Compressive Strength of Plate Elements with Welding Residual Stresses and Deformation†

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Abstract

When steel structures are constructed by welding, the structural elements are always accompanied by welding residual stresses and distortion. Usually plate structure is considered to consist of plate elements of various sizes. Among these elements, deck and bottom plating play a most important role in the strength of ships and bridges. These plates are reinforced and sub-divided by stiffeners into narrow, strip plate elements. These sub-divided plate elements sustain mostly in-plane axial forces, which are either tensile and/or compressive. Accurate assessment of compressive strength of these plate elements containing welding residual stresses and distortion is very important to insure the safety of welded structures. The recent rapid development of high speed computers has enabled us to conduct nonlinear analyses. Using this advantage in this paper, buckling and ultimate strengths of rectangular plates and stiffened plates with welding residual stresses and/or deformation under compression have been comprehensively investigated both theoretically and experimentally. For theoretical analyses, several methods were applied including the finite element method which the authors have developed. From the results of the investigation, it was found that welding residual stresses and deformation reduce the buckling and ultimate strengths and their effects become greater for thicker plates up to $(b/t)\sqrt{\sigma_Y/E}<1.8$. *The one exception is the case when a stiffened plate buckles in the over-all mode.*

1. Introduction

In general, steel structures are fabricated by welding, and the structural elements are always accompanied by welding residual stresses and usually also by deformation. These influence the performance of the welded structures. For this reason the research on the mechanical aspect of welding can be divided into the following two parts: (1) mechanism of production of residual stresses and deformation and dynamic conditions to prevent weld cracking, and (2) effects of these residual stresses and deformation on the strength of welded structures.

For the first part of the problem, one of the authors has established a method of analysis for the thermal elastic-plastic behavior of metal during welding and stress relief annealing, based on the finite element method.[1)-5)] By applying his method, he and his colleagues have performed many kinds of analyses and provided useful information on the mechanics of production of residual stresses by welding[6)-8)] and reduction of welding stresses by annealing.[7)] Furthermore, they developed the general principle of measurement of three dimensional residual stresses and showed the validity of the principle by using several numerical models and applying the finite element method.[9),10)]

The aim of the above research is: (1) to prevent weld cracking from the dynamic point of view and (2) to estimate and predict welding residual stresses and deformation in structural elements. Based on the information from this research, investigations into the effects of residual stresses and deformation upon the performance (mainly strength) of the welded structures can be conducted and the results should be useful in evaluating the reliability of the welded structures.

In this report, the authors will describe their recent research on this subject, that is, the effects of welding imperfection on the strength of welded structures, by applying several methods of analyses.

Steel structures are composed of many structural components. Particularly, large size structures are often constructed by using plates to reduce their weight and cost. These are characterized as plate structures. Their structural components are decks, bottoms, girders, etc., which are mainly fabricated by welding. Accordingly, welding produces the welding residual stresses and deformation in their plate

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elements. Among these, compressive plate elements, such as decks and bottoms of ships and steel bridges of some types, flanges of large size girders, etc. are the primary structural members whose strengths are decisive in determining the entire strengths of the structures.

As fundamental research, the buckling strengths and ultimate strengths of a rectangular plate and a stiffened plate containing welding imperfection (residual stresses and deformation) were investigated both theoretically and experimentally and the results of the investigation will be shown.

2. Methods of Theoretical Analyses on Elastic-plastic Buckling and Large Deflection of Plates with Welding Residual Stresses and Deformation

Ever since welding was used for construction of structures, the effect of welding on performance of the structures has been one of the main research subjects among the engineers in this field. Although rapid development of the research has been observed recently, the main reason it has taken so many years is attributed to the complexity of the behavior of the structural element containing the welding imperfection and the difficulty in the theoretical analysis of this behavior.

Here, the outline of the analytic methods which the authors mainly employed in their research is described. First, the following four typical behaviors of plates is considered.

(1) When a plate containing welding residual stresses is subjected to a tensile force above a certain magnitude, some portion of the plate will be in the plastic range, because the stress distribution is not uniform due to the existence of the welding residual stresses. The deformation and strains of the plate are small, but the stress distribution includes the plastic portion. This type of behavior is the so called elastic-plastic one and is characterized as the material nonlinear problem. This nature is the same as that when a compressive force is applied to the plate where buckling of the plate is prevented.

(2) When a thin flat plate is loaded by a compressive force, the plate buckles; either with an elastic, an elastic-plastic, or a plastic stress distribution and this is called either elastic, elastic-plastic or plastic buckling, respectively. The buckling is considered as a bifurcation from a flat equilibrium to a bent equilibrium. Then, the analysis is performed to examine the possibility of an equilibrium state of the plate for a small deflected shape under the stress distribution for the flat shape. This problem exhibits the simplest, geometric nonlinearity. When the stress distribution is elastic-plastic, the problem also includes the material nonlinearity.

(3) After a plate buckles under compression, the plate can hold an additional load, increasing its lateral deflection. On the other hand, if a plate is initially deflected, lateral deflection of the plate increases at the beginning of application of compression. For theoretical analysis of these phenomena, the equilibrium of the plate against the external load should be formulated at the deflected shape. This is a typical geometric nonlinear problem.

(4) When the plate discussed in (3) is loaded further, the stress distribution becomes elastic-plastic. The problem is a combined nonlinear problem of material and geometry, and is highly complicated.

Concerning the theoretical analyses of the above mentioned behaviors, there are some conditions which must be satisfied. First, for an analysis of small deflection of a plate within the elastic range, the following three conditions should be satisfied: (1) displacement-strain relations; (2) stress-strain relations; (3) equilibrium conditions of stresses. The solutions from these must also satisfy the boundary conditions of the individual problem.

For elastic-plastic problems, which have material nonlinearities, the stresses must satisfy the yield condition of the material, and the stress-strain relations (2) should be replaced by those in the plastic range based on the appropriate theory of plasticity. For large deflections of a plate that is geometric nonlinearity, the internal equilibrium is considered at the deflected shape and the displacement-strain relations (1) should be replaced by those which include the effect of large deflections. These relations are nonlinear.

In order to describe the various analytic methods, it is most convenient to use the energy theorems. The principle of virtual work furnishes the equilibrium conditions for an assumed stress distribution by using the displacement-strain relations (1). As this principle does not require the stress-strain relations, this is applicable to both elastic and elastic-plastic problems. On the other hand, the principle of minimum potential energy provides the equilibrium conditions by using the displacement-strain relations (1), and the stress-strain relations. For elastic-plastic problems, only an incremental form of the equilibrium conditions can be derived by specifying the stress-strain relation for a small increment of strain.

In application of these principles, two different forms of the resulting equilibrium conditions are obtained: (1) When displacements of the plate are treated as being unknown, the resulting conditions are differential equations. (2) When the displacements are represented by continuous functions with constant coefficients which satisfy the boundary conditions, the resulting conditions produce a set of simultaneous equations in the unknown coefficients. The solution by either method is quite equivalent in quality.

For the differential equations, it is usually difficult to obtain the exact solutions, and by using Galerkin's method (denoted by Method 1–1) approximate solutions (including the exact ones) should be obtained in most cases. For elastic problems, even for large deflection of a plate, this method furnishes fairly accurate results, with only a small amount of numerical calculation.

For the latter type, the Rayleigh-Ritz method (Method 2–1) is well known and used extensively. When displacements are assumed in small domains and they are continuous at the interfaces of the adjacent domains, the method used is the finite element method, (Method 2–2) whose unknown quantities are the nodal displacements. Along the same line, for simpler geometry, the finite strip element method assumes displacements so as to cover larger domains. The methods in the latter group are advantageous to analyze general nonlinear problems; the finite element method especially is most powerful. However, it should be noted that the analysis of combined nonlinear problems by the finite element method requires much computer time.

In addition to the methods mentioned above, the finite difference method is used. This method has been a capable method, but the finite element method is better in its formulation and applicability to general cases.

3. Welding Residual Stresses and Deformation of a Rectangular Plate

For welding residual stresses in a rectangular plate, two typical patterns are considered, as shown in **Figures 3.1, 3.2.** One is that produced in a plate whose pair of parallel sides are welded, such as a panel between stiffeners, webs of columns or beams, etc. (**Fig. 3.2**(a)). The other is that induced in flanges or stiffeners whose one side is welded and the opposite side is free. (**Fig. 3.2**(b)). The stress distributions vary in the individual cases, depending on the type of welding, the welding conditions, the size of a plate, the sequence of welding, etc. There is generally a close relation between welding residual stresses and deformation (deflection). First, a plate furnished

