg T_i$ (as is the case in typical glow discharges), we have $\lambda_D \simeq \lambda_i$.

On the other hand, if the ion flow velocity is large compared to the ion thermal velocity, i.e., $v_{i0} \gg v_{i,\text{th}}$, the sheath thickness is given by the electron Debye length λ_e. In this case, the ion density profile around the particulate is almost uniform, and the sheath comprises only a deficiency of electrons. The conditions for such fast ion flows arise when particulates are trapped (due to a balance between electrostatic forces and drag forces) at the plasma–sheath boundaries of glow discharges, where the ion flow velocity equals the sound speed c_s. Under such conditions, λ_e should be used for λ in Eq. (42).

Strictly speaking, Eq. (42) holds only in these two limiting cases. For intermediate values of v_{i0}, as is clear from Eq. (38), the contribution from the ion flow gradient to the polarization force may be comparable to that from the density gradient. A more accurate evaluation of the total force at arbitrary v_{i0} may be obtained by performing the integration (29) numerically, using the complete expression for $\hat{\psi}^{(1)}(\mathbf{k})$ (which is valid for any $v_{i,0}$), given by Eqs. (31)–(34).

Note also that, as indicated in Eqs. (39) and (41), the errors incurred by using Eq. (42) in the two limiting cases are only of second order, i.e., $\mathcal{O}(v_{i0}^2/v_{i,\text{th}}^2)$ for $v_{i0} \ll v_{i,\text{th}}$ and $\mathcal{O}(v_{i,\text{th}}^2/v_{i0}^2)$ for $v_{i,\text{th}} \ll v_{i0}$. [The ion-flow-gradient polarization force is represented explicitly by the third term of Eq. (38) when the ion flow is relatively small.]

It is interesting to note that, regardless of the sign of the particulate charge, the polarization force is always in the direction of decreasing Debye length. From Eq. (42), we may calculate the (Helmholtz) free energy $F_H=-\int^z F_z\,dz$ (neglecting the ion drag force) as

$$F_H=-Q\Psi_0-\frac{Q^2}{8\pi\epsilon_0\lambda}.$$

It is evident from this expression that the particulate has a lower free energy when it has a thinner Debye sheath (i.e., smaller λ).

The magnitude of the force given by Eq. (42) may vary significantly, depending on the plasma conditions. For typical glow discharges used in industrial applications, however, the total force $\mathbf{F}$ was estimated in Ref. 1, where the polarization force was found to be typically a fraction of the external electrostatic force.

As noted in Sec. II, in most particulate systems observed in laboratory or space plasmas, the sizes of the particulates and their Debye sheaths are small compared to the ion mean-free path for the background plasma. Therefore the fluid approximation that we have employed here is not strictly valid, especially when it is applied to phenomena that involve significant variation of the ion distribution function. The most important aspect of such kinetic effects is the ion drag force, which needs to be obtained separately from kinetic calculations and added to the fluid force, as indicated in Eq. (42).

As shown in Sec. IV, the drag force for an infinitesimal particulate vanishes in the fluid approximation. However, the polarization force, with which we have been concerned, is well described by the fluid model. Although a systematic kinetic treatment of nonuniform plasmas is rather formidable, the question of whether such kinetic effects substantially modify the polarization force obtained from the fluid approximation deserves attention.

For expressions of the ion drag force, the reader is referred to Ref. 17, where large-angle Coulomb scattering,[22] dynamical friction,[23,24] and collective effects[20] are treated systematically. [In this treatment, however, the background plasma is assumed uniform, so the effects of external fields and density gradients are excluded. Thus, to obtain the total force in a nonuniform plasma, one needs to add the first two terms of Eq. (42), which represent the external electrostatic force and polarization force, to the drag force—assuming that such a superposition is a valid approximation.] The final expression obtained in this manner, which is a function of local plasma conditions, such as the electric field, plasma density gradients, etc., may be used in conjunction with macroscopic simulations to determine the overall particulate transport characteristics in a discharge.