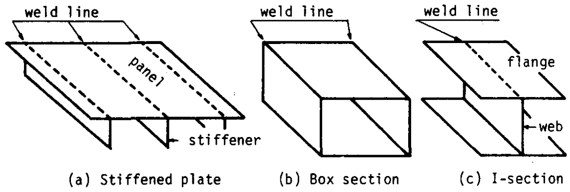


Fig. 3.1 Plate elements in welded plate structures

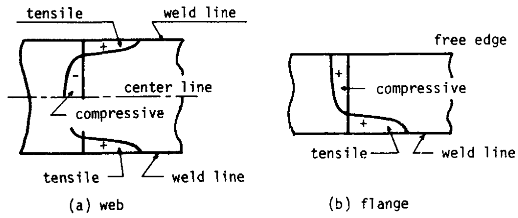


Fig. 3.2 Two typical patterns of distribution of welding residual stresses

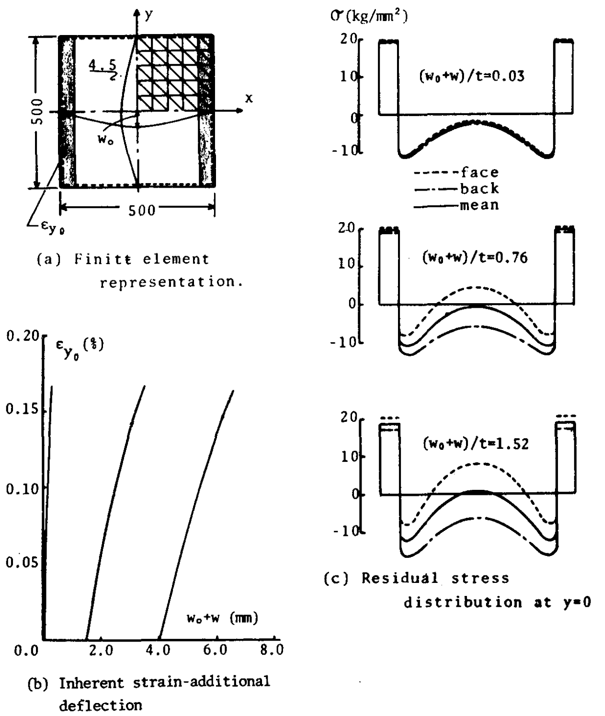


Fig. 3.3 Additional deflection and residual stress distribution caused by inherent strain. (F.E.M. analysis)

from the factory has an initial deflection. Additionally, welding deformation is produced primarily by thermal angular distortion of a fillet weld of stiffeners welded to the panels along their edges. Simultaneously additional deflection is produced by the compressive residual stresses due to shrinkage of the weld beads and the adjacent portions of the plate. The following examples will demonstrate this relation.[11] It is assumed that a plate of 500×500 mm has an initial deflection of a sinusoidal wave, and then weld metal is laid along opposite edges. In the analysis using the finite element method, the shrinkage is replaced by inherent strains ε_{Y_0} which are imposed in the portions 1/10 of the plate breadth along the two edges as shown in **Fig. 3.3**(a). The inherent strains are applied incrementally until these edge portions become plastic in tension. The relations between the imposed inherent strains and the central deflection, and the distribution of the residual stresses along y=0 are shown in **Fig. 3.3**(b) and (c) for the plate 4.5 mm thick. The calculated distribution of residual stresses coincide well with the measured ones, both in sense and magnitude. As seen by the residual stresses on the face and back of the plate, the deflction changes the residual stress distribution and is accompanied by local bending stresses.

4. Fundamental Elastic-Plastic In-Plane Behavior of a Rectangular Plate Containing Welding Residual Stresses

For the next study on the strength of a rectangular plate containing welding residual stresses, it is worthwhile to examine the fundamental elastic-plastic behavior of the plate by applying tension and compression in the plane and preventing any out-of-plane deformation. To make the discussion simple, it is assumed that (1) the pattern of welding residual stress distribution is as shown in **Fig. 3.2**(a); (2) it is uniform along the length of the plate (x axis); and (3) the material is elastic-perfectly plastic. First, if a plate is completely free from welding residual stresses, the elastic-plastic behavior of the plate is the same as the original material; that is, the average stress-strain relation is shown by the solid line in **Fig. 4.1.** Next, the behavior of a plate containing welding residual stresses is considered. When the plate is subjected to a tensile load producing a uniform displacement, the portions under the tensile residual stresses are not effective and the plate behaves in an elastic-plastic manner at the beginning of loading, as if the breadth of the plate were b_c. Then the apparent rigidity of the plate is lower than the original one and the average stress-strain relation is shown by the dashed line. However, as the welding residual stresses are assumed to be self-equilibrating, the compressive residual stress is $\sigma_c=\sigma_Y \cdot b_t/b_c$ and the apparent yield stress in this portion σ_{Yc} is $\sigma_Y+\sigma_{Yc}$. Therefore, the total load carrying capacity is $(\sigma_Y+\sigma_{Yc})(b_c)(t)=\sigma_Y bt$, which is the same as that of the plate free from residual stress. When the plate is unloaded, the relationship takes the path indicated by the arrows in the same figure 4.1, and the line is parallel to the solid one. The plate becomes free from residual stress. This is the basic idea behind mechanical stress relieving.

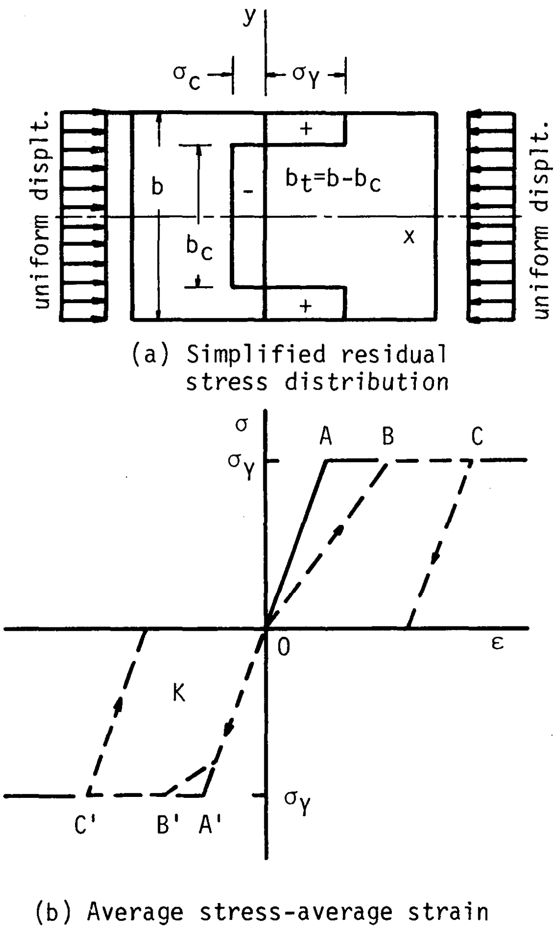


Fig. 4.1 In-plane rigidity of a plate containing residual stresses under a uniaxial load

When a compressive force is applied to the plate, the stress-strain relation is just the same as that of the plain plate until the stress reaches point K, where the portion under the compressive residual stresses yields. Then, the behavior of the plate is elastic-plastic and the rigidity of the plate decreases greatly. The load carrying capacity is the same as before.

5. Buckling of a Plate with Residual Stresses

5.1 Introduction

When a column buckles elastically under a concentrated load, the critical load is not affected by residual stresses. In contrast with this, elastic buckling of a plate is always influenced by residual stresses. The effect of residual stresses may be understood by comparing the buckling of the following two plates. The two plates are subjected to the same total amount of compressive load, but the distributions of the load along the breadth of the plates are different as shown in **Figs. 5.1**(b) and (c). The critical load in the case (c) is lower than that of the case (b). Therefore, if the distribution of the load is forced to change from a uniform one (**Fig. 5.1**(a)) to either that of case (b) or case (c), the critical load becomes either higher or lower, since more or less compressive force is applied in the middle portion of the plate.

As explained in Chapter 3, when a uniform compressive load is applied to the plate with residual stresses, the middle portion of the plate under compressive residual stresses becomes plastic before the average stress reaches the yield stress. In this case, as the rigidity of the middle portion decreases, the critical stress also decreases. Therefore, residual stresses have two different effects on the elastic-plastic buckling of the plate.

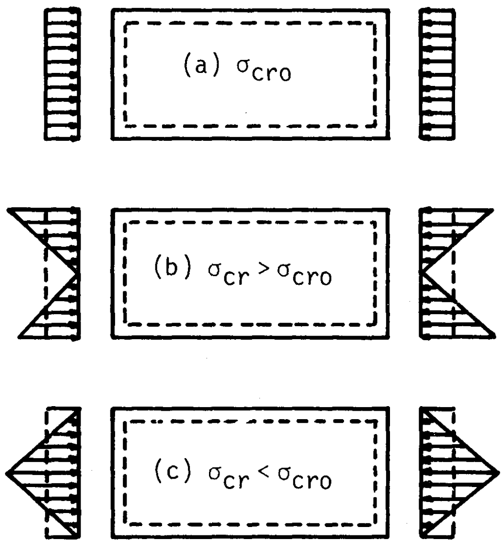


Fig. 5.1 Buckling of rectangular plates subjected to different distributions of compressive forces (simply supported along all edges)

5.2 Analytical method used

The anlysis of buckling of a plate consists of two parts: One is the stress analysis in the plate with consideration of both residual stresses and applied stresses at each step of the loading. The other is buckling analysis for the calculated stress distribution.

When the same residual stresses are assumed to exist along the length of the plate, the calculation of stress distribution involves simple manipulation. When the residual stresses are general, the finite element method is most suitable.

In the buckling analysis of a plate with simple geometry, any method mentioned in chapter 2 is applicable. For complicated geometry, the finite element method again shows the best applicability. In the elastic-plastic buckling analysis, the bending rigidity of the plastic portion of the plate must be evaluated based on an appropriate theory of plasticity. The incremental theory of plasticity is more rational from the mathematical theory of plasticity than from the total strain theory. However, the latter is thought to predict the critical load closer to that obtained by experiments. In this report, a rectangular plate, which is geometrically the simplest, is considered. Then the buckling analysis is performed by the conventional energy method (Method 2–1 in Chapter 2) which is based on the principle of minimum potential energy using a series of trigonometric functions. For the plastic portion, the total strain theory of plasticity is applied.

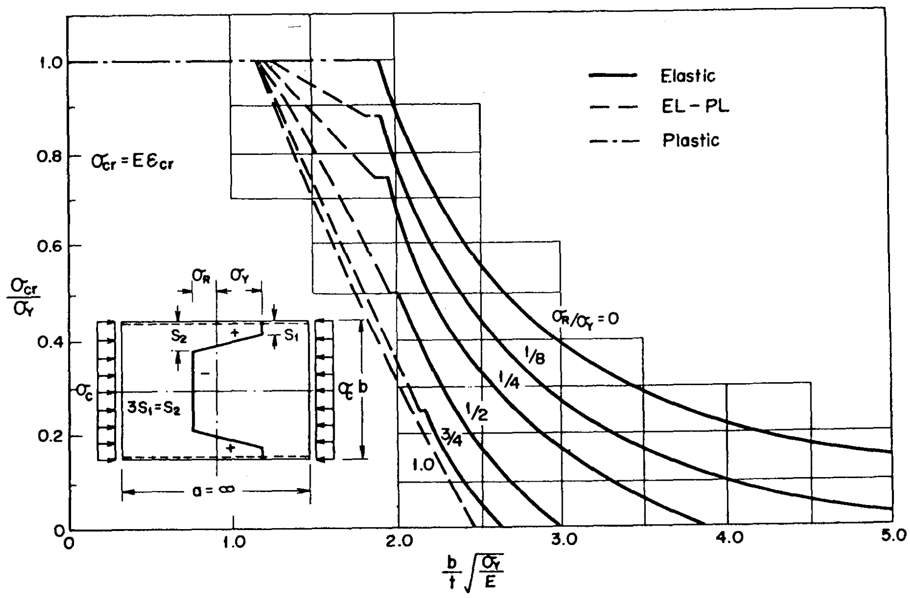


Fig. 5.2 Compressive buckling strength of infinite strips simply supported along opposite sides (theory)

5.3 Theoretical and experimental compressive buckling strengths of rectangular plates with residual stresses

(a) Simply supported rectangular plates[12),13)]

For a simplified pattern of residual stress distribution, the buckling strengths of finite strips are calculated and the results are shown in **Fig. 5.2.** The strips correspond to plates between one stiffener space of a stiffened plate, webs of Columns of I-section, etc. The buckling strength of the plate was calculated for such a wave length as to indicate the minimum critical load. The wave length in this case is very close to the width of the plate.

In order to verify the theoretical results, experiments on local buckling of columns of square sections were carried out, applying a concentrated compressive force. The unloaded edges of the plate elements are regarded as being simply supported. The wave length is súch as to produce the minimum critical load. Both results show good correlation with theory as presented in **Fig. 5.3.**

(b) Local buckling of the webs of built-up I-sections[14)]

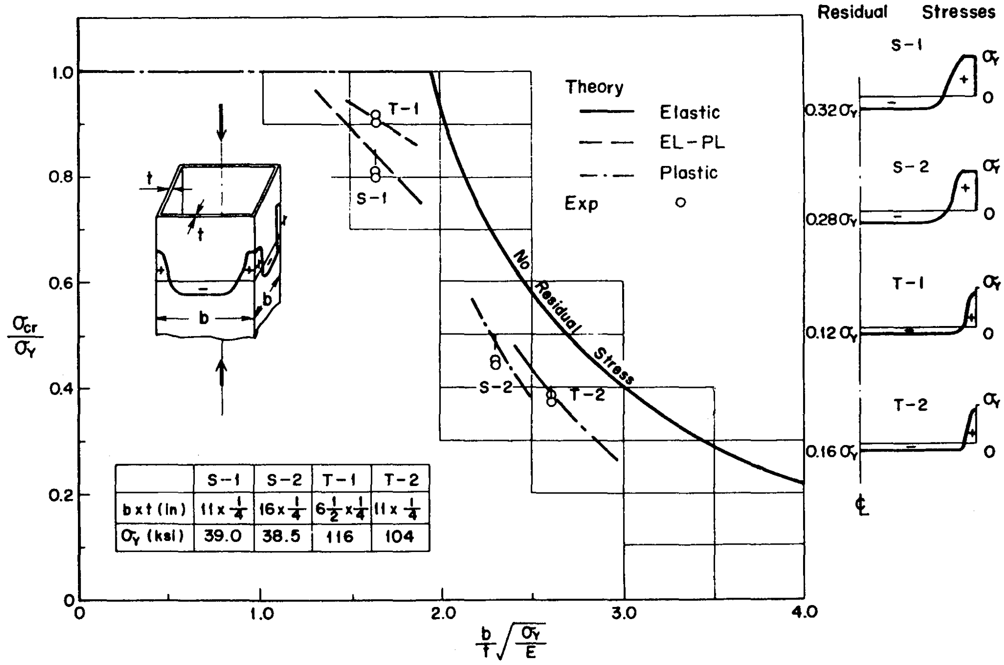


Fig. 5.3 Compressive local buckling strength of built-up columns with square cross sections (theory and experiment)

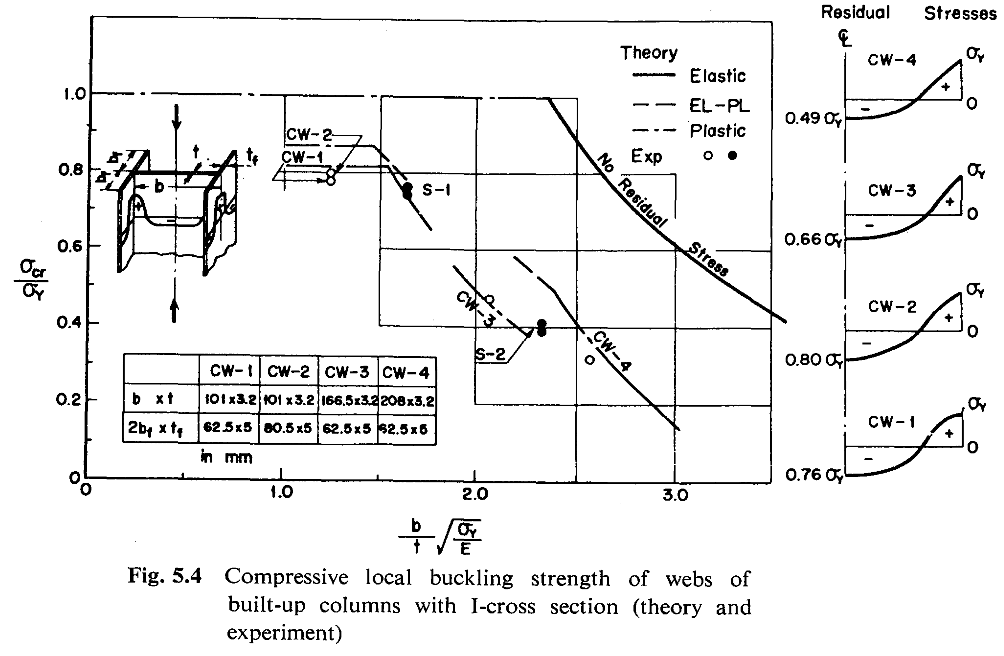


Fig. 5.4 Compressive local buckling strength of webs of built-up columns with I-cross section (theory and experiment)

When the web of a built-up I-section buckles, the web is elastically restrained. The buckling strength of such webs were studied both theoretically and experimentally. The results are presented in **Fig. 5.4** with their measured welding residual stresses. These results agree with each other. It should be noted that the residual stresses are self-equilibrating over the entire cross section, but not in either the web or flanges. In the usual cases, more compressive residual stresses are induced in the weaker element, which is the one to buckle.

(c) Rectangular plate simply supported along one side and free along the opposite side[14]

When a flange buckles, the flange is elastically restrained by the web. If the degree of restraint of the flange is small, the flange can be regarded as a rectangular plate simply supported along one side and free along the opposite side. Applying the same method as (a), the results of the theoretical analysis of buckling of the plate are shown in **Fig. 5.5**.

(d) Local buckling of the flanges of built-up I-sections[14]

In a way similar to (b), the investigation was carried out for local buckling of the flanges of built-up I-sections. The results of the theoretical analyses correlate well with experiments, as shown in **Fig. 5.6.** In these cases, the more compressive welding residual stresses are induced in the weaker elements which are the flanges.

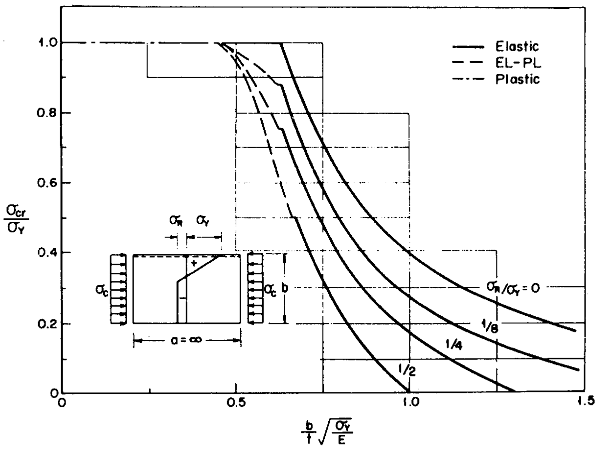


Fig. 5.5 Compressive buckling strength of infinite strips simply supported along one side and free along the opposite side

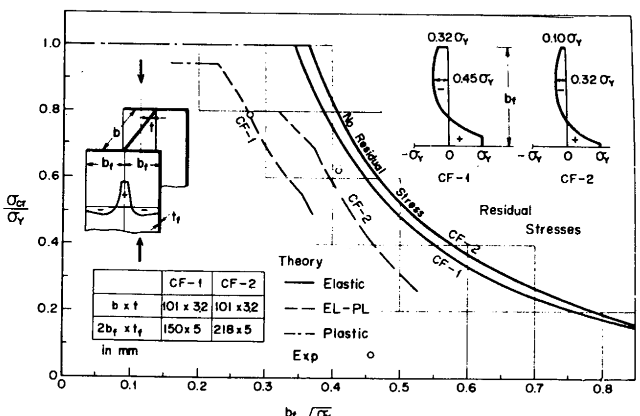


Fig. 5.6 Compressive local buckling strength of flanges of built-up columns with I-cross section