APPENDIX A: DERIVATION OF EQ. (27)

If $u_0 \ll 1$, Eq. (25) may be written as

$$\hat{\psi}^{(0)}(\mathbf{k})=Q^*\left(\frac{1}{k^2+1+\alpha}-\frac{\alpha u_0^2\cos^2\theta}{(k^2+1+\alpha)^2}\right).$$

The first term evidently yields the screened Coulomb potential [i.e., the first term of Eq. (27)], so we are now only concerned with the inverse Fourier transform of the second term, i.e.,

$$\delta\phi^{(0)}(\mathbf{r})=\frac{-\alpha u_0^2 Q^*}{q\beta_i(2\pi)^3}\int\frac{\cos^2\theta}{(k^2+c^2)^2}\exp\left(\frac{-i\mathbf{k}\cdot\mathbf{r}}{\lambda_e}\right)d\mathbf{k}, \tag{A1}$$

where $c^2=\alpha+1$.

To perform the integration of Eq. (A1), we choose the direction of the position vector $\mathbf{r}$ as the polar direction, rather than that of $\mathbf{v}_{i,0}$. Denoting the angle between $\mathbf{r}$ and $\mathbf{v}_{i,0}$ by Θ, and that between $\mathbf{r}$ and $\mathbf{k}$ by $\tilde{\theta}$, we may write

$$\cos\theta = \sin\Theta \sin\tilde\theta \cos\tilde\phi + \cos\Theta \cos\tilde\theta,$$

$$\mathbf{k}\cdot\mathbf{r} = kr\cos\tilde\theta,$$

where the azimuthal angle $\tilde\phi$ of $\mathbf{k}$ around $\mathbf{r}$ is chosen appropriately.

After integrating over the angles $\tilde\theta$ and $\tilde\phi$, the following formulas may be used to perform the integration over k in Eq. (A1):

$$\int_0^\infty \frac{x\sin ax}{(x^2+b^2)^2}\,dx = \frac{\pi a}{4b}\exp(-ab),$$

$$\int_0^\infty \frac{\cos ax}{(x^2+b^2)^2}\,dx = \frac{\pi(1+ab)}{4b^3}\exp(-ab), \tag{A2}$$

$$\int_0^\infty \frac{\sin ax}{x(x^2+b^2)^2}\,dx = \frac{\pi}{2b^4}\left[1-\left(1+\frac{ab}{2}\right)\exp(-ab)\right],$$

where $a,b>0$.

APPENDIX B: DERIVATIONS OF EQS. (36) AND (37)

To obtain Eq. (36) by the inverse Fourier transform, one needs to evaluate the integrals,

$$J_1 = \frac{1}{(2\pi)^3}\int \frac{k_z}{(k^2+c^2)^3}\exp(-i\mathbf{k}\cdot\boldsymbol{\rho})d\mathbf{k},$$

$$J_2 = \frac{1}{(2\pi)^3}\int \frac{k_z}{k^2(k^2+c^2)^2}\exp(-i\mathbf{k}\cdot\boldsymbol{\rho})d\mathbf{k},$$

where $k^2=k_\perp^2+k_z^2$ and $c^2=\alpha+1$ $(c>0)$. Unlike Appendix A, here we choose the z axis (i.e., the direction of the ion flow $\mathbf{v}_{i,0}$) as the polar direction.

To perform the integration of J_1, we first write

$$g_1(k) = \frac{-1}{4(k^2+c^2)^2}.$$

Then, using spherical polar coordinates in $\mathbf{k}$ space, we have

$$J_1 = \frac{1}{(2\pi)^3}\int \frac{dg_1(k)}{dk_z}\exp(-i\mathbf{k}\cdot\boldsymbol{\rho})d\mathbf{k}$$

$$= \frac{i\zeta}{(2\pi)^3}\int g_1(k)\exp(-i\mathbf{k}\cdot\boldsymbol{\rho})d\mathbf{k}$$

$$= -\frac{i\zeta}{8\pi^2\rho}\int_0^\infty \frac{k}{(k^2+c^2)^2}\sin k\rho\,dk$$

$$= -\frac{i\zeta}{32\pi c}\exp(-c\rho),$$

where ζ is the z component of $\boldsymbol{\rho}$. Here we have used Eq. (A2).