6. Buckling of a Stiffened Plate with Residual Stresses[15]

When stiffeners are fitted to a plate by welding, residual stresses are produced both in the plate and stiffeners. In order to investigate a fundamental feature of the effect of welding residual stresses upon buckling of the stiffened plate, a simple model was considered. The model was a square plate and had one stiffener fitted along one centerline, which was symmetric with respect to the middle plane of the plate (**Fig. 6.1**). Using a plate of mild steel, the

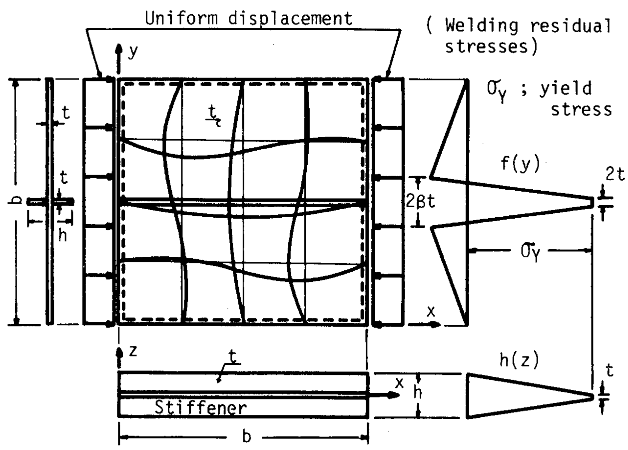


Fig. 6.1 Loading condition, welding residual stresses, and deformation of a stiffened plate

model was constructed by staggered incremental welding. The welding residual stresses were measured by applying the sectioning method. The distribution of the observed residual stresses can be approximated with two straight lines as shown in **Fig. 6.1** and when the ratio for the distribution, β, is equal to 5, the approximate distribution shows a very good accuracy.

For the buckling analysis of the stiffened plate, under a uniform compressive load, it is assumed that the above mentioned distribution of the residual stresses is uniform along the stiffener. The theoretical analysis was carried out by applying Galerkin's method (Method 1-1 in Chapter 3), and the result is presented in **Fig. 6.2.**

The magnitude of bending rigidity of the stiffener

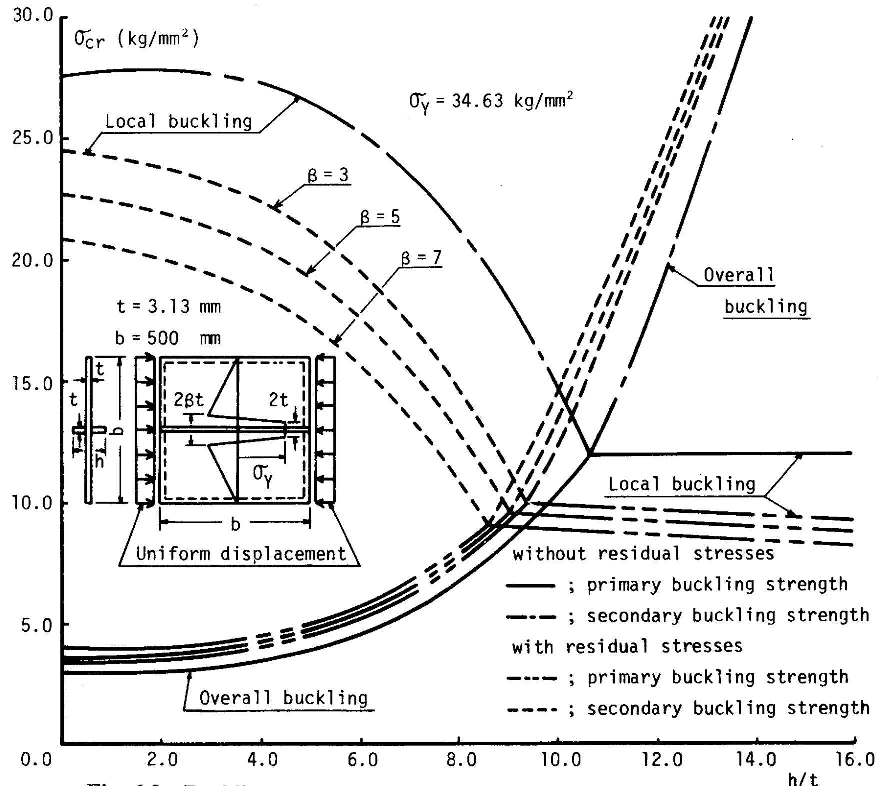


Fig. 6.2 Buckling strength of a stiffened plate with welding residual stresses

plays an important role to determine the buckling mode. When the stiffness ratio $\gamma^{*)}$ is smaller than a certain value, γ^{B}_{min} , the primary buckling occurs in the over-all mode. In this case, the welding residual stresses increase the critical stress, since more tensile residual stresses exist in the stiffener. When the ratio γ is greater than γ^{B}_{min} , primary local buckling of the plate occurs. Since more compressive residual stresses are induced in the plate, the local buckling stress decreases. The intersection of these two buckling strength curves furnishes the value of γ^{B}_{min} . The value of γ^{B}_{min} decreases and the maximum critical stress of primary buckling decreases, when there are the welding residual stresses. The figure also shows the effect on the secondary buckling.

7. Ultimate Strength of a Plate with Residual Stresses and Initial Deflection

7.1 Introduction

When a plate element in a welded structure is initially deflected, from the beginning, the in-plane loading increases the deflection and there is no clear bifurcation (buckling). On the other hand, a flat plate will still sustain a load after buckling and will reach its ultimate strength. Therefore, ultimate strength is an important standard for evaluation of the strength of a plate. In this Chapter, the results of an investigation into the effect of welding imperfection (welding residual stresses and deformation) on the ultimate strength of a rectangular plate under compression will be discussed.

Before the plate reaches its collapse state, the plate experiences large deflections and undergoes plastification of the material. As these are geometric nonlinearities and material nonlinearities, it is advantageous to use the finite element method or the finite strip element method. In this paper, the finite element method is applied based on the formulation made by the author.[17),11)]

7.2 Accuracy of the results obtained by the finite element method[11)]

In order to examine the validity of the analysis, the

*) $\gamma = EI_s/D_b$

where $I_s = \dfrac{t_s(h^3 - t_p^3)}{12}$, $D = \dfrac{Et_p^3}{12(1-\nu^2)}$

E=Modulus of elasticity, ν=Poisson's ratio

b, h: defined in Fig. 6.1

t_p : thickness of a plate

t_s : thickness of a stiffener

results of elastic-plastic large deflection analysis by the finite element method on square plates with sinuisoidal shape of initial deflection are compared with the results of experiments. The experiments were performed for simply supported square plates, applying a compressive load with a set of specially designed loading frames (as shown in **Fig. 7.1**). **Figure 7.2** shows the relation between the mean compressive stress and central deflection based on the results of both experiments and analyses for some specimens. In the case of thicker plates, both results are almost coincident. On the other hand, for the thin plate in these examples, some differences are observed between the results of experiment and analysis. This may be attributed to the difference of the shape of actual initial deflection from the assumed one in the analysis, since the formation of initial deflection of the specimen is not well controlled in the case of thin plates. It may be concluded that this analysis can accurately describe the actual behavior of the plate.

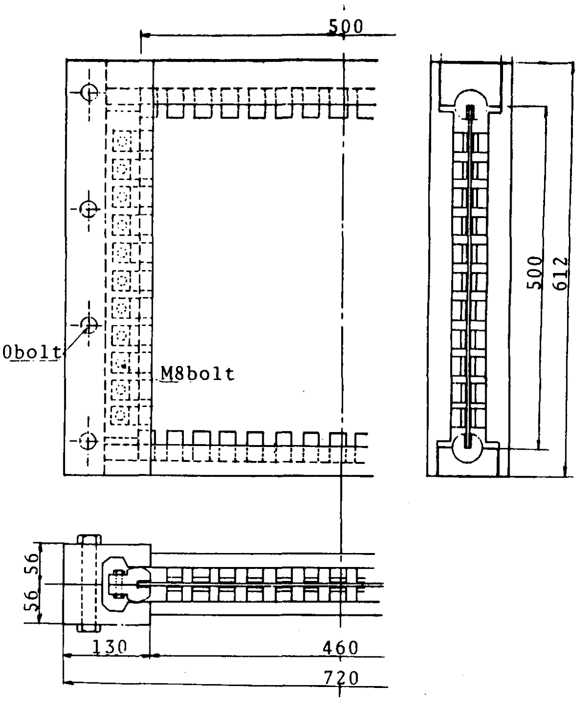


Fig. 7.1 Loading frame and end fixture for compression

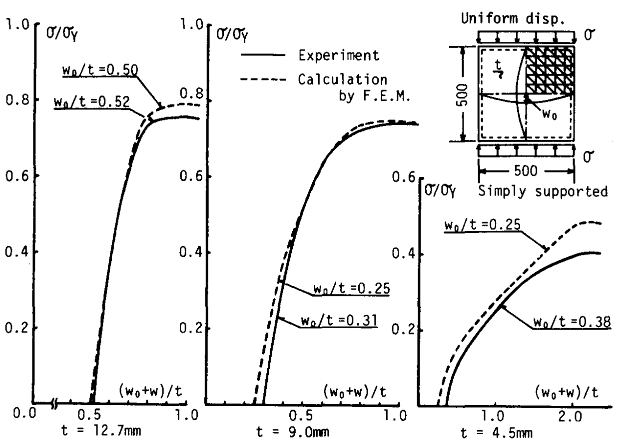


Fig. 7.2 Comparison of the results of F.E.M. analysis with experimental results

7.3 Theoretical and experimental results[11),16)]

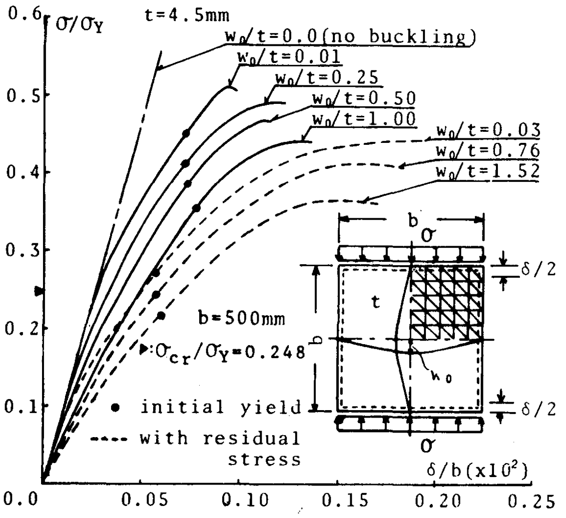
Important compressive plate elements such as plates between stiffeners of decks and bottoms, etc. are usually long strips. Under excessive compressive forces, these strips, when simply supported, buckle with a half wave length approximately equal to the width of the strips. Therefore, for investigation into compressive strength of the strips, a square plate is considered to be one of fundamental geometry.

In order to clarify the effect of the magnitude of initial deflection, a series of the elastic-plastic large deflection analyses was performed on 500×500 mm square plates of which the thicknesses were 4.5 mm, 9.0 mm and 12.7 mm. The plates were assumed to be simply supported along all four edges. The shape of initial deflection was assumed to be a sinusoidal wave, and the ratios of the maximum deflection to the plate thickness were chosen as 0.01, 0.25, 0.50 and 1.00 for each plate thickness.

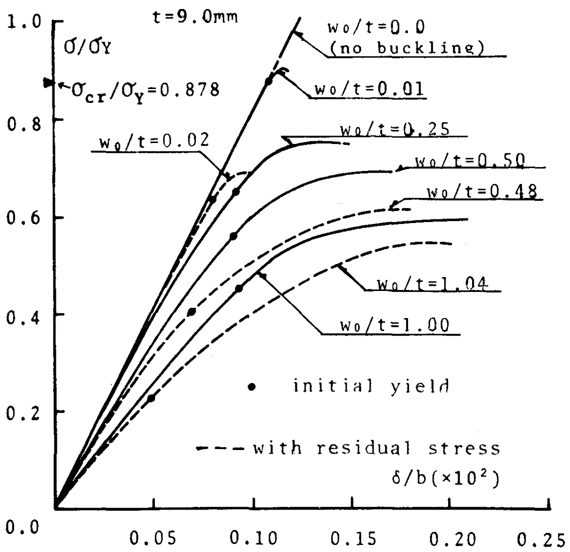
Load-displacement curves and load-deflection curves with a variation in the magnitude of initial deflection are shown for each plate thickness in **Figs. 7.3**(a) to (c), and **Figs. 7.4**(a) to (c), respectively. The finite element representation used for the analysis is also shown in these figures. The full line curves represent the results considering only the initial deflection and the dotted line curves containing both the welding residual stresses and initial deflection. The ultimate strength of square plates under compression obtained by the analyses and the experiments are represented in **Figs. 7.5**(a), (b) and (c), with respect to the ratio of the maximum initial deflection to the plate thickness, w_0/t. The plate thicknesses were 4.5 mm, 9.0 mm and 12.7 mm, respectively. The calculated curves are a little higher than the experimental ultimate strength for the plates of 4.5 mm and 12.7 mm thickness. However, taking into account the sensitive nature of the behavior of the plate, it can be said that both results coincide well for all cases. The individual effects of welding residual stresses and deformation on the ultimate strength can be recognized.

7.4 Effect of the shape of initial deflection[11)]

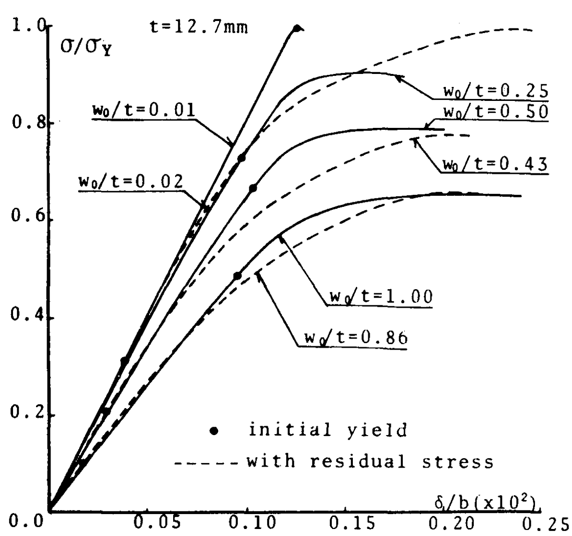
There are two factors which should be considered concerning the effect of initial deflection on the rigidity and strength of plates: the shape and the magnitude



(a) t= 4.5mm

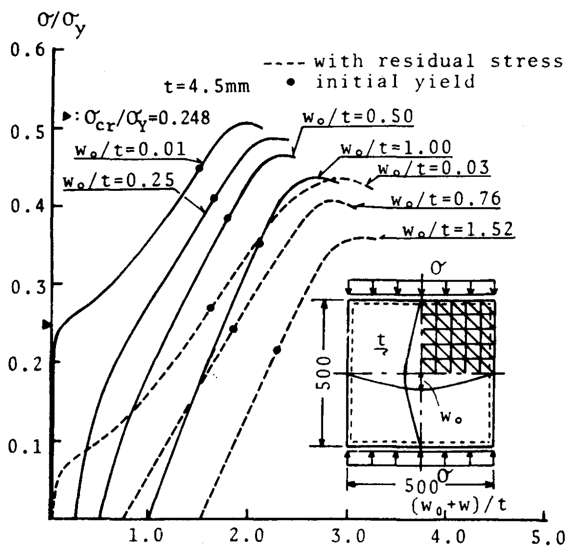


(b) t= 9.0mm

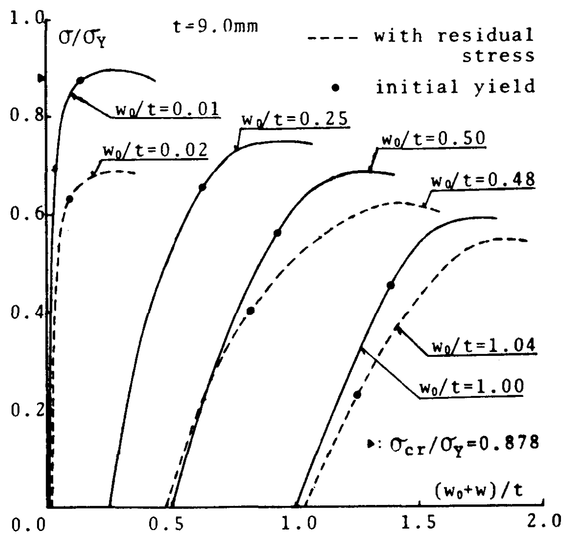


(c) t=12.7mm

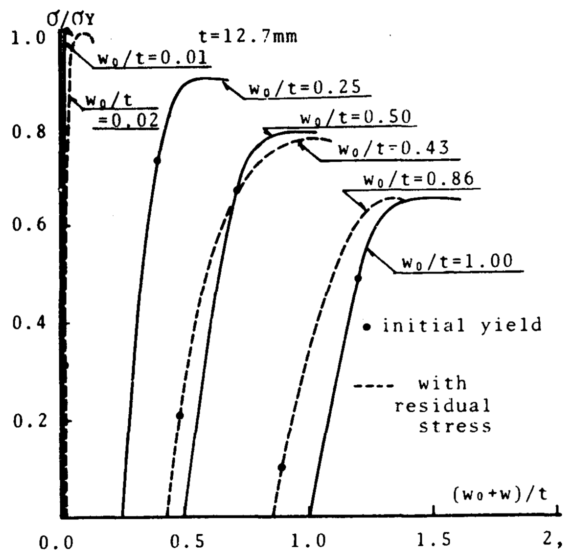
Fig. 7.3 Average stress-displacement for square plate under compression (F.E.M. analysis)



(a) t= 4.5mm



(b) t= 9.0mm



(c) t= 9.0mm

Fig. 7.4 Average stress-central deflection for square plate under compression (F.E.M. analysis)