Similarly, to evaluate J_2, we use

$$g_2(k) = -\frac{1}{2c^4}\left(\log\frac{k^2+c^2}{k^2} - \frac{c^2}{k^2+c^2}\right),$$

where

$$\frac{dg_2(k)}{dk_z} = \frac{k_z}{(k^2+c^2)^3}.$$

Then we have

$$J_2 = \frac{1}{(2\pi)^3}\int \frac{dg_2(k)}{dk_z}\exp(-i\mathbf{k}\cdot\boldsymbol{\rho})d\mathbf{k}$$

$$= \frac{i\zeta}{(2\pi)^3}\,\frac{4\pi}{\rho}\int_0^\infty g_2(k)k\sin k\rho\,dk$$

$$= \frac{-i\zeta}{4\pi c^4\rho^3}\left[1-\exp(-c\rho)\left(1+c\rho+\frac{1}{2}c^2\rho^2\right)\right].$$

Here we have used the formulas

$$\int_0^\infty \frac{x\sin ax}{x^2+b^2}\,dx = \frac{\pi}{2}\exp(-ab), \tag{B1}$$

$$\int_0^\infty \log\left(\frac{x^2+a^2}{x^2+b^2}\right)x\sin xy\,dx = \frac{\pi}{y^2}[\exp(-by)(1+by) - \exp(-ay)(1+ay)], \tag{B2}$$

where $a,b\geqslant 0$ and $y>0$. Note that Eq. (A2) may be obtained by differentiating Eq. (B1) with respect to b. Similarly, Eq. (B2) may be derived by differentiating the following identity with respect to y:

$$\int_0^\infty \log\left(\frac{x^2+a^2}{x^2+b^2}\right)\cos xy\,dx = \frac{\pi}{y}[\exp(-by)-\exp(-ay)], \tag{B3}$$

where, again, $a,b\geqslant 0$ and $y>0$. In deriving Eq. (36) from J_1 and J_2, we have also used $c\rho=\kappa_{\rm D}r$, $\kappa_e c=\kappa_{\rm D}$, and $(\alpha\mu+\mu_e)=\sigma\lambda_e^3$.

To obtain Eq. (37) directly from Eq. (29), we evaluate the integrals,

$$J_3 = \frac{1}{(2\pi)^3}\int \frac{k_z^2}{(k^2+c^2)^3}\,d\mathbf{k},$$

$$J_4 = \frac{1}{(2\pi)^3}\int \frac{k_z^2}{k^2(k^2+c^2)^2}\,d\mathbf{k}.$$

Again, using spherical polar coordinates in the k space and writing $\eta=k/c$, we may write

$$J_3 = \frac{1}{6\pi^2 c}\int_0^\infty \frac{\eta^4}{(\eta^2+1)^3}\,d\eta$$

and

$$J_4 = \frac{1}{6\pi^2 c}\int_0^\infty \frac{\eta^2}{(\eta^2+1)^2}\,d\eta.$$

Using the formula

$$\int_0^\infty \frac{dx}{x^\alpha(1+x^\lambda)^\beta} = \frac{1}{\lambda}B\left(\beta-\frac{1-\alpha}{\lambda},\frac{1-\alpha}{\lambda}\right), \tag{B4}$$

if $\alpha<1$, $\lambda,\beta>0$, and $\lambda\beta>1-\alpha$, we obtain $J_1=1/32\pi$ and $J_4=1/24\pi$. In Eq. (B4), B is the beta function, related to the gamma function Γ by

$$B(p,q)=B(q,p)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$

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