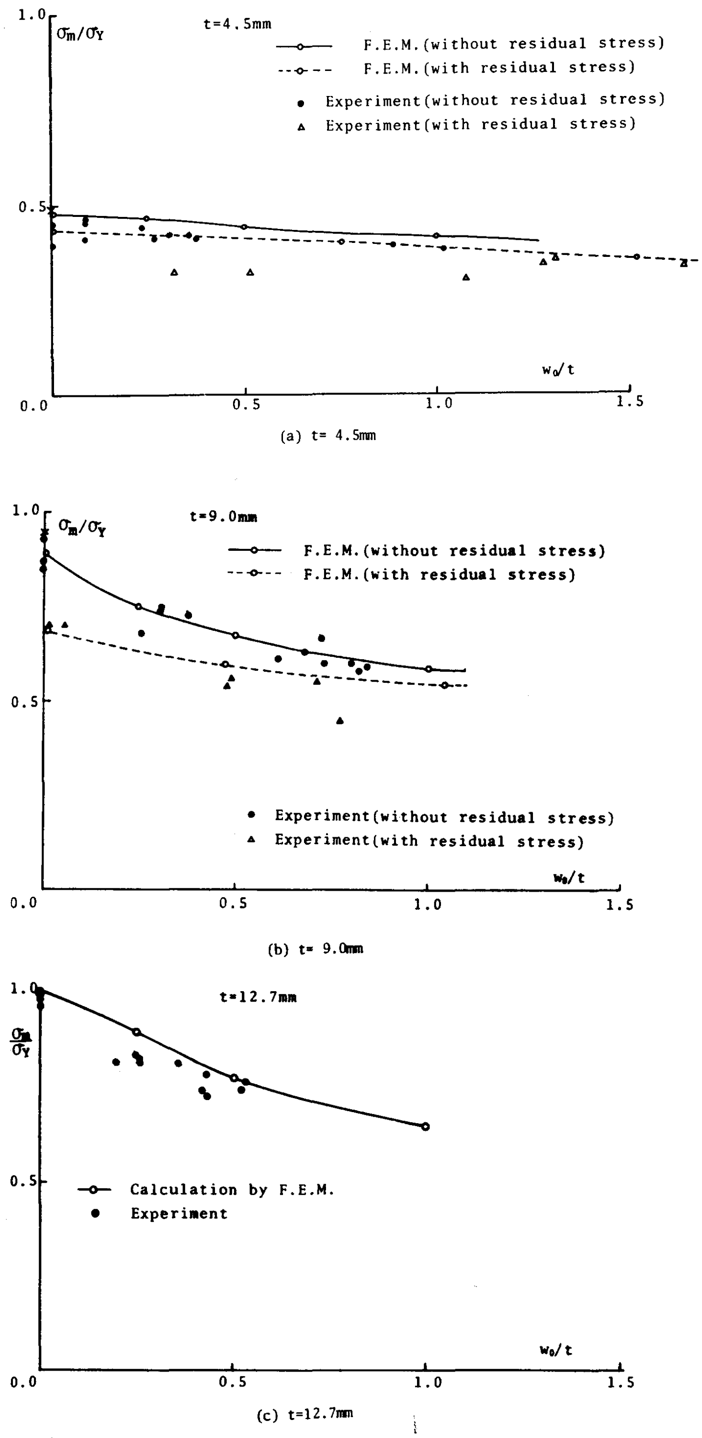


Fig. 7.5 Relation between compressive ultimate strength and initial deflection including residual stresses (F.E.M. analysis and experiment)

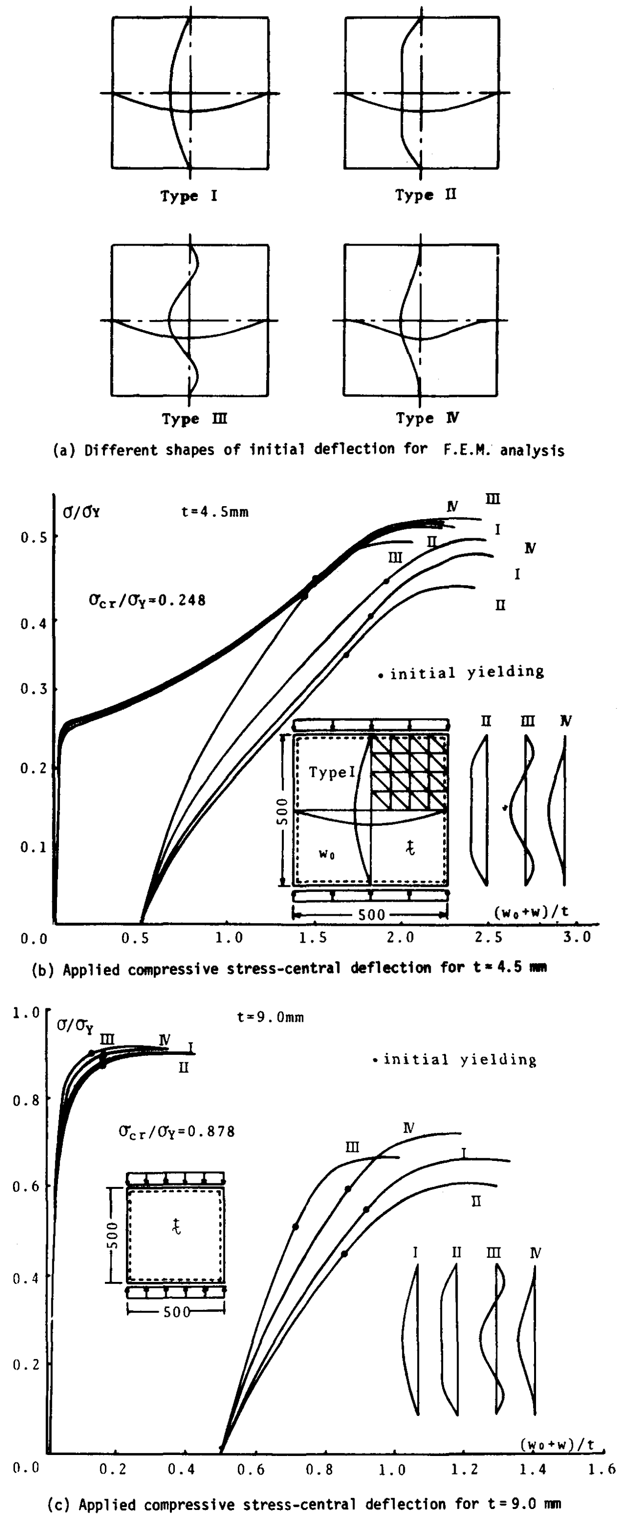


Fig. 7.6 Effect of the shape of intial deflection on compressive ultimate strength of square plates (F.E.M. analysis)

of initial deflection. In order to clarify the influence of the shape of initial deflection, four types of initial deflections for plates 4.5 mm and 9.0 mm thick were used in the analysis shown in **Fig. 7.6**(a). The magnitude of deflection at the center of the plate was kept constant at 0.01 and 0.5 times the plate thickness. The plates are assumed to be simply supported along all four edges, and the loads were applied so as to give a uniform displacement on the loading edges.

Fig. 7.6(b) shows the relationship between the average compressive stress and the deflection at the center for a 4.5 mm thick plate. In the same figure, the finite element representation is also shown.

When the magnitude of initial deflection is small, the difference in the shape of initial deflection has very little influence on the rigidity and strength of the plates. However, when it is large, there exists noticeable differences in the behavior of the plates such that, the larger the volume occupied between the initial surface of the middle plane of the plate and the flat plane containing its four corners is, the lower the rigidity and the ultimate strength become. This is due to the fact that a larger bending moment at each point of the plate is produced by a larger deflection for a given compressive load. However, the ultimate strength for all types of shape and for any magnitude of initial deflection would never exceed the ultimate strength of the flat plate. **Fig. 7.6**(c) also shows the results for a plate of 9.0 mm thick, and the same tendency can be observed as for the previous case.

7.5 Effect of the magnitude of initial deflection[11),16)]

The theoretical ultimate strength for a square plate under compression (uniform displacement) is plotted by ○[11)] and △[16)] against $(b/t)\sqrt{\sigma_Y/E}$ in **Fig. 7.7** where b/t, σ_Y and E are the breadth to thickness ratio, yield stress and Young's modulus of the material, respectively. The plate is assumed to have an initial deflection of the form

$$w = w_0 \cos\frac{\pi x}{b} \cos\frac{\pi y}{b} \qquad (1)$$

where w_0 is the magnitude of the initial deflection at the center of the plate, and w_0/t is chosen as 0.01, 0.25, 0.50 and 1.00. The origin of the x and y-coordinates is taken at the center of the plate. The computed ultimate strength indicated by ○ and △ are smoothly connected by solid lines. These curves can be expressed in the following form applying the method of least squares:

$$\frac{\sigma_u}{\sigma_Y} = \frac{1.338\eta^2 + 4.380\eta + 2.647}{\xi + 6.130\eta + 0.720} - 0.271\eta - 0.088 \qquad (2)$$

where

$\xi = (b/t)\sqrt{\sigma_Y/E}$ and $\eta = w_0/t$

The ultimate strength curves given by Eq. (2) are represented by chain lines in **Fig. 7.7**, and in good agreement with the solid lines, especially for the range of $(b/t)\sqrt{\sigma_Y/E} > 1.5$.

When the plate is thin, the deflection due to the compressive load is large compared with its thickness. But plastification is delayed because the thin plate is more flexible. In contrast to this, the deflection of the thick plate due to the compressive load is not as large compared with its thickness. But plastification occurs for a relatively smaller deflection. This plastification causes the remarkable decrease in the ultimate strength of thick plates due to initial deflection as compared to thin plates.

7.6 Effect of local bending stresses[11),16)]

Initial deflection in the plate elements of welded structures is produced by angular distortion of the welded portion and it is promoted by welding residual stresses in compression. However, in this section, it is assumed that initial deflection is produced only by the angular distortion. In the theoretical analysis, this angular distortion is replaced by the action of

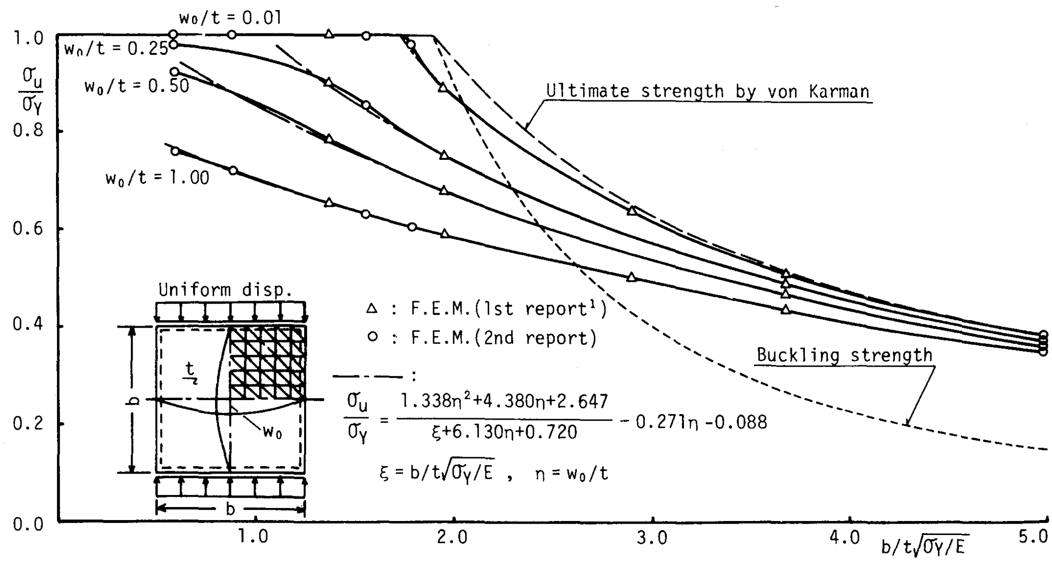


Fig. 7.7 Ultimate strength of square plates with initial deflection under compression (F.E.M. analysis)

uniformly distributed bending moment along the edges, and the resulting initial deflection is accompanied by local bending stresses. When the plate is subjected to a compressive force, the behavior of the square plate with initial deflection accompanied by local bending stresses is illustrated in **Fig. 7.8**, together with that excluding the effect of local bending stresses. It is observed that the local stresses reduce remarkably the ultimate strength, especially in the case of thick plates.

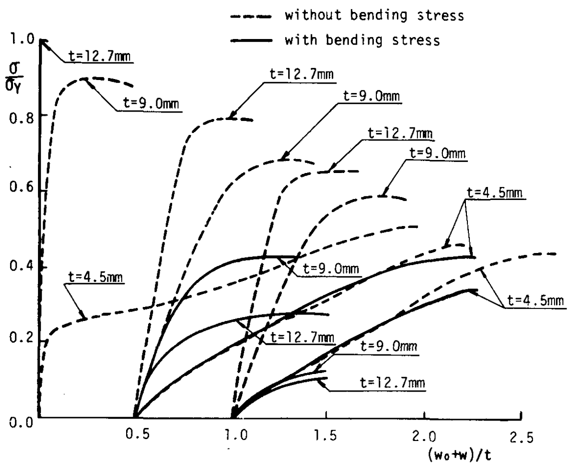


Fig. 7.8 Effect of local bending stresses caused by initial deflection on the compressive ultimate strength of square plates (F.E.M. analysis)

7.7 Effect of welding residual stresses[11),16)]

In this section, a plate is considered to be taken out between one stiffener space of a stiffened plate, and the plate is assumed to be simply supported along all edges. The initial imperfection of the plate due to welding is assumed to be produced by angular distortion of the weld (uniformly distributed bending moment) along the two parallel weld lines, and the welding residual stresses. The magnitude of the initial deflection at the center of the plate is specified as 0.01 and 0.25 times the plate thickness. As possible distributions of welding residual stresses, two simplified patterns are assumed in the plate. These patterns are illustrated in **Fig. 7.9**, where the stresses are self-equilibrating. The notation of the pattern is read as follows: for example, taking Type A/5, the pattern of the distribution is type A, and the width in which the residual stresses are tensile is 1/5 times the halt plate width. The distributions used for the welding residual stresses are Type A/5 and Type B/20 with $w_0/t=0.01$, and Type A/5 with $w_0/t=0.25$. The ultimate strength for each case is illustrated in **Fig. 7.10** by the chain lines with one dot, together with that with only initial deflection, which is represented by solid lines. As is known from the analytic results for the case of $w_0/t=0.01$, the welding residual stresses reduce the

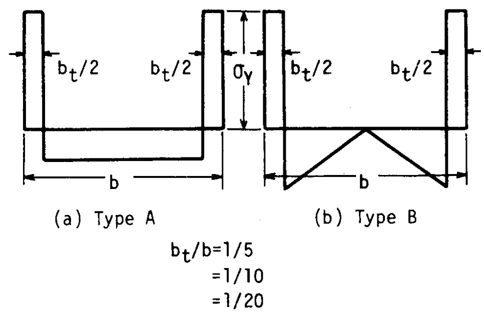


Fig. 7.9 Assumed distribution of welding residual stresses

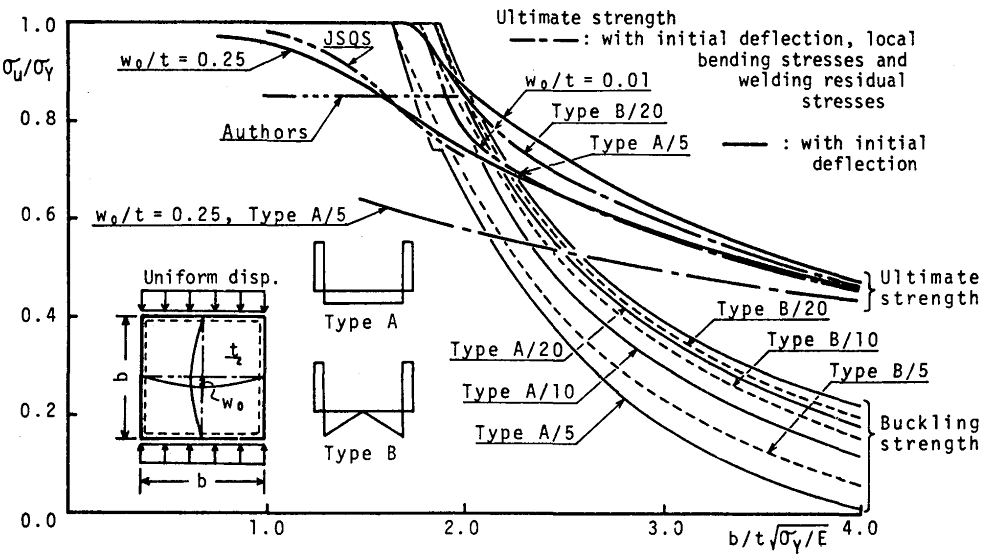


Fig. 7.10 Compressive buckling and ultimate strengths of square plates with initial imperfection due to welding

ultimate strength in the range of $(b/t)\sqrt{\sigma_Y/E}>1.8$, but not as much for the case of buckling strength. It can also be said that the welding residual stresses have little effect on the ultimate strength for $(b/t)\sqrt{\sigma_Y/E}>1.8$. In the case of $w_0/t=0.25$, the ultimate strength is remarkably reduced due to initial imperfection when the plate is thick. This reduction is mainly due to the existence of the local bending stresses, judging from the following fact: when the plate has both initial deflection and welding residual stresses, but not local bending stresses, in the case of thick plates, the ultimate strength does not decrease more than that for the plate with initial deflection only.

7.8 These effects on the in-plane rigidity of the plate[11]

The slope of the load-displacement (applied stress and average strain) curve indicates the compressive rigidity of a plate. In the case of a thin flat plate as seen in **Fig. 7.3**(a), the rigidity is equal to Young's modulus, E, of the material for small loads. But as the load increases, buckling occurs and the rigidity decreases to about half of E. A plate with small initial deflection exhibits similar behavior. When the initial deflection is large, the rigidity is smaller than E from the beginning of the loading. After the buckling load is reached, again the rigidity becomes about half of its normal value of E. Then as the load increases, local yielding occurs, and the rigidity gradually decreases to zero when the plate reaches its ultimate strength.

Fig. 7.3(a) also shows the decrease of the rigidity due to the welding residual stresses. In the case of a thicker plate with small initial deflection, shown in **Figs. 7.3**(b) and (c), the rigidity is first equal to Young's modulus, E, of the material, but after yielding occurs at a high value of σ/σ_Y, it suddenly decreases and the fatal collapse occurs. On the other hand when the initial deflection is large, the rigidity is less than E from the beginning of the loading, but keeps nearly constant values according to the magnitude of initial deflection. After yielding, which occurs at a lower value of σ/σ_Y, it gradually decreases to zero, and the load reaches the maximum. The welding residual stresses also decrease the rigidity of the plates.

8. Ultimate Strength of a Stiffened Plate with Residual Stresses and Initial Deflection

In buckling of a stiffened plate, it is well known that there exists a minimum stiffness ratio for the stiffener, γ^B_{min}. If the value of the stiffness ratio γ*) is smaller than γ^B_{min}, the buckling of the stiffened plate is in the overall mode, and when γ is greater than γ^B_{min}, local buckling of the plate occurs first.

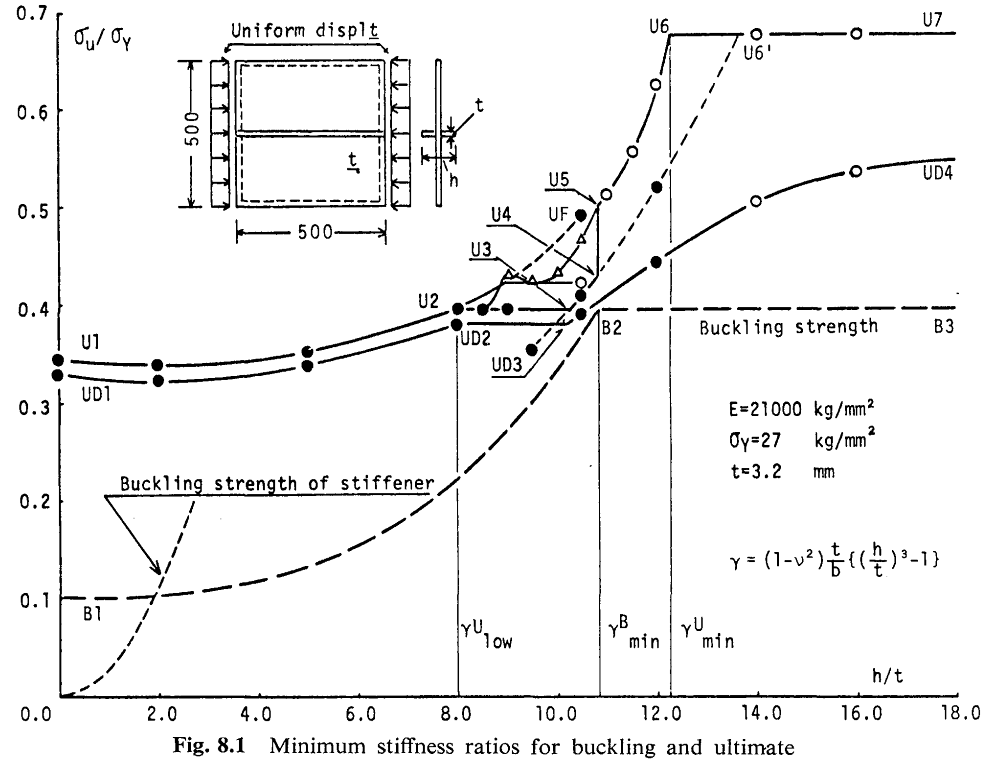


Fig. 8.1 Minimum stiffness ratios for buckling and ultimate strength of a stiffened plate (γ^B_{min}, γ^U_{low}, and γ^U_{min})

*) γ should be referred to in Chapter 6.

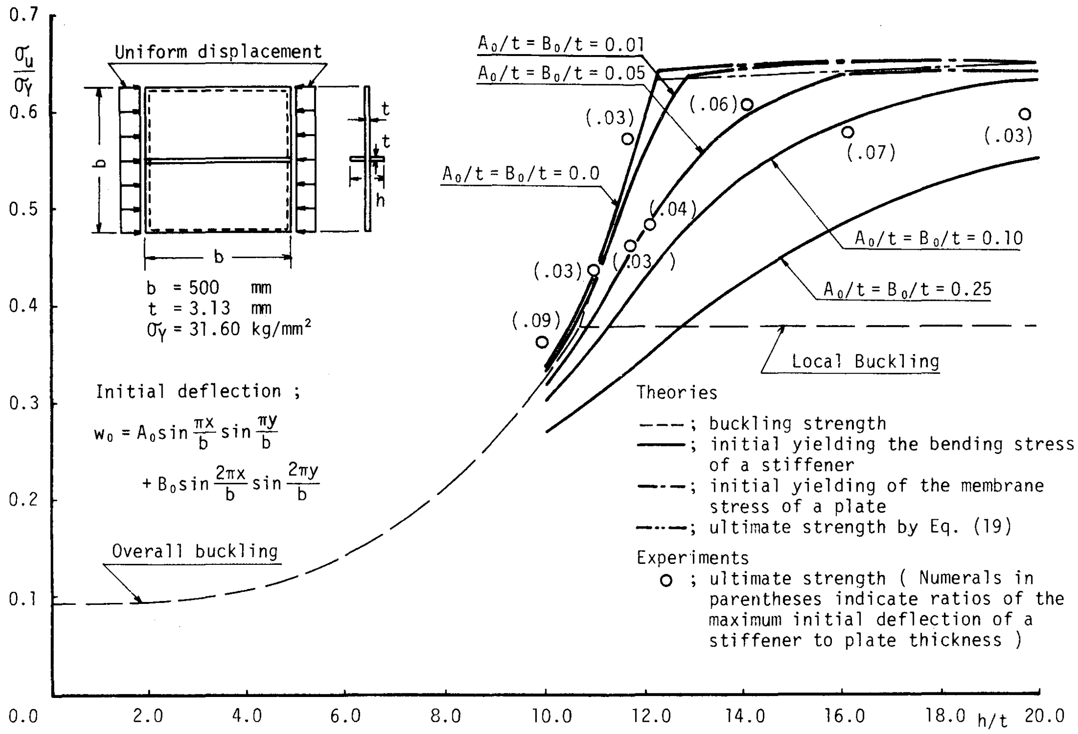


Fig. 8.2 Compressive ultimate strength of a stiffened plate

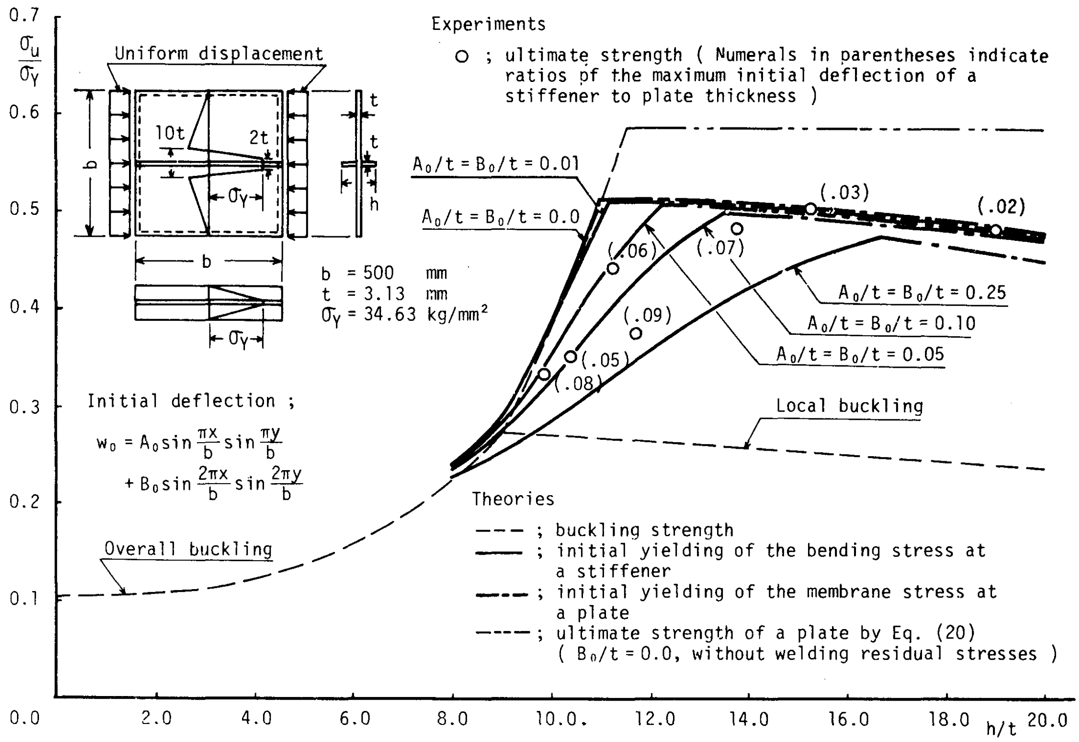


Fig. 8.3 Compressive ultimate strength of a stiffened plate with welding residual stresses

Concerning the ultimate strength of a stiffened plate, the authors have found and confirmed both theoretically and experimentally that there are two significant stiffness ratios for the ultimate strength, γ^{U}_{low} and γ^{U}_{min}. The ratio γ^{U}_{low} is smaller than γ^{B}_{min}, while γ^{U}_{min} is larger than γ^{B}_{min}. **(Fig. 8.1).**[18),19)] If γ is larger than γ^{U}_{min}, the ultimate strength of a stiffened plate reaches its maximum value; therefore it is not worthwhile to use a stiffener for which γ is greater than γ^{U}_{min}. The authors have named this value of the stiffness ratio as the minimum stiffness ratio for ultimate strength.[18),19)]

It is in this connection that the effects of welding residual stresses and deflection upon the ultimate strength of a stiffened plate have been investigated. The model is the same as that in Chapter 6. The stiffened plate has one stiffener along one center line, which is symmetric with respect to the middle plane of the plate.

For a theoretical analysis of the behavior of the plate, the finite element method and Gelarkin's method (Methods 2–2, and 2–1, in Chapter 2, respectively) were applied. A series of tests were conducted on the as-welded models and the annealed models which had various magnitudes of initial deflection. The results of the theoretical analyses and experiments are represented in **Figs. 8.1, 8.2,** and **8.3.**[18),19),15)] The theoretical predictions of the ultimate strength by Gelarkin's method, which were calculated using the loads at initial yielding of the stiffened plates, correspond well with the ultimate strength obtained by the experiments.[14)-15)]

There are two types of collapse modes, the overall mode and the local mode, which occur depending on the magnitude of initial deflection. First, consider a case where the stiffened plate is free from welding residual stresses and has initial deflection.

If the magnitudes of initial deflections of the stiffener and the panel plate are the same, the collapse of the stiffened plate is in the over all mode for smaller values of γ and the ultimate strength decreases as the magnitude increases (**Fig. 8.2**). If the stiffness ratio γ becomes greater than a certain value, the collapse occurs in the local mode and the ultimate strength increases approximately as that of the stiffened plate without initial deflection. The authors have named this specific value of the stiffness ratio as the "effective minimum stiffness ratio" for ultimate strength. When the welding residual stresses exist in the model, they greatly reduce the ultimate strength as observed in **Fig. 8.3**.

9. Concluding Remarks

In this paper, the authors have described the influences of welding residual stresses and deflections on the buckling and ultimate strengths of plate elements and stiffened plates. The theoretical analyses on the nonlinear behaviors were carried out by the methods explained in Chapter 2. The precise analyses using those methods have become possible by the remarkable development of high speed digital computers. With these tools and the information they provided, detailed discussions about the separate influences of welding residual stresses, deformation, etc. on the strength have come practical.

The results of the investigation provide the following information: Both welding residual stresses and deformation reduce the compressive buckling and ultimate strengths of plate elements and stiffened plates and their effects become greater for thicker plates up to $(b/t)\sqrt{\sigma_Y/E}<1.8$. The one exception is the case when a stiffened plate buckles in the overall mode. The authors and their colleagues have investigated to a certain extent the effects of these imperfections on the strength of structural elements under bending and shear. The analyses were conducted in a manner similar to the above explained methods.

